Effective Field Theory for BSM studies

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References

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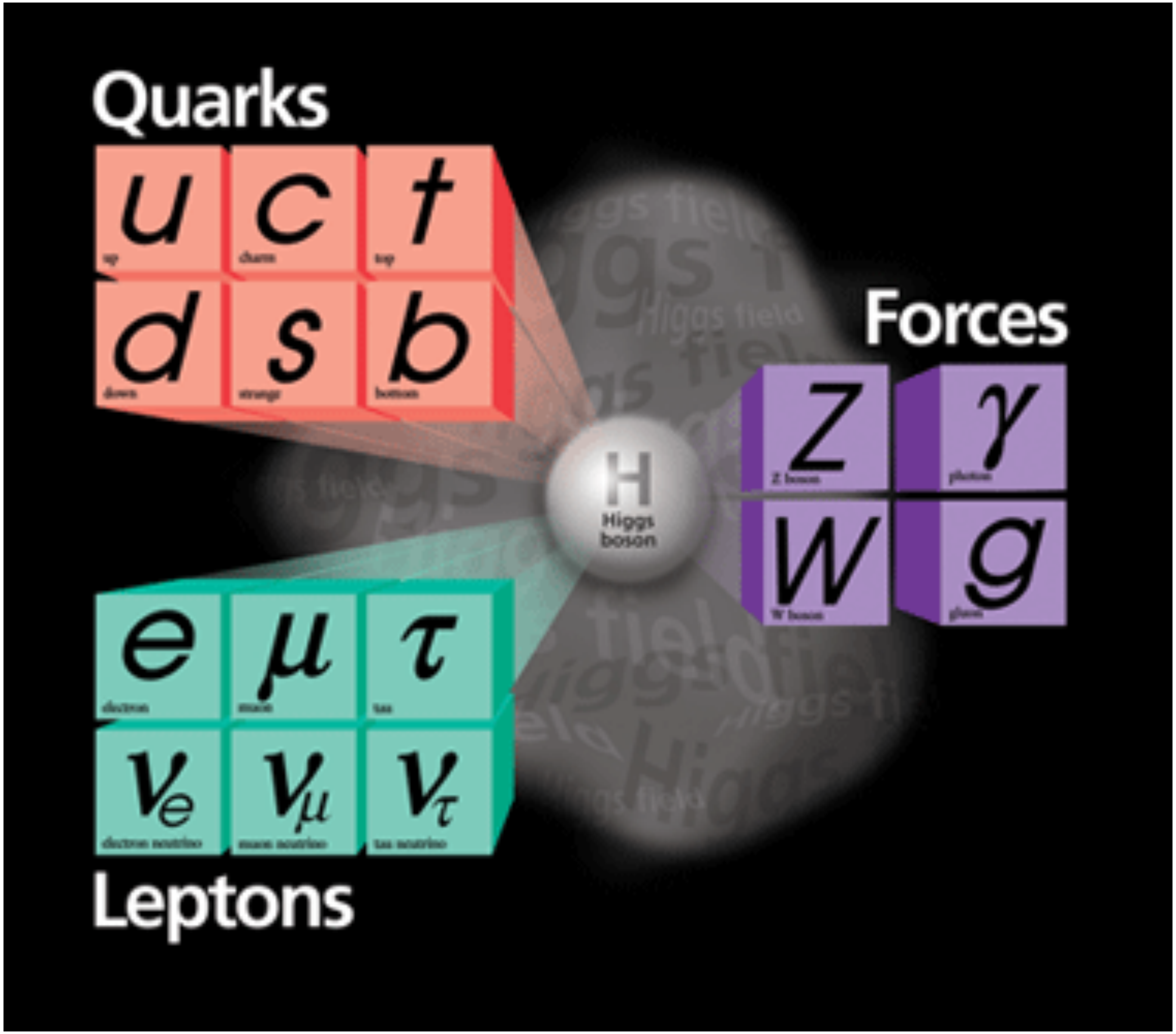
- D. Kaplan, nucl-th/9506035, nucl-th/0510023
- H. Georgi, DOI: 10.1146/annurev.ns.43.120193.001233
- W. Skiba, arXiv:1006.2142
- And many original papers on this topic in various physics problems

Standard Model of Particle Physics

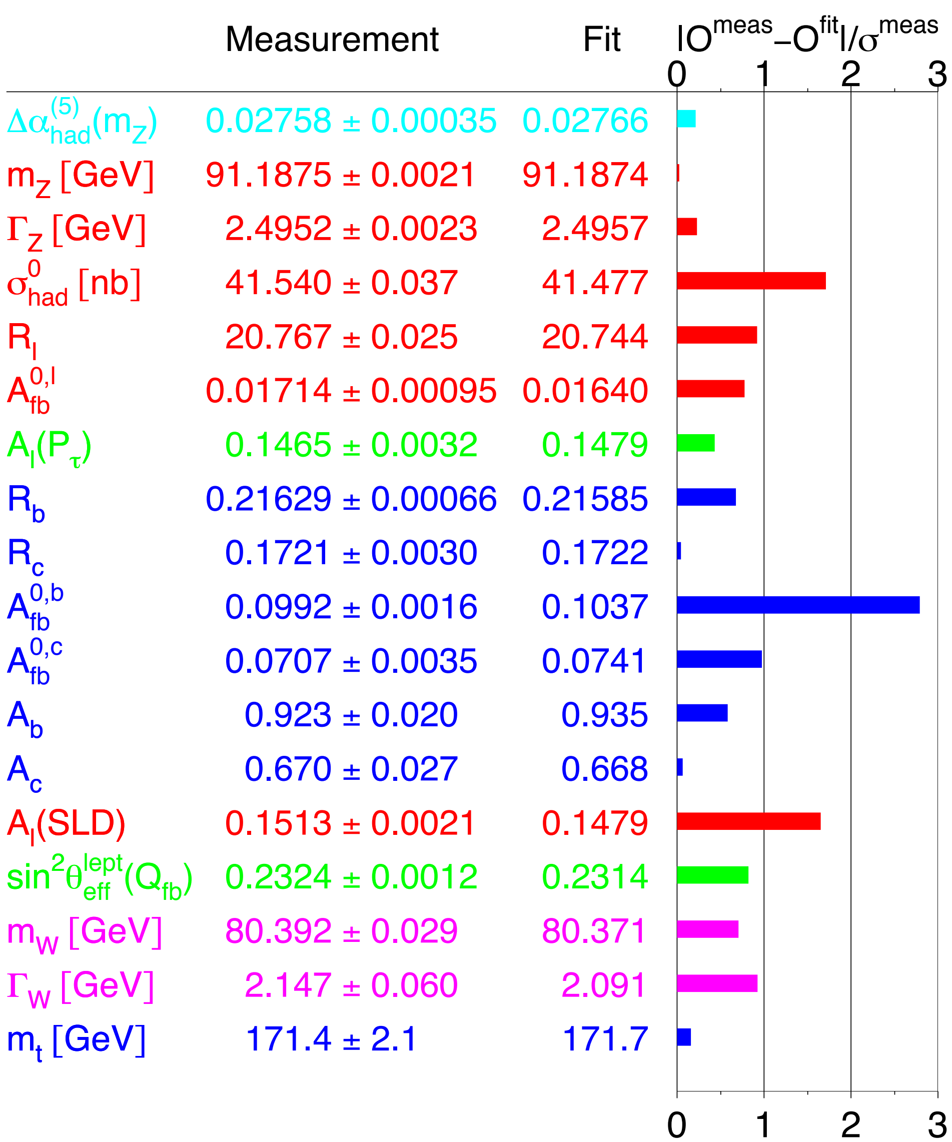


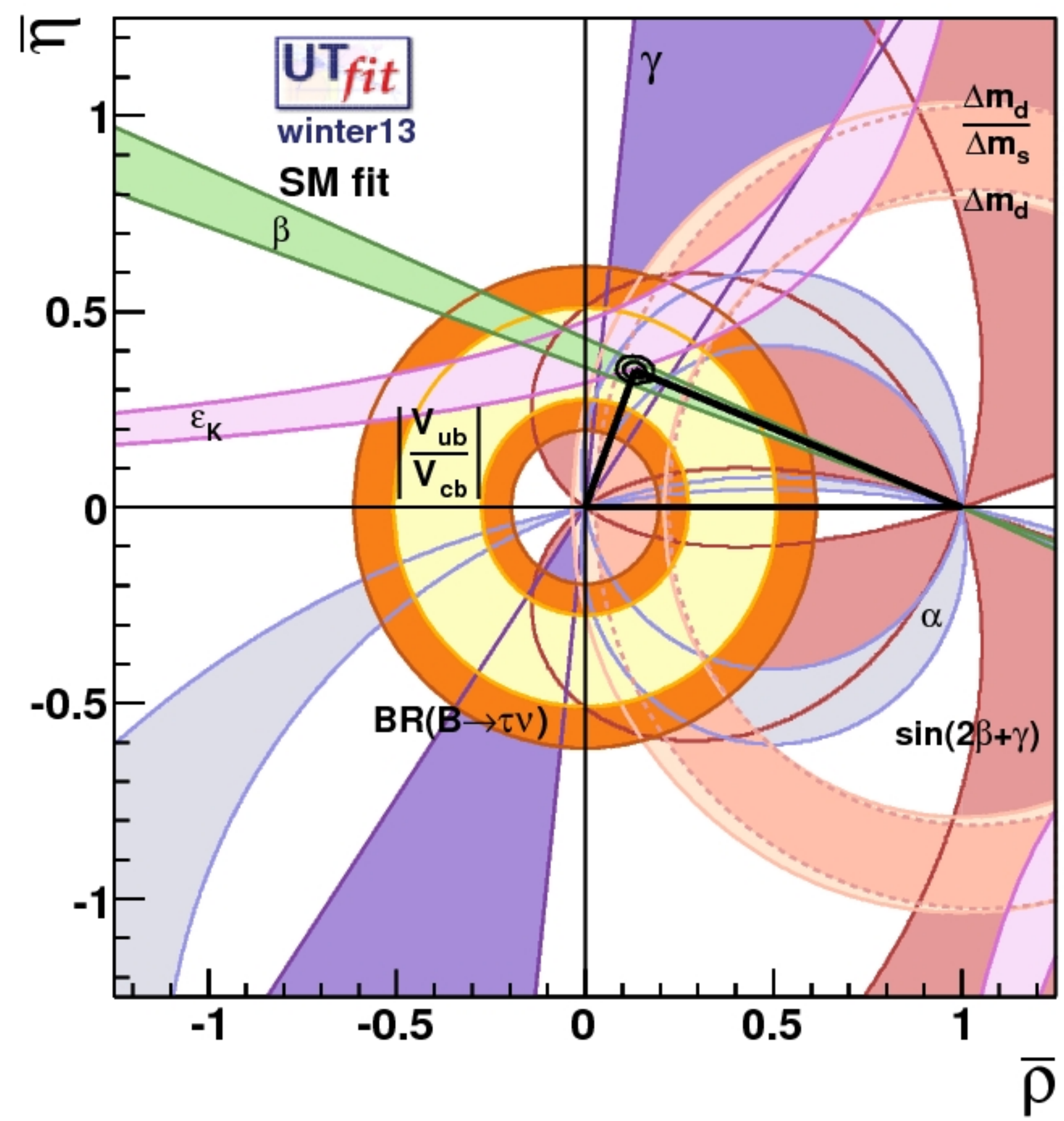
SM Lagrangian

$$
\begin{aligned}
\mathcal{L}_{MSM} = & -\frac{1}{2g_s^2}\mathrm{Tr}G_{\mu\nu}G^{\mu\nu} - \frac{1}{2g^2}\mathrm{Tr}W_{\mu\nu}W^{\mu\nu} \\
& -\frac{1}{4g'^2}B_{\mu\nu}B^{\mu\nu} + i\frac{\theta}{16\pi^2}\mathrm{Tr}G_{\mu\nu}\tilde{G}^{\mu\nu} + M_{Pl}^2 R \\
& +|D_\mu H|^2 + \bar{Q}_i i\not{D} Q_i + \bar{U}_i i\not{D} U_i + \bar{D}_i i\not{D} D_i \\
& +\bar{L}_i i\not{D} L_i + \bar{E}_i i\not{D} E_i - \frac{\lambda}{2}\left(H^\dagger H - \frac{v^2}{2}\right)^2 \\
& -\left(h_u^{ij} Q_i U_j \tilde{H} + h_d^{ij} Q_i D_j H + h_l^{ij} L_i E_j H + c.c.\right). (1)
\end{aligned}
$$

Based on local gauge principle

EWPT & CKM





Almost Perfect !

Only Higgs (~SM) and Nothing Else So Far at the LHC

All the interactions except for gravity are described by Quantum Gauge Theories !

Still Many Why's !!

- Neutrino masses and mixings ?
- Nonbaryonic DM ? DE ?
- Why is top much heavier than other fermions ?
- Why Q(e) = - Q(p) ?
- Do all forces unify at high energy scale ?
- Why 3 generations ?

Occam's razor principle ?

- Is our spacetime 4-dim ?

There are many more, including your own Q's

Building Blocks of SM

- Lorentz/Poincare Symmetry
- Local Gauge Symmetry : Gauge Group + Matter Representations from Experiments
- Higgs mechanism for masses of weak gauge bosons and SM chiral fermions
- These principles lead to unsurpassed success of the SM in particle physics

Lessons from SM

- Specify local gauge sym, matter contents and their representations under local gauge group
- Write down all the operators upto dim-4
- Check anomaly cancellation
- Consider accidental global symmetries
- Look for nonrenormalizable operators that break/conserve the accidental symmetries of the model

- If there are spin-1 particles, extra care should be paid : need an agency which provides mass to the spin-1 object
- Check if you can write Yukawa couplings to the observed fermion
- One may have to introduce additional Higgs doublets with new gauge interaction if you consider new chiral gauge symmetry (Ko, Omura, Yu on chiral U(1)' model for top FB asymmetry)
- Impose various constraints and study phenomenology

(3,2,1) or SU(3)cXU(1)em ?

- Well below the EW sym breaking scale, it may be fine to impose SU(3)c X U(1)em
- At EW scale, better to impose (3,2,1) which gives better description in general after all
- Majorana neutrino mass is a good example
- For example, in the Higgs + dilaton (radion) system, and you get different results
- Singlet mixing with SM Higgs

Towards BSM

Bottom-Up

- Precision Calculations
- Experimental Anomalies
- Construct phenomenological model and try to explain the anomaly
- If successful, try to construct more complete theories
- Otherwise one gives up

Top-Down

- Hierarchy problem (SUSY,X-Dim, etc.)
- GUT, String Theory etc.
- Start from (beautiful) high energy theory, then RG run down to low energy scale and do phenomenology
- If fails, modify the high energy theory and repeat the whole procedure

We are living in a data-driven era, and so I will follow the bottom-up approach !

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We have to rely on effective field theory (EFT)

How to construct EFT ?

- Top-Down : If a high energy scale theory is given, you integrate out the heavy d.o.f. and RG run down to the next heaviest mass scale, and repeat (Match and Run) until you reach the energy scale you are interested in

- Bottom-Up : At energy scale E you are interested in, identify dynamical fields and symmetries (local or global), and write down all possible interactions with Lorentz/Poincare symmetry

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This is the most difficult part !
Only ext's can help us !

Effetive Field Theory (EFT)

- Why EFT ?
- SM (Ren + Nonren) as an EFT
- EFT for Dark Matter Physics

Weinberg’s theorem (1979)

- To any given order in perturbation theory, and for a given set of asymptotic states, the most general possible Lagrangian containing all terms allowed by the assumed symmetries will yield the most general S-matrix elements consistent with analyticity, perturbative unitarity, cluster decomposition and assumed symmetry principles

- Originally this was to conjecture the equivalence of current algebra results and effective Lagrangian method for pion physics

Why EFT ? (weak coupling case)

- We don't know what happens at energy higher than it is affordable
- High Energy physics can leave footprints in low energy regime, which can be adequately described by effective lagrangian with an infinite tower of local operators
- If new physics scale is much higher than experimental energy scale, the lowest dim nonrenormalizable operators will give the dominant corrections to the SM prdictions
  Fermi's theory of weak interaction is a good example
- One can do meaningful phenomenology with a few number of unknown parameters
- Existing proof : the very most successful SM down to $r \lesssim 10^{-18}$ m
- In any case, we are living with EFT any way in real life

Why EFT ? (strong coupling case)

- In a strongly coupled theory such as QCD where nonperturbative aspects are very important, it is ususally very difficult to solve a problem
- Very often physical dof is different from fields in the lagrangian
  (quarks and gluon vs. hadrons in QCD)
- Useful (often critical) to construct EFT based on the symmetries of the underlying strongly interacting theory, using the relevant dof only
- Most important to identify the relevant dof and relevant symmetries
- Many examples in QCD: chiral lagrangian [$+$ (axial) vector mesons, heavy hadrons], NRQCD for heavy quarkonium, HQET for heavy hadrons, SCET etc.

EFT is not new at all !

- Newton's gravity near the earth's surface (R = radius of the earth) : $V(r) = -GMm/(R+h) \approx const + mgh + \ldots$

- Classical E&M : multipole expansions for local charge/ current distributions, Good for $r \gg R$ (R: the size of local charge distribution)

- Potential for +q and -q separate by a distance d: Good for

$$V(\vec{r}) = q\left(\frac{1}{|\vec{r} + \overrightarrow{d}/2|} - \frac{1}{|\vec{r} - \overrightarrow{d}/2|}\right) \approx \frac{q\overrightarrow{d} \cdot \vec{r}}{r^3} + \ldots = \frac{\overrightarrow{p} \cdot \vec{r}}{r^3} + \ldots$$

However this is not good for d < r , especially near r = 0

Dim. Analysis : Area of Ellipse

- Consider an ellipse given by $\left(\frac{x^2}{a^2}\right)+\left(\frac{y^2}{b^2}\right)=1$
- What is the area of this ellipse ? Ans = πab
- Dimensional Analysis : [a]=[b]=L, [Area]=L^2
- Area ~ a^2, b^2, ab (one can think of more complicated forms)
- Area is symm under $a \leftrightarrow b$, vanishes when $a, b \to 0$: Area = Cab
- For $a = b$ (circle), we know $C = \pi$ (Area = πa^2)
- Therefore we get the area of an ellipse is πab
- HW 0: What is the volume of 3-dim ellipsoid ?

Naive Dimensional Analysis

- Natural Units in HEP:

$$c = \hbar = 1 \to [\vec{L} = \vec{r} \times \vec{p}] = 0$$

$$[L] = [T] = [\vec{p}]^{-1}$$

$$E = \sqrt{(pc)^2 + (mc^2)^2} \quad \longrightarrow \quad E = \sqrt{p^2 + m^2},$$

$$\text{QM Amp} \sim \int_{\text{path}} e^{iS/\hbar} \quad \longrightarrow \quad [\text{Action}] = 0 = [\int d^4x\mathcal{L}]$$

- $[E] = [p] = [M] = [L]^{-1} = [T]^{-1}$
- Everything will be in mass dimensions:

$$[\mathcal{L}] = 4, \quad [\sigma(= \text{Area})] = -2, \quad [\tau(= \Gamma^{-1})] = -1$$

- Both the decay rate ($\Gamma \equiv \tau^{-1}$) and the cross section (σ) are given by

  Fermi's Golden Rule

  with suitable flux facors

$$|\mathcal{M}|^2 \times \text{phase space} \left(\equiv \Pi_{i=1^n} \frac{d^3\vec{p}_i}{(2\pi)^3 2E_i}) \right) \times (2\pi)^4 \delta(\sum_i p_i - \sum_f p_f)$$

- Note that $[\Gamma] = +1$ and $[\sigma] = -2$
- It is often enough to do the dimensional analysis for Γ and σ, when there is only one important mass scale from the phase space integration
- A number of easy examples will be given in this lecture

Scalar fields

- Lagrangian for a real scalar field:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2 - \mu\phi^3 - \frac{\lambda}{4}\phi^4 + \sum_{i=1}^{\infty}\frac{C_{4+i}}{\Lambda^i}\phi^{4+i}$$

- $[\partial] = +1, [\mathcal{L}] = 4 \to [\phi] = 1$
- $[m] = [\mu] = +1$ and $[\lambda] = [C_i] = 0$
- C_i terms are nonrenormalizable interaction terms ($\phi^{d>4}$: Irrelevant operators $\to$ Will discuss shortly)
- Field op ϕ create or annihilate a particle of mass m:

$$\phi \sim a(p)e^{-ip\cdot x} + a^\dagger(p)e^{+ip\cdot x}$$

- Complex scalar $\phi \sim a + b^\dagger$ with a and b relevant to particle and antiparticle

Fermion fields

- Lagrangian for fermion fields :

$$\mathcal{L} = \overline{\psi}(i\partial\cdot\gamma - m_\psi)\psi + \frac{C}{\Lambda^2}(\overline{\psi}\psi)^2 + \ldots.$$

- $[\psi] = 3/2\ ,\ [m] = 1\ ,\ [C] = 0$
- C term: nonrenormalizable (irrelevant at low energy)
- Dirac field operator:

$$\begin{aligned}\psi &\sim bu + d^\dagger v\\ \overline{\psi} &\sim b^\dagger\overline{u} + d\overline{v}\end{aligned}$$

- Fermi's theory of weak interaction is the classic example

- Dimensional analysis for $\psi\overline{\psi}$ scattering

$$\mathcal{M}(\psi(p_1, s_1)\overline{\psi}(p_2, s_2) \to \psi(p_3, s_3)\overline{\psi}(p_4, s_4)) \sim \frac{1}{\Lambda^2}$$

$$\sigma \sim \left(\frac{1}{\Lambda^2}\right)^2 \times (phasespace) \sim \left(\frac{1}{\Lambda^2}\right)^2 \times s$$

- Mandelstam variables for $2 \to 2$ scattering:

$$s \equiv (p_1 + p_2)^2, t = (p_3 - p_1)^2, u = (p_4 - p_1)^2$$

$$s + t + u = \sum_{i=1}^{4} m_i^2$$

- Cross section becomes zero as $s \to 0$: Irrelevant

Unitarity Violation

- What happen at high energy ?

$$\sigma \to \infty \to$$

Violation of perturbative Unitarity near $\sqrt{s} \sim \Lambda/\sqrt{C}$
$\to$ New dof's will come into play for cure (e.g., $W^{\pm}$ or Z^0)

- This is the wonder of Nature with special relativity and quantum mechanics
- In the SM, the pointlike interaction is replaced by the $W^{\pm}, Z^0$ propagator, which cuts off the bad high energy behavior
- $\sigma \sim 1/s$ at very high energy scale $\sqrt{s} \gg m_{W,Z}$

Vector fields

- Lagrangian for abelian gauge field with a charged particle (QED):

$$\begin{aligned}
\mathcal{L} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}(iD\cdot\gamma - m_\psi)\psi \\
F_{\mu\nu} &\equiv \partial_\mu A_\nu - \partial_\nu A_\mu \\
D_\mu\psi &\equiv (\partial_\mu + ieA_\mu)\,\psi
\end{aligned}$$

- $[A_\mu] = 1, [F_{\mu\nu}] = 2, [e] = 0$
- Dimensionless coupling $e \rightarrow$ Renormalizable interaction (marginal operator, meaning that it is important at all energy scales)
- RG equation for e may run into a Landau pole, above which the coupling diverge $\rightarrow$ Either new theory before/around Landau pole, or low energy theory is free field theory

Heavy Particle EFT

- If the energy scale is so low that the particle cannot be created or destroyed, the particle number will be conserved
- Heavy particle EFT

$$p^\mu = mv^\mu + k^\mu, \quad |k| << m$$

- Remove e^{-imt} factor from the field : $\phi = e^{-imv\cdot x}\psi_v(x)$
- Lagrangian (with Lorentz sym restore by v^μ) :

$$\mathcal{L}(\psi_v, v^\mu) = \psi_v^\dagger v \cdot D\psi_v + ...$$

- Can be applied to baryon ChPT, heavy meson ChPT, etc..

Renormalizable Opertors

- dim 0 : $I_{\rm op}$ (cosmological constant)
- dim 1 : S (scalar tadpole)
- dim 2 : S^2 , $A_\mu A^\mu$ (mass terms for bosons)
- dim 3 : $\overline{\psi}\psi$ (Fermion mass term) , S^3 (self interaction of singlet scalar)
- dim 4 : $S\overline{\psi}\psi$ (Yukawa interaction) , S^4 (Scalar self coupling) , A_μ^4 , $\partial_\mu A_\nu A^\mu A^\nu$ (self interactions of gauge fields)

NB: S, S^3 etc possible only for gauge singlet S

nabelian Gauge Symmetry and Renormalizability

- Renormalizable Interactions are only 3 types:

$$B^3,\ B^4,\ \overline{F}FB$$

- Power counting renormalizable interactions for spin-1:

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)^2 + m_A^2 \frac{1}{2} A_{\mu a} A^{\mu a} + \partial_\mu A_\nu^a A^{\mu b} A^{\nu c} + + A_\mu^a A_\nu^b A^{\mu c} A^\nu$$

(all possible contraction over group indices)

- Although this is power counting renormalizable, it is not
- Only special type of lagrangian consistent with local Nonabelian gauge symmetry is renormalizable
- Local gauge symmetry is really a powerful principle for a spin-1 object

Some remarks on QFT

- QFT is the basic framework for particle physics, and is a marriage of QM and Special Relativity
- Spin-Statistics theorem
  - Bosons : totally symmetric wavefunction
  - Fermions : totally antisymmetric wavefunction
  - Intrinsic $P(B, F) = (+B, -F)$
- CPT is a symmetry of any local QFT
  $\rightarrow CP$ violation implies T (time-reversal) violation
- CPT theorem: $m_n = m_{\bar{n}}$ and $\tau_n = \tau_{\bar{n}}$, $\mu_n = \mu_{\bar{n}}$
- However, a partial width of n and $\bar{n}$ can be different $\rightarrow$ Direct CP Violation :

$$\Gamma(n \rightarrow f) \neq \Gamma(\bar{n} \rightarrow \bar{f})$$

Heavy Quarknia Quantum Numbers

- Bound State of spin-1/2 Q and $\bar{Q}$ with ${}^{2S+1}L_J$:

$$P = (-1)^{L+1}, \quad C = (-1)^{L+S} \to 0^{-+}, 1^{--}, 1^{++}, 1^{+-},$$

- Bound State of spin-0 Q and $\bar{Q}$ with ${}^{2S+1}L_J$ (with $S = 0$ and $L = J$):

$$P = (-1)^{L}, \quad C = (-1)^{L} \to 0^{++}, 1^{--}, 2^{++}, \mathrm{etc.}$$

- No place for π (with 0^{-+})
- Observed J^{PC} clearly says that quarks are spin-1/2 fermions, not scalars
- Exotic mesons don't follow the above assigment

Effective Lagrangian Approach

- If new physics scale is high enough, it is legitimate to integrate out the heavy d.o.f.
- The low energy physics can be described in terms of effective lagrangian :

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm ren} + \sum_{d=5}^{\infty} \frac{\mathcal{O}^{(d)}}{\Lambda_d^{d-4}}$$

  where all the operators in $\mathcal{L}_{\rm eff}$ are made of light d.o.f. with their local gauge symmetries
- Effects of the nonrenormalizable operators $\sim (E/\Lambda_d)^{d-4}$ relative to the amplitude from $\mathcal{L}_{\rm ren}$
- EFT is useful, as long as $E \ll \Lambda_d$, since we can keep only a few of the NR operators

SM as an EFT: Below e^+e^- Threshold

- Only relevant quantum dof is photon A_μ
- If E increases, we need to include more and more NR operators
- Eventually, unitarity will be broken $\rightarrow$ We have to include new d.o.f.'s in the EFT, and redefine the EFT with more d.o.f.
- QED at $E \ll 2m_e$: A_μ, local $U(1)$ and P, C

$$\mathcal{L}_{\mathrm{EET}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{e^4}{(4\pi)^2 \Lambda^4} F^4 + \ldots$$

  where $\Lambda \sim m_e$
- This effective lagrangian describes $\gamma\gamma$ scattering, the cross section of which will break unitarity when E reaches $2m_e$

SM as an EFT: Below e^+e^- Threshold

- The cross section grows like $\sim s^3$:

$$\sigma(\gamma\gamma \to \gamma\gamma) \sim \frac{e^8}{\Lambda^8} s^3$$

  and see at which energy scale unitarity is violated
- Unitarity will be restored due to a new process that opens up: $\gamma\gamma \to e^+e^-$
- One has to redefine the effective lagrangian near e^+e^- threshold, by including the electron/positron fields explicitly

Digress on Unitarity

- Unitarity is the most profound thing in QM
- Scattering Operator S is unitary:

$$\langle f|S|i\rangle = S_{fi} = \delta_{fi} + i(2\pi)^4\delta^4(p_i - p_f)T_{fi}$$

- Unitarity: $S^\dagger S = SS^\dagger = 1$

$$T_{fi} - T^*_{fi} = i(2\pi)^2\sum_n \delta^4(p_f - f_n)T_{fn}T^*_{in}$$

- If interaction is weak, we can ignore the RH $\rightarrow$ T becomes Hermitian $T_{fi} = T^*_{if}$
- Optical theorem for $f = i$:

$$2\mathrm{Im}T_{ii} = (2\pi)^4\sum_n |T_{in}|^2\delta^4(P_i - P_n)$$

Rayleigh Scattering: Why is Sky Blue ?

- Photon scattering with neutral atom A where

$$E_\gamma \ll \Delta E_{n1} \equiv E_n - E_1$$

$\rightarrow$ Elastic scattering of light on neutral atoms

- Atom is described by nonrelativistic Schrödinger wave function ψ_A with dim 3/2:

$$\mathcal{L} = \psi_A^\dagger \left(i\frac{\partial}{\partial t} - H \right) \psi_A + \frac{e^2}{\Lambda^3} \psi_A^\dagger \psi_A F_{\mu\nu} F^{\mu\nu} + \ldots.$$

- $\Lambda \sim \Delta E_{21}, r_0$??
- Note that photon couples to a neutral atom. How ???

- No coupling of photon to neutral objects only at renormalizable level
- Photon couples to neutral particle at nonrenormalizable level due to quantum fluctuation can involve charged particles in the loop
- Likewise, gluons can couple to photons
- γA scattering cross section :

$$\sigma(\gamma A \to \gamma A) \sim \frac{e^4}{\Lambda^6} E_\gamma^4 \sim \frac{1}{\lambda_\gamma^4}$$

  for $E_\gamma \ll \Delta E_{2,1}$
- Blue light scatters more than red light $\to$ Sky is blue, and we can enjoy the beautiful sunrise/sunset in red

Hydrogen Atom

- Hydrogen atom : 3 energy scales (m_e, $a_0^{-1} \sim m_e\alpha$, $R \sim m_e\alpha^2$) with R Rydberg constant
- Consider $H(2P) \to H(1S) + \gamma$
- Fields : ϕ for $H(1S)$, χ_μ for $H(2P)$, $F_{\mu\nu}$ for photon
- Lorentz inv, Parity : only one operator for the radiative transition: $\mathscr{L} = \dfrac{Ce}{\Lambda}\phi F^{\mu\nu}\chi_{\mu\nu}$ $(\chi_{\mu\nu} \equiv \partial_\mu\chi_\nu - \partial_\nu\chi_\mu)$

- $g \sim O(1)$ dimensionless constant, encoding microscopic physics for the radiative transitions (electric dipole,…)

- Now the new physics scale relevant to this case is $\Lambda \sim a_0^{-1}$, since we are ignoring fine structure now

- HW 1: Show that $\Gamma(H(2P) \to H(1S) + \gamma) = \left(\frac{4}{3} g^2\right) \alpha a_0^2 \omega^3$

- QM textbook : $\Gamma(H(2P) \to H(1S) + \gamma) = \frac{2^{17}}{3^{11}} \alpha a_0^2 \omega^3$

- $g = 0.74$: is really ~O(1)

Useful relations

- For photon polarization $\epsilon_\mu(\gamma)$, the pol sum is given by
  $$\sum_{pol} \epsilon_\mu(\gamma)\epsilon_\nu^*(\gamma) = -g_{\mu\nu}$$

- For massive spin-1 pol. $\epsilon_\mu(2P)$, the pol sum is given by
  $$\sum_{pol} \epsilon_\mu(2P)\epsilon_\nu^*(2P) = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_{2P}^2}\right)$$
  [$q^\mu = (M_{2P}, \overrightarrow{0})$: 4-momentum of H(2P)]

- $\Gamma = \frac{1}{2M_{2P}} \int d\Pi_2 \overline{|M|^2}$ ($d\Pi_2$: 2-body phase space element, and sum over final spin, average over the initial spin)

- HW 2: Show that $H(2S) \to H(1S) + \gamma$ is forbidden by showing that there are no effective operators allowed by symmetries. What is the allowed decay mode of $H(2S)$ into $H(1S)$ with emitting photons ?

- HW 3: In Nature, one observes $\rho^0(770) \to \pi^+\pi^-$, but not $\rho^0 \to \pi^0\pi^0$. Can you explain why this is the case ? Which symmetry forbids the latter decay mode ? [This is a kind of theorem similar to Landau-Yang theorem: Spin-1 particle can not decay into a pair of identical scalar particles]

- HW4: Prove Landau-Yang theorem [a spin-1 particle can not decay into a pair of photons]. How about the case a colored spin-1 particle decays into a pair of gluons in QCD ?

Van der Waals Force (or Quantum Force)

- Potential between neutral atoms are described by two-photon exchange diagrams from the previous lagrangian $\psi_A^\dagger \psi_A F^2$
- Additional contact interaction has to be considered:

$$\frac{1}{\Lambda^2}\left(\psi_A^\dagger \psi_A\right)^2$$

- Calculate the two contributions and discuss what is the form of the force between two neutral atoms (Van der Waals interaction) ?
- What is a in the exponent in $V(r) \sim r^a$?
- What if we consider the neutral atom relativistically ? (Itzykson and Zuber, QFT)

QED as an EFT below $\mu^+\mu^-$ threshold

- QED at $2m_e \leq E \ll 2m_\mu$: A_mu, e, $\bar{e}$, local $U(1)$ and P, C

$$\mathcal{L}_{\rm Eff} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{e}(iD - m_e)e + \frac{e^4}{(4\pi)^2\Lambda_1^4}F^4 + \frac{e}{(4\pi)^2\Lambda_2}\overline{e}\sigma^{\mu\nu}eF_{\mu\nu}$$

  where $\Lambda_1 \sim m_\mu$, and $\Lambda_{2,3} \sim O(1)$ TeV or larger (see later discussions on these points)

- NP scale in each NR operator is independent (different from each other) in general, since the origin can be different
- Scale for F^4 is now $\sim m_\mu$, unlike the previous case

QED as an EFT below $\mu^+\mu^-$ threshold

- Additional $1/(4\pi)^2$ suppression for NR operators generated at one-loop level, compared with NR operators generated at tree level, even if their operator dim's are the same
- If we impose $SU(2)_L \times U(1)_Y$ instead of $U(1)_{\rm em}$, the Λ_2 term should be replaced by

$$\frac{e}{(4\pi)^2\Lambda_2^2}\overline{e_L}\sigma^{\mu\nu}He_R F_{\mu\nu} \rightarrow \frac{ev}{\sqrt{2}(4\pi)^2\Lambda_2^2}\overline{e_L}\sigma^{\mu\nu}e_R F_{\mu\nu}$$

  and the effect becomes smaller for the same Λ_2, or the bound on Λ_2 becomes stronger
- Chiraliry flip operator

QED as an EFT above $\mu^+\mu^-$ threshold

- QED at $E \ll 2m_\pi$: A_μ, e, $\bar{e}$, μ, $\bar{\mu}$, local $U(1)$ and P, C

$$
\begin{aligned}
\mathcal{L}_{\text{Eff}} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{e}(iD - m_e)e + \overline{\mu}(iD - m_\mu)\mu \\
&+ \frac{e^4}{(4\pi)^2\Lambda_1^4}F^4 + \frac{e}{(4\pi)^2\Lambda_2}\overline{e}\sigma^{\mu\nu}eF_{\mu\nu} + \frac{e}{(4\pi)^2\Lambda_3}\overline{\mu}\sigma^{\mu\nu}\mu F_{\mu\nu} \\
&+ \frac{e}{(4\pi)^2\Lambda_4}\overline{e}\sigma^{\mu\nu}\mu F_{\mu\nu} + \frac{e^2}{\Lambda_5^2}(\overline{e}e)(\overline{e}\mu) + H.c.
\end{aligned}
$$

where $\Lambda_1 \sim m_\pi$, $\Lambda_{2,3} \gtrsim XX$ TeV , and $\Lambda_{4,5} \gtrsim XX$ TeV or larger

- $\Lambda_{2,3}$ terms contribute to $(g-2)_{e,\mu}$
- $\Lambda_{4,5}$ generate $\mu \to e\gamma$ and $\mu \to 3e$

Muon Decay $\mu \to e\overline{\nu_e}\nu_\mu$

- Apply the Fermi's theory of weak interaction with replacing (p, n) by (ν_μ, μ)

$$\mathcal{L}_{CCweak} = -\frac{G_F}{\sqrt{2}}(\overline{\nu_\mu}\gamma^\mu \mu)(\bar{e}\gamma_\mu \nu_e) + H.c.$$

- Muon lifetime :

$$\tau^{-1} = \Gamma_\mu = \frac{G_F^2}{2(4\pi)^3} m_\mu^5$$

cf. Compare with the exact expression:

$$\tau^{-1} = \Gamma_\mu \sim \frac{G_F^2}{192\pi^3} m_\mu^5 \propto m_\mu^5$$

- $\Gamma \propto m^5$ is a generic behavior of a fermion decaying through 4-fermion (dim 6) operators (τ, proton decays etc.)

Weinberg operator for neutrino mass

- If we impose $SU(2)_L \times U(1)_Y$ local gauge symmetry instead of $U(1)_{\rm em}$, the above neutrino mass terms will be replaced by dim-5 Weinberg operator breaking with $\Delta L = 2$:

$$\frac{y_{\alpha\beta}}{\Lambda_{\alpha\beta}} (L_\alpha H)(J_\beta H) + H.c.$$

with $\Lambda_{\alpha\beta} \sim 10^{12-16}$ GeV $\sim M_N$ (RH Majorana mass scale in seesaw mechanism)

- This is the only dim-5 operator which is invariant under the full SM gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$
- This nonrenormalizable terms can be made renormalizable (UV complete) by introducing the RH singlet neutrinos (Type-I seesaw), or by triplet Higgs fields (Type-II seesaw)

Proton Decay

- These decays are kinematically allowed, but never been observed

$$\begin{aligned} \tau(p \to e^+\pi^0) &> 8.2 \times 10^{33}\mathrm{yr} \\ \tau(p \to K^+\nu) &> 6.7 \times 10^{32}\mathrm{yr} \end{aligned}$$

- Why proton is so stable ?

$$\tau_p > \tau_{\mathrm{universe}} = 4 \times 10^{17}\ \mathrm{sec}$$

- Consider operators $\bar{e}p\pi^0$ (dim 4), and $\bar{e}\gamma^\mu p\partial_\mu\pi^0$(dim 5), both give dangerously short lifetime for proton

Proton Decay

- One possible way out: p and π are composite of quarks, and B and L violation occurs at very high energy scale, where proton is no longer a good description with the following dim-6 operators:

$$\frac{g^2}{\Lambda^2} uude$$

  (ignoring Dirac structure)
- SUch operators can be generated in (SUSY) GUT, or MSSM with $R-$parity violation
- Calculate the lower bound on the scale Λ from the lower bound on the proton lifetime.