Kinetic Isocurvature Perturbations

w/K.J. Bae, J.-O. Gong, K. Harigaya, C.S. Shin

[2603.22394]

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@ Yonsei HEP-COSMO Seminar

Research Background / Interests

Ph.D. (Seoul, S. Korea)

연세대학교
YONSEI UNIVERSITY

Postdoc. (Chicago, USA)

THE UNIVERSITY OF CHICAGO

ENRICO FERMI INSTITUTE

KICP

THE UNIVERSITY OF CHICAGO
LEINWEBER INSTITUTE FOR THEORETICAL PHYSICS

Dark Matter + Cosmology

- Wave Dark Matter
- Warm Dark Matter
- Cosmological Perturbations

Quantum..?

- Axion + GW Detection
- Axion Learning
- Quantum State of Axions


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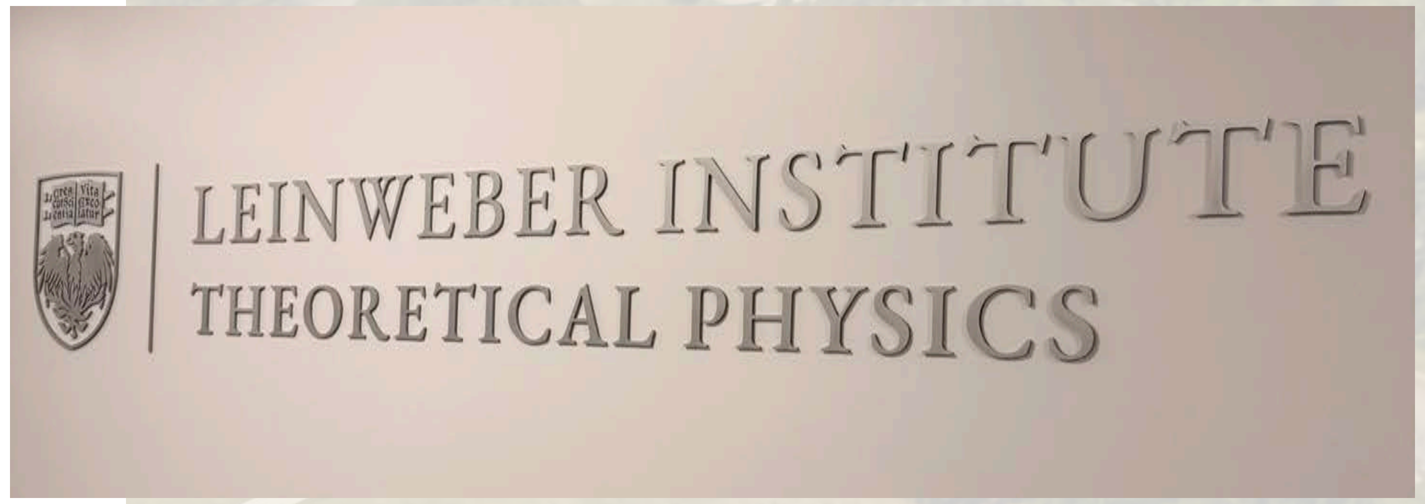
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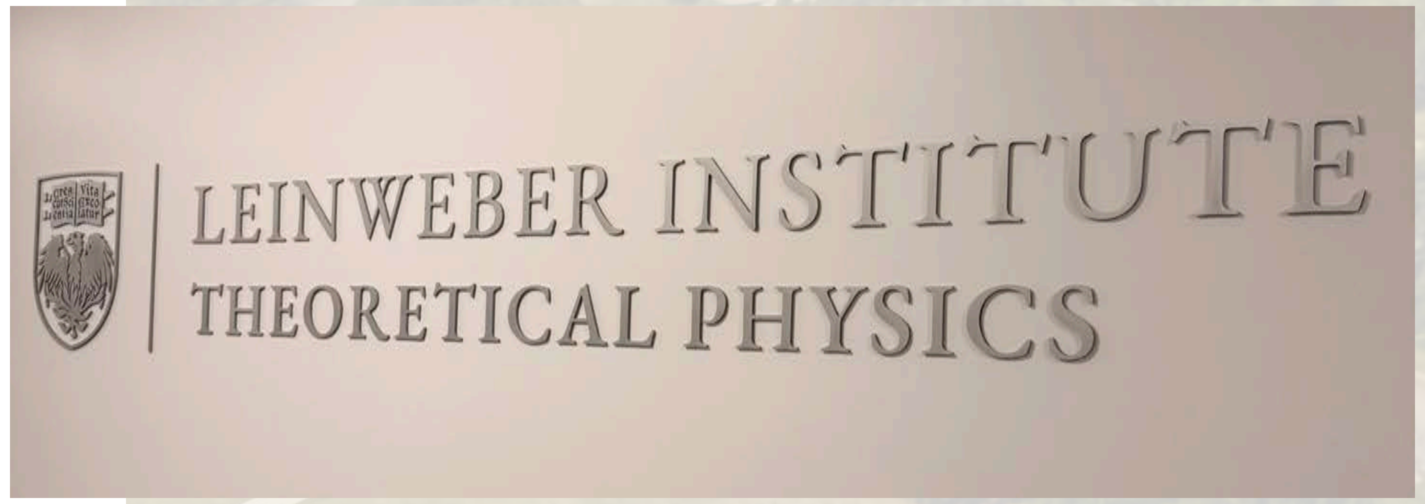
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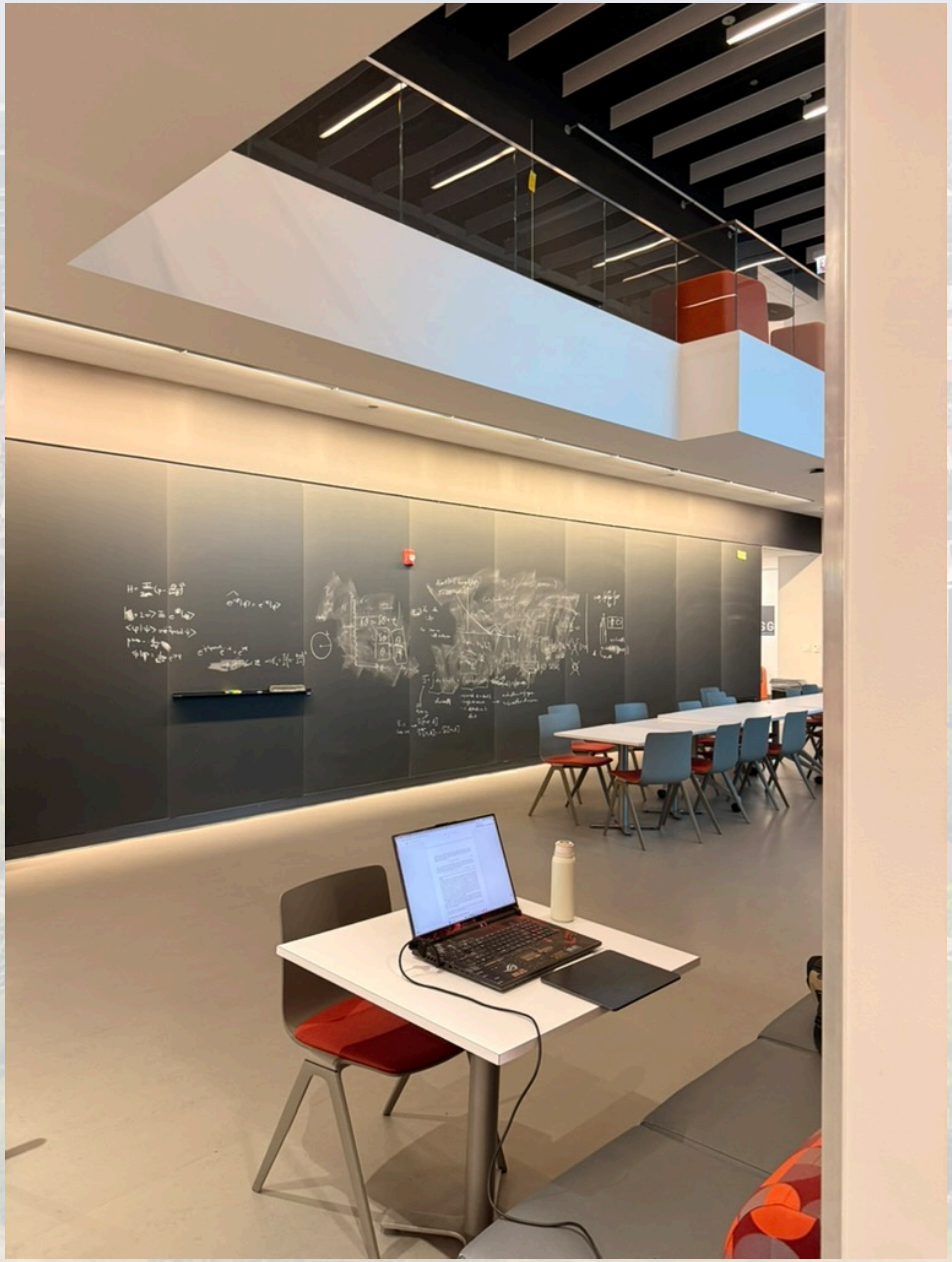


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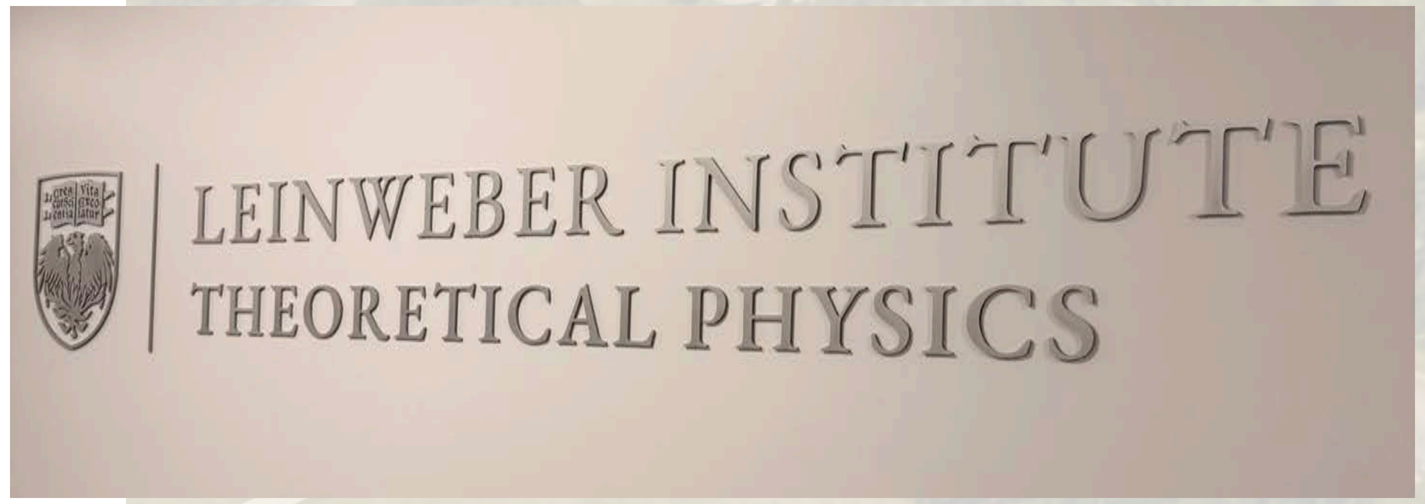
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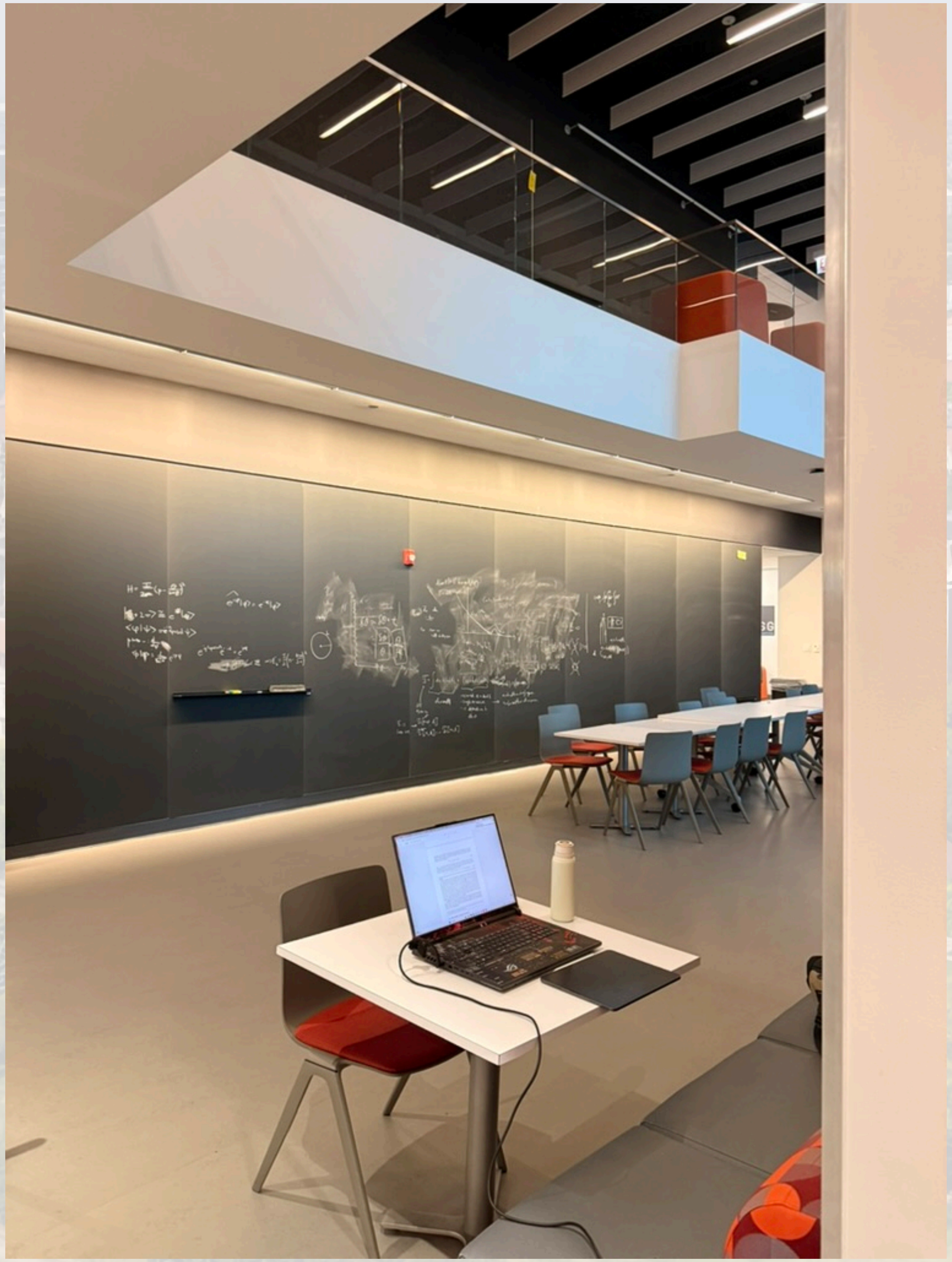


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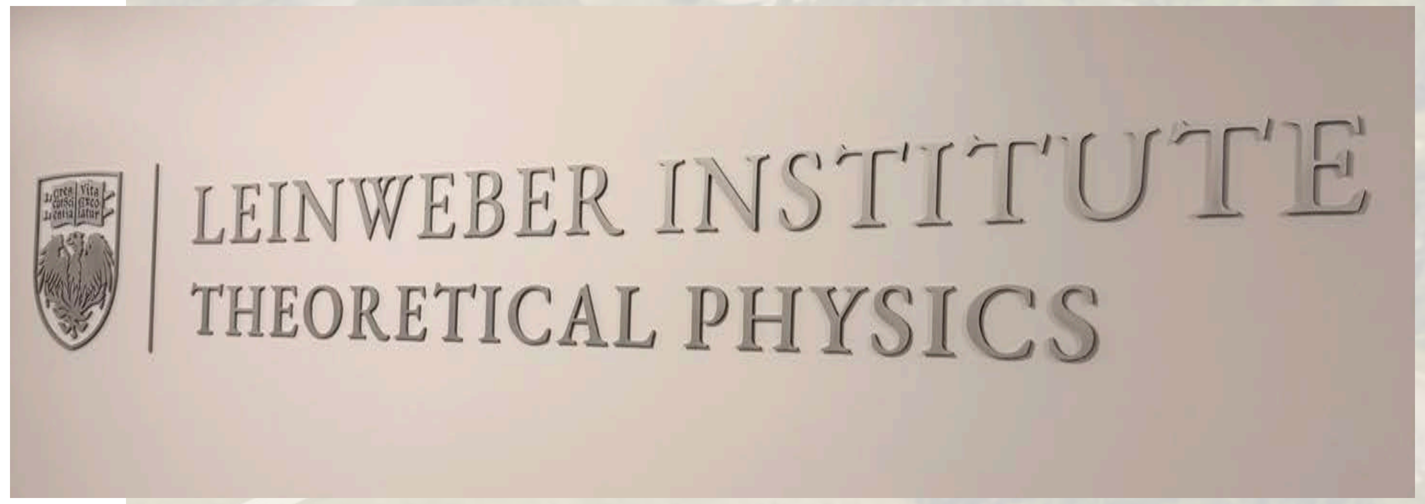
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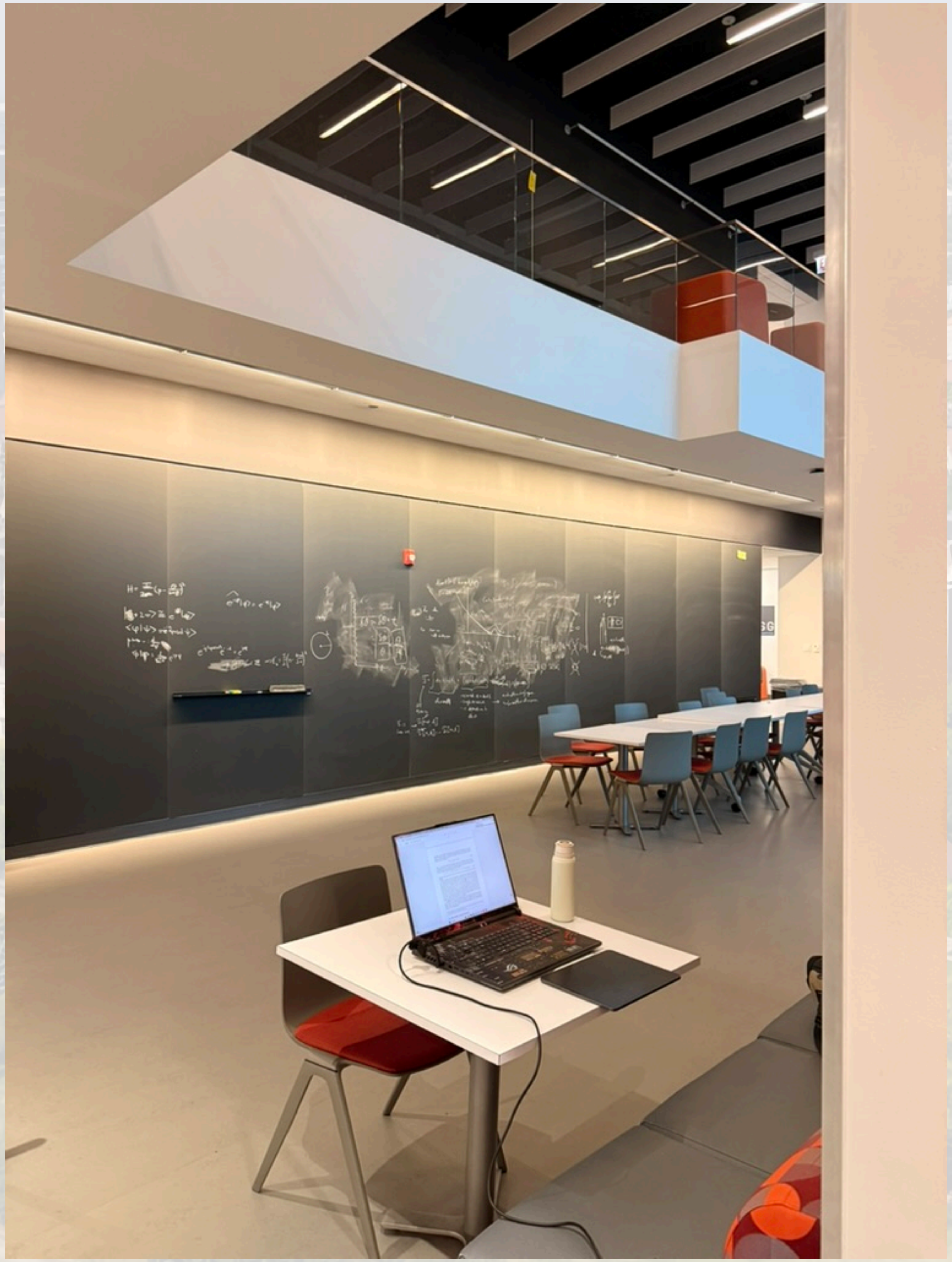

Motivation

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$$\Omega_{\mathrm{DM}} \neq 0$$

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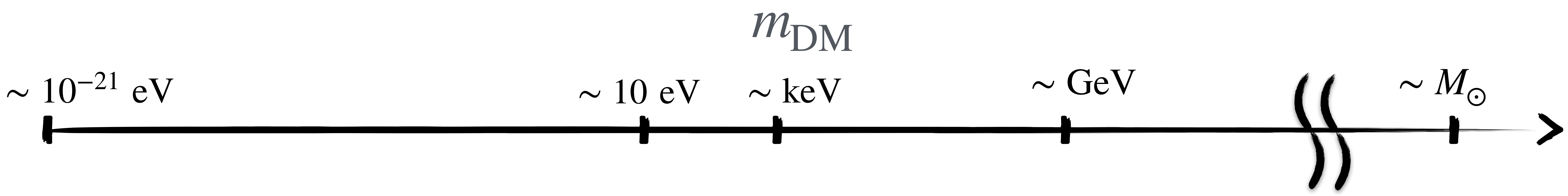

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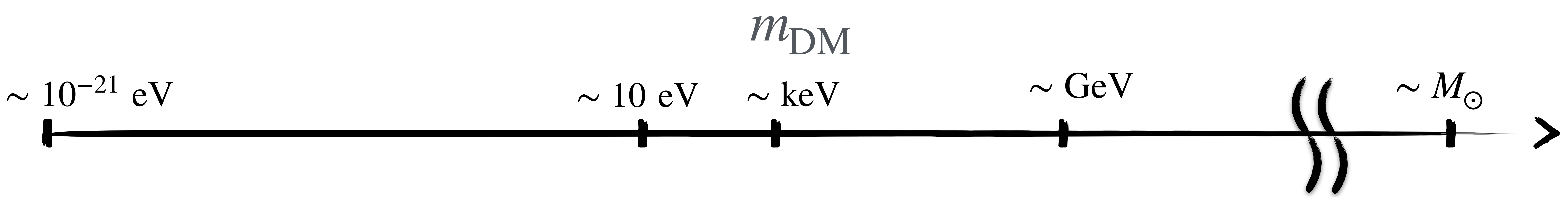

$$\Omega_{\rm DM} \neq 0$$

Many possibilities : origin, properties, features

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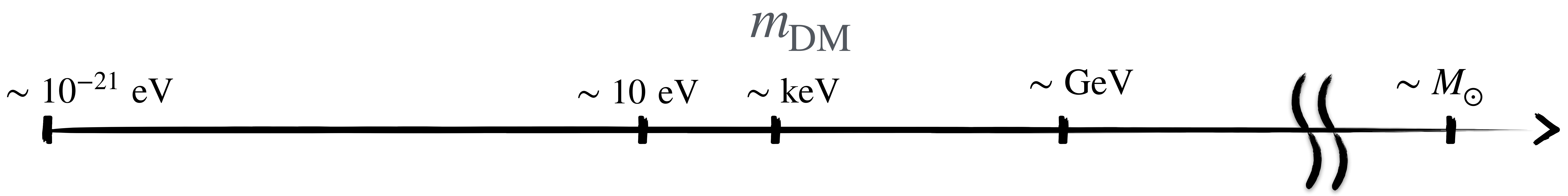

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Many possibilities : origin, properties, features

Important to identify unique (cosmological) signatures

Crash Course on (Isocurvature) Perturbations

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Curvature Perturbations <—> Density Perturbations —> Structure/Effects!

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Adiabatic Perturbations	Isocurvature Perturbations

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- Adiabatic Number + Non-Adiabatic Momentum

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- Modulation of the Free Streaming Length
- Modulation of the Matter Power Spectrum

1. Kinetic Isocurvature Perturbation

DM Energy Density Perturbations

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The energy density of DM

$$\rho_{\rm DM} = E_{\rm DM}\, n_{\rm DM}, \quad E_{\rm DM} \equiv \sqrt{m_{\rm DM}^2 + p_{\rm DM}^2}$$

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Kinetic Isocurvature Perturbation

Fluctuations imprinted at the production of DM, inheriting p modulation.

Kinetic Isocuvature Perturbation

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For (warm) DM, as DM becomes non-relativistic

$$v_{\rm DM} = \frac{p_{\rm DM}}{E_{\rm DM}} \propto \frac{1}{a} \qquad\qquad \frac{\delta\rho_{\rm DM}}{\rho_{\rm DM}} \propto \frac{1}{a^2}$$

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Setup to realize kinetic isocurvature perturbations?

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2. Setup

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Key : adiabatic number perturbations + modulated kinetic distribution

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$$\mathcal{L} \supset \begin{cases} A\phi\chi^2 & \chi: \text{ scalar} \\ y\phi\bar{\chi}\chi & \chi: \text{ fermion.} \end{cases}$$

$$\Gamma = \begin{cases} \dfrac{A^2}{16\pi m_\phi}\left(1-\dfrac{4m_\chi^2}{m_\phi^2}\right)^{1/2} \\ \dfrac{y^2 m_\phi}{8\pi}\left(1-\dfrac{4m_\chi^2}{m_\phi^2}\right)^{3/2} \end{cases}$$

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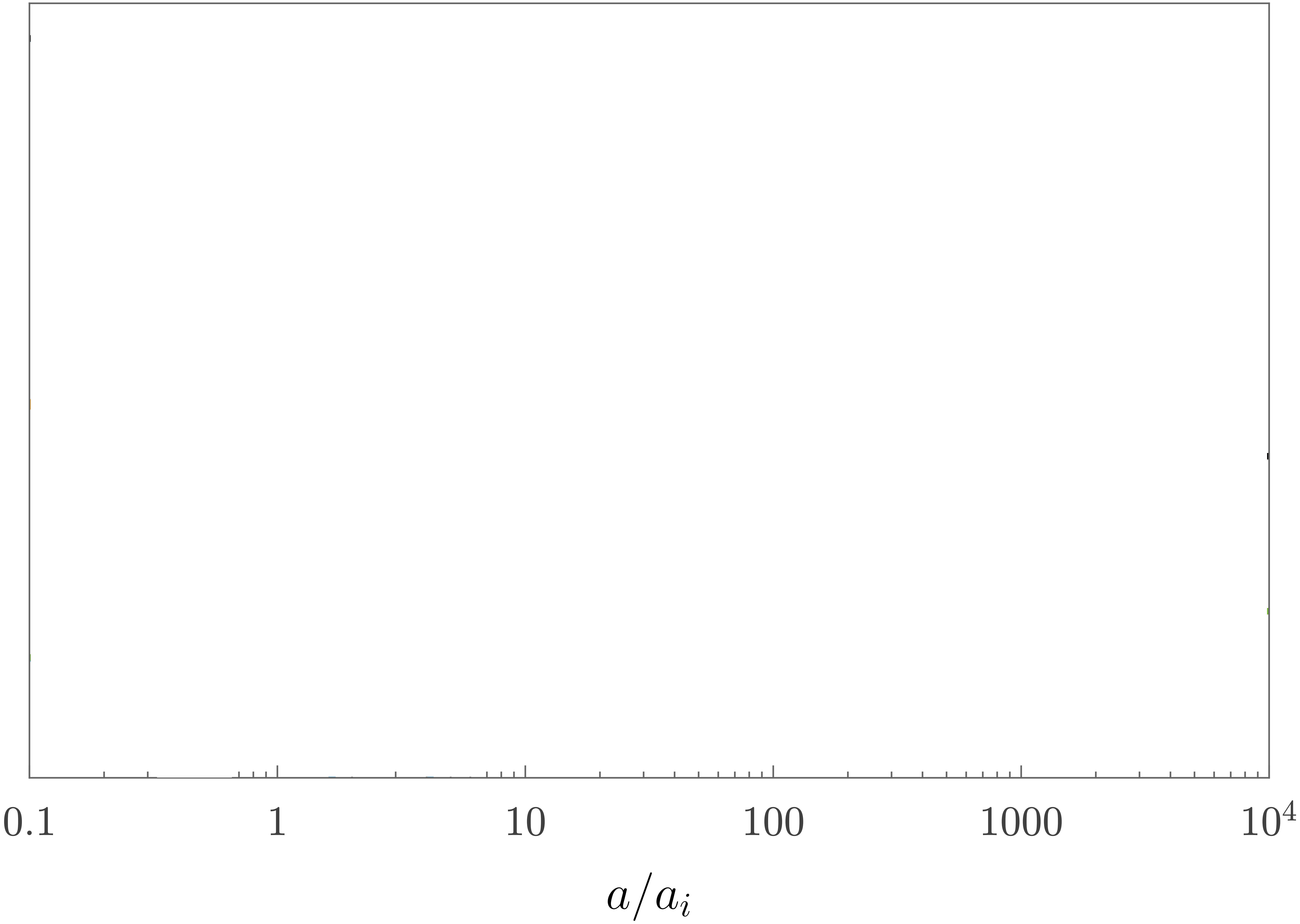
$$\boxed{\delta\sigma \to (\delta A, \delta y) \to \delta\Gamma}$$

Energy density modulation



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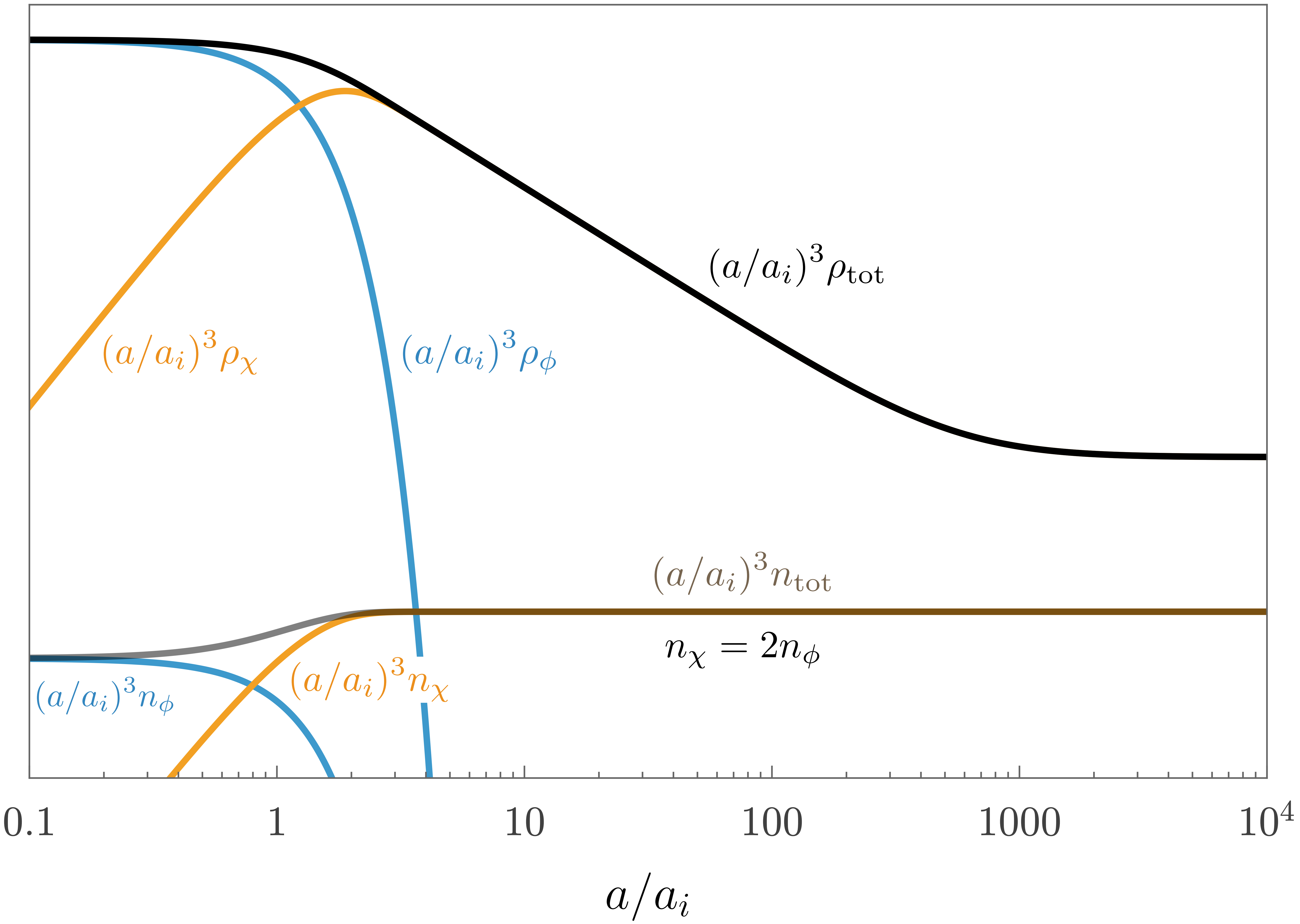

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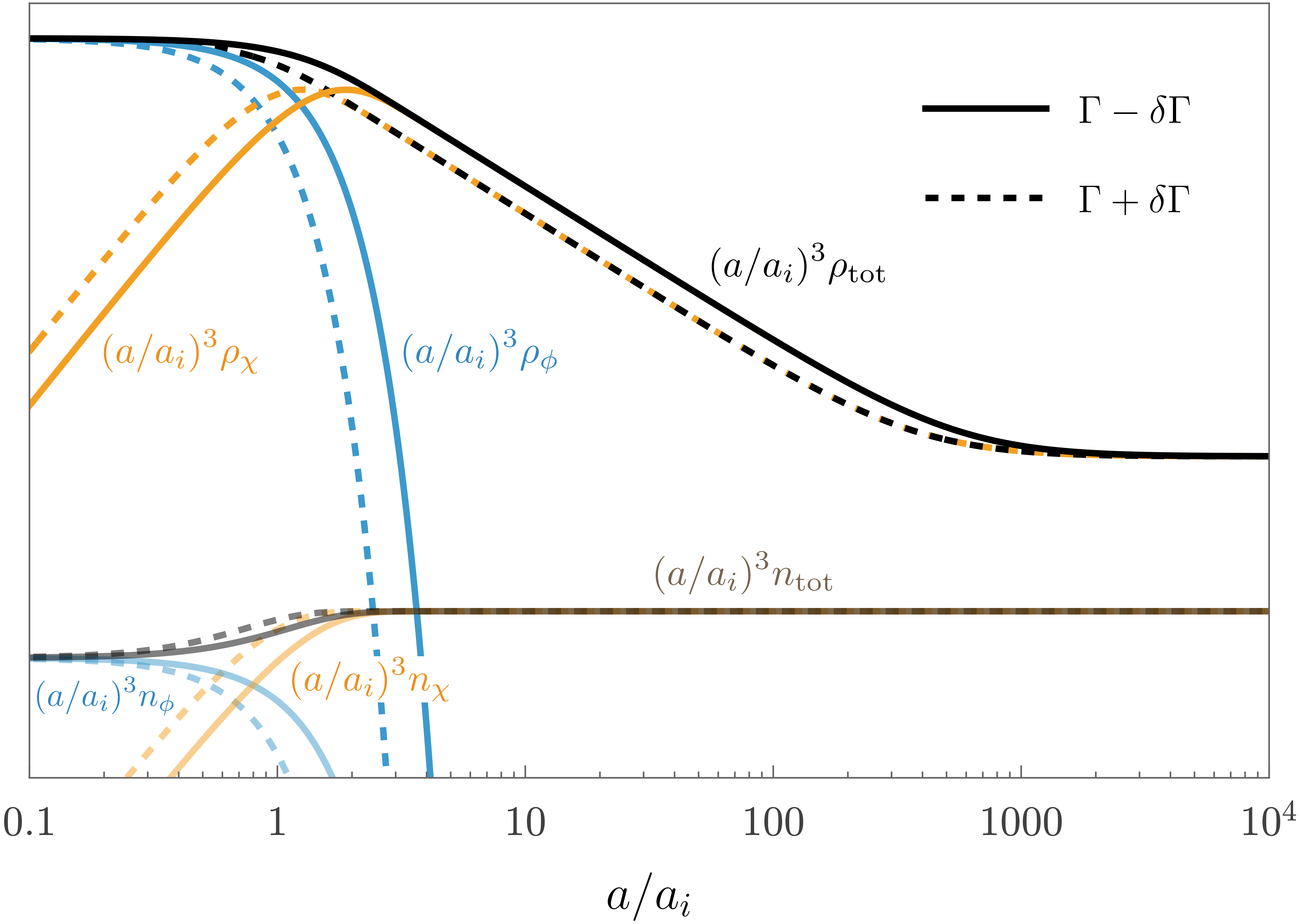

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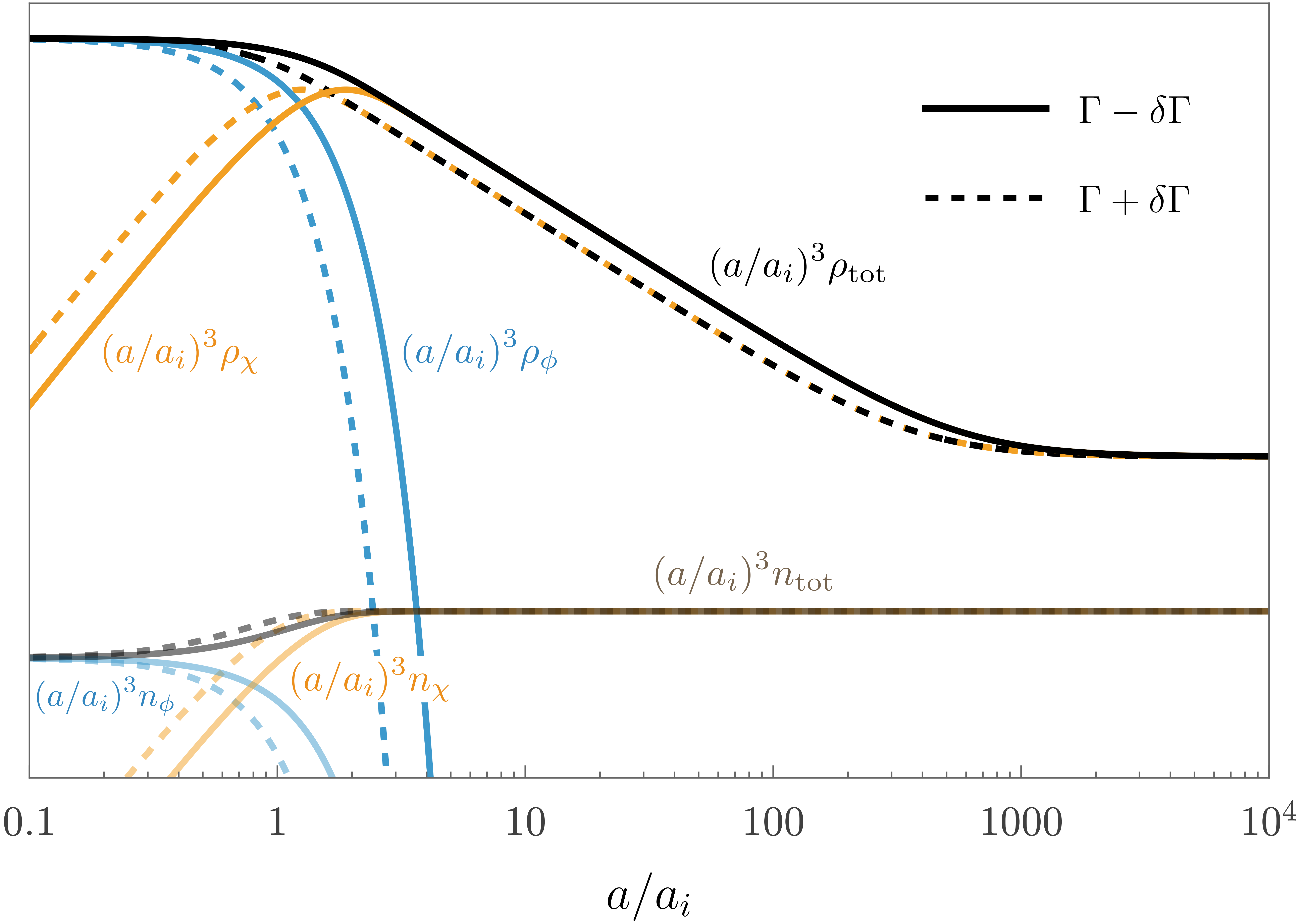
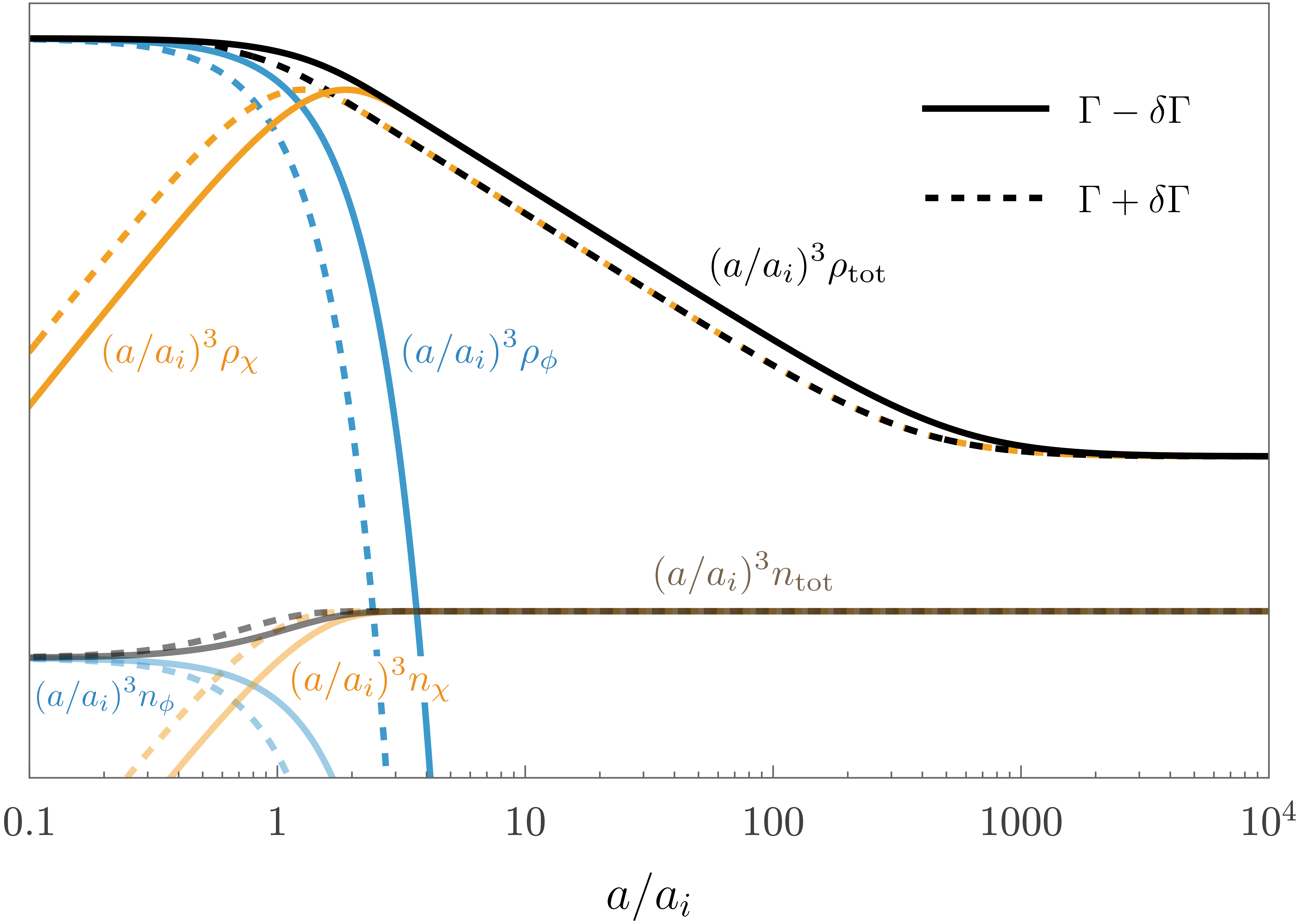


Decay yield

$$n_\chi(a_i) = 2n_\phi(a_i)\,.$$

Fixed, no δn

Energy density modulation



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Fixed, no δn

Different decay times t_i

Momentum redshift

$$p(a) = p_i\left(a_i/a\right)$$

Induced $\delta\rho_\chi$

Relation between $\delta\rho$ and δ_λ

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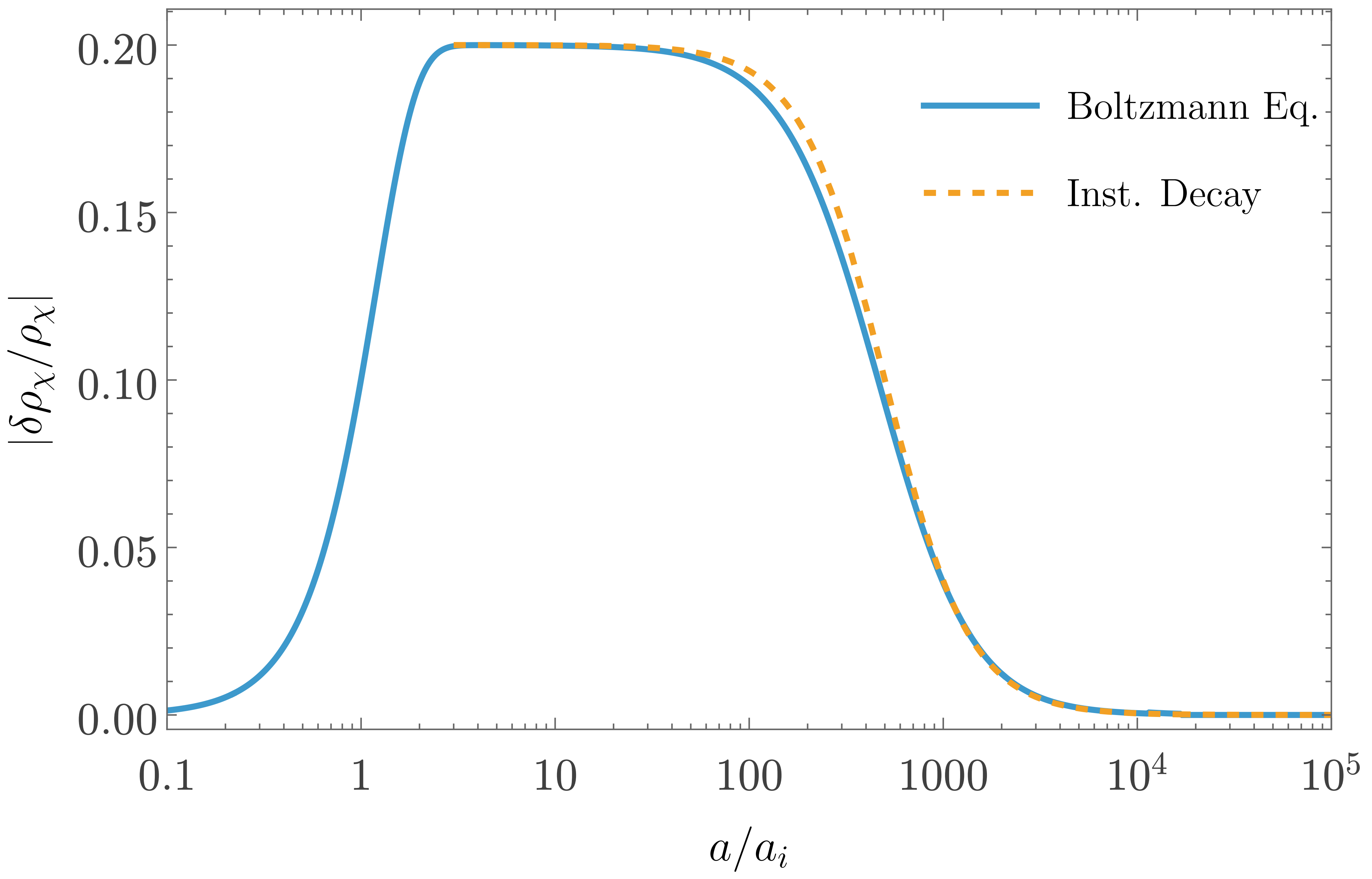

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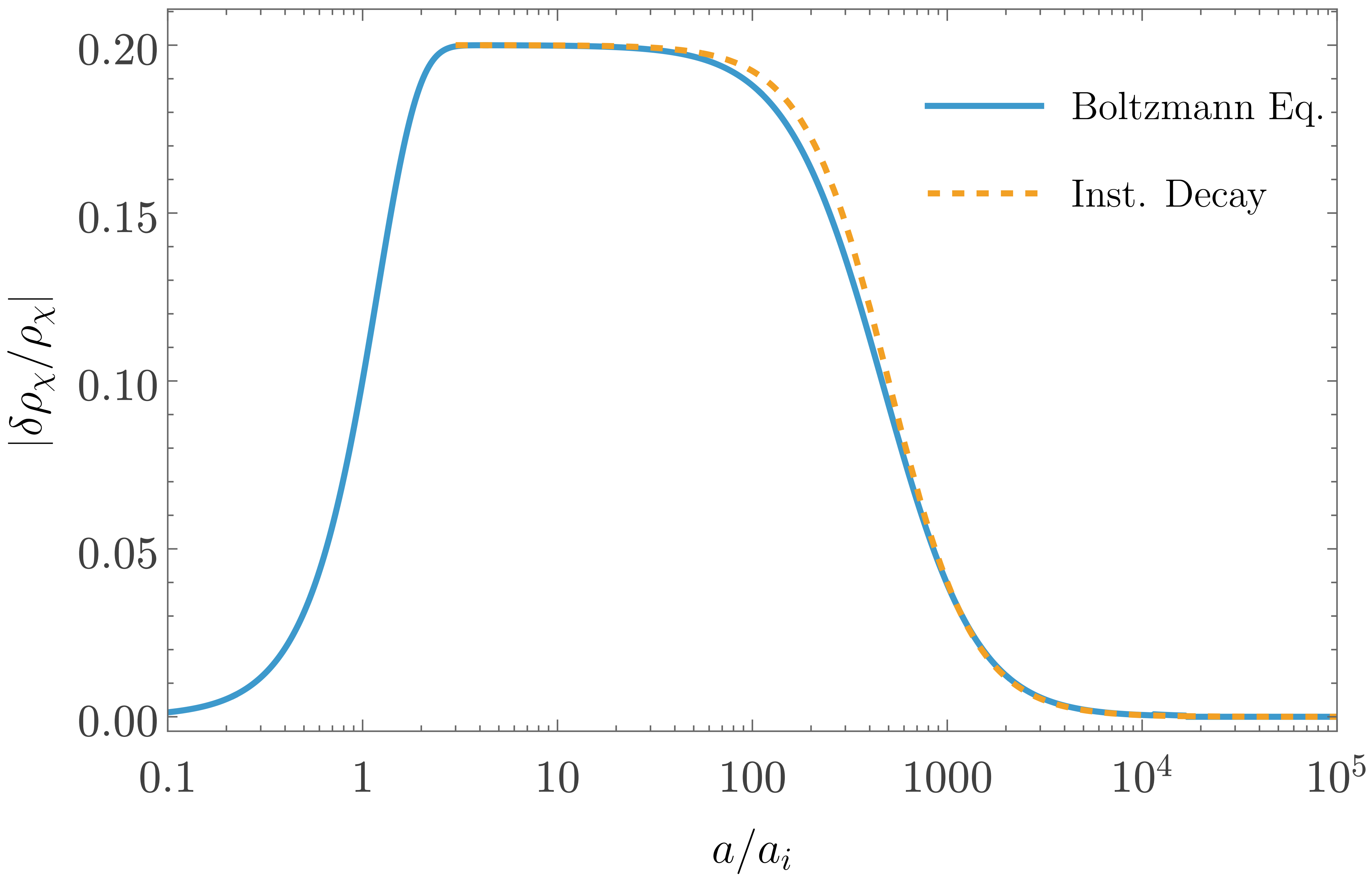

Analytic relation w/ instantaneous decay

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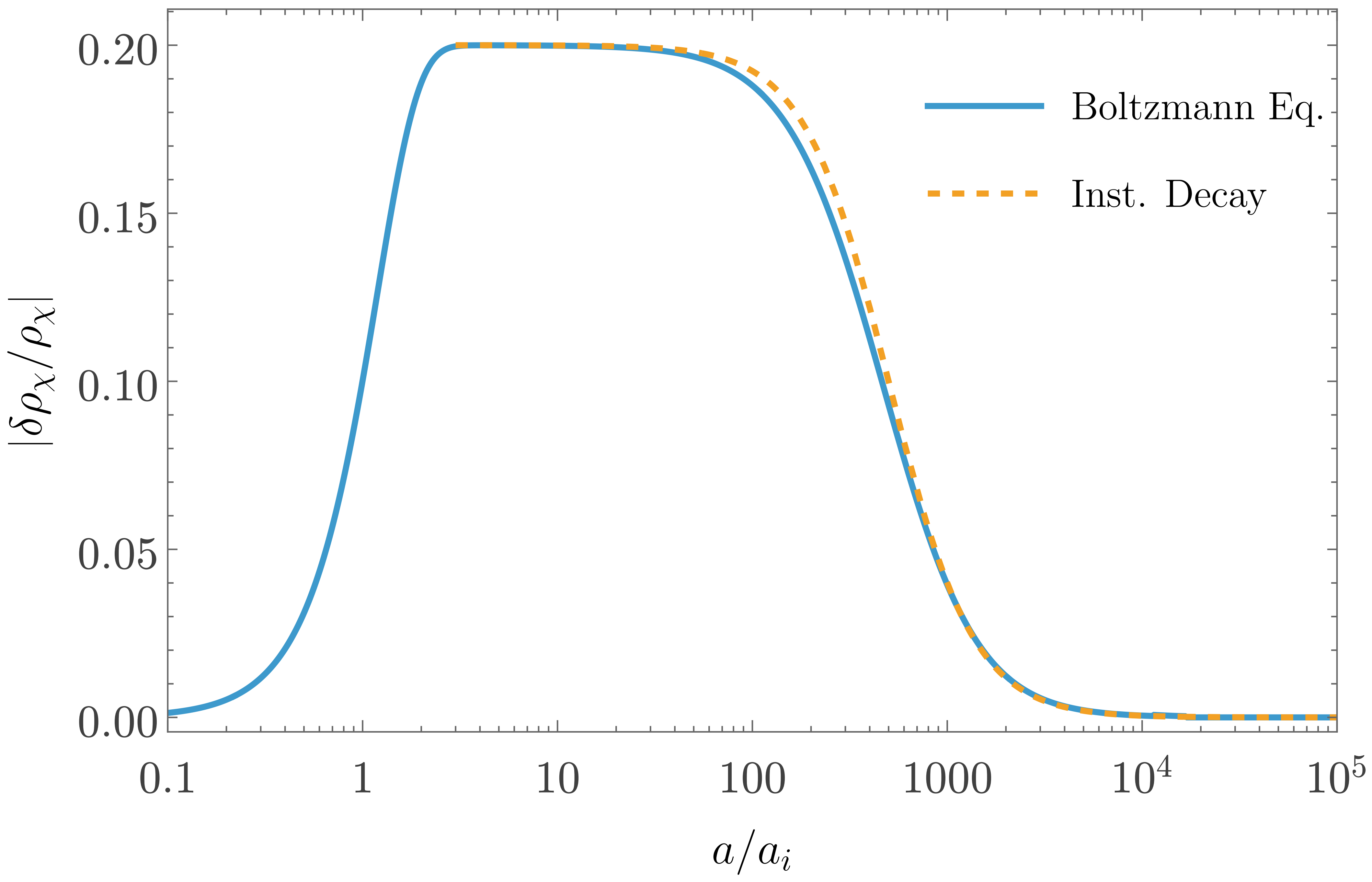

Analytic relation w/ instantaneous decay

$$\frac{\delta\rho_\chi}{\rho_\chi} \simeq -\frac{1}{2}\left[\frac{\left(m_\phi^2/4\right)(a_i/a)^2}{m_\chi^2 + \frac{m_\phi^2}{4}\left(\frac{a_i}{a}\right)^2}\right]\frac{\delta\Gamma}{\Gamma}.$$

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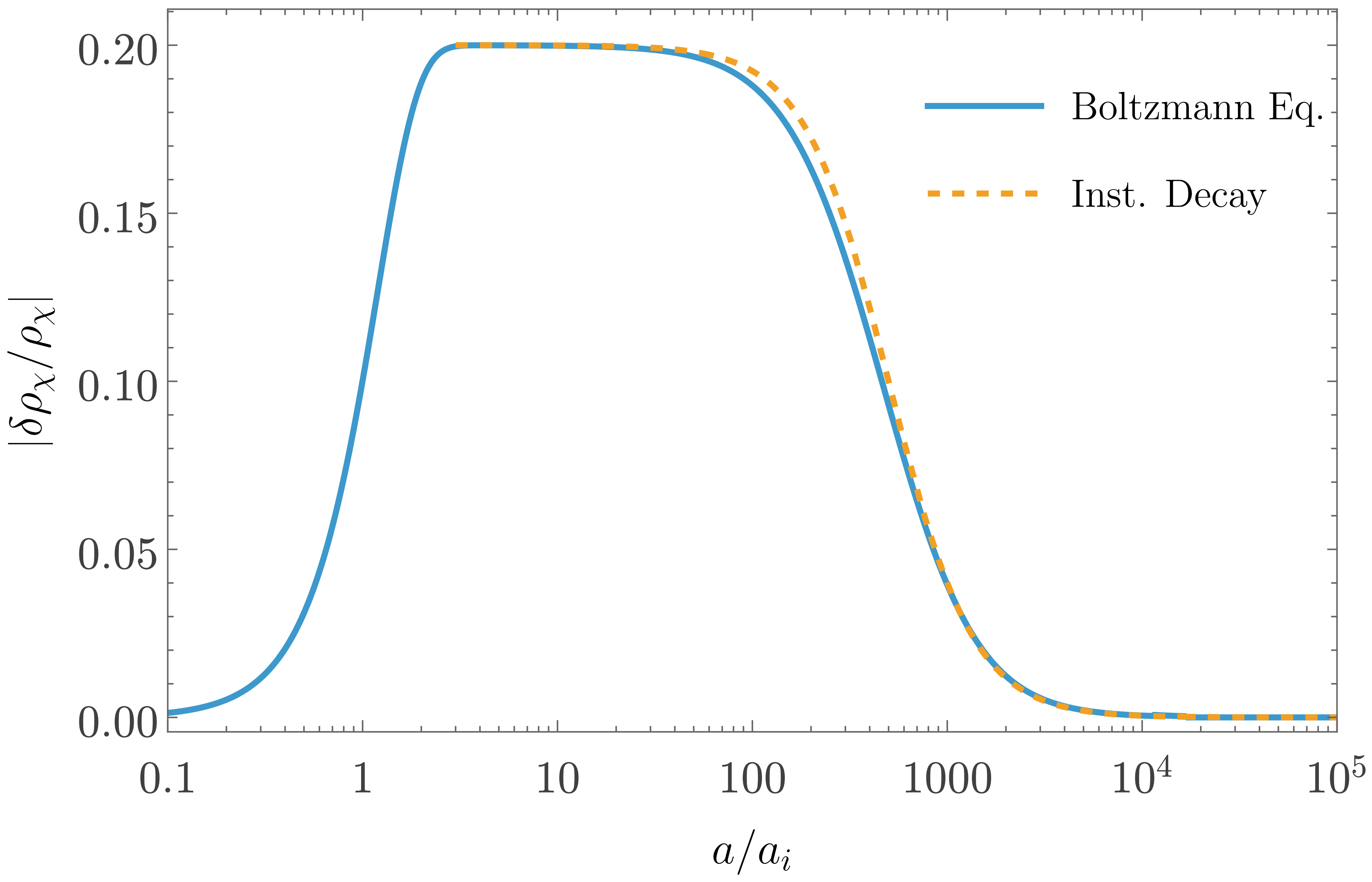

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$$\delta_\lambda \simeq -\frac{1}{2}\frac{\delta\Gamma}{\Gamma}$$

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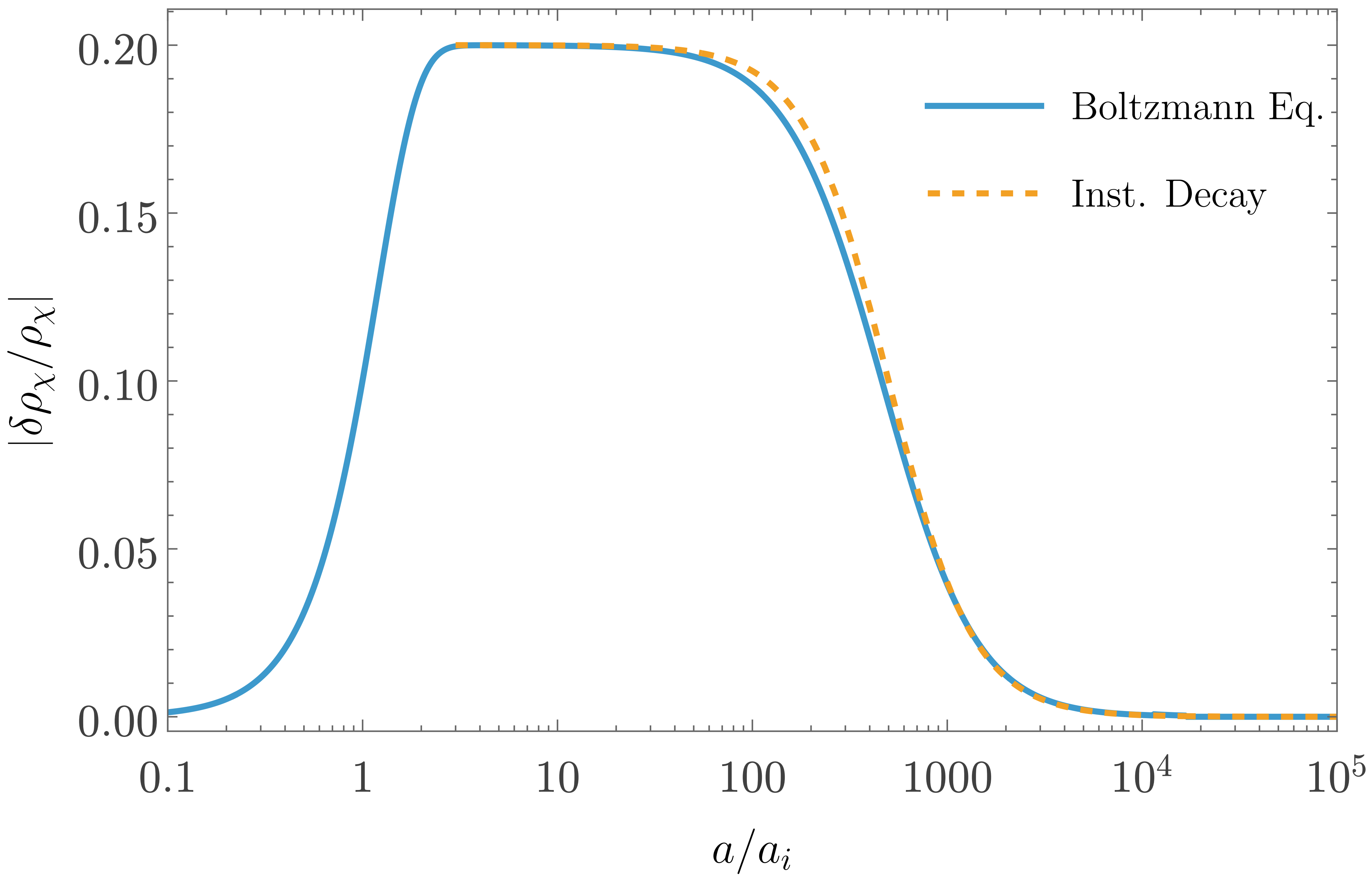

Analytic relation w/ instantaneous decay

$$\frac{\delta\rho_\chi}{\rho_\chi} \simeq -\frac{1}{2}\left[\frac{\left(m_\phi^2/4\right)(a_i/a)^2}{m_\chi^2 + \frac{m_\phi^2}{4}\left(\frac{a_i}{a}\right)^2}\right]\frac{\delta\Gamma}{\Gamma}.$$

$$\delta_\lambda \simeq -\frac{1}{2}\frac{\delta\Gamma}{\Gamma}$$

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Relation between $\delta\rho$ and δ_λ

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Within our region of interest,

$$u_i = p_i/m_\chi$$

$$\lambda_{\rm FS} \simeq 1{\rm Mpc}\left(\frac{u_i^2 t_i}{10^6{\rm sec}}\right)^{1/2}\left[1 - 0.07\ln\left(\frac{u_i^2 t_i}{10^6{\rm sec}}\right)\right]$$

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Any constraints on $\delta\rho_\chi/\rho_\chi \leftrightarrow \delta_\lambda$?

3. Isocurvature Constraints

Kinetic Isocurvature Constraints

Kinetic Isocurvature Constraints

We consider kinetic isocurvature perturbations entering the horizon with

$$\langle \delta_\lambda(k)\delta_\lambda(k')\rangle = (2\pi)^3\delta(k+k')\frac{2\pi^2}{k^3}P_{\delta_\lambda}(k)$$

$$P_{\rm kin}(k) \simeq 2\times 10^{-4} g_* \left(\lambda_{\rm FS}k\right)^4 P_{\delta_\lambda}(k).$$

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Two representative profiles :

$$P^{(1)}_{\delta_\lambda}(k) = A_{\delta_\lambda}\delta\left(\ln k - \ln k_0\right)$$
$$P^{(2)}_{\delta_\lambda}(k) = A_{\delta_\lambda}$$

Kinetic Isocurvature Constraints

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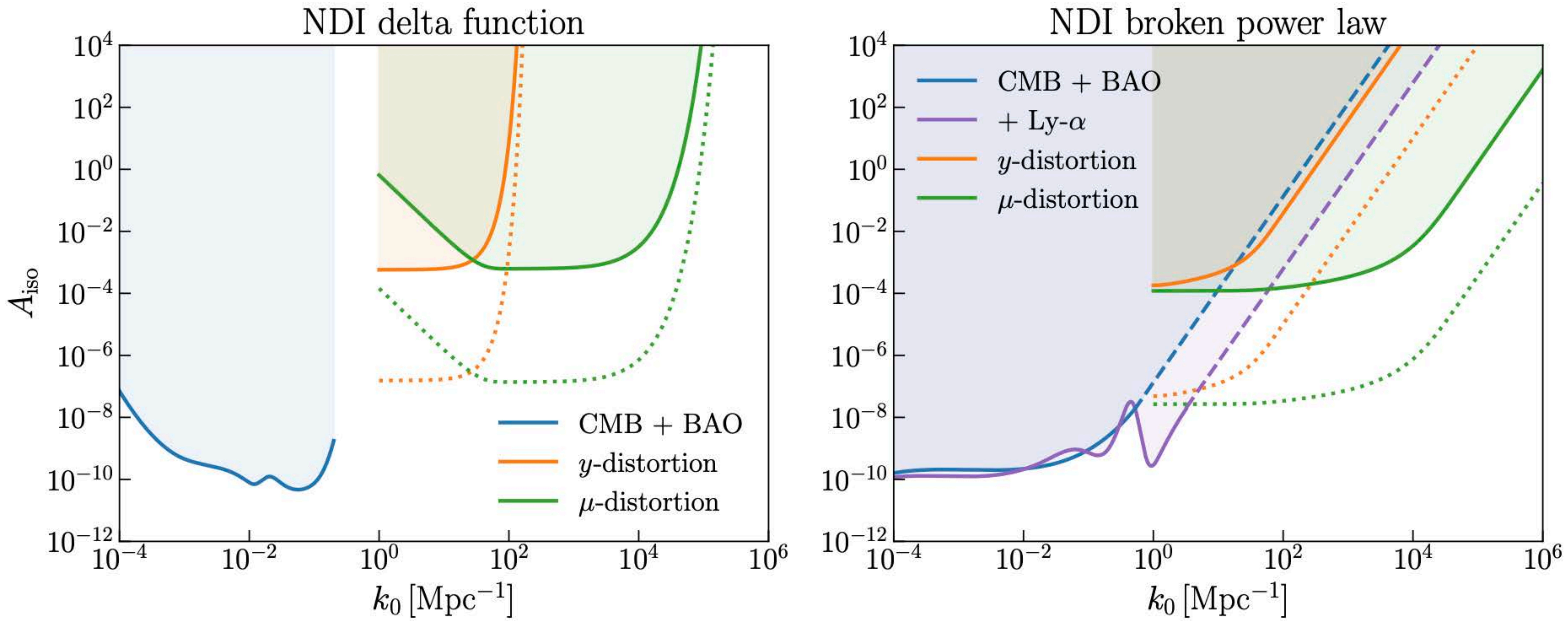
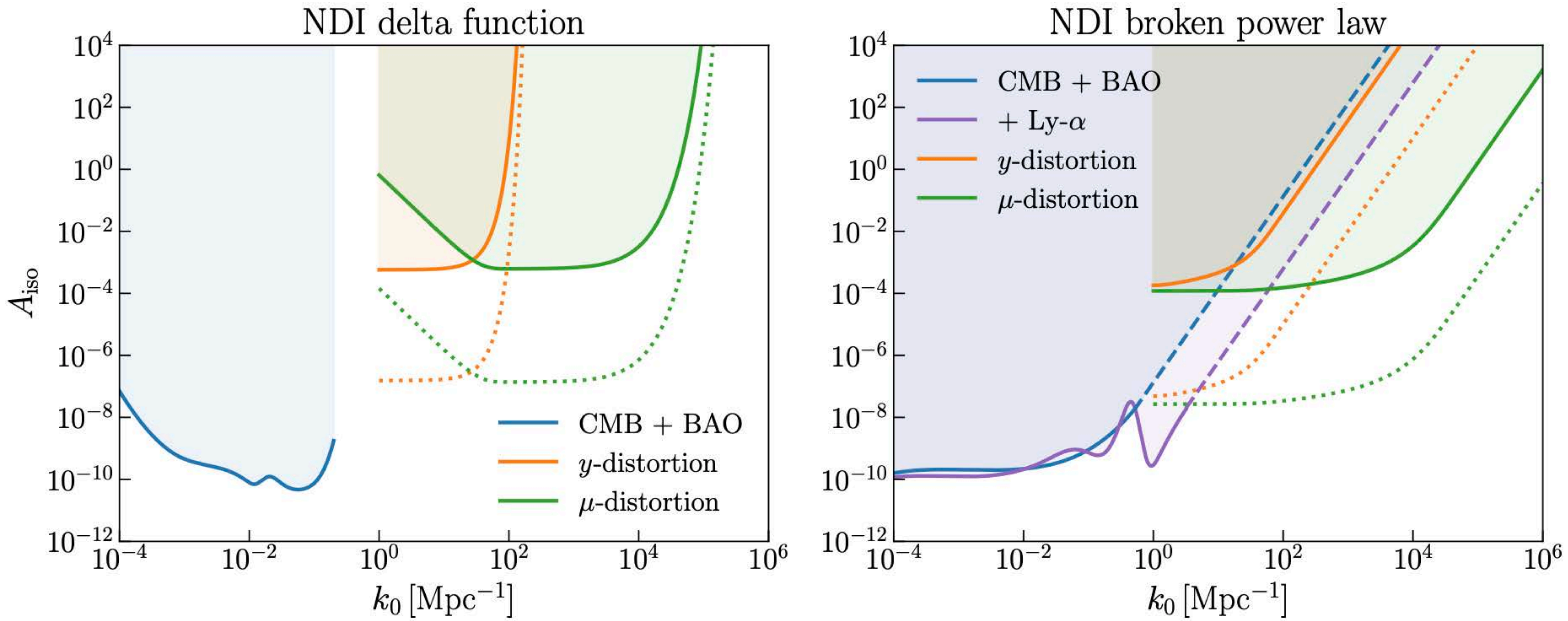


[Buckley et. al., 2025]

Kinetic Isocurvature Constraints



[Buckley et. al., 2025]

We rescale NDI constraints with $A_{\rm kin} = A_{\rm NDI}\left(\frac{\rho_\nu}{\rho_\chi}\right)^2$

Kinetic Isocurvature Constraints

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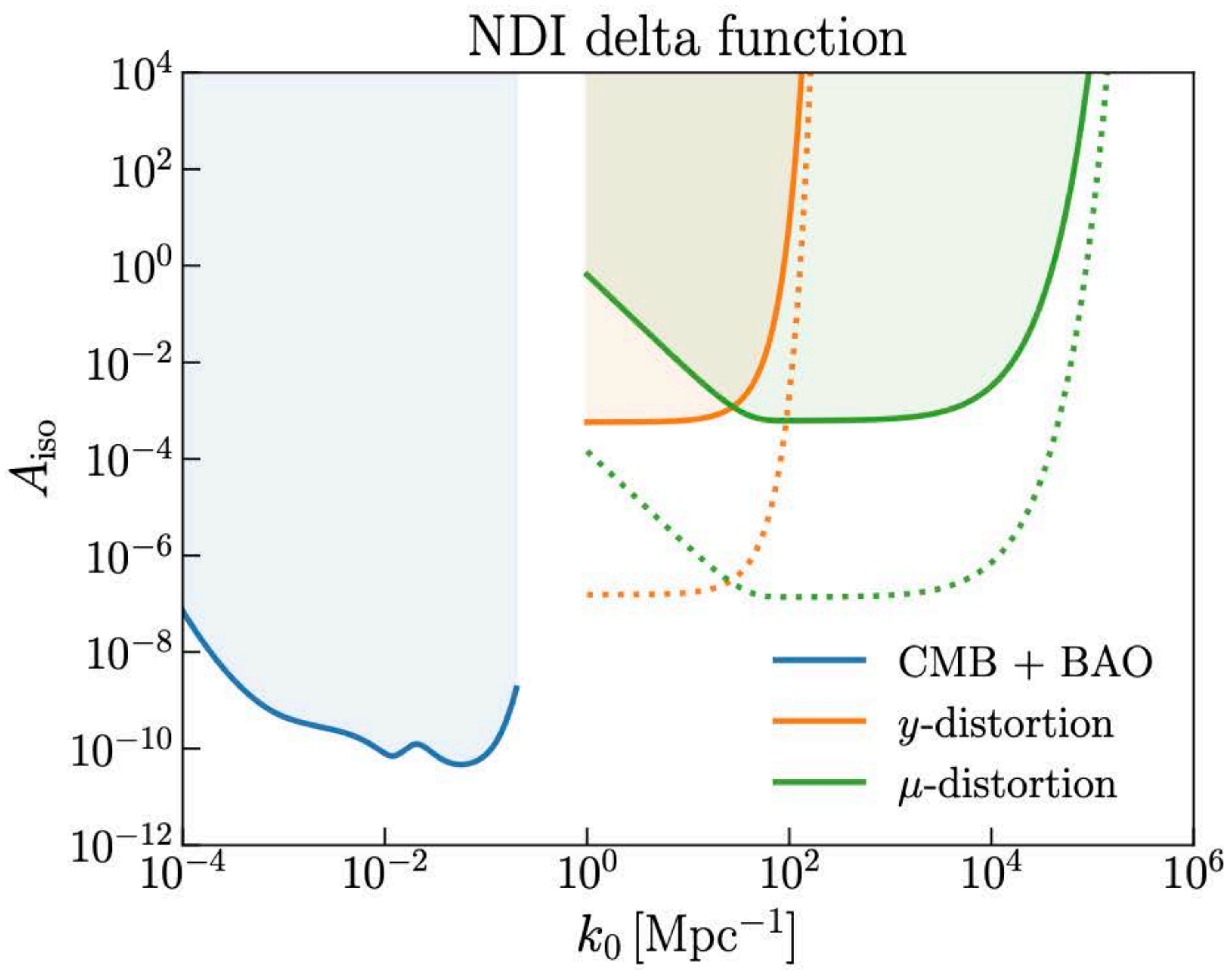

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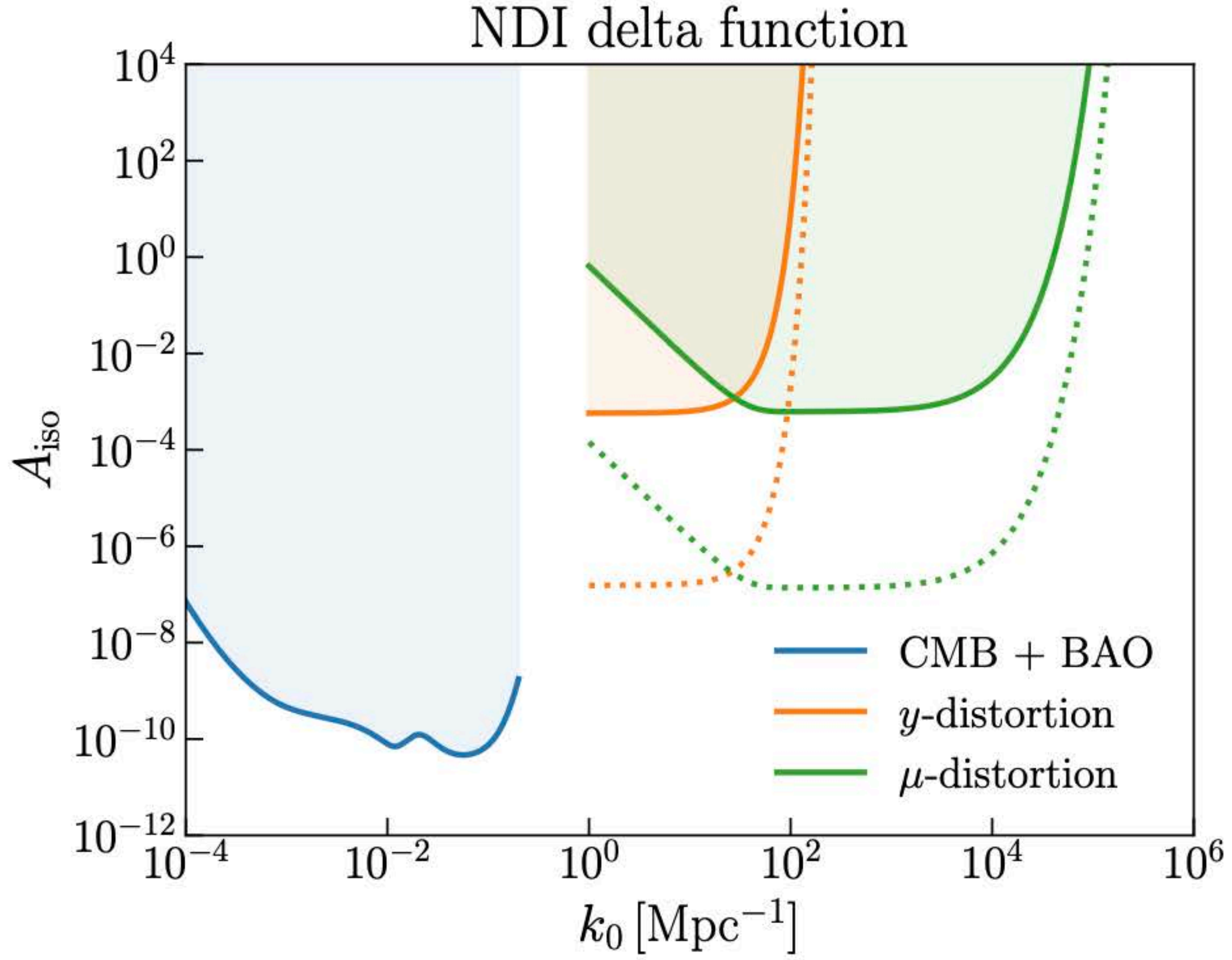

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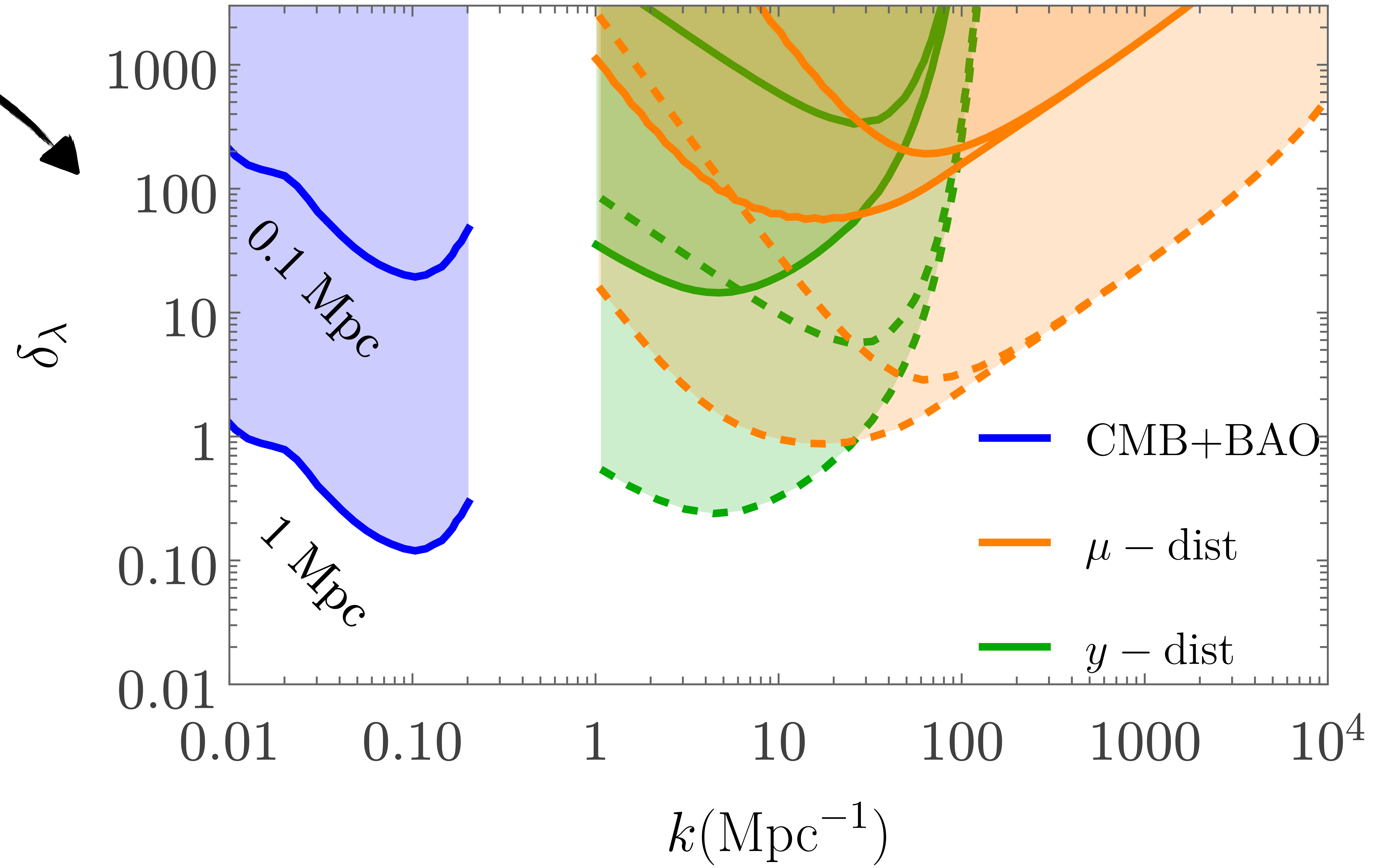

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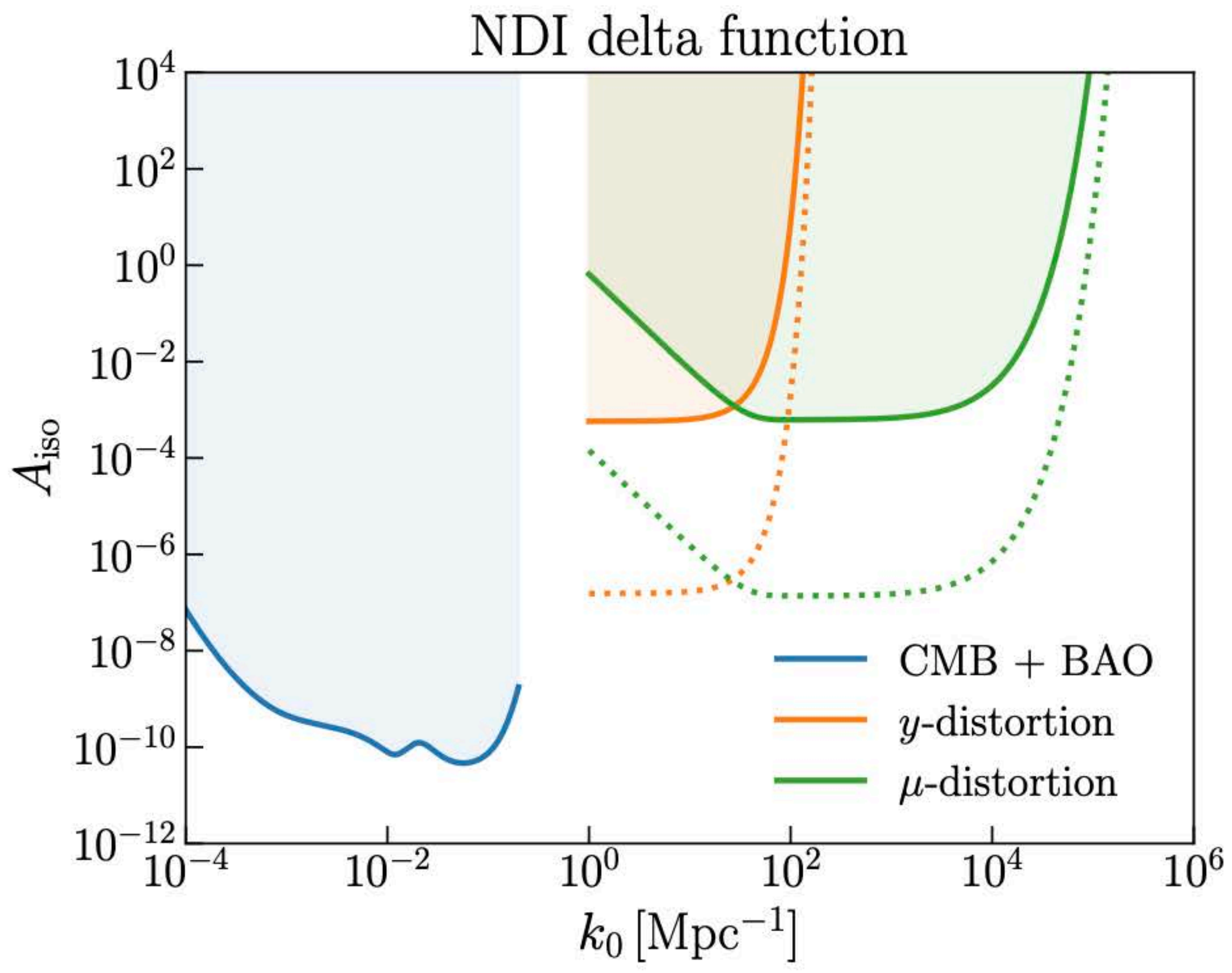

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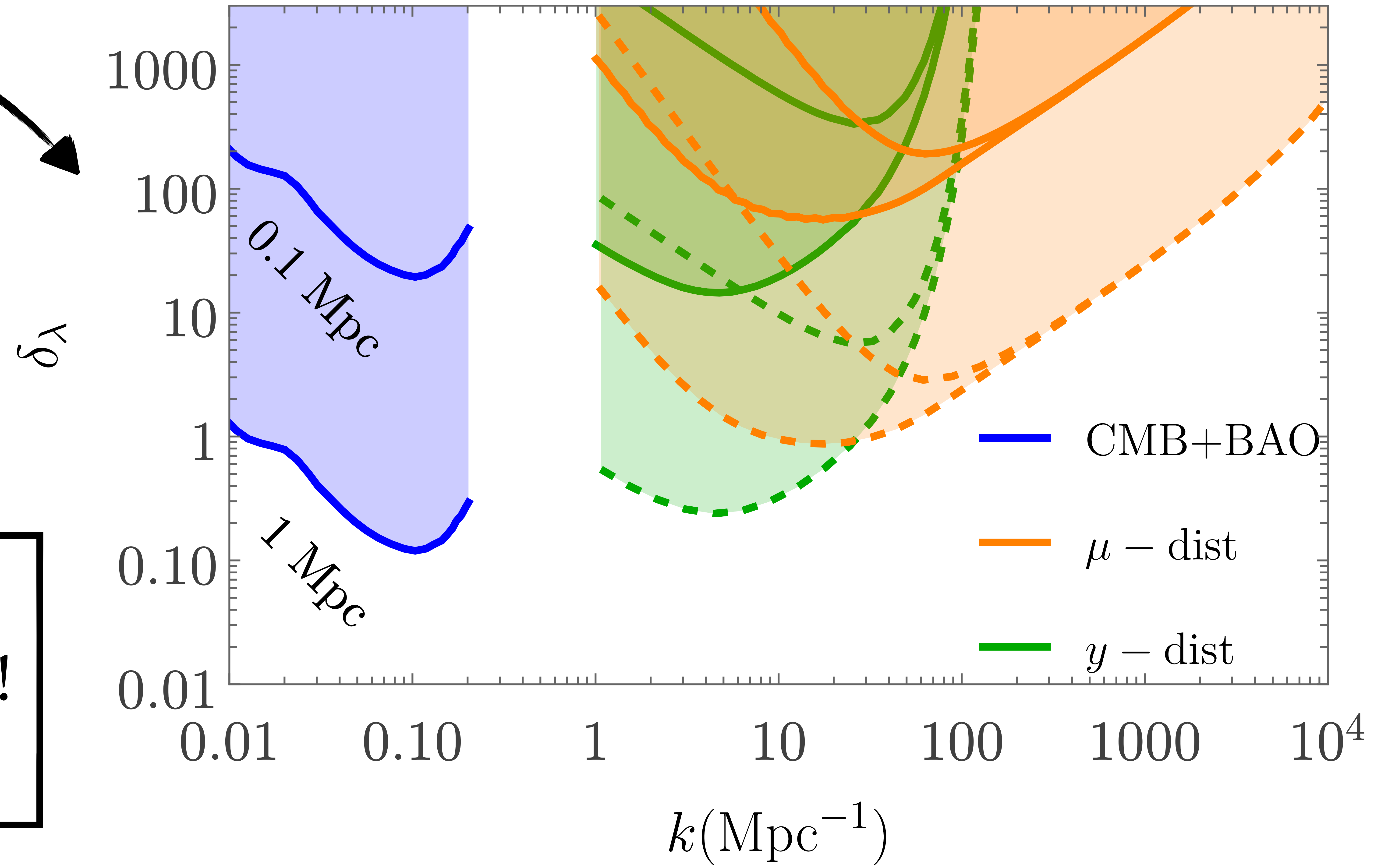

No constraints for $A_{\delta_\lambda} \lesssim 1$!

Kinetic Isocurvature Constraints

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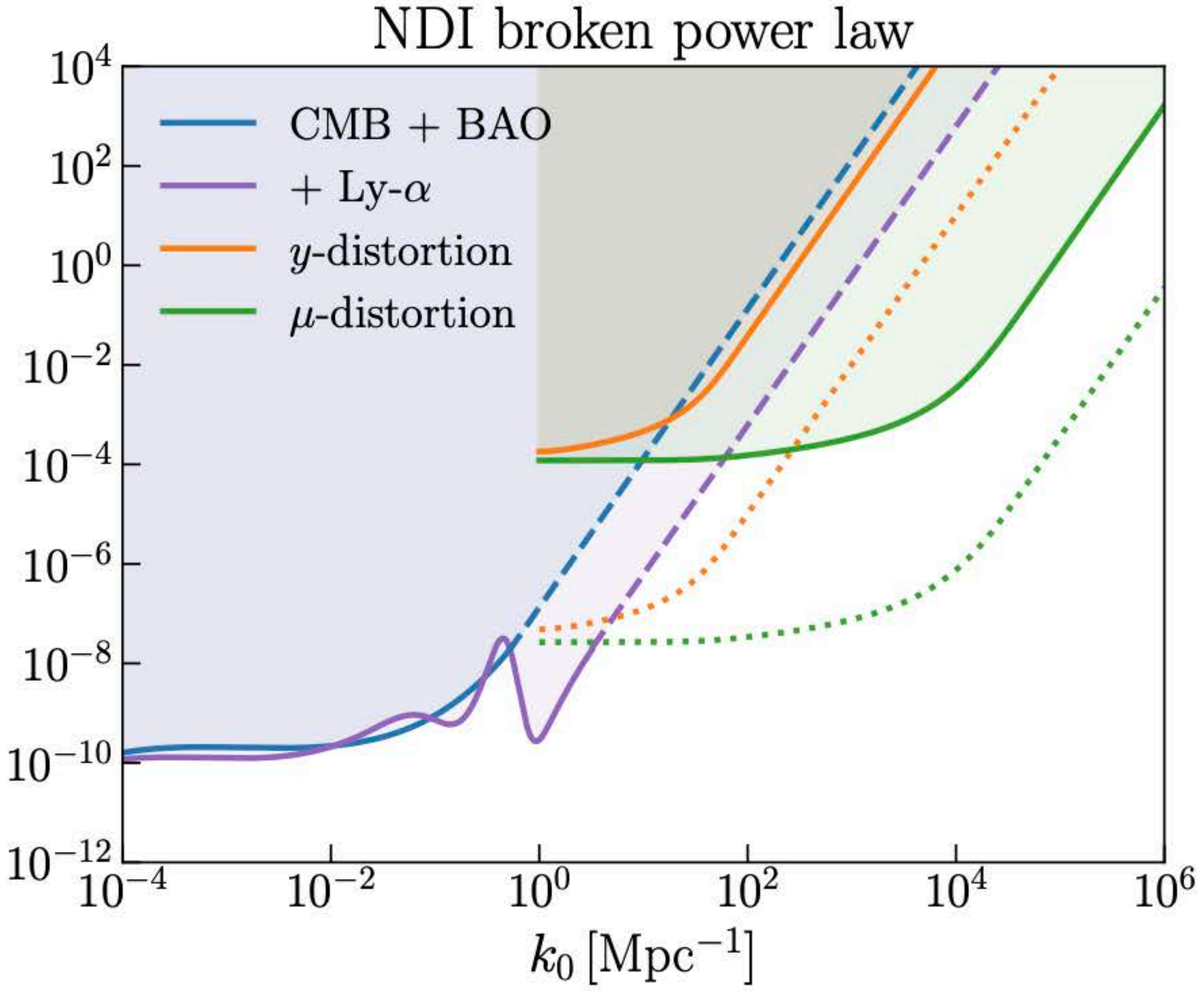

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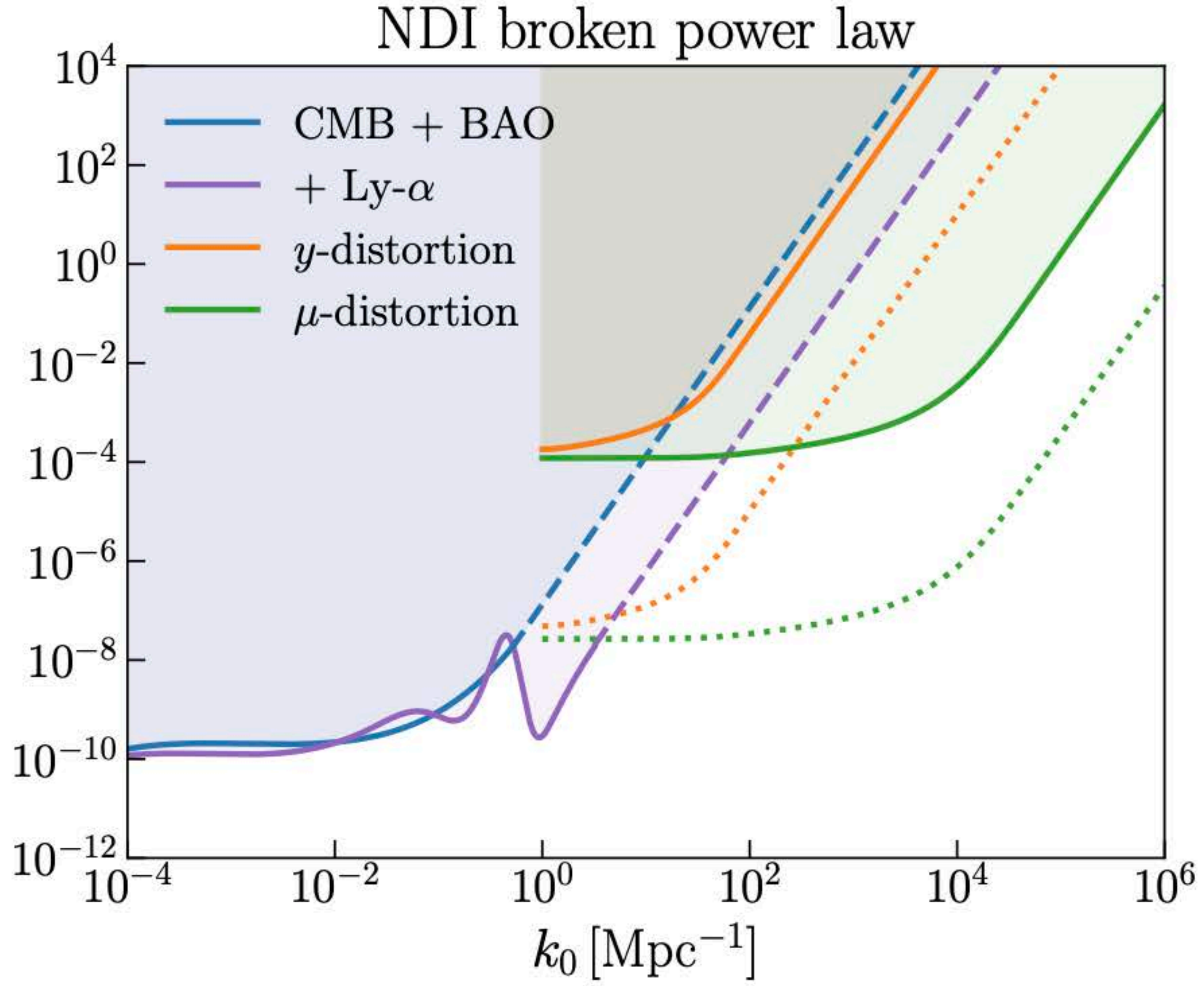
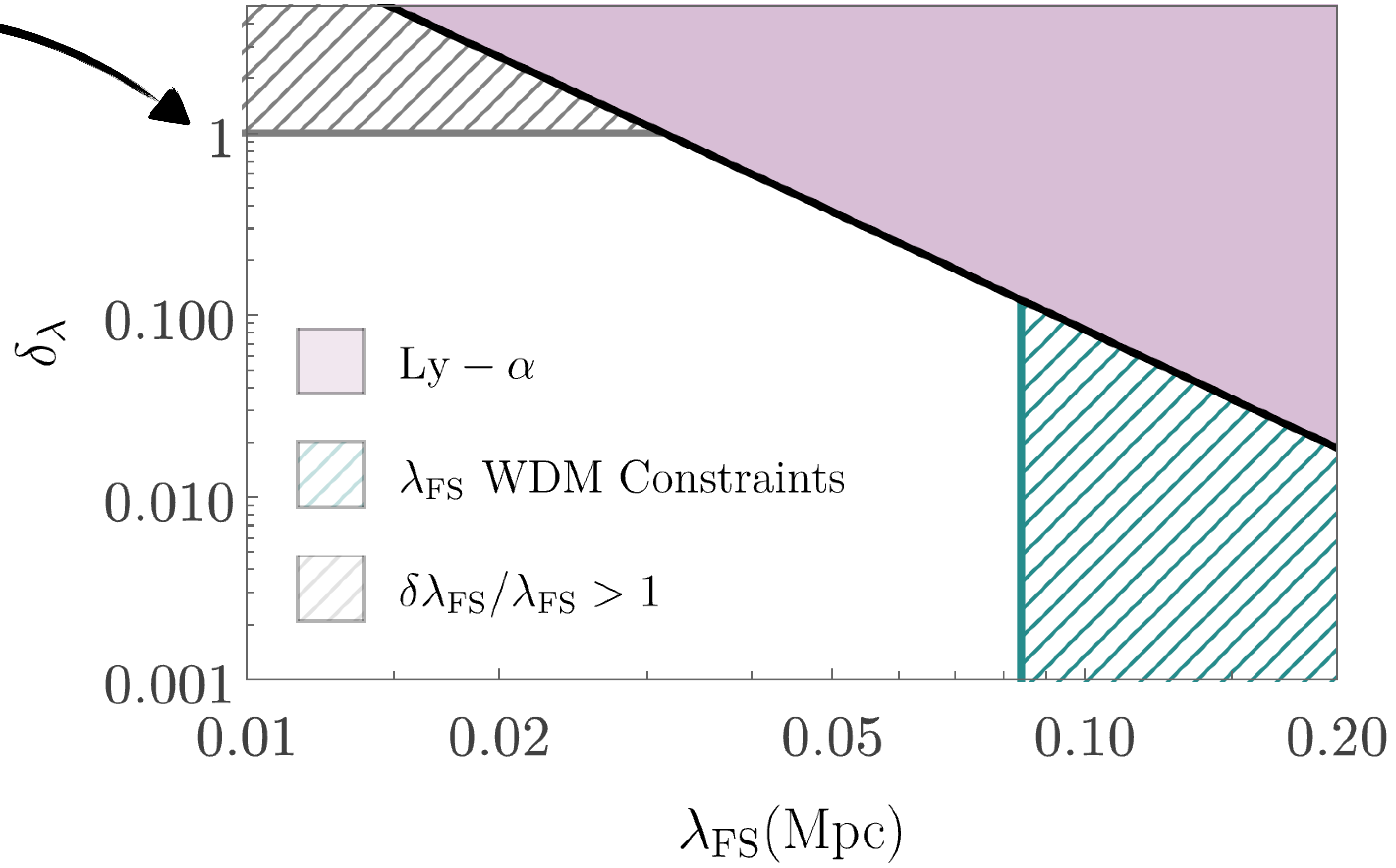


$$P^{(2)}_{\delta_\lambda}(k) = A_{\delta_\lambda}$$



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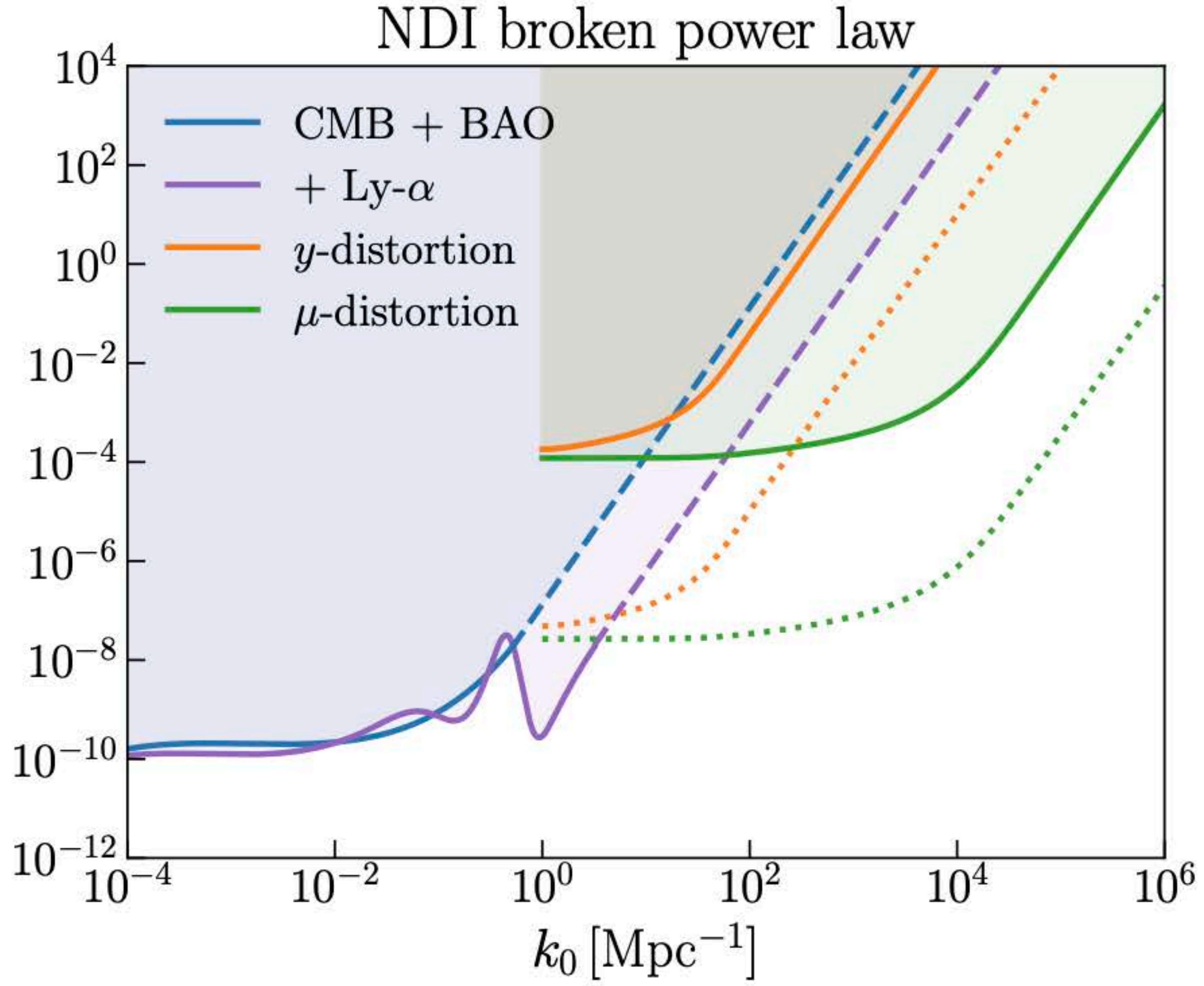

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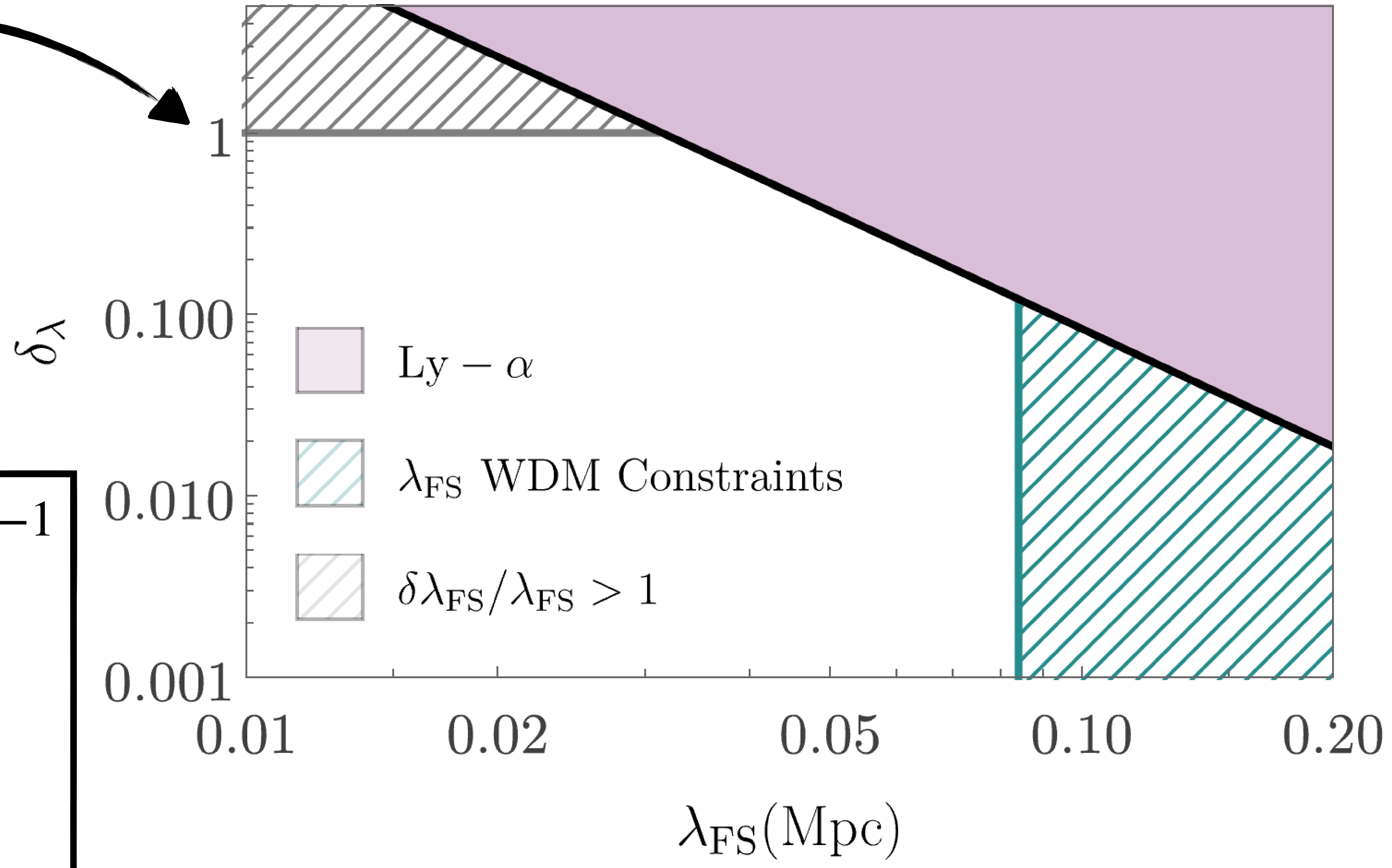
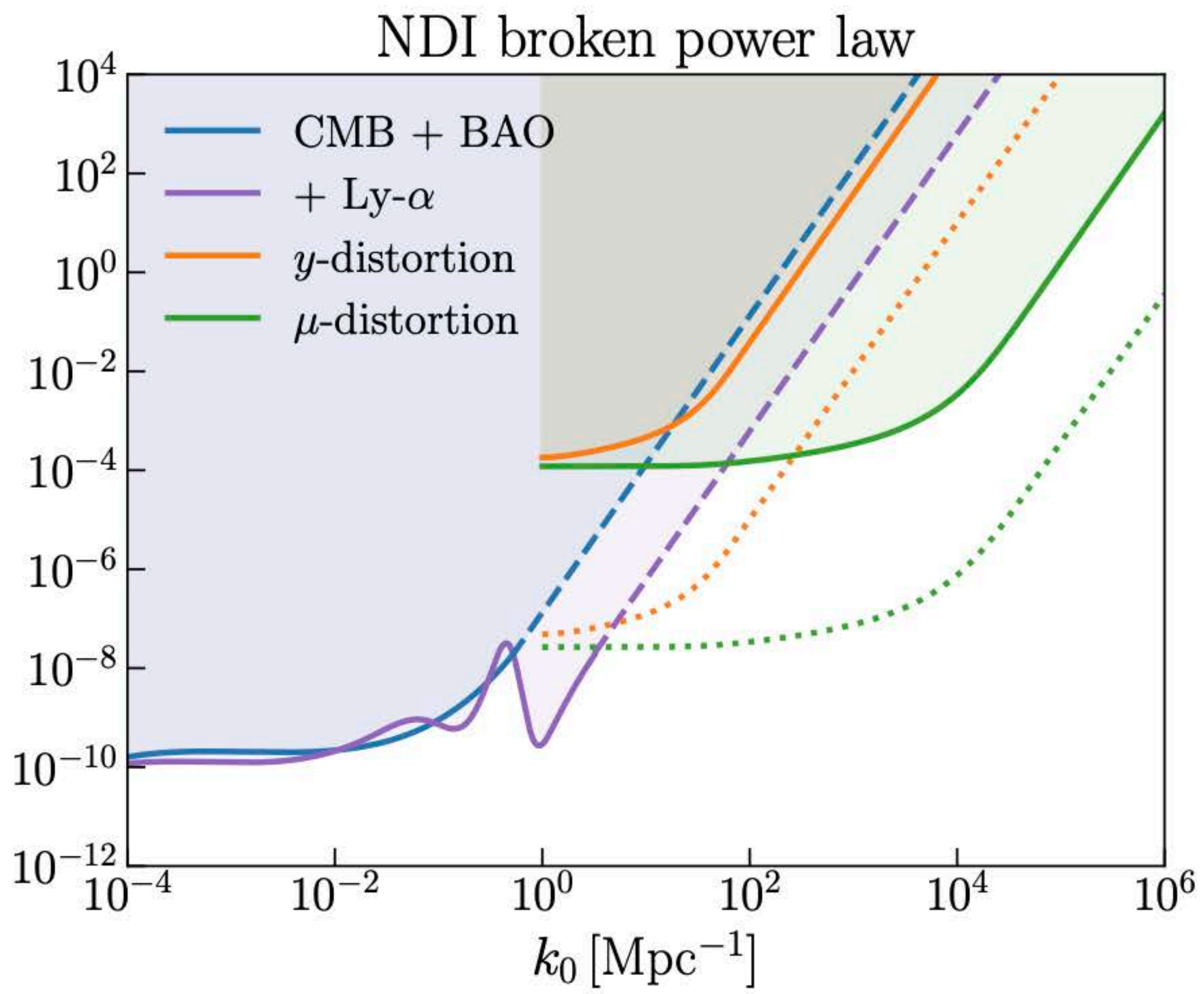


Ly$-\alpha$ bounds at $k \sim 1\,\mathrm{Mpc}^{-1}$

Kinetic Isocurvature Constraints



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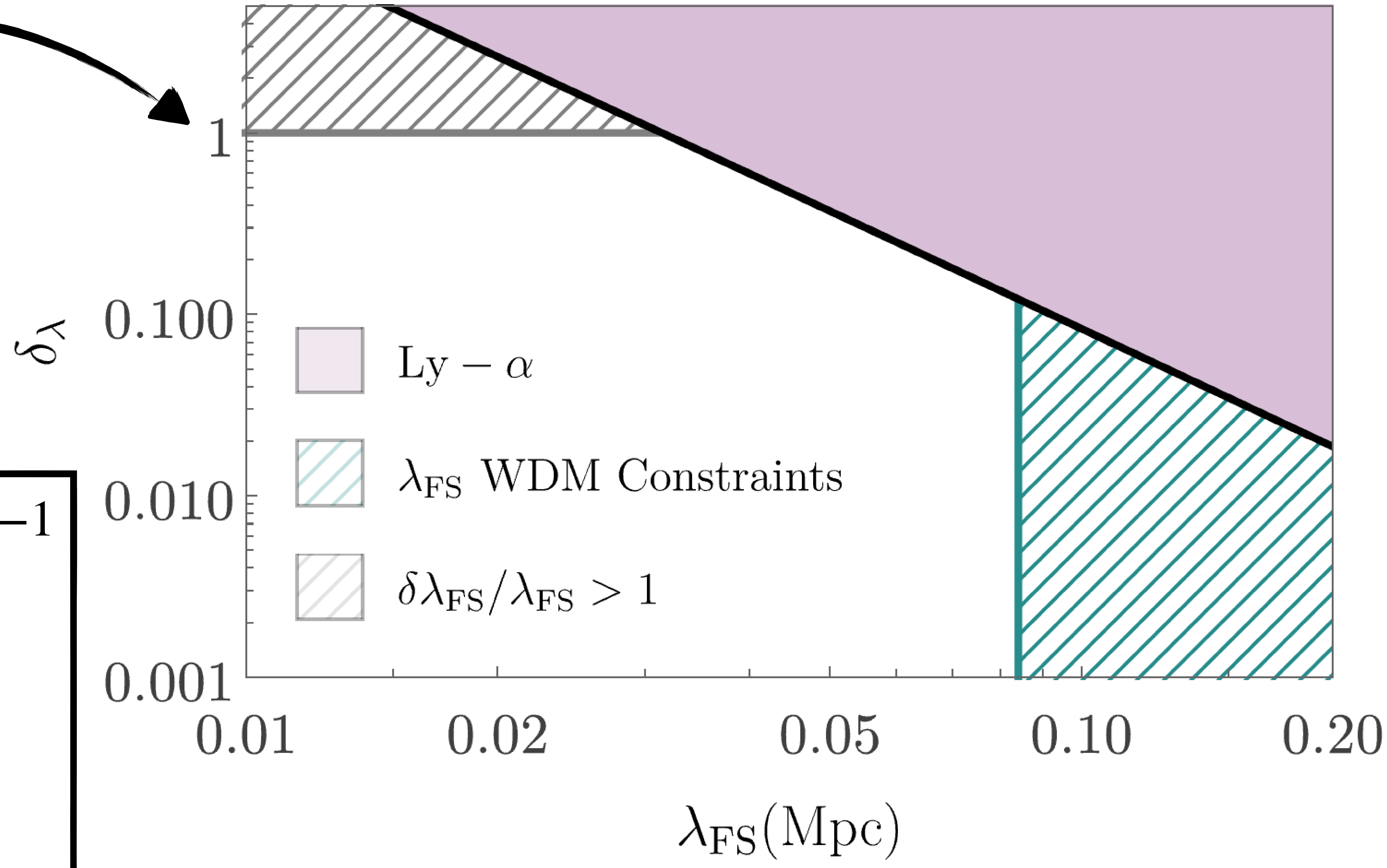

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$\delta_\lambda = \mathcal{O}(0.1-1)$ allowed!

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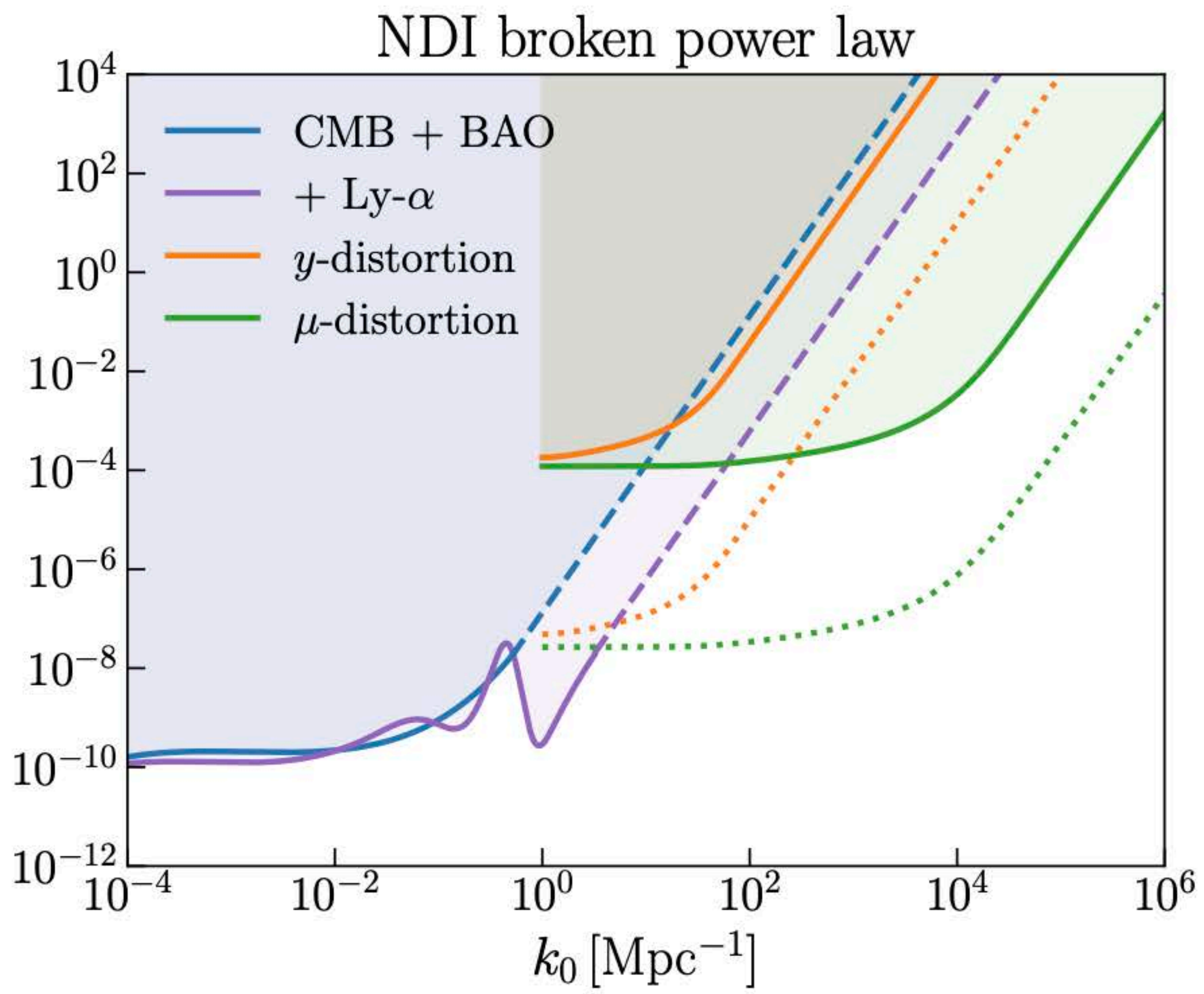

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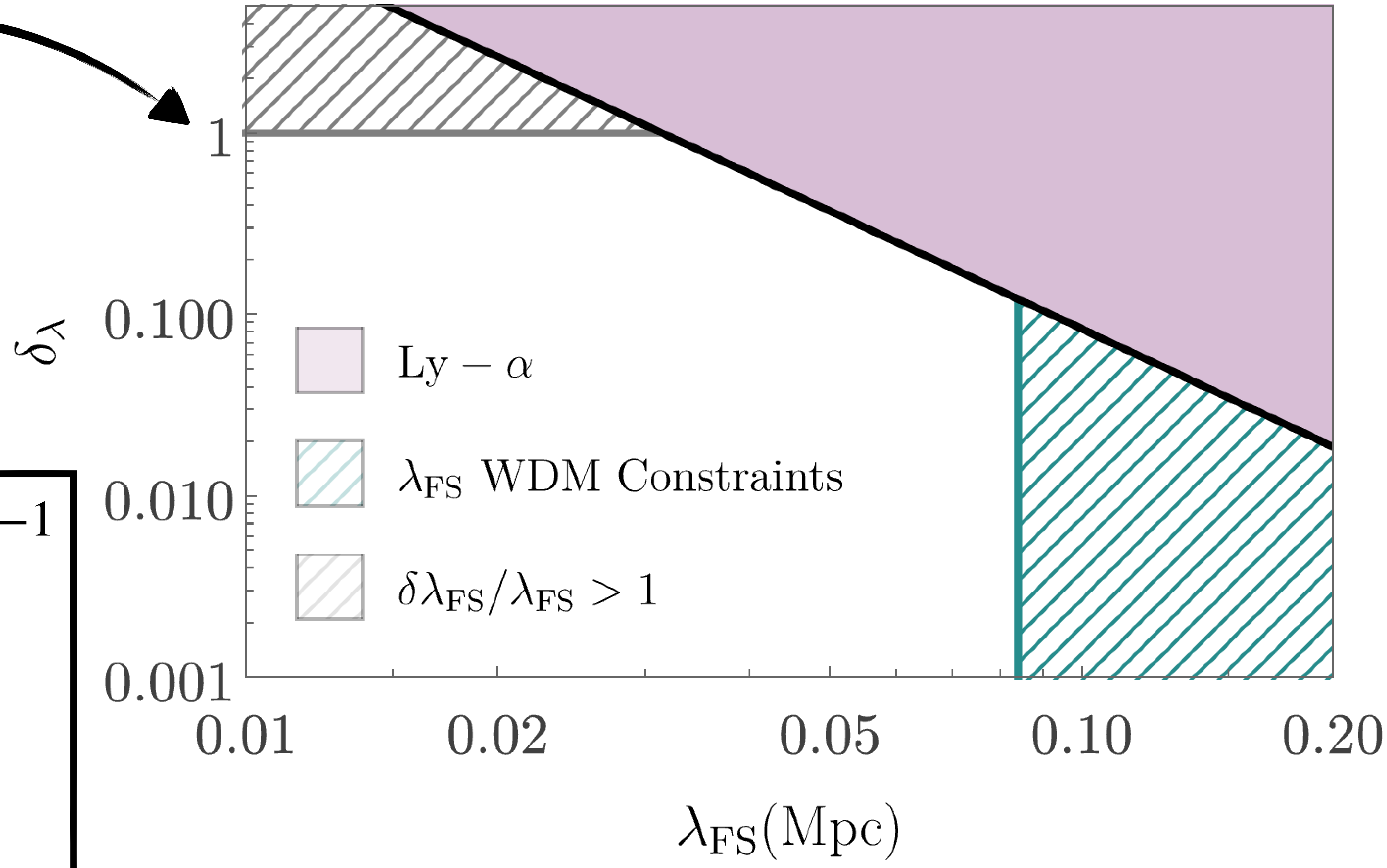

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Any observational effect?

3. Modulating Matter Power Spectrum
& Free Streaming Length

Modulating Free Streaming Length

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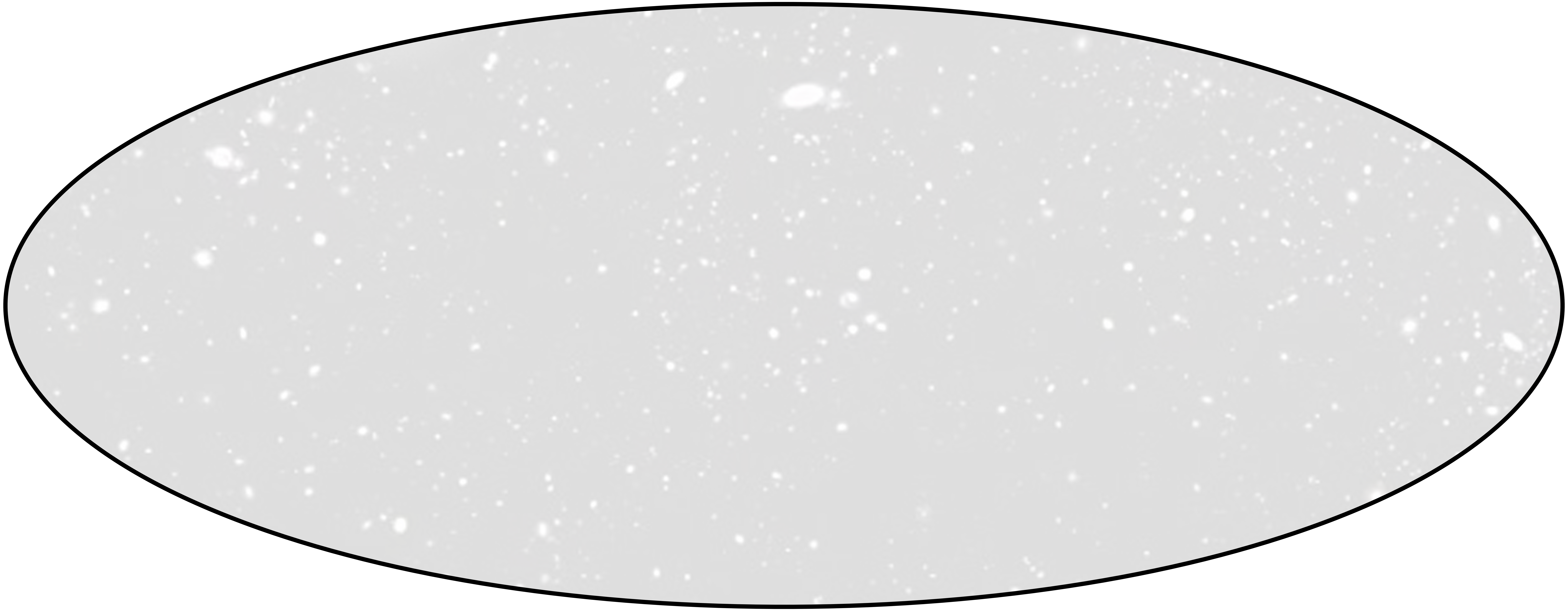
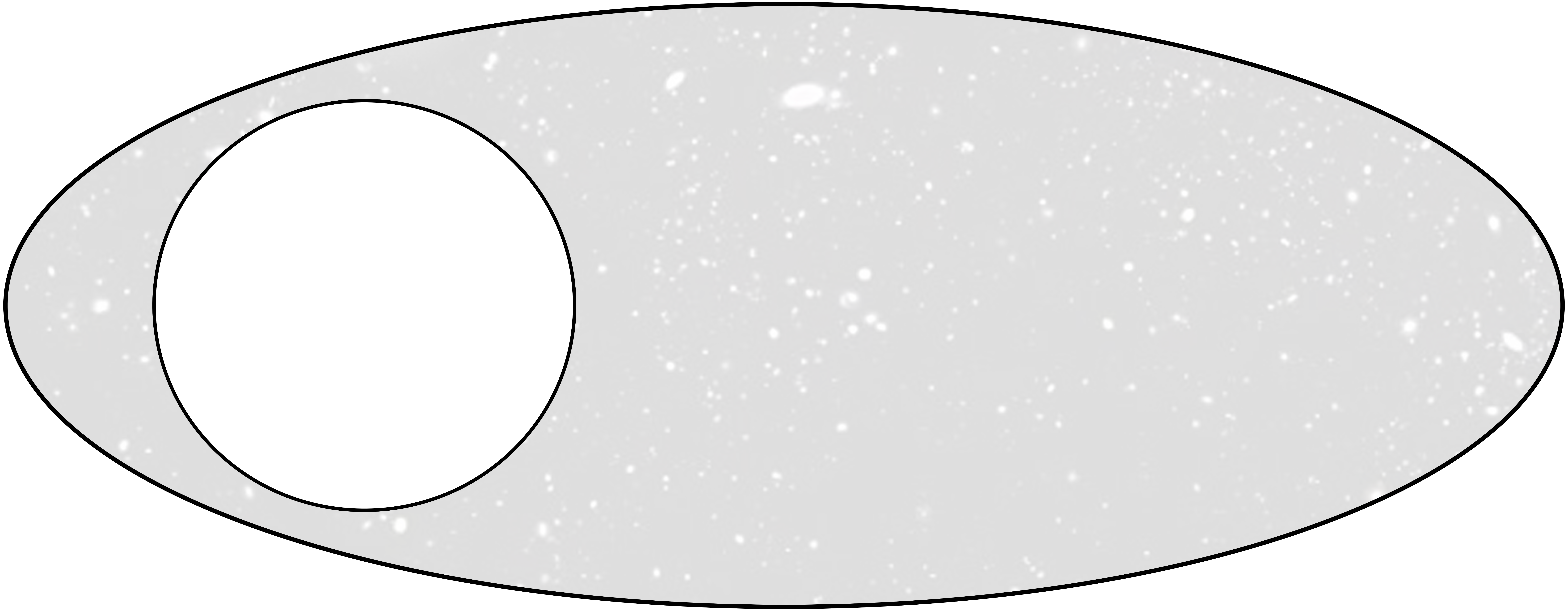


Modulating Free Streaming Length



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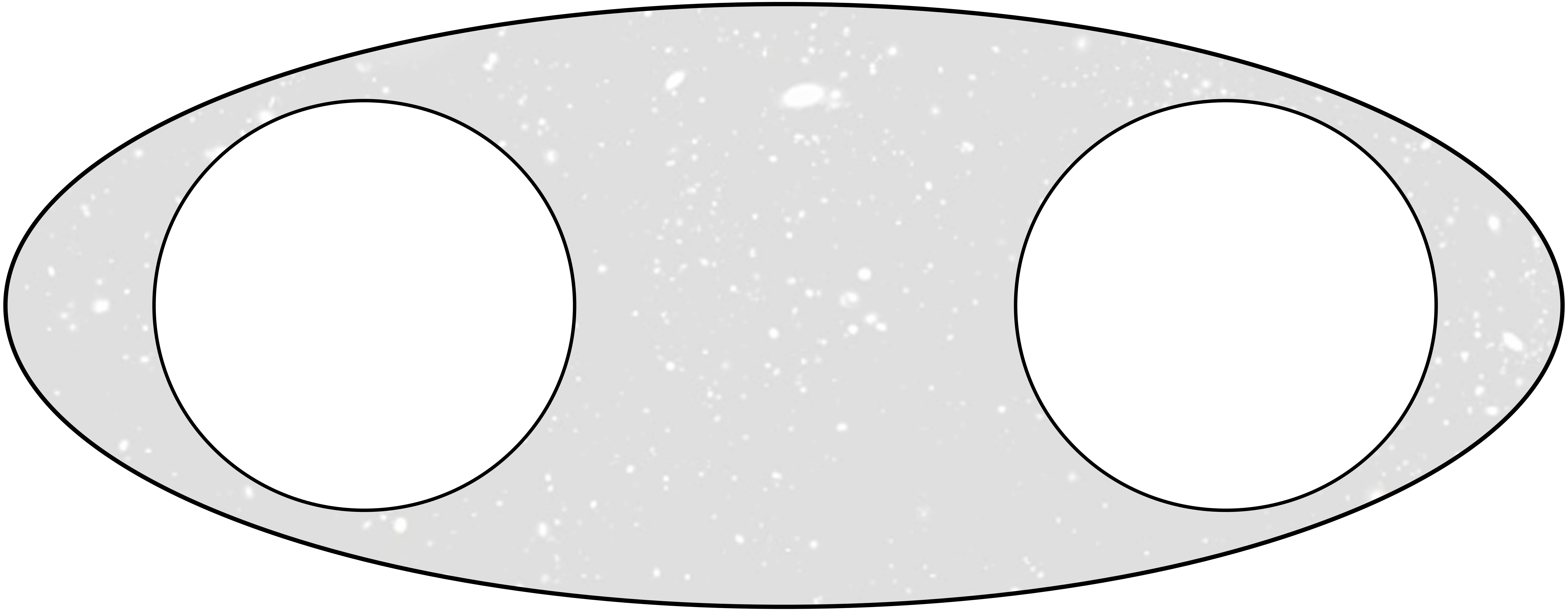

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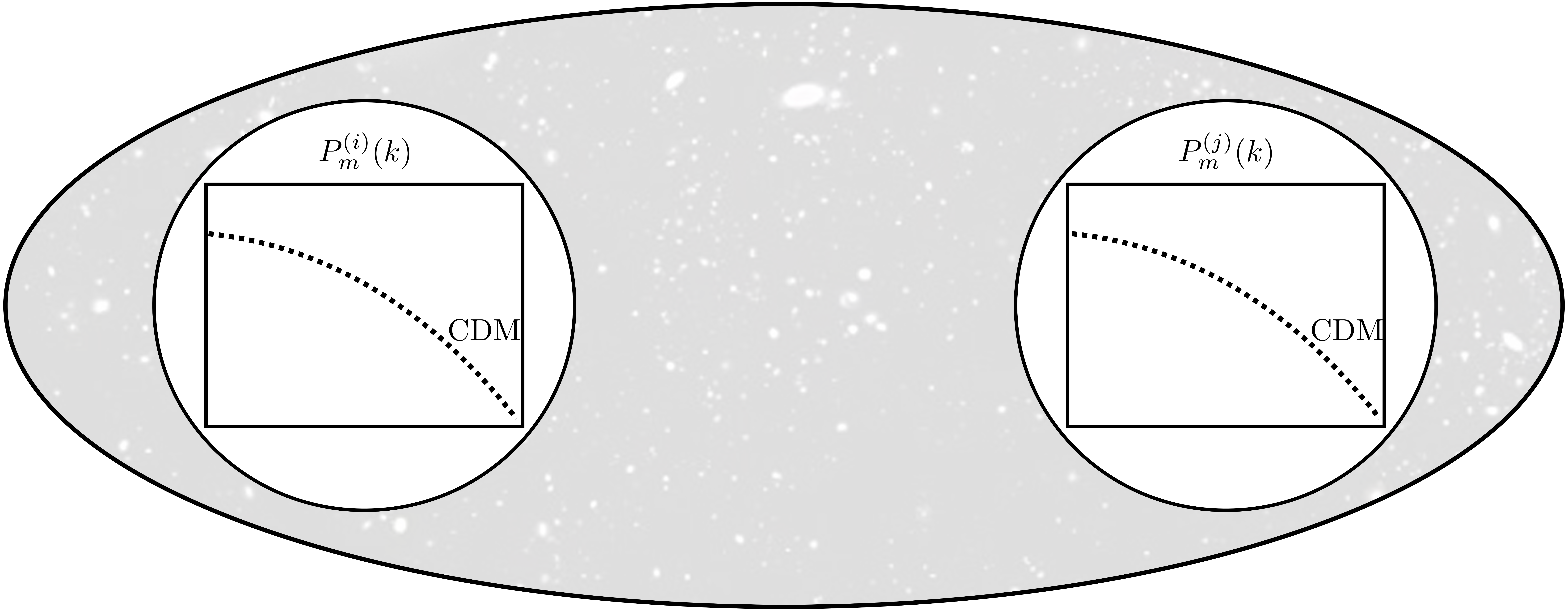

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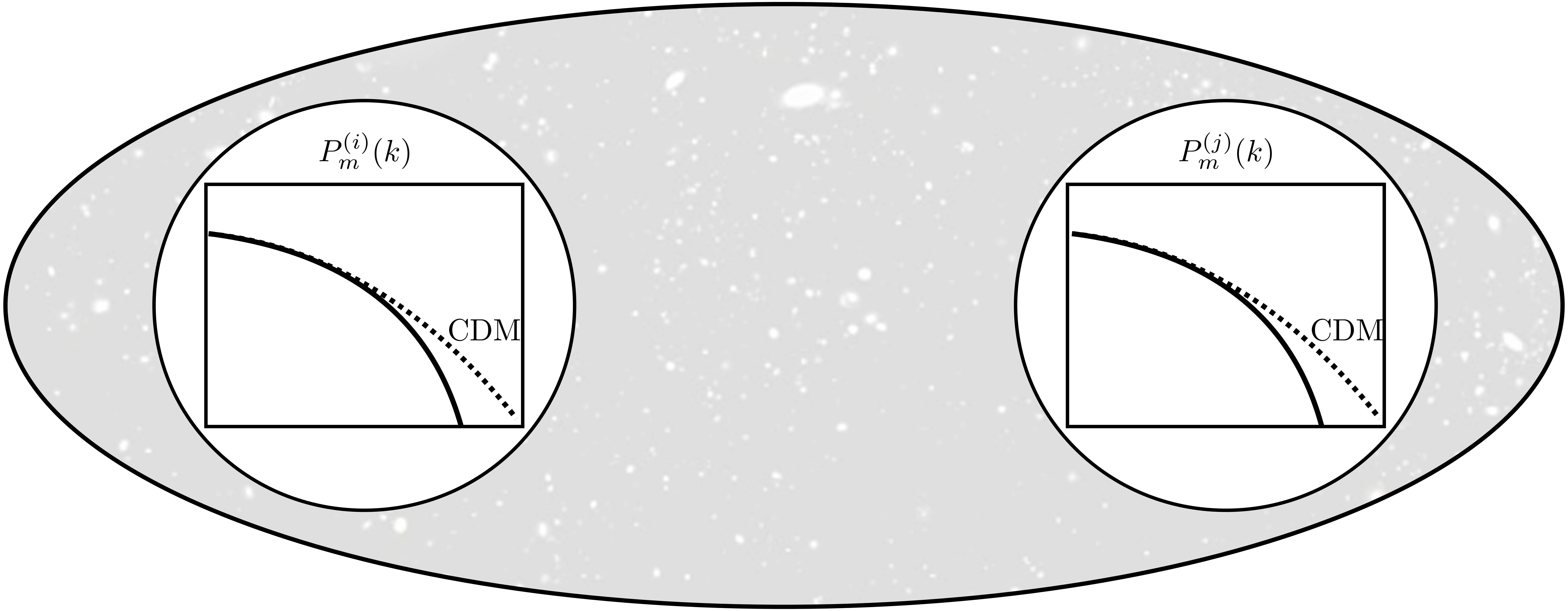

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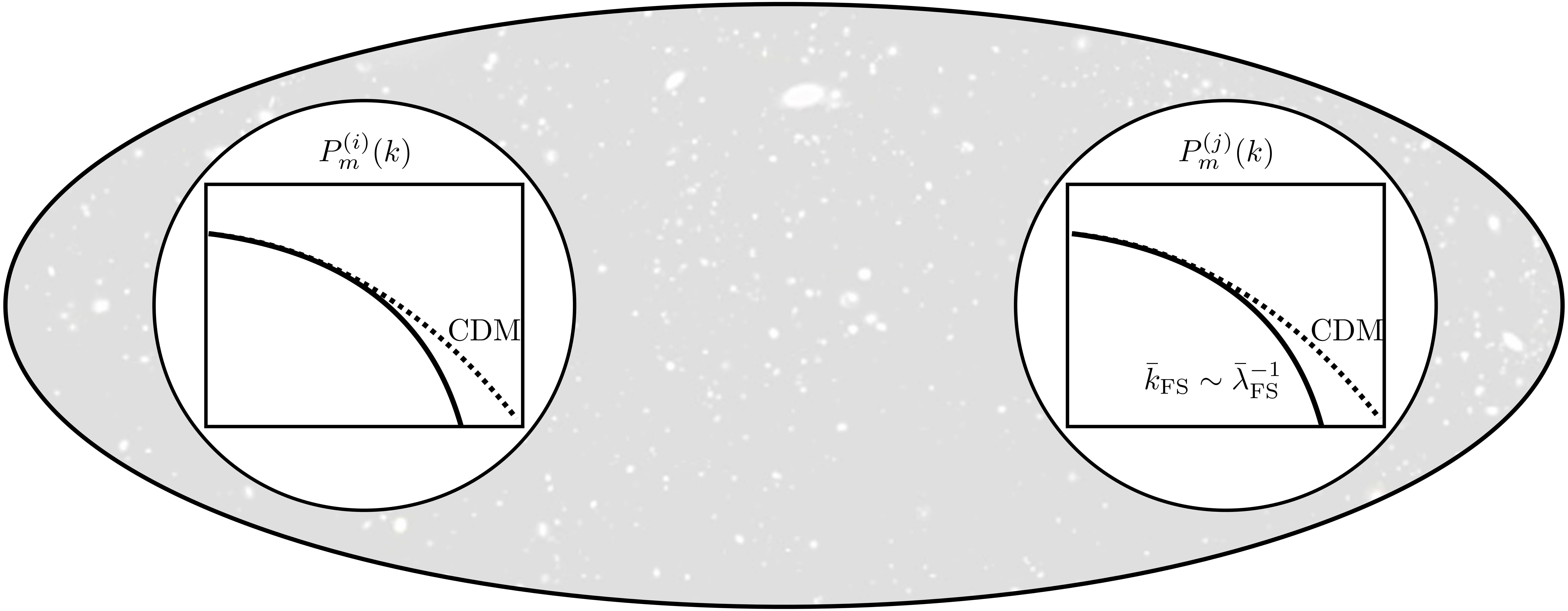

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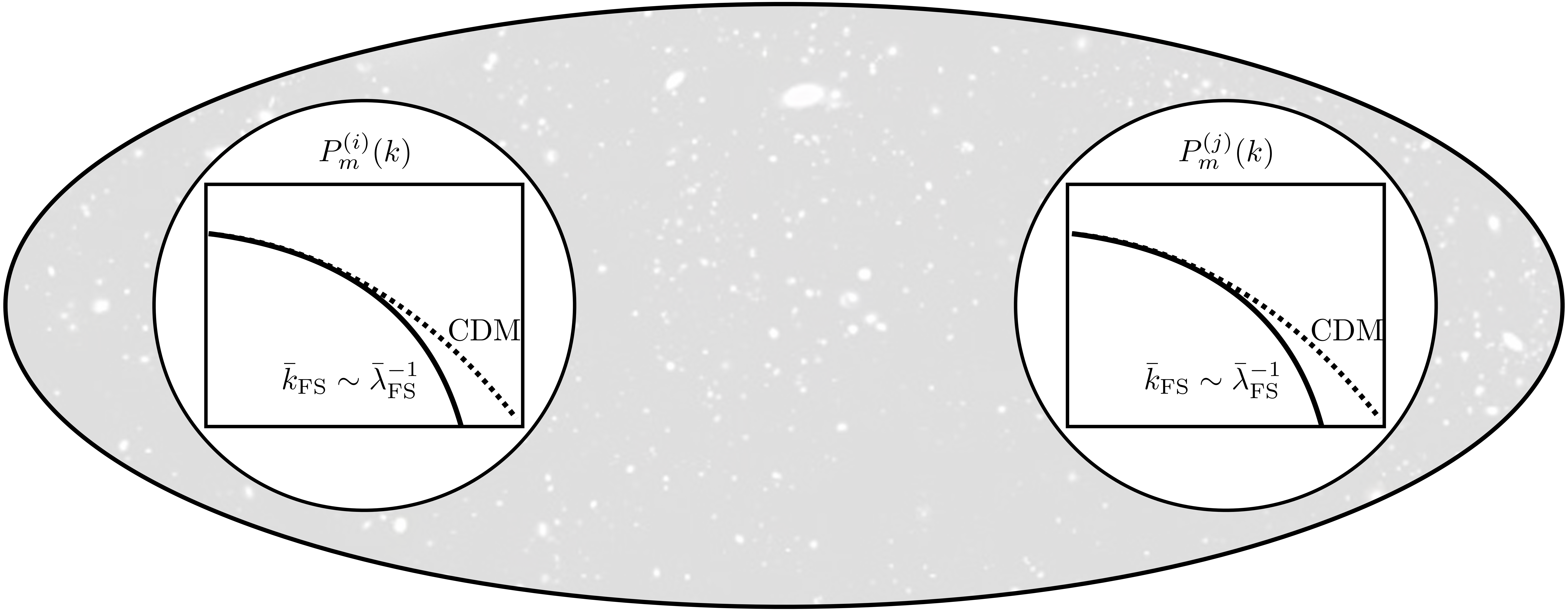

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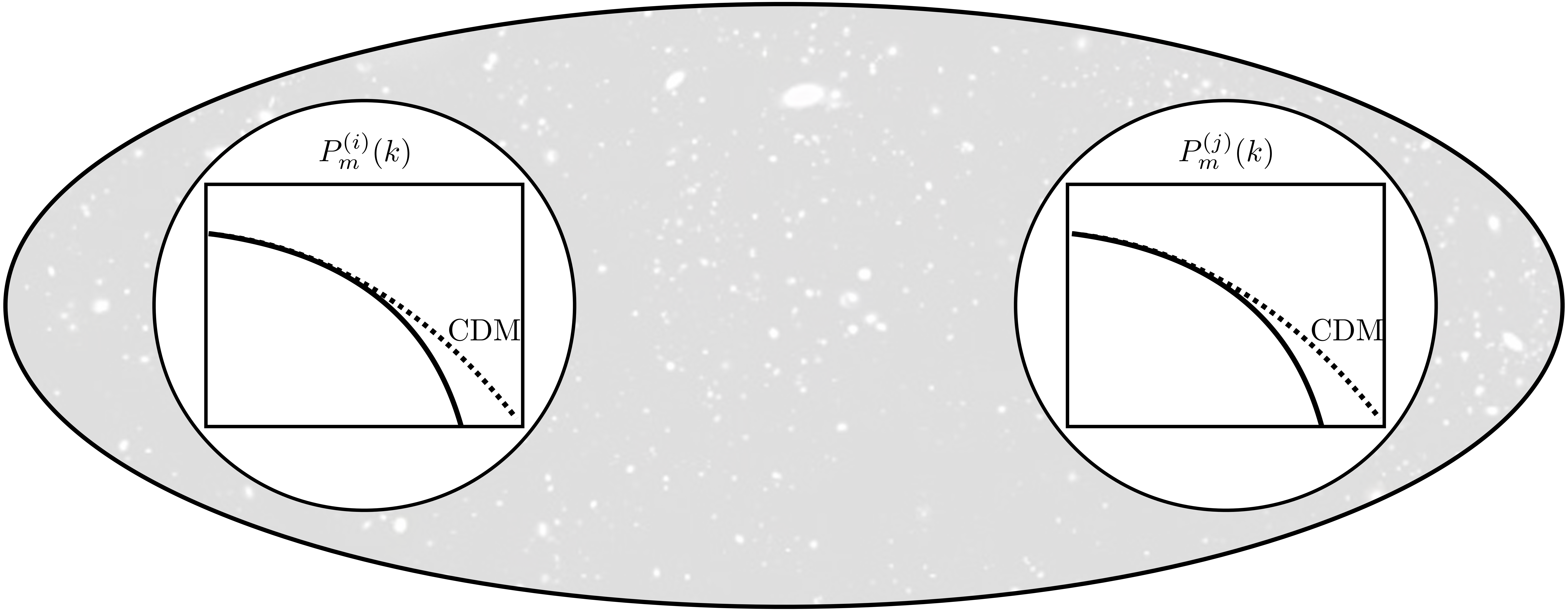

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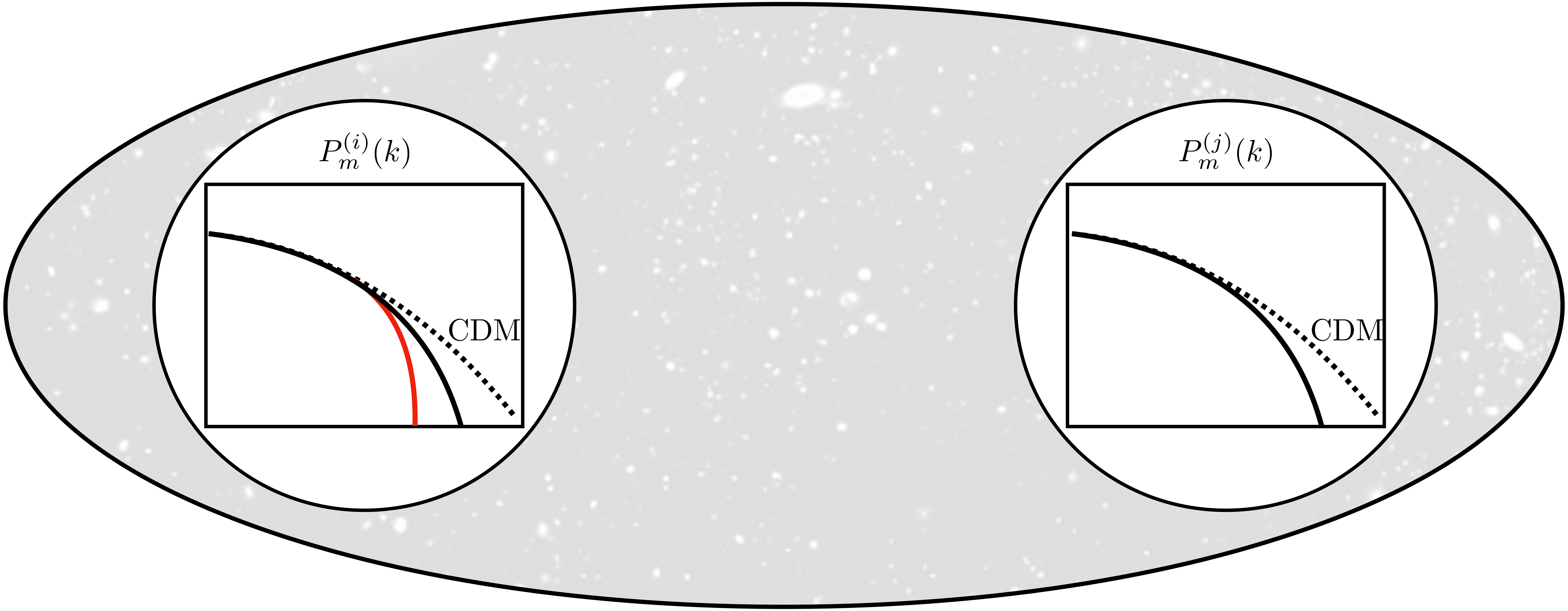

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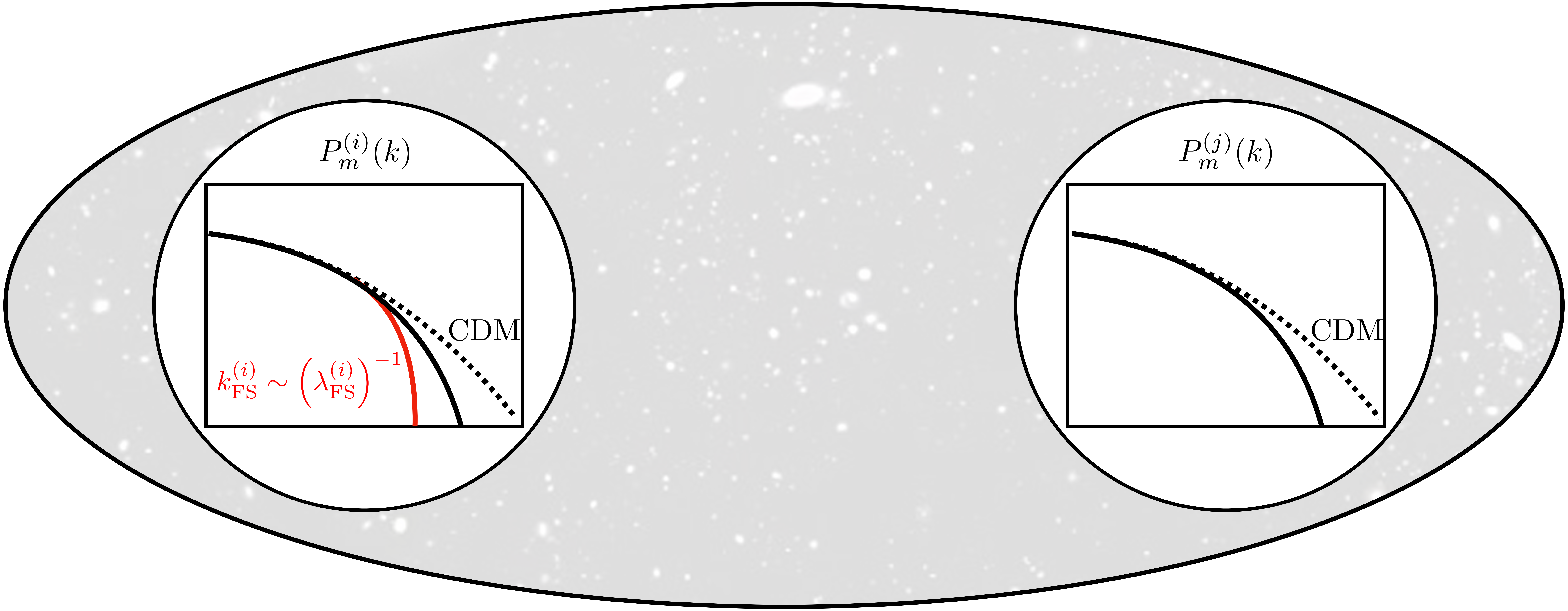

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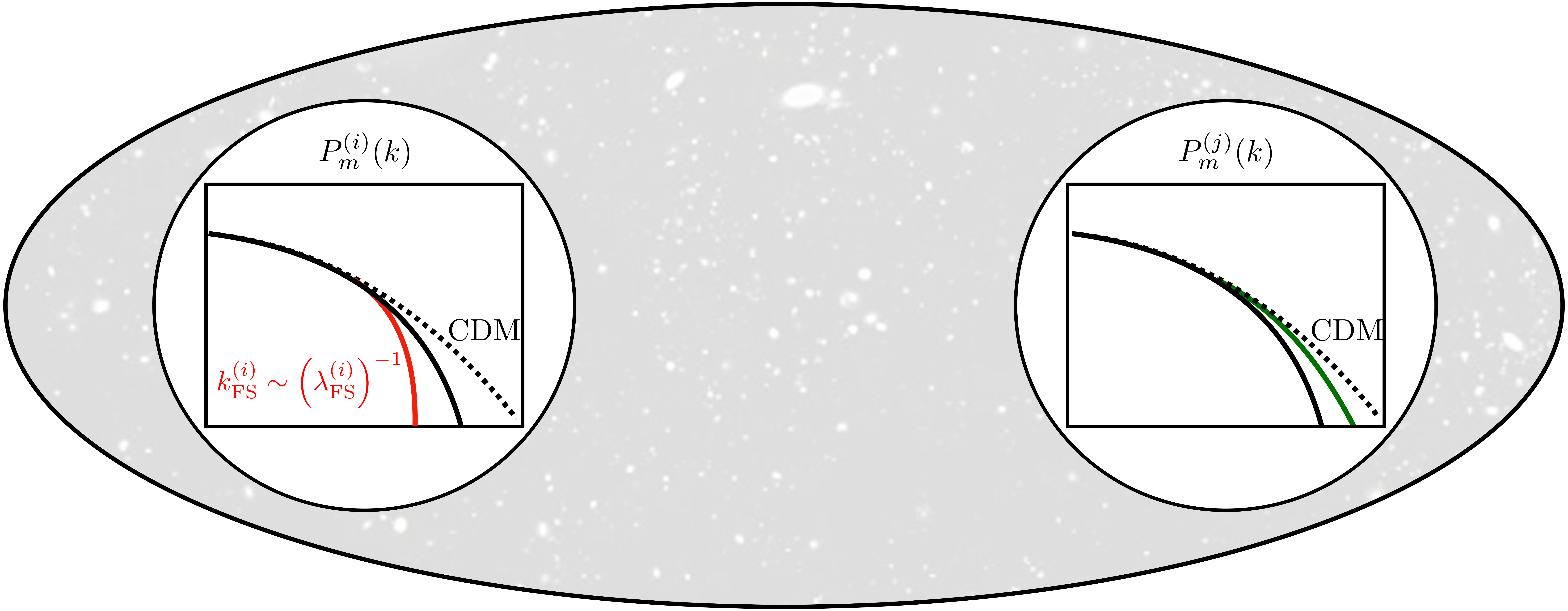

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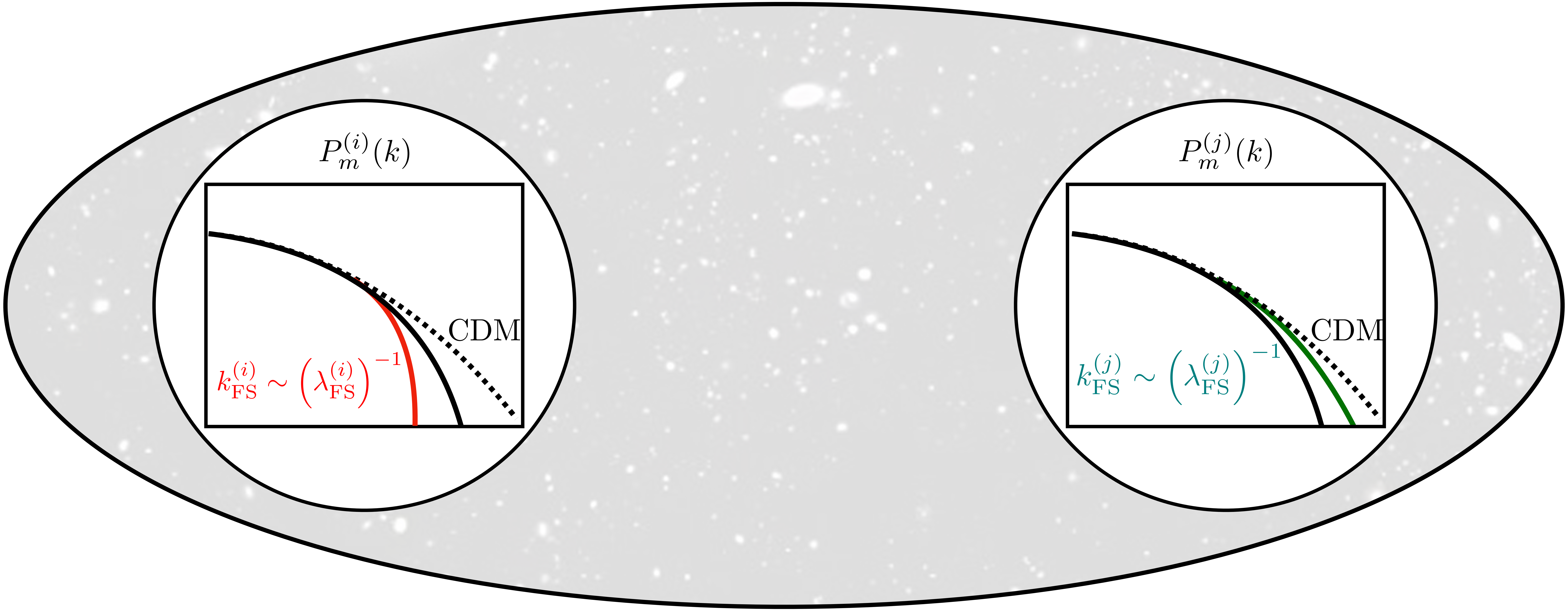

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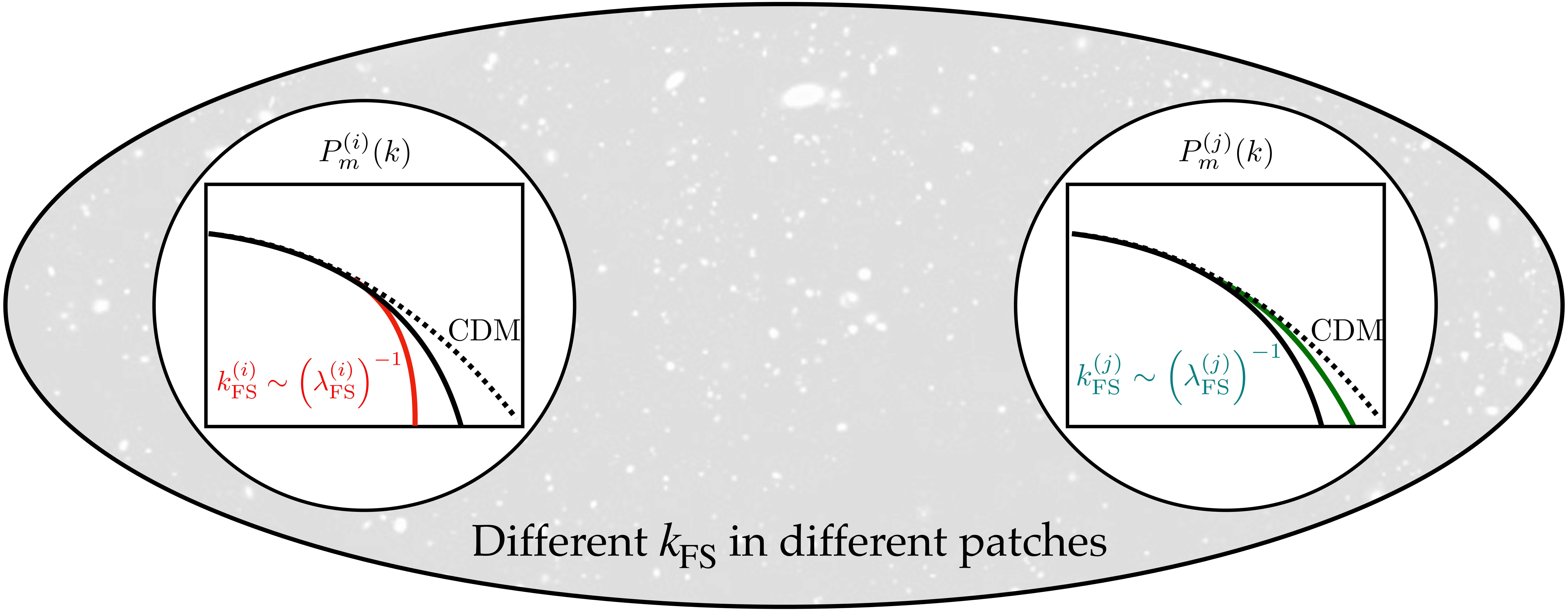

Position Dependent Power Spectrum

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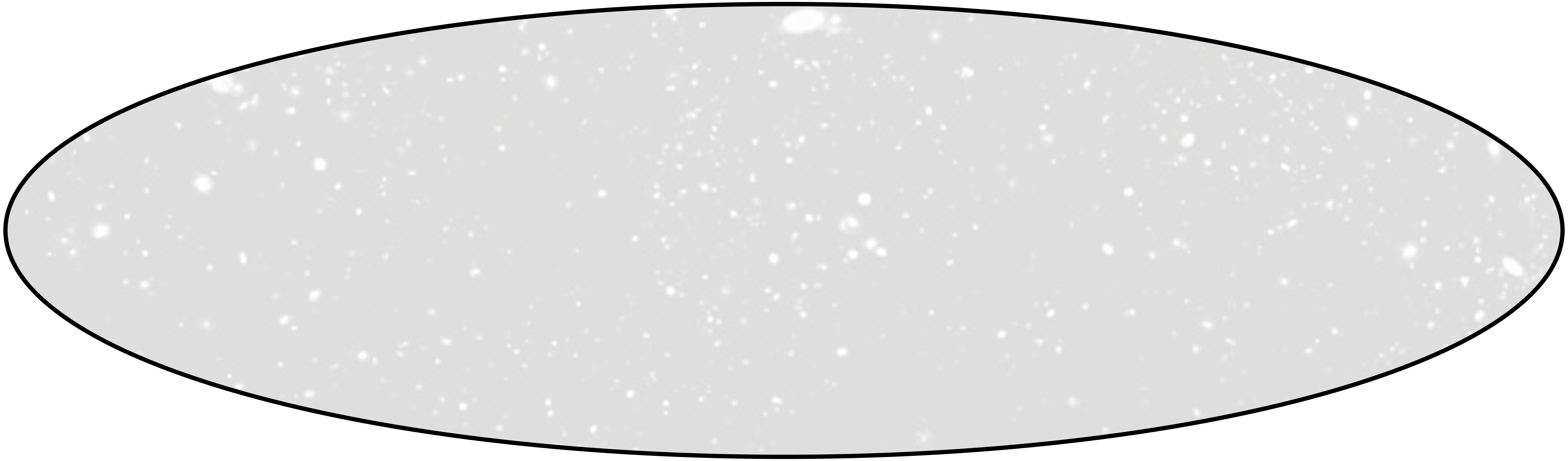

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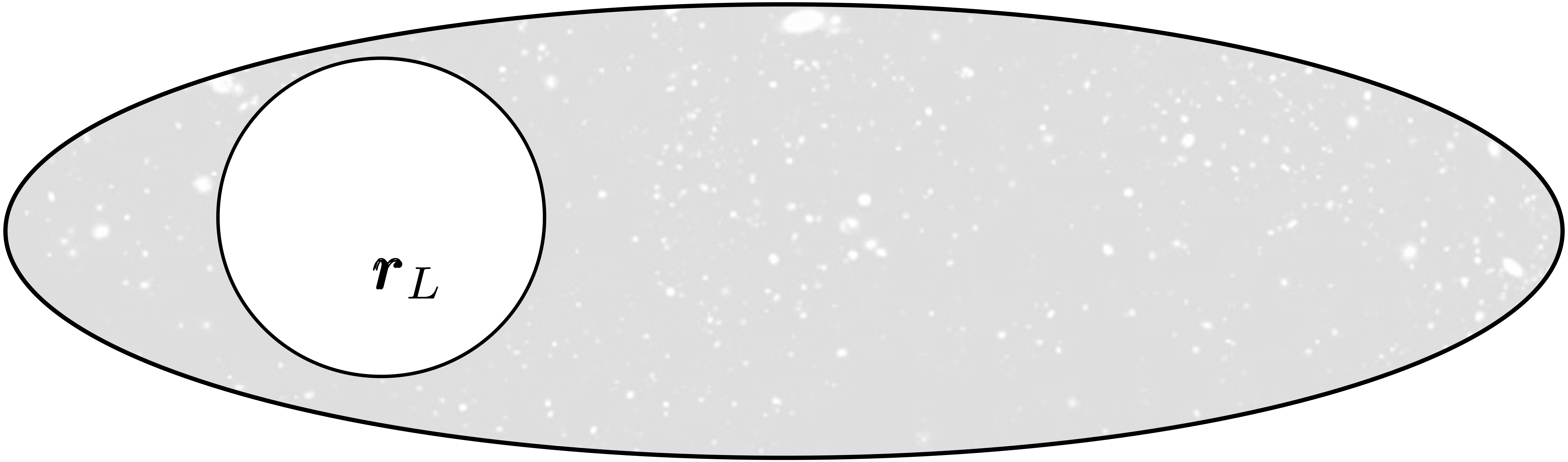

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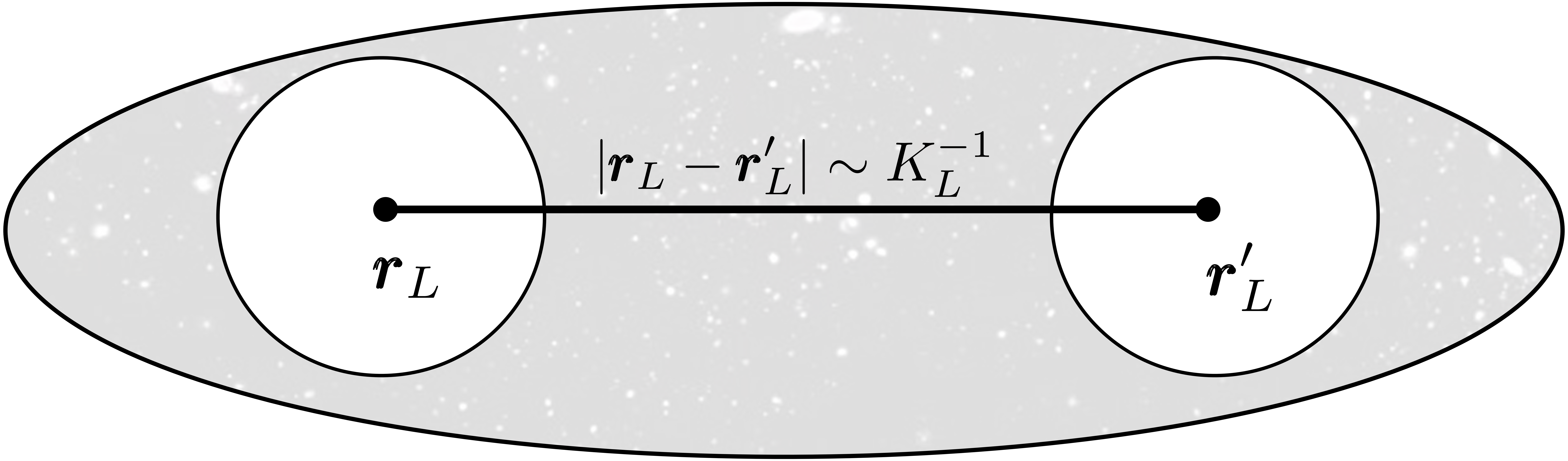

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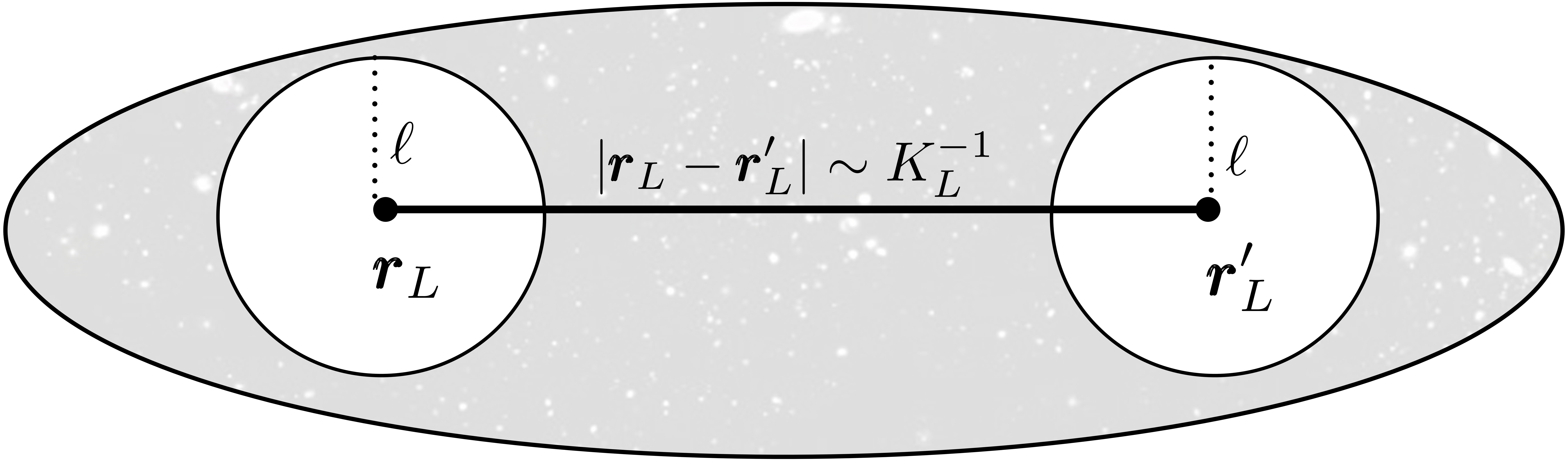

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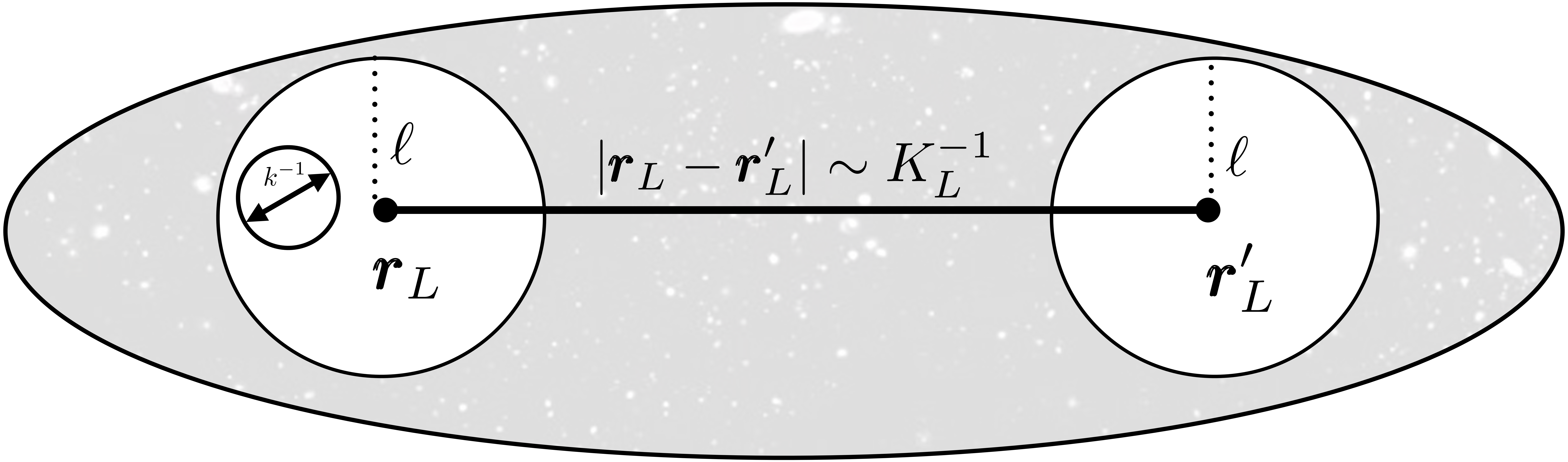

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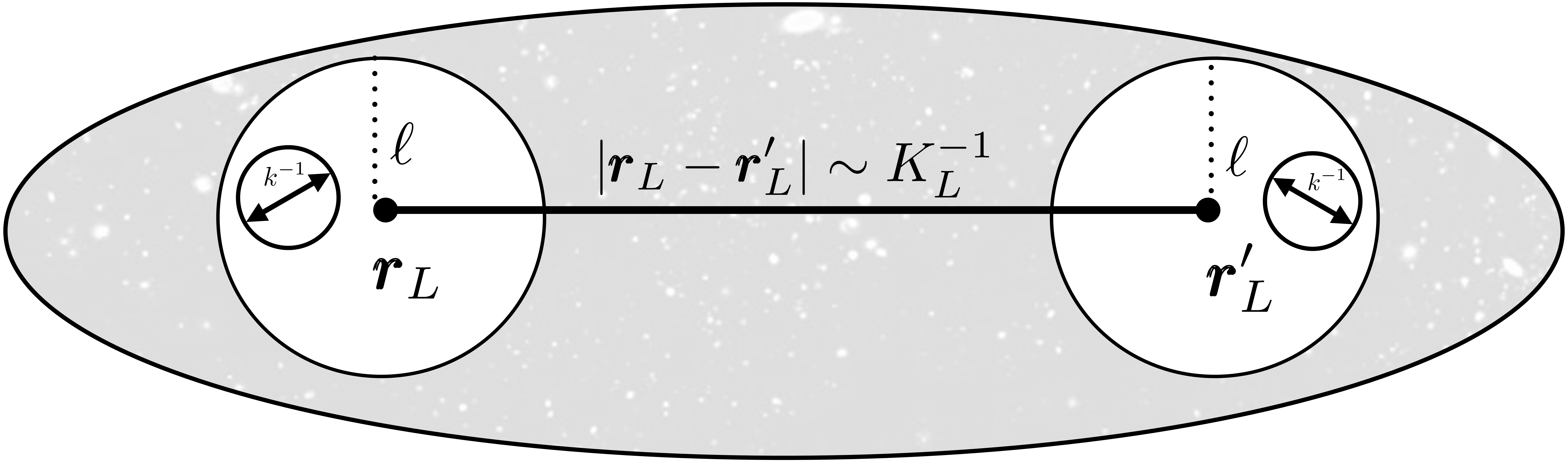

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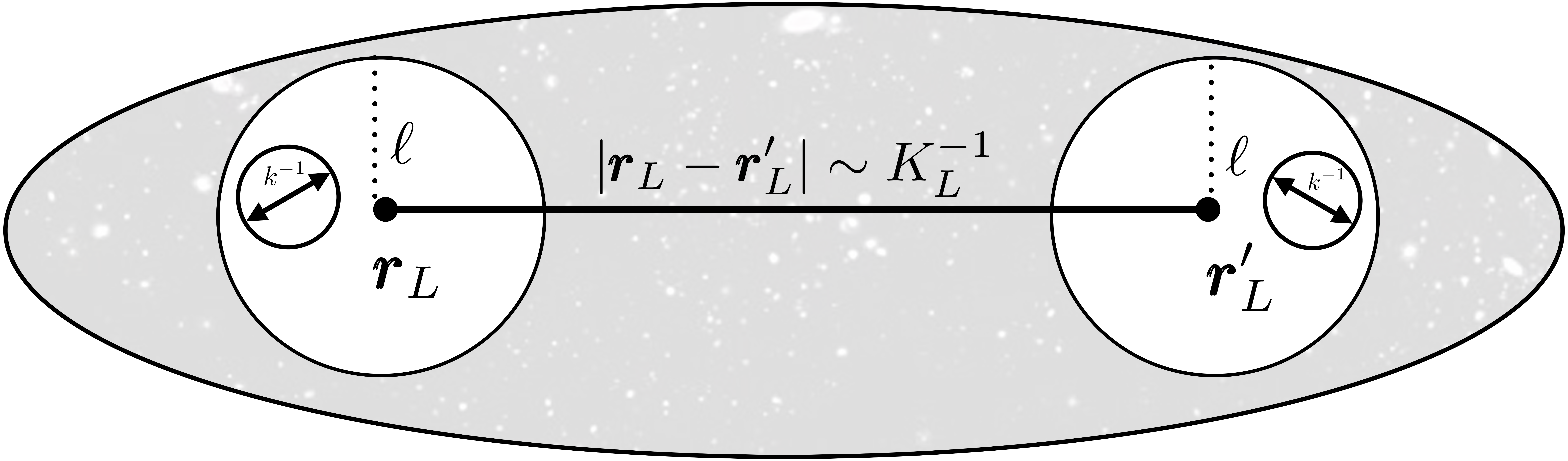

[Takada & Wu (2013), Chiang et.al. (2014)]

Position dep. power spectrum within sub volume V_s

$$P_\delta\left(\boldsymbol{k}, \boldsymbol{r}_L\right) \equiv \frac{1}{V_s}\left|\delta\left(\boldsymbol{k}, \boldsymbol{r}_L\right)\right|^2,$$

$$\delta\left(\boldsymbol{k}, \boldsymbol{r}_L\right) \equiv \int d^3 r \delta(\boldsymbol{r}) W_s\left(\boldsymbol{r}-\boldsymbol{r}_L\right) e^{-i \boldsymbol{k} \cdot \boldsymbol{r}}=\int \frac{d^3 q}{(2 \pi)^3} \delta^{(3)}(\boldsymbol{k}-\boldsymbol{q}) W_s(\boldsymbol{q}) e^{-i \boldsymbol{q} \cdot \boldsymbol{r}_L}$$

Position Dependent Power Spectrum

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Modulation in λ_{FS}

$$\lambda_{\mathrm{FS}}(\boldsymbol{x}) = \bar{\lambda}_{\mathrm{FS}} + \delta\lambda_{\mathrm{FS}}(\boldsymbol{x}).$$

Position Dependent Power Spectrum

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Separate universe approach : $\lambda_{\rm FS}$ constant within a V_s patch.

Position Dependent Power Spectrum

Modulation in $\lambda_{\rm FS}$

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$$\Delta P_m\left(\boldsymbol{k}, \lambda_{\rm FS}\left(\boldsymbol{r}_L\right)\right) \simeq \lambda_{\rm FS} \left.\frac{dP_m}{d\lambda_{\rm FS}}\right|_{\lambda_{\rm FS}=\bar{\lambda}_{\rm FS}} \delta_\lambda\left(\boldsymbol{r}_L\right)$$

$$\Delta P_m\left(\boldsymbol{k}, \lambda_{\rm FS}\left(\boldsymbol{r}_L\right)\right) \equiv P_m\left(\boldsymbol{k}, \lambda_{\rm FS}\left(\boldsymbol{r}_L\right)\right) - P_m\left(\boldsymbol{k}, \bar{\lambda}_{\rm FS}\right)$$

Position Dependent Power Spectrum

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Correlation of ΔP_m governed by P_{δ_λ}

Position Dependent Power Spectrum

Correlation of ΔP_m governed by P_{δ_λ}

$$P_{PP}\left(\boldsymbol{k}, \boldsymbol{K}_L\right) = \lambda_{\rm FS}^2 \left(\frac{dP_m}{d\lambda_{\rm FS}}\right)^2 P_{\delta_\lambda}\left(\boldsymbol{K}_L\right)$$

$$\langle \Delta P_m(\boldsymbol{k}, \lambda_{\rm FS}; \boldsymbol{K}_L) \Delta P_m(\boldsymbol{k}, \lambda_{\rm FS}; \boldsymbol{K}'_L) \rangle \equiv (2\pi)^3 \delta^{(3)}(\boldsymbol{K}_L + \boldsymbol{K}'_L) P_{PP}(\boldsymbol{k}, \boldsymbol{K}_L)$$

Position Dependent Power Spectrum

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Size of effect governed by P_m dependence on $\lambda_{\rm FS}$, through the T_{FS} !

Modulating Power Spectrum

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$P_m(k) = T_{\rm FS}(k)^2 P_{\mathcal{R}}(k)\,|W_s(k)|^2$

Modulating Power Spectrum

$P_m(k) = T_{\mathrm{FS}}(k)^2 P_{\mathcal{R}}(k) \left| W_s(k) \right|^2$

Representative form $T_{\mathrm{FS}}\left(k, \lambda_{\mathrm{FS}}\right) = \left[1 + \left(c \lambda_{\mathrm{FS}} k\right)^{\beta}\right]^{\gamma}$

Modulating Power Spectrum

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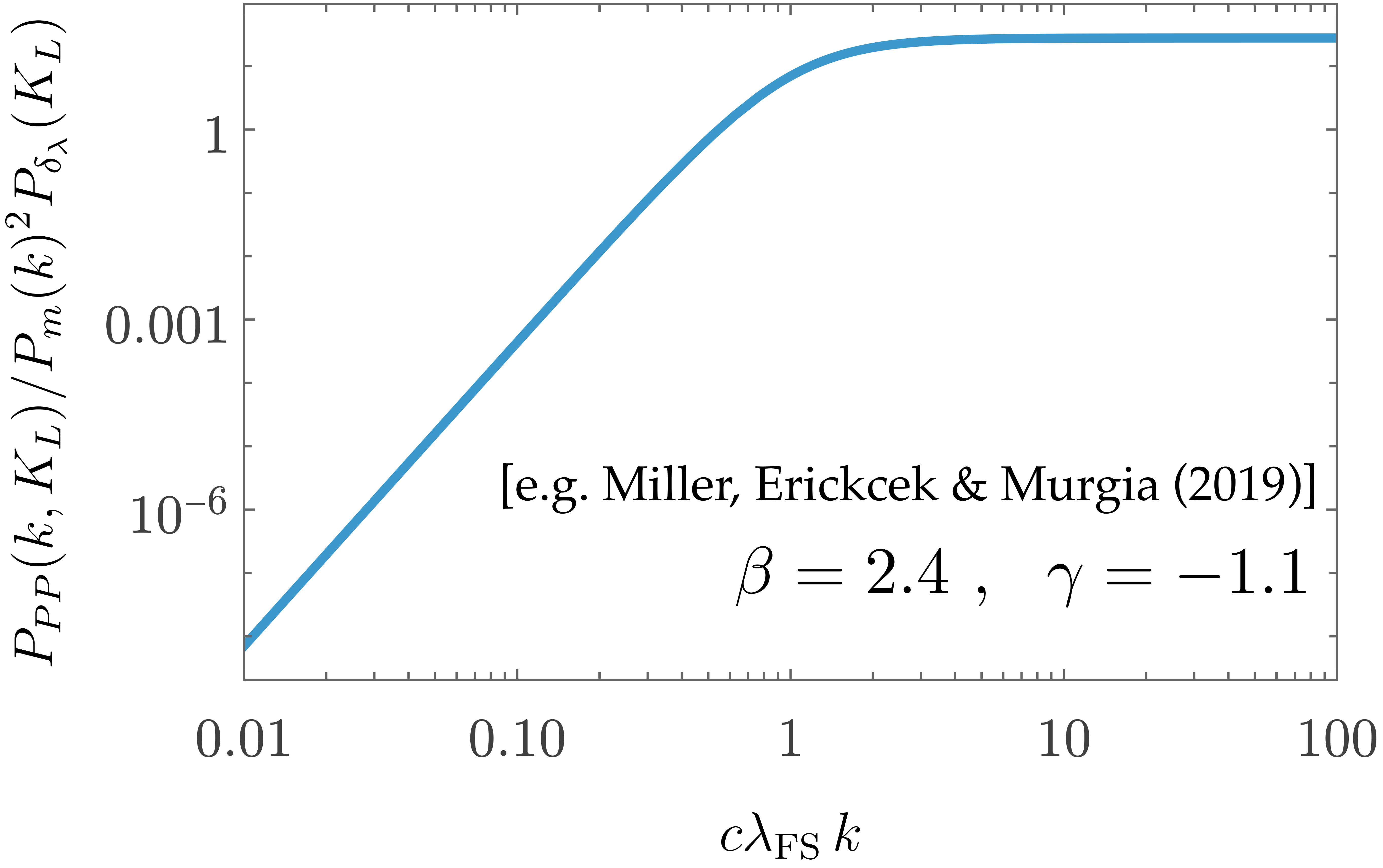

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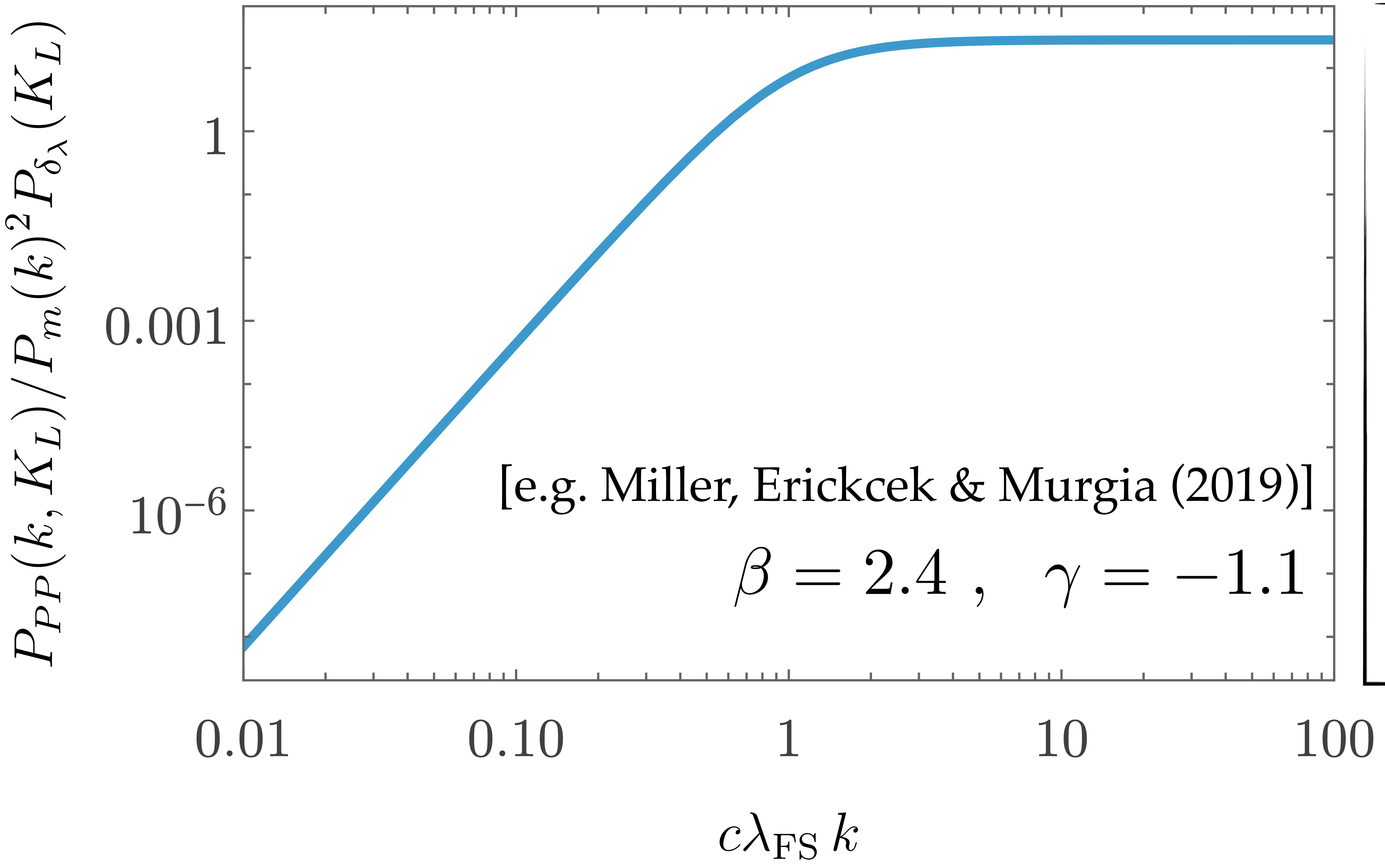

$$\frac{P_{PP}\left(k_{\rm FS}, K_L\right)}{P_m\left(k_{\rm FS}\right)^2} \simeq 10 P_{\delta_\lambda}\left(K_L\right)$$

$\mathcal{O}(1)$ fluctuations in normalized correlation

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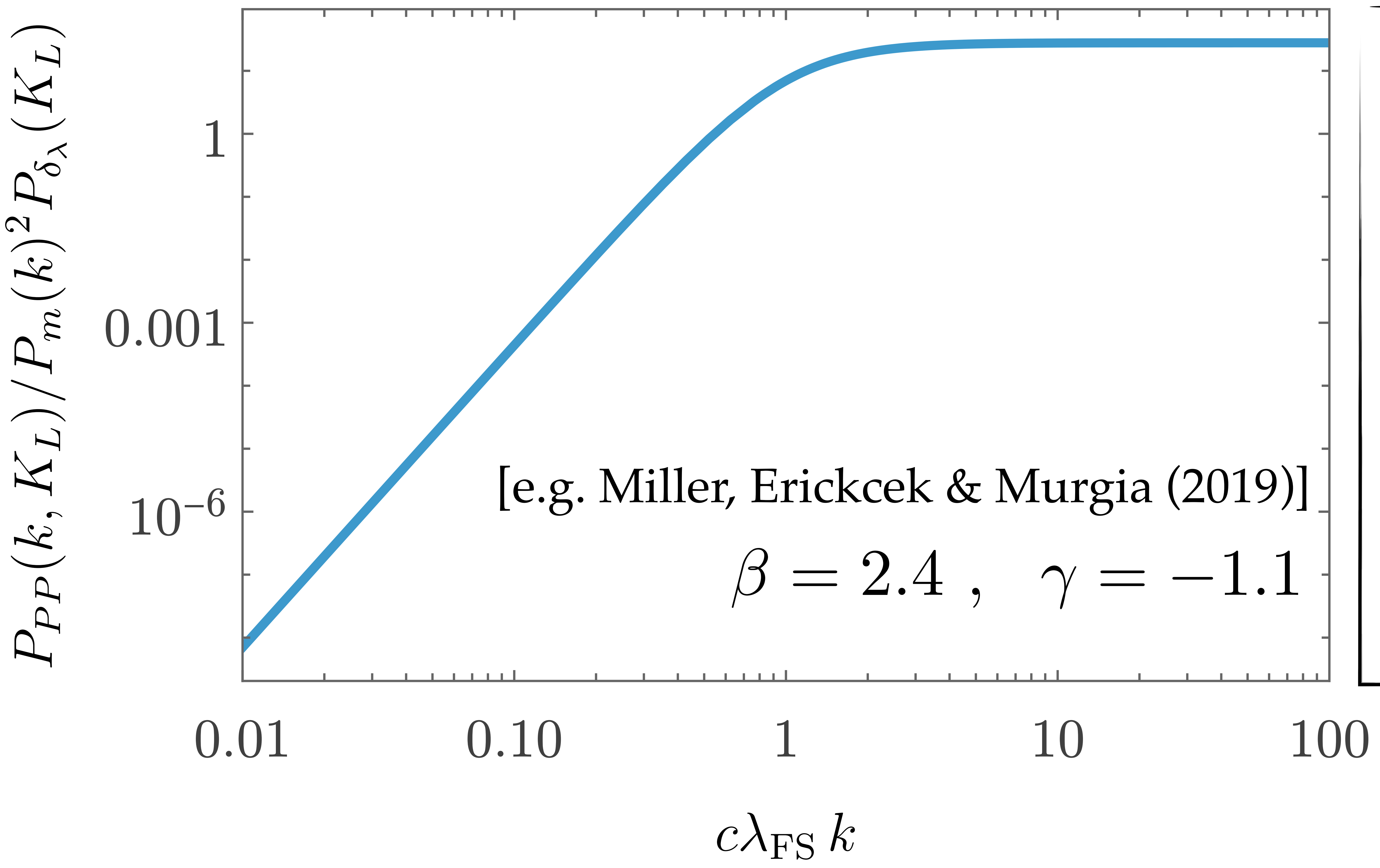

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$\mathcal{O}(1)$ fluctuations in normalized correlation

Modulations in galaxy surveys, small scale observations?

Conclusions and Discussions

- Kinetic Isocurvature Perturbations as a new class of isocurvature!

- Governed by p fluctuation, $O(1)$ fluctuations compatible with isocurvature constraints.

- $O(1)$ modulations in the patch-by-patch free streaming length.

Open questions :

UV complete model, optimized observables, observational test?

Backup Material

A Concrete Model

Massive scalar field : ϕ (zero mode), scalar DM χ, modulating scalar σ.

$$\mathcal{L}_{\rm int} = g\sigma\phi\chi^2 \ , \quad \Gamma_\phi \simeq \frac{1}{8\pi}\frac{g^2\sigma_i^2}{m_\phi}$$

Temperature scales : $T_{\rm dec}$, $T_{\rm osc}$, $T_{\rm NR}$, assuming RD at $T_{\rm dec}$ and $T_{\rm osc}$.

$$T_{\rm dec} \simeq \sqrt{\Gamma M_P} \ , \quad T_{\rm osc} \simeq \left(\frac{90}{\pi^2 g_*}\right)^{1/4}\sqrt{m_\phi M_P} \ , \quad T_{\rm NR} \simeq T_{\rm dec}\frac{m_\chi}{m_\phi}$$

$$\frac{\rho_\chi}{s} \simeq \frac{m_\chi n_\chi}{s} \simeq m_\chi \frac{n_\phi}{s} \sim \frac{m_\chi m_\phi \phi_i^2}{g_* \left(m_\phi M_P\right)^{3/2}}$$

$$\phi_i \sim \left(\frac{g_* T_{\rm dec} T_{\rm eq}}{T_{\rm NR}}\right)^{1/2}\frac{M_P^{3/4}}{m_\phi^{1/4}} \sim 10^{15}{\rm GeV} \times \left(\frac{T_{\rm dec}}{10^9{\rm GeV}}\right)^{1/2}\left(\frac{10{\rm keV}}{T_{\rm NR}}\right)^{1/2}\left(\frac{10^8{\rm GeV}}{m_\phi}\right)^{1/4}$$

A Concrete Model

Requirements on m_σ

In order for large Γ modulation, σ should start oscillating at $T < T_{\rm dec}$

$$m_\sigma < \frac{T_{\rm dec}^2}{M_P} = 1\text{GeV}\left(\frac{T_{\rm dec}}{10^9\text{GeV}}\right)^2$$

In order for $\delta m_\sigma^{\text{1-loop}} \sim \dfrac{g}{4\pi}\phi < \dfrac{T_{\rm dec}^2}{M_P}$, $g \lesssim 10^{-8}\left(\dfrac{T_{\rm NR}}{10\text{keV}}\right)^{1/2}\left(\dfrac{m_\phi}{10^8\text{GeV}}\right)$

A Concrete Model

σ eventually starts oscillations and dissipated at T_{dis}, needs suppressed

$$\mathcal{R}_\sigma \sim \frac{\delta\sigma}{\sigma_i}\frac{\sigma_i^2}{M_P^2}\frac{\sqrt{m_\sigma M_P}}{T_{\text{dis}}}$$

$$\sim 10^{-6} \times \frac{\delta\sigma}{\sigma_i}\frac{m_\phi}{10^8\text{GeV}}\left(\frac{T_{\text{dec}}}{10^9\text{GeV}}\right)^2\left(\frac{m_\sigma}{1\text{GeV}}\right)^{1/2}\frac{10^3\text{GeV}}{T_{\text{dis}}}\left(\frac{10^{-8}}{g}\right)^2$$

$\mathcal{R}_\sigma$ suppressed for sufficiently large T_{dis}

e.g. If $\mathcal{L} \supset A_H\sigma|H|^2$, σ is dissipated with a rate $\sim \dfrac{A_H^2}{8\pi T}$

$$T_{\text{dis}} \sim 10^3\text{GeV}\left(\frac{A_H}{10^{-3}\text{GeV}}\right)^{2/3}$$

$m_\sigma \gg \text{MeV}$ for suppressing the ΔN_{eff}