Physics of Neutrinos

Yonsei-Saga Workshop
January 22, 2021

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Outline

- Understanding of neutrino oscillations
- Origin of neutrino mixing matrix
- Origin of neutrino masses
- New Physics in Neutrino Oscillations
- Conclusion

Understanding of neutrino oscillations

- Two big discoveries over past two decades :

  - Neutrinos are massive

  - Leptons mix

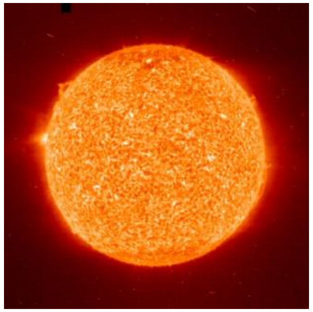
T. Kajita

A. McDonald

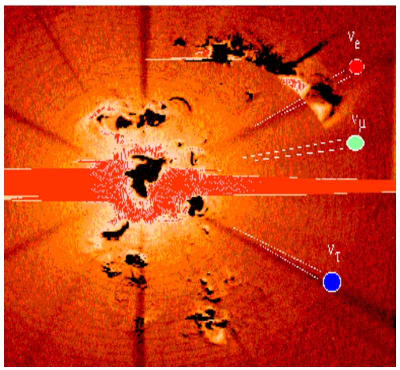
- They have been achieved by the observation of neutrino oscillations

2015 Nobel Prize

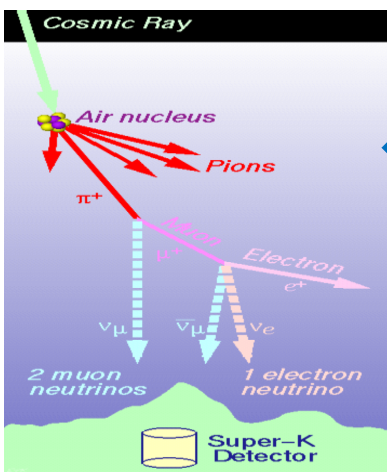
Neutrino Sources



← Sun



← Cosmology



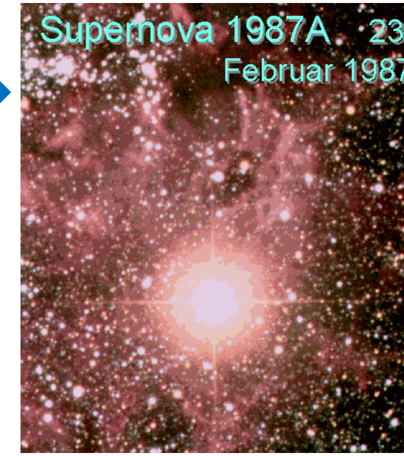
← Atmosphere

Astronomy: →

Supernovae

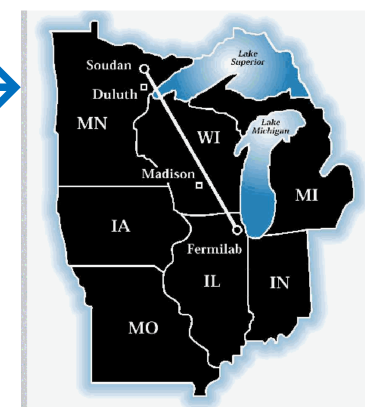
GRBs

UHE ν's



Reactors →

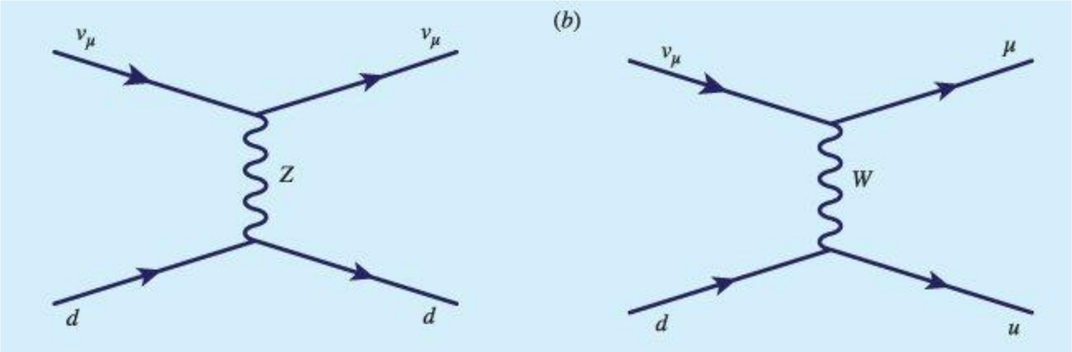
Accelerators →



← Earth

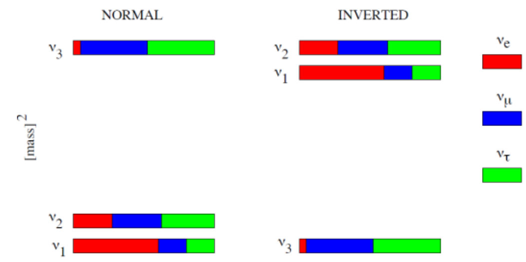
Why neutrinos mix ?

- Neutrino eigenstates involved in weak interactions



are not mass eigenstates:

3-flavor paradigm



$$\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_L \quad \begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}_L \quad \begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}_L \ :$$

weak eigenstates

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}=\begin{pmatrix}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3}\\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

mass eigenstates

- Specific parameterization of lepton mixing matrix

$$U_{PMNS}=\begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & -s_{23}\\ 0 & s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}c_{13} & 0 & -s_{13}e^{-i\delta}\\ 0 & 1 & 0\\ s_{13}e^{i\delta} & 0 & c_{13}\end{pmatrix}\begin{pmatrix}c_{12} & -s_{12} & 0\\ s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}e^{i\alpha} & 0 & 0\\ 0 & e^{i\beta} & 0\\ 0 & 0 & 1\end{pmatrix}$$

How do we probe neutrino mixing ?

➔ neutrino oscillation

In vacuum, $\nu_\alpha \rightarrow \nu_\beta$ transition probability :

=0 if δ =0 in U,
$\alpha = \beta$

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4\sum_{j\neq i}^{n} \mathrm{Re}[U^\star_{\alpha i}U_{\beta i}U_{\alpha j}U^\star_{\beta j}]\sin^2\left(\frac{\Delta_{ij}}{2}\right) + 2\sum_{j\neq i} \mathrm{Im}[U^\star_{\alpha i}U_{\beta i}U_{\alpha j}U^\star_{\beta j}]\sin(\Delta_{ij})$$

$$\frac{\Delta_{ij}}{2} = \frac{(E_i - E_j)L}{2} = 1.27\frac{(m_i^2 - m_j^2)}{\mathrm{eV}^2}\frac{L/E}{\mathrm{Km/GeV}}$$

- Experimentally, what is measured is

$$\langle P_{\alpha\beta}\rangle = \frac{\int dE \frac{d\Phi}{dE}\sigma(E)P_{\alpha\beta}(E)\epsilon(E)}{\int dE \frac{d\Phi}{dE}\sigma_{CC}(E)\epsilon(E)}$$

- From ν oscillation exps. we can determine
  - Δm^2_{21} , Δm^2_{31}
  - θ_{12} , θ_{23} , θ_{13}
  - δ

- Electron neutrino survival probability

$$P_{\nu_e \to \nu_e} = \left| \sum_{k=1}^{3} |U_{ek}|^2 e^{-i m_k^2 L/2E} \right|^2$$

$$|U_{e3}|^2 \ll |U_{e1}|^2, |U_{e2}|^2 \quad \Longrightarrow \quad |U_{e1}|^2 \simeq \cos \vartheta_{12}\,, \; |U_{e2}|^2 \simeq \sin \vartheta_{12}$$

$$P_{\nu_e \to \nu_e} = 1 - \sin^2 2\vartheta_{12} \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

⇨ It is decoupled from atmospheric ν osc.

- In the case that $\Delta m^2_{21} \ll |\Delta m^2_{31}| \approx |\Delta m^2_{32}|$

$$P_{\nu_\alpha \to \nu_\beta} = \left| U^*_{\alpha 1} U_{\beta 1} + U^*_{\alpha 2} U_{\beta 2} + U^*_{\alpha 3} U_{\beta 3} \exp\left(-i \frac{\Delta m^2_{31} L}{2E} \right) \right|^2$$

$$U^*_{\alpha 1} U_{\beta 1} + U^*_{\alpha 2} U_{\beta 2} = \delta_{\alpha\beta} - U^*_{\alpha 3} U_{\beta 3}$$

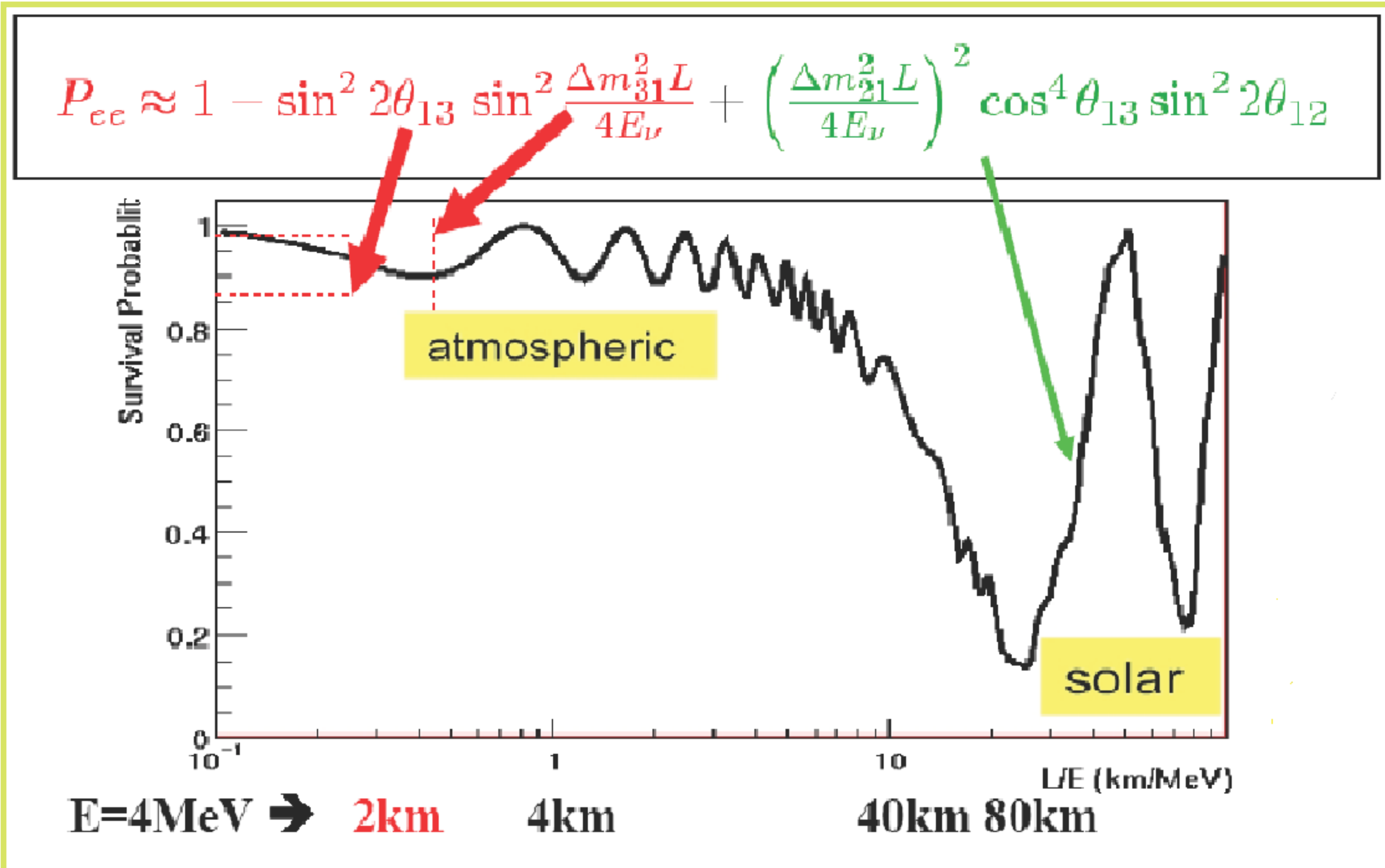
$$= \delta_{\alpha\beta} - 4|U_{\alpha 3}|^2 \left(\delta_{\alpha\beta} - |U_{\beta 3}|^2 \right) \sin^2 \frac{\Delta m^2_{31} L}{4E}$$

$$\alpha \neq \beta \quad \Longrightarrow \quad P_{\nu_\alpha \to \nu_\beta} = 4|U_{\alpha 3}|^2 |U_{\beta 3}|^2 \sin^2 \left(\frac{\Delta m^2_{31} L}{4E} \right)$$

$$\alpha = \beta \quad \Longrightarrow \quad P_{\nu_\alpha \to \nu_\alpha} = 1 - 4|U_{\alpha 3}|^2 \left(1 - |U_{\alpha 3}|^2 \right) \sin^2 \left(\frac{\Delta m^2_{31} L}{4E} \right)$$

Atmospheric & reactor exp.

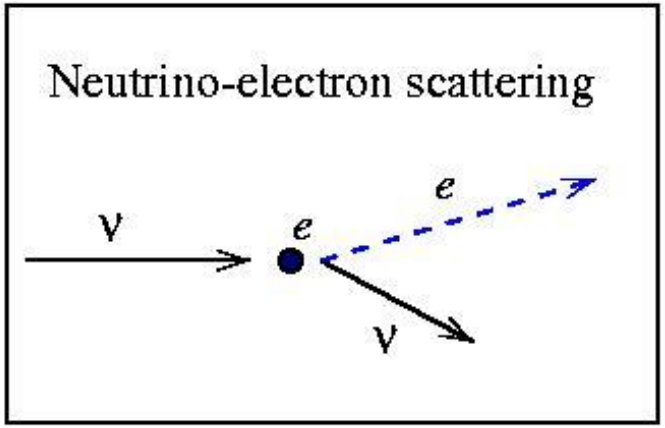
- Plot of P_{ee} for reactor neutrinos

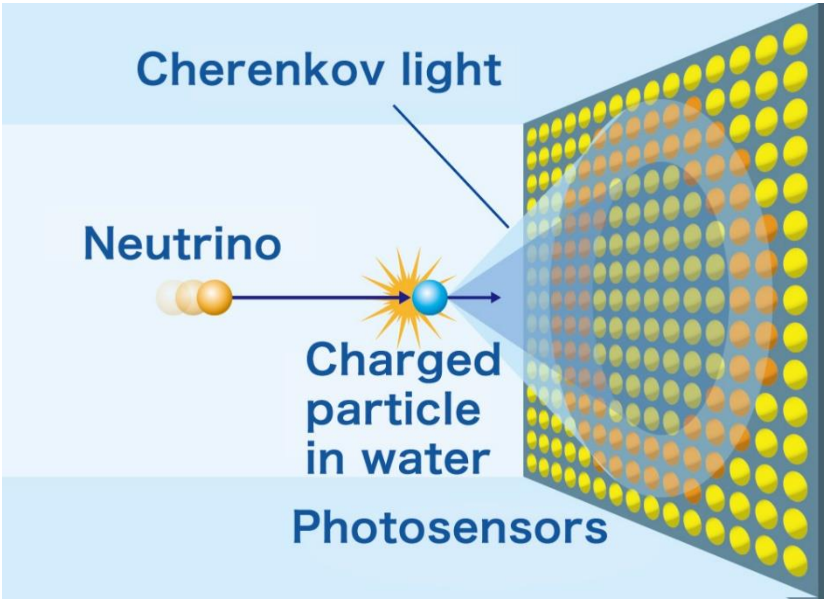


How can we detect Neutrinos?

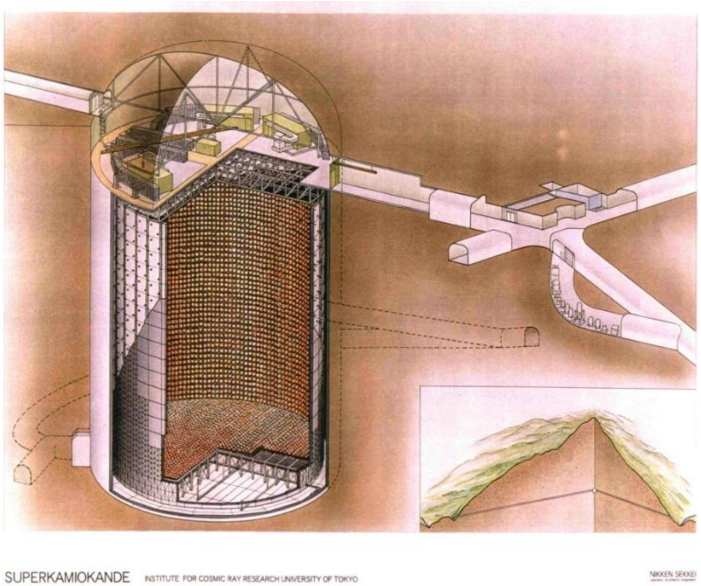
- **Cherenkov detectors : Using H_2O, D_2O**

Cerenkov radiation : emitted when a charged particle passes through a medium at a speed greater than phase velocity of light

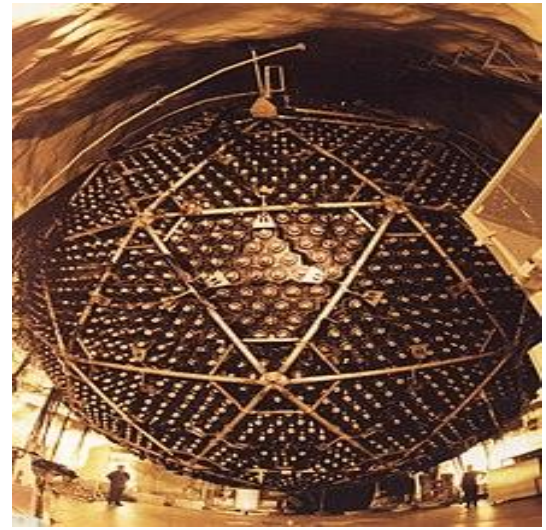




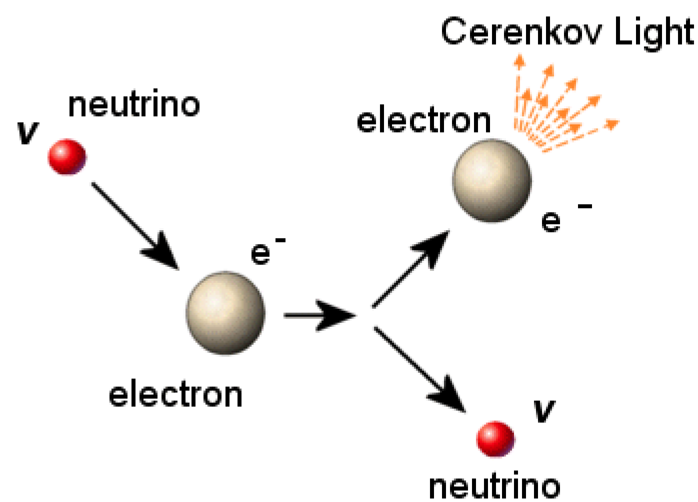
Super Kamiokande



SNO

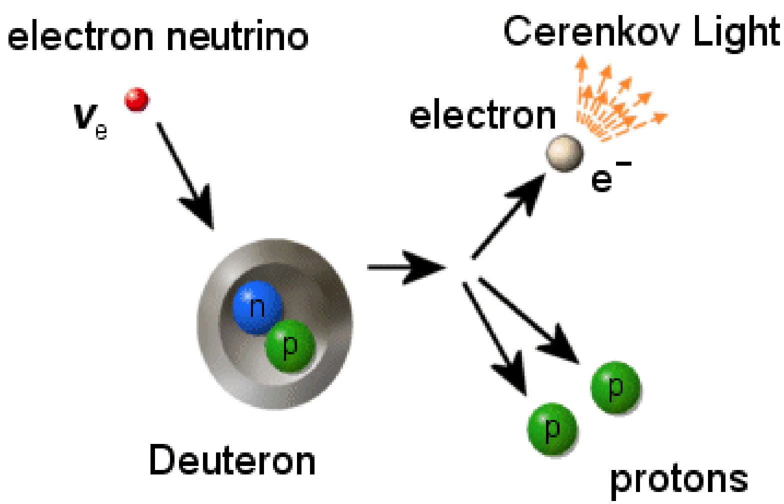


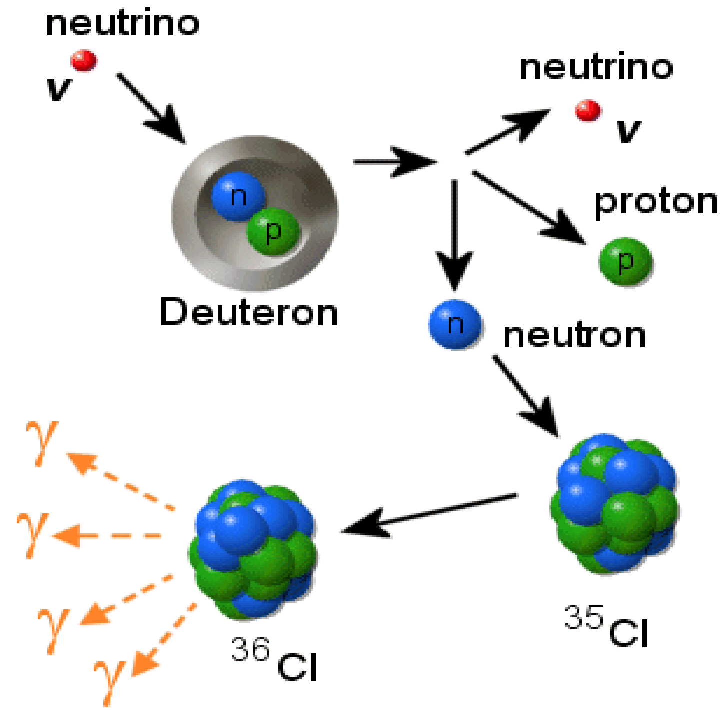
Electron Scattering Reaction

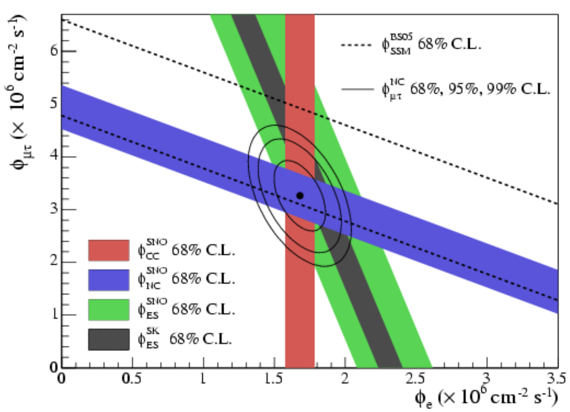


Charged-Current Reaction

Neutral-Current Reaction



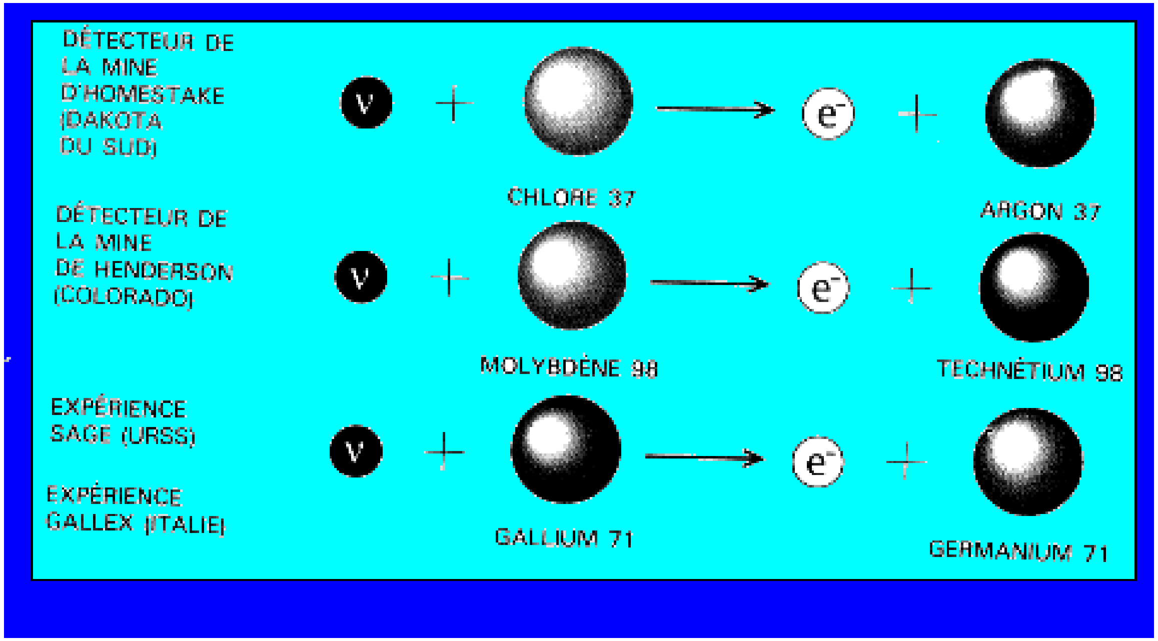




$$\frac{\Phi_{CC}}{\Phi_{NC}} = 0.301 \pm 0.033$$

Radiochemical detectors

(Homestake, SAGE, GALLEX)



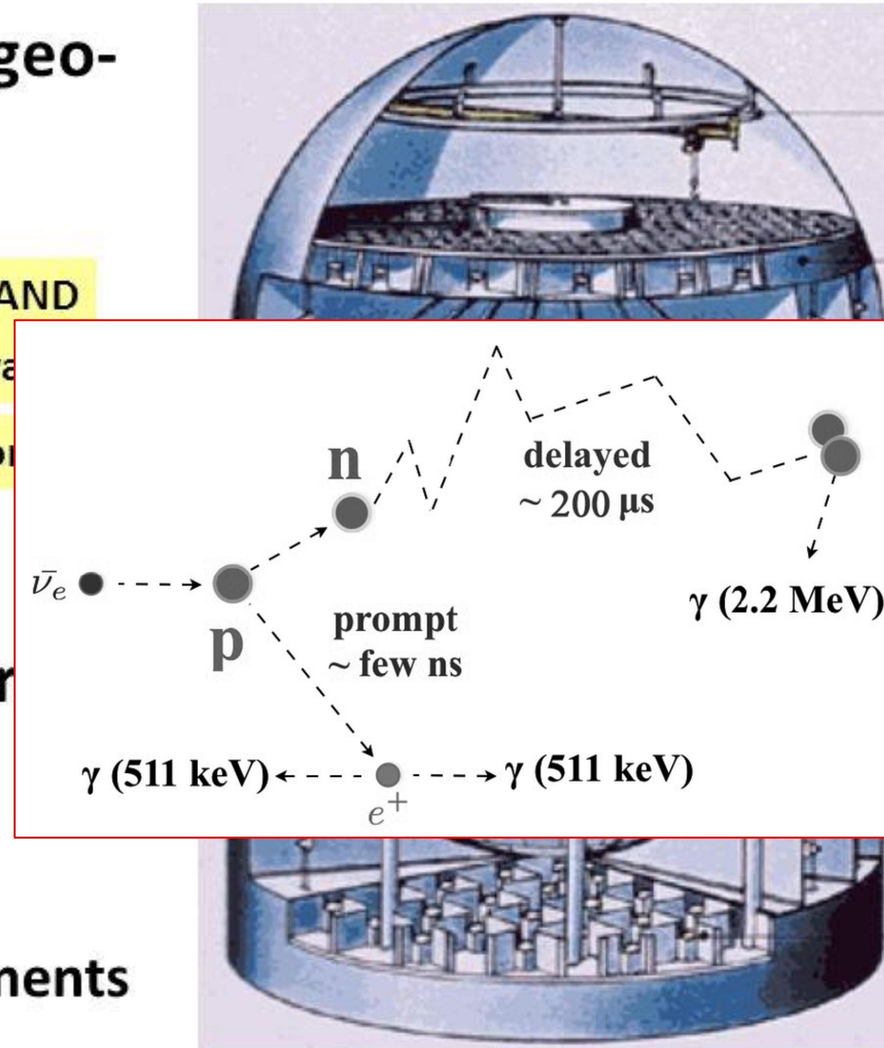
Liquid scintillator detectors

- Successful for reactor and geo-neutrinos
- Current benchmark:
  - Mass: 1 kt
  - Gd-loading LS: ~200t
  - Threshold: (0.1-0.3) MeV
  - Light yield: ~500 PE/MeV
  - PMT coverage: up to 80%
- Future ➔ (10-50)t detector
  - LBNE
  - Supernova/geo-neutrinos
  - Mass hierarchy
  - Precision mixing matrix elements

KamLAND

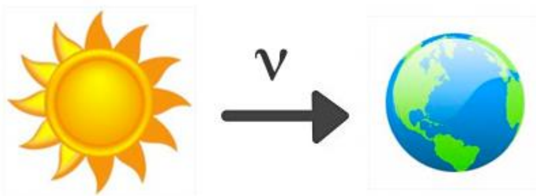
Daya

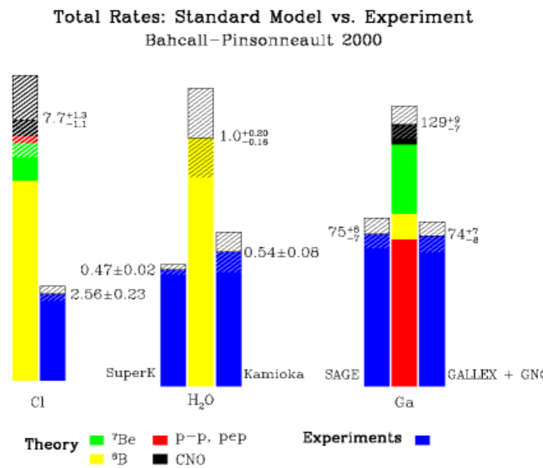
Bor



θ_{12} & Δm^2_{21}

- Well measured by solar neutrino experiments and KamLAND





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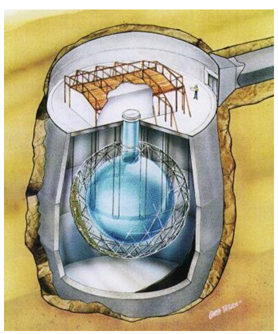
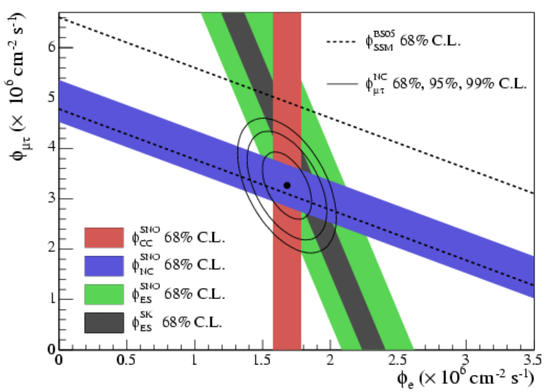
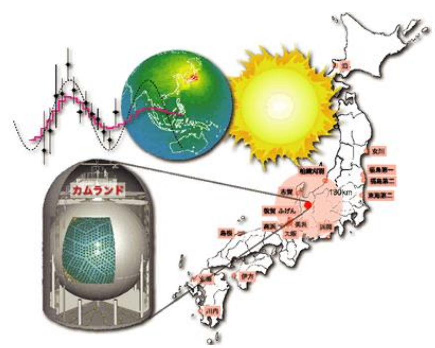
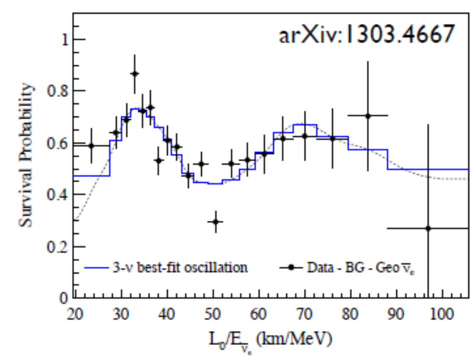




$$\frac{\Phi_{CC}}{\Phi_{NC}} = 0.301 \pm 0.033$$

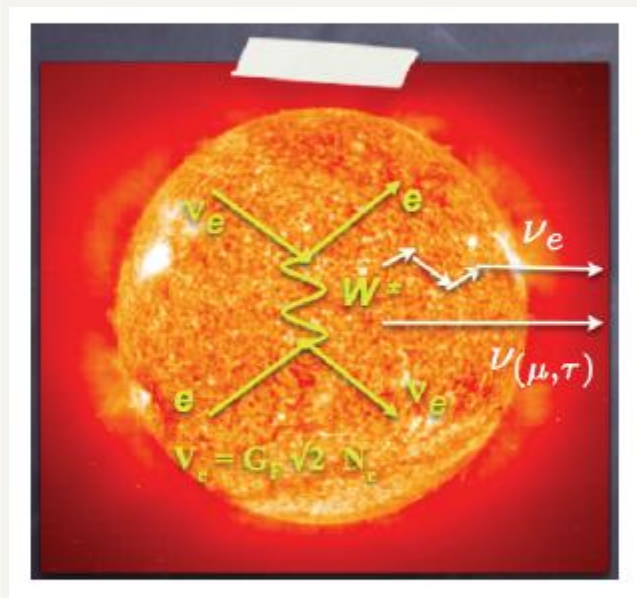
KamLAND

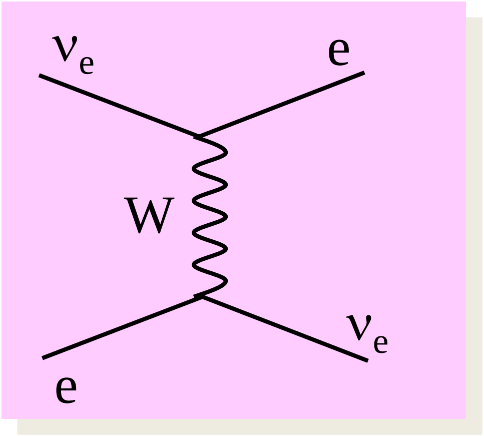




• MSW matter effect

When neutrinos travel through a medium, they interact with the background of electron, proton and neutron and acquire effective mass.





Such a interaction modifies Hamiltonian causing shifts of neutrino flavor mixing and masses.

$$i\frac{d}{dt}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + \sqrt{2}G_F N_e & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}$$

$$\tan 2\theta_M = \frac{\tan\theta}{1 - \frac{A_{CC}}{\Delta m^2 \cos 2\theta}} \qquad \Delta m_{\mathrm{M}}^2 = \sqrt{(\Delta m^2 \cos 2\vartheta - A_{\mathrm{CC}})^2 + (\Delta m^2 \sin 2\vartheta)^2}$$

$$A_{CC} = 2\sqrt{2}G_F N_e E$$

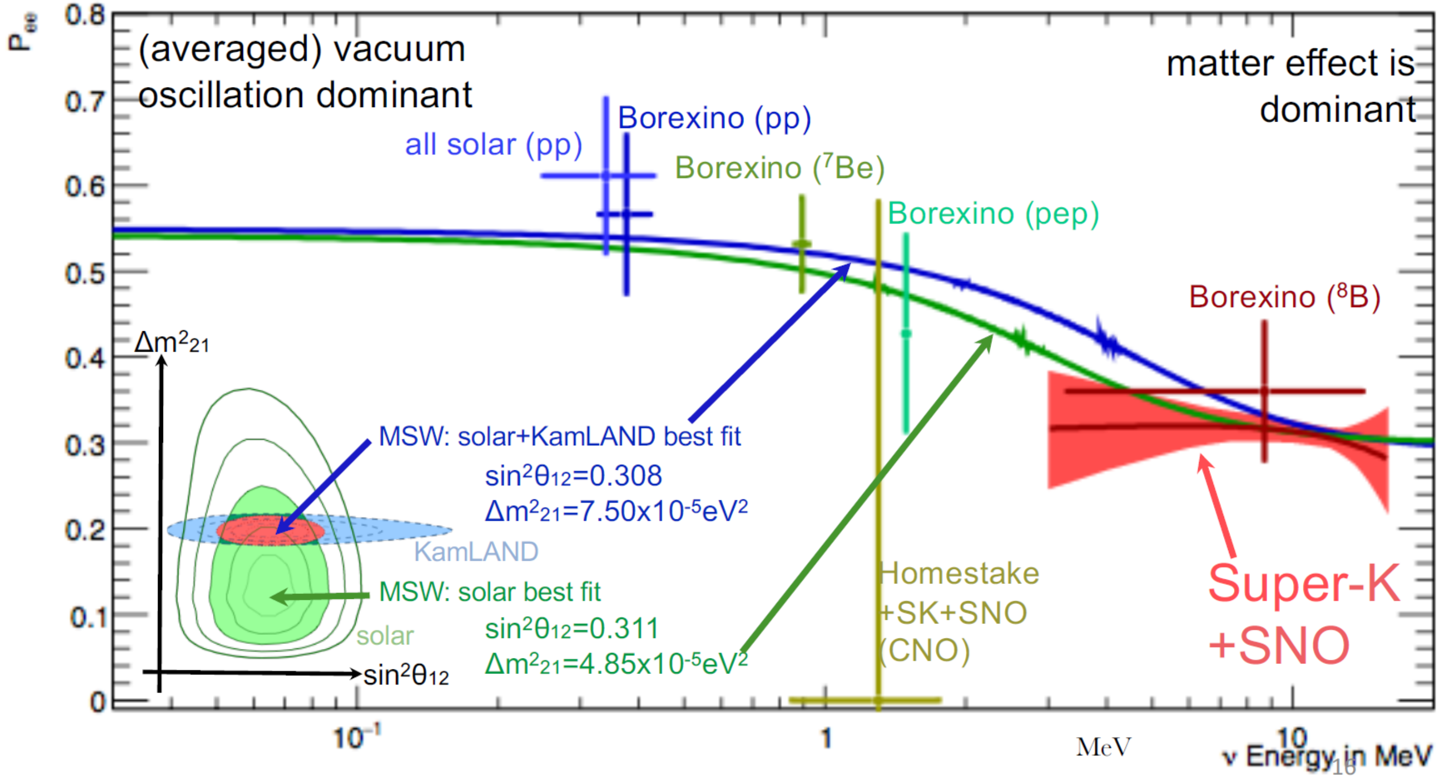
Probability for ν_e production in matter and detection in vacuum

$$P(\nu_e \to \nu_e; t) = \cos^2\theta_m \cos^2\theta + \sin^2\theta_m \sin^2\theta + \frac{1}{2}\sin 2\theta_m \sin 2\theta \cos\left(\frac{\delta(t)}{2E}\right),$$

- **For solar neutrinos produced in matter**

Averaging out oscillation phase in the probability

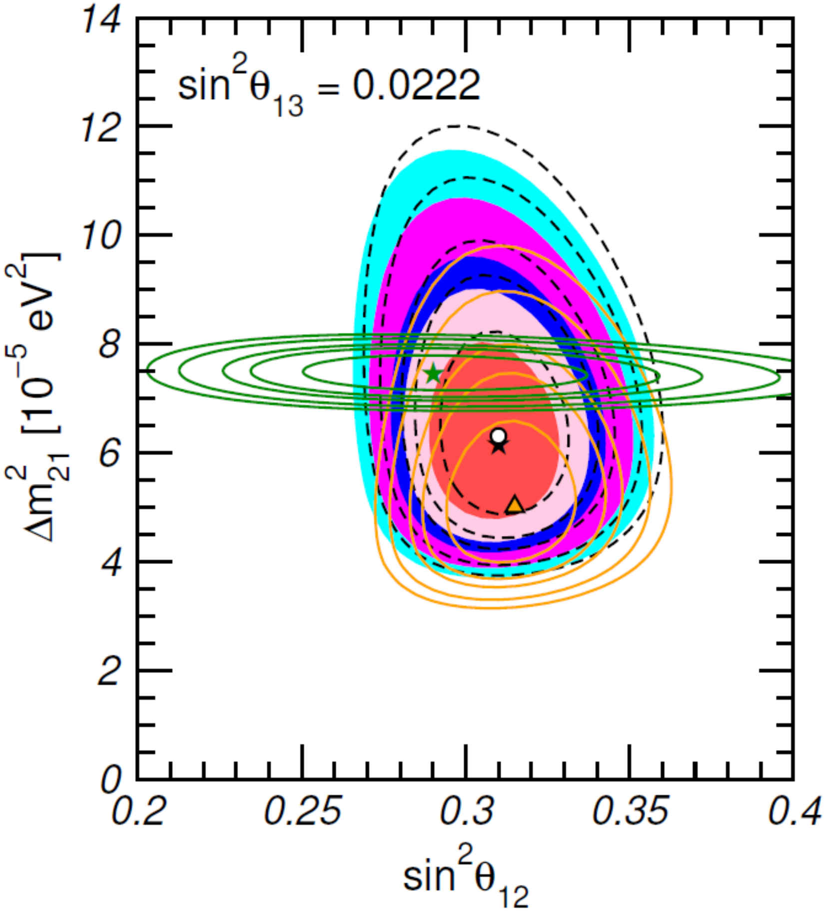
$$P(\nu_e \rightarrow \nu_e; t) = \frac{1}{2}[1 + \cos 2\theta_m \cos 2\theta].$$



M. Ikeda, Neutrino 2018

BOREXINO (*Barbara Caccianiga* 2019)

θ_{12} & Δm_{21}^2

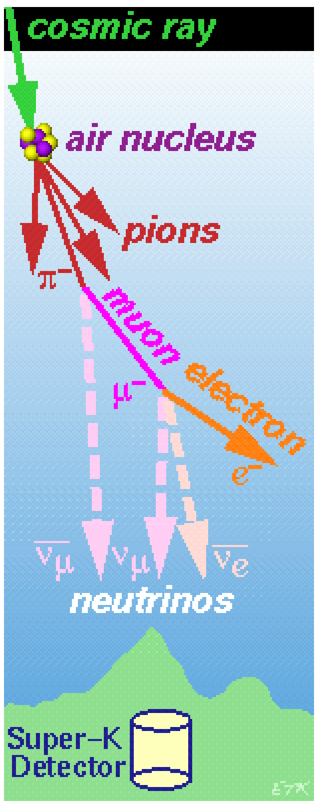


NuFIT 5.0 (2020)

- Both solar neutrino data & KamLAND data are compatible at 1.1σ

θ_{23} & Δm^2_{31}

- Measured by atmospheric ν experiments (SK), ν telescope (IceCube, ANTARES), accelerator experiments (MINOS, T2K,NovA)



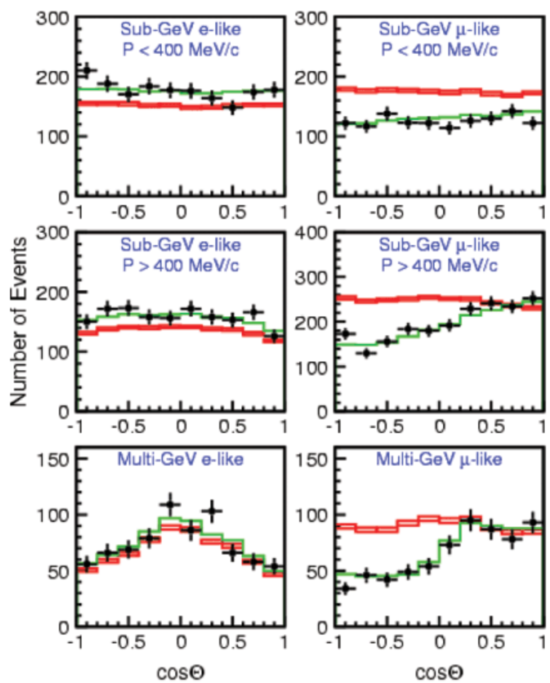
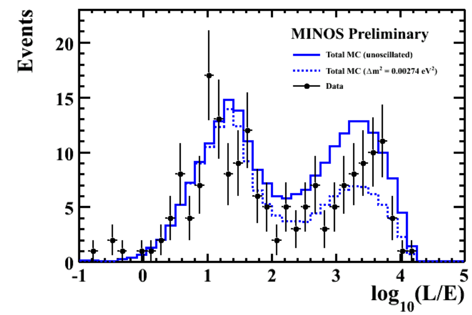
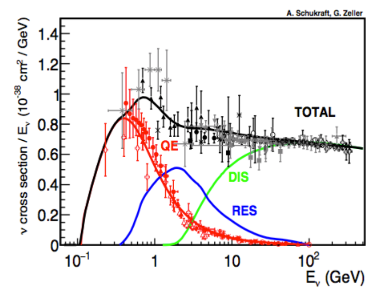
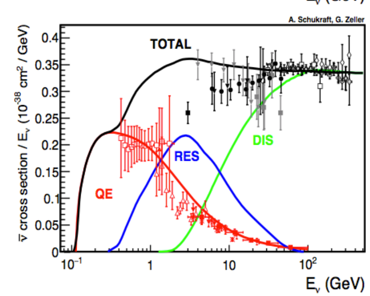


FIG. 1: Atmospheric neutrino data from Super-Kamiokande experiment, the left panel is the electron type events and the the right panel is the muon type events. The energy range increase from top to down. From Ref. [8].



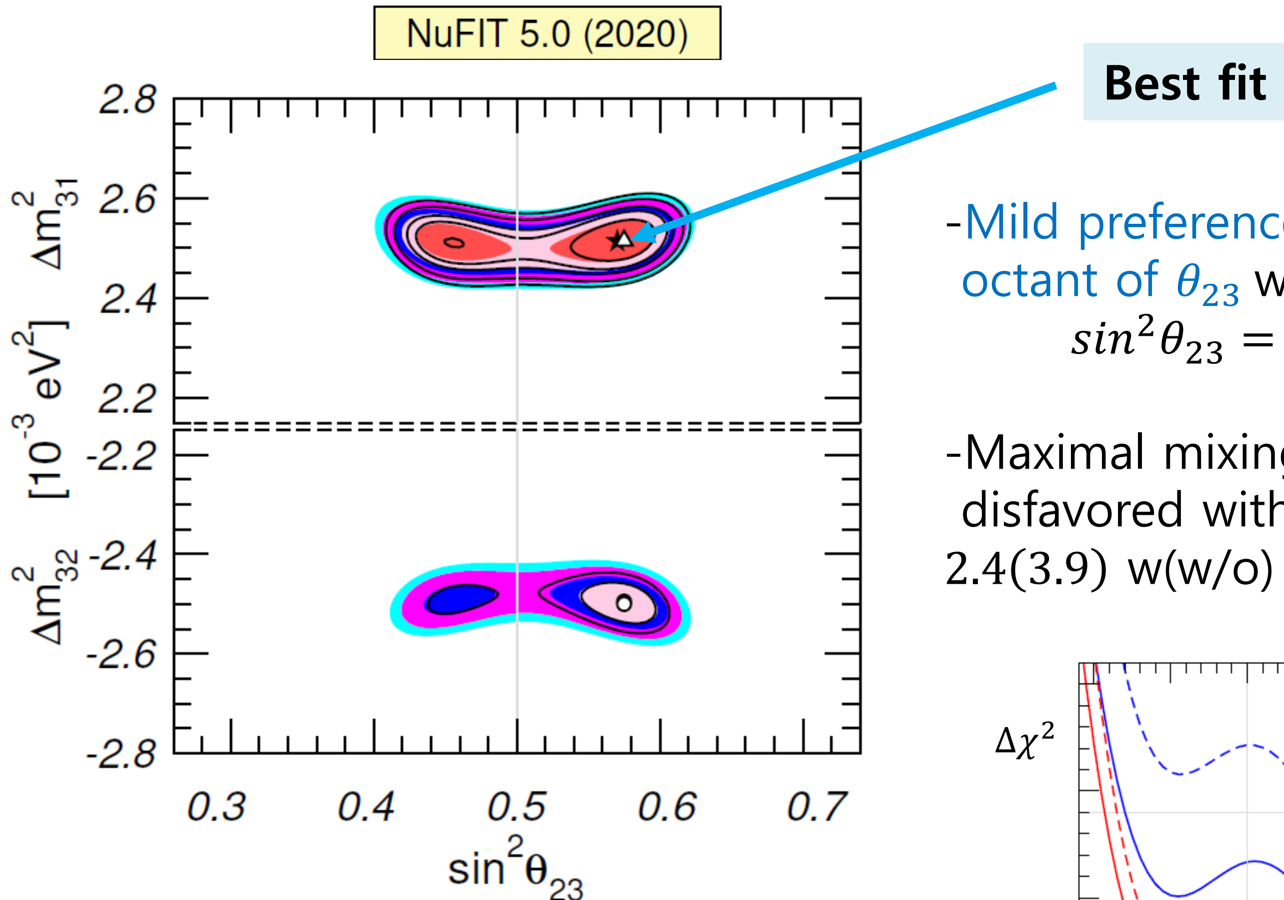




θ_{23} & $\Delta m^2_{31(2)}$

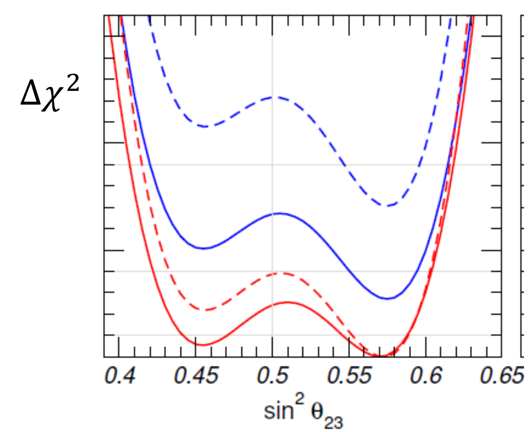
NO

IO



-Mild preference for 2nd octant of θ_{23} with bf at $sin^2\theta_{23} = 0.57$

-Maximal mixing is disfavored with $\Delta\chi^2 =$ 2.4(3.9) w(w/o) SK-atm



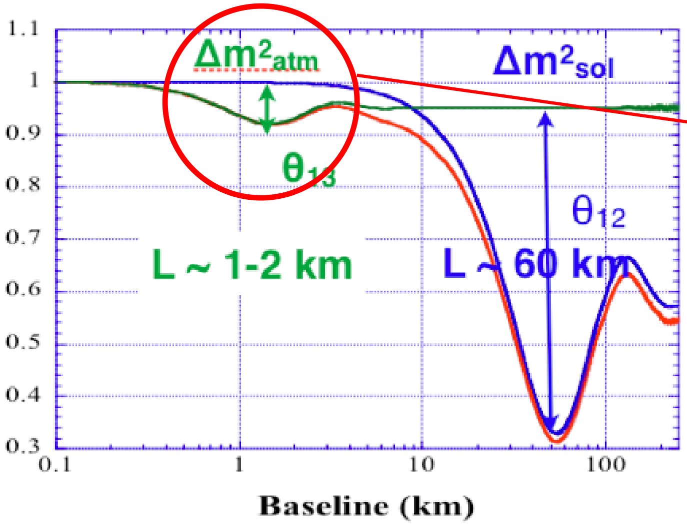
θ_{13}

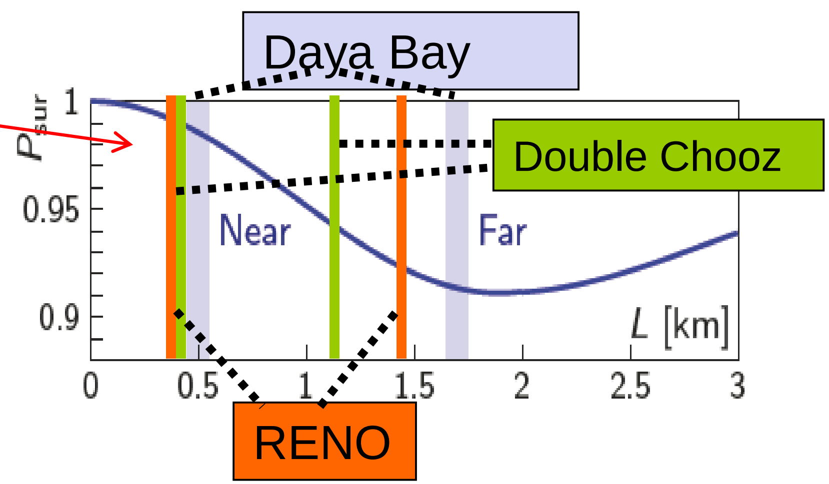
- Measuring θ_{13} : important role in determining CP violation & mass hierarchy

$$J_{CP} = \mathrm{Im}(U_{\mu 3}U_{e3}^{*}U_{e2}U_{\mu 2}^{*}) = \frac{1}{8}\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13}\sin\delta$$

- Using reactor antineutrino oscillation

$$P(\nu_e \to \nu_e) \approx 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\left(\frac{1.27\Delta m_{12}^2 L}{E_\nu}\right) - \sin^2 2\theta_{13}\sin^2\left(\frac{1.27\Delta m_{13}^2 L}{E_\nu}\right)$$

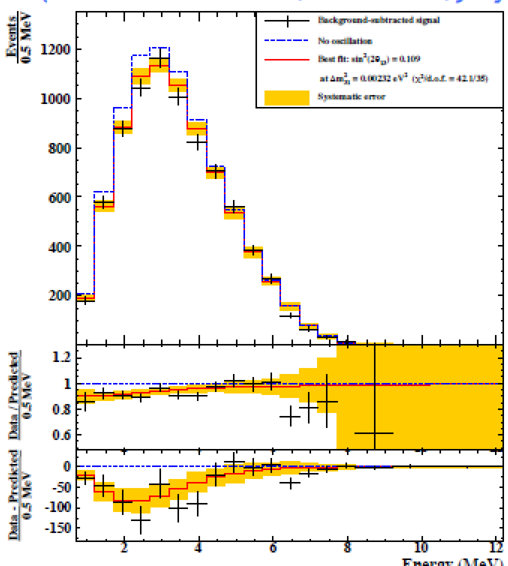


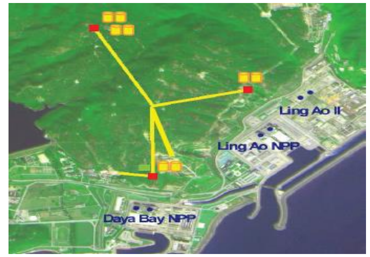


θ_{13}

Double Chooz

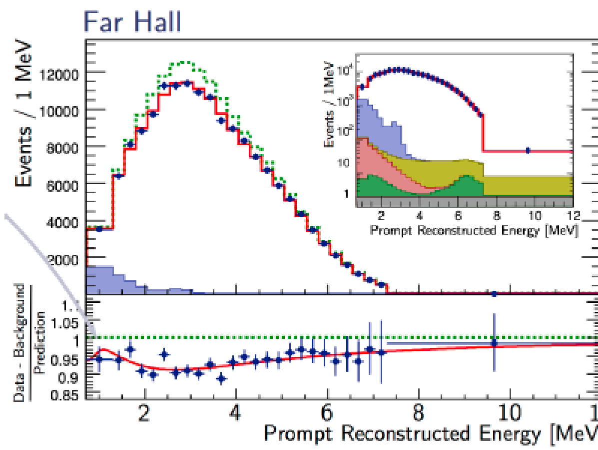
(Gd+H combined, EPS 2013, July)





Daya Bay

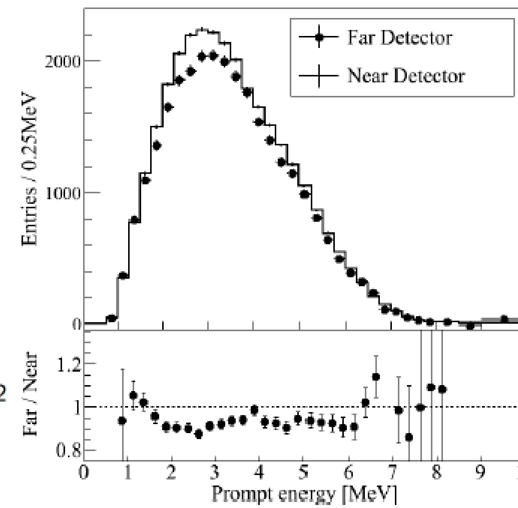
(NuFact 2013, August)



New results from Daya Bay
Rate+Shape fit just delivered

RENO

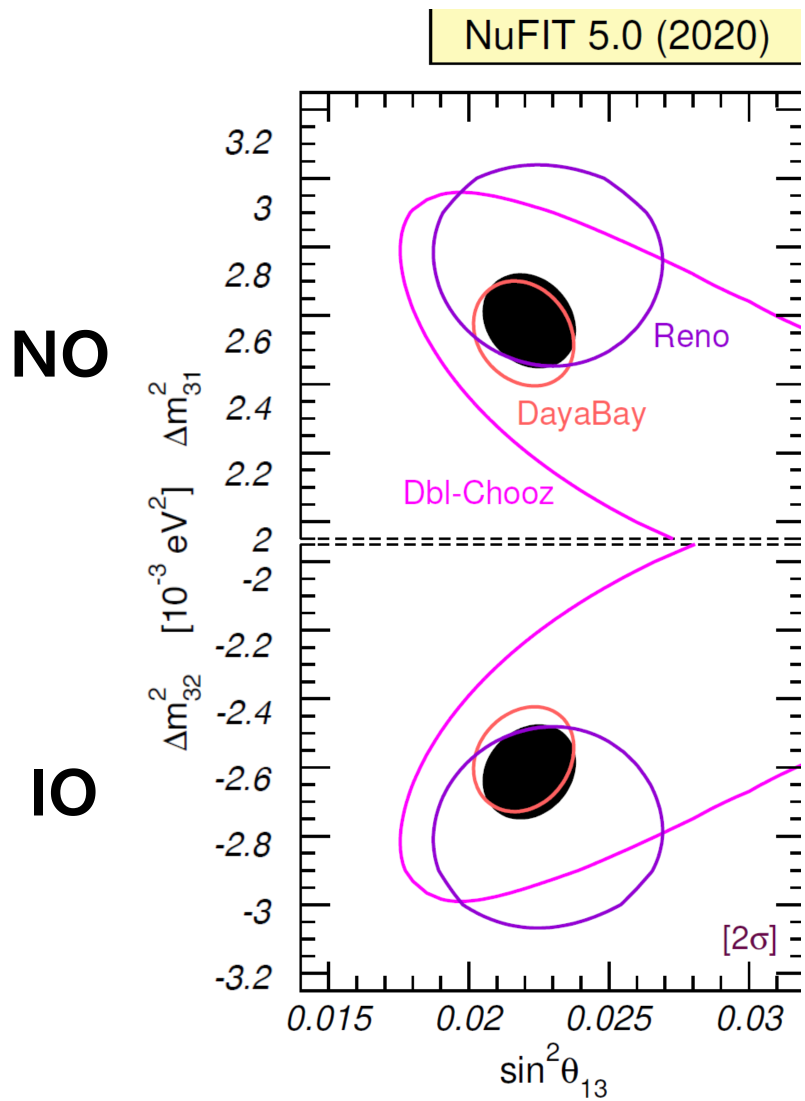
(NuTel 2013, March)



$\sin^2 2\theta_{13}$

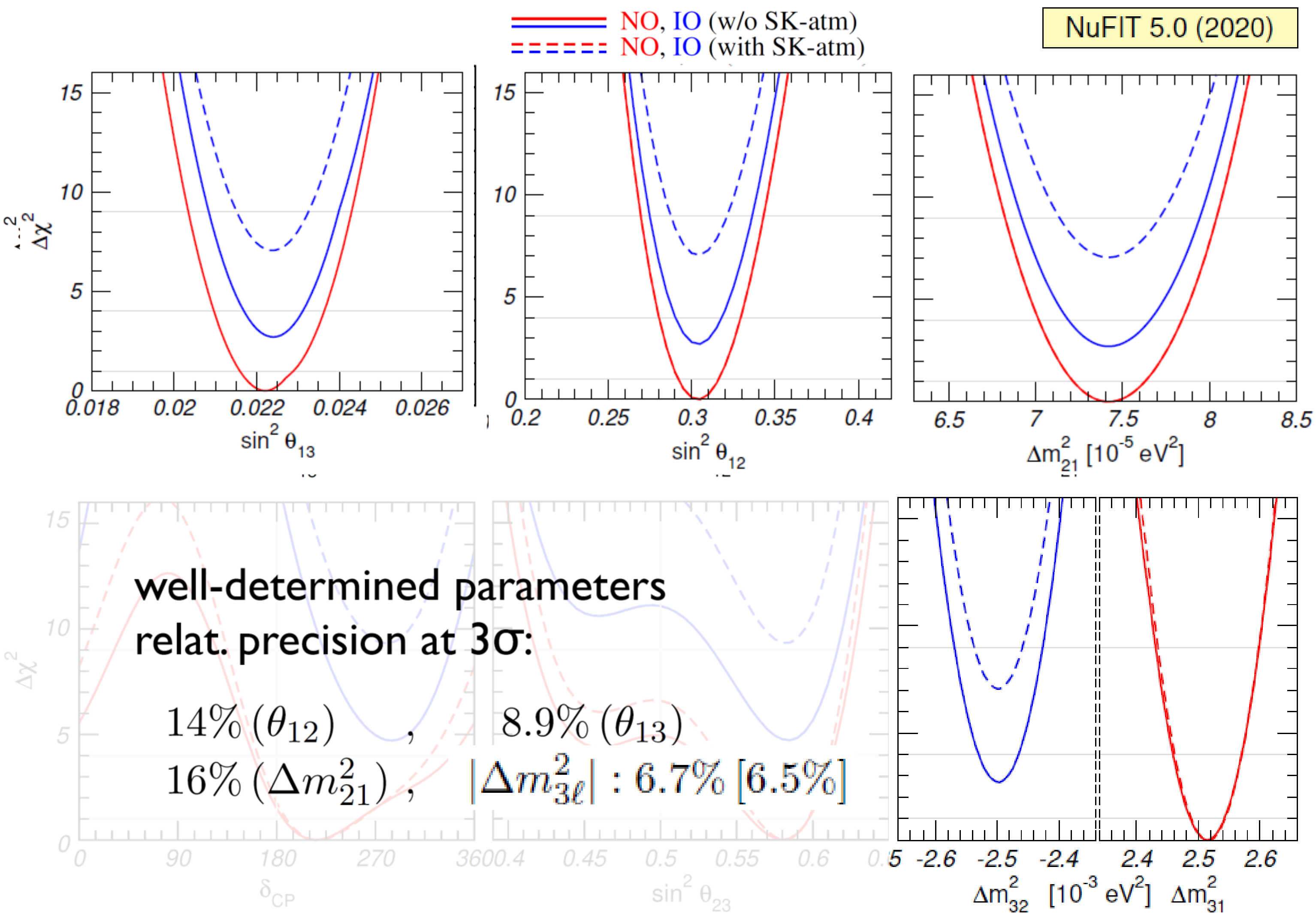
Double Chooz	Daya Bay	RENO
0.109±0.035	0.090+0.008-0.009	0.100±0.018

θ_{13}



NuFIT 5.0 (2020)
NO
IO
Reno
DayaBay
Dbl-Chooz
[2σ]
3.2
3
2.8
2.6
2.4
2.2
2
-2
-2.2
-2.4
-2.6
-2.8
-3
-3.2
0.015
0.02
0.025
0.03
sin²θ13
Δm²32 [10⁻³ eV²] Δm²31

How precisely determined?

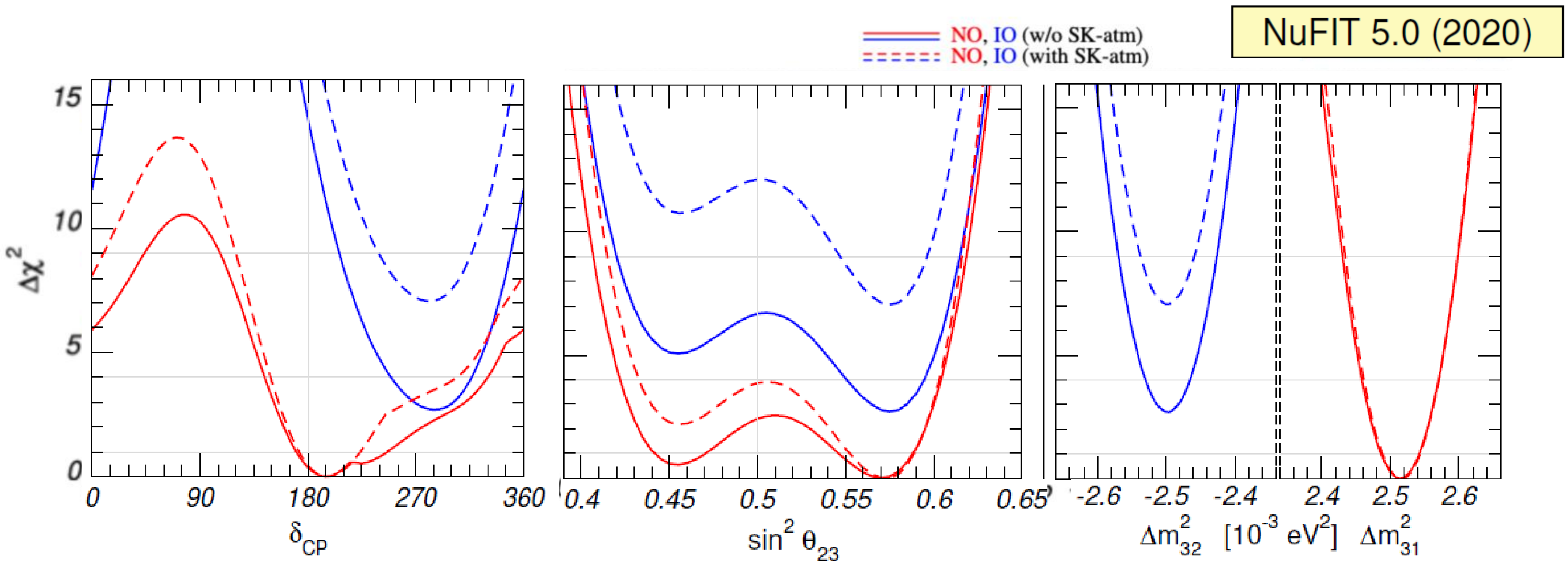


well-determined parameters
relat. precision at 3σ:

$14\% \, (\theta_{12})$, $8.9\% \, (\theta_{13})$

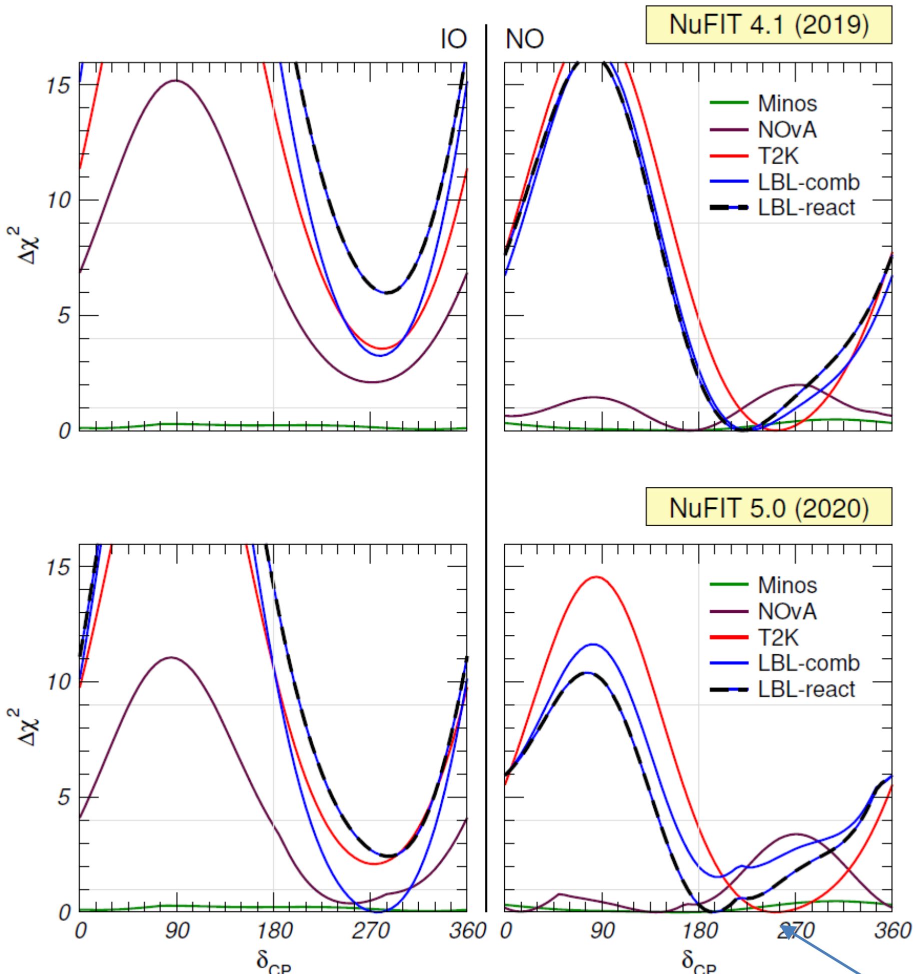
$16\% \, (\Delta m^2_{21})$, $|\Delta m^2_{3\ell}| : 6.7\% \, [6.5\%]$

Not-so-well measured



- CP conservation allowed at $\Delta\chi^2 = 0.38(0.6\ \sigma)$ but bf at $\delta = 195°$
- Octant of θ_{23} : 2nd octant preferred, bf at $sin^2\theta_{23} = 0.57$
- Mass ordering : NO is preferred over IO.
  - adding SK I-IV to the global fit → IO disfavored at $\Delta\chi^2 = 7.3\ (2.7\sigma)$
  - without SK I-IV to the global fit → IO disfavored at $\Delta\chi^2 = 2.7\ (1.6\sigma)$

Leptonic CP violation



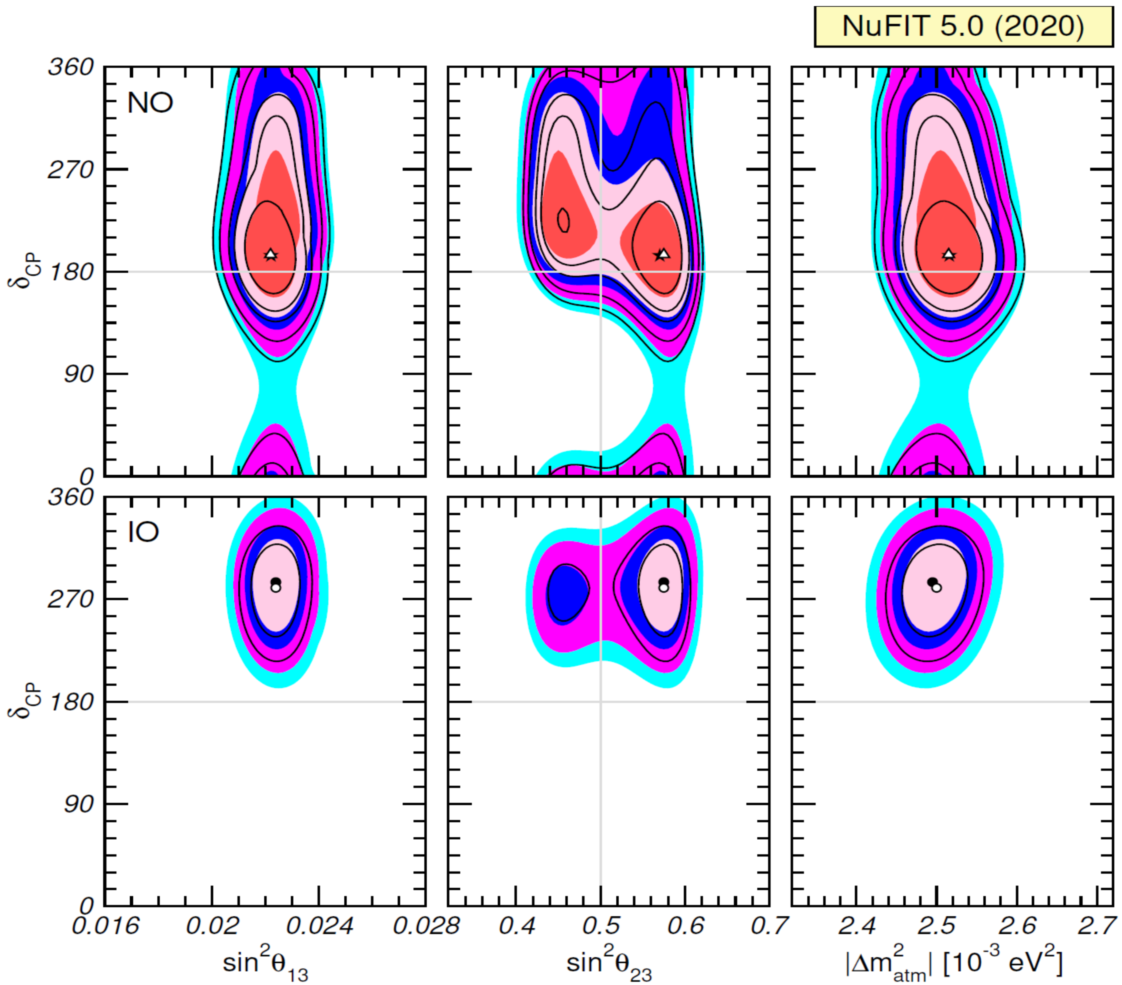
- Both NovA & T2K are fitted best at δ~3π/2 for both NO & IO but, NO is preferred.

-For IO, Both NovA & T2K are fitted best at δ~3π/2

-For NO, T2K is fitted best at δ~3π/2 while NovA is fitted best at δ~0.8π (tension)

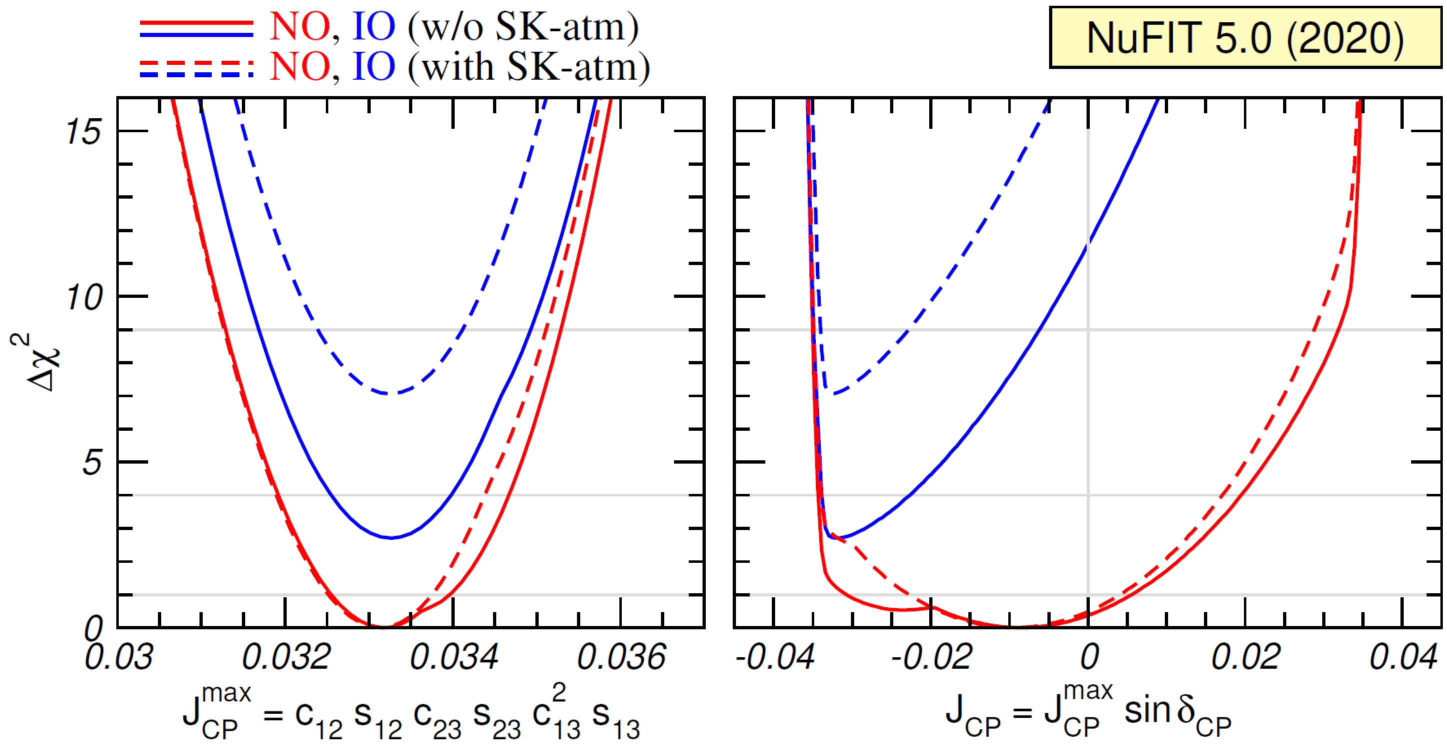
Best fit from global analysis

Leptonic CP violation



- Best fit : $\delta_{CP} = 195°$
- Comparing to previous results the allowed range is pushed towards CPC~π.
- If we restrict to IO, bf of δ_{CP} is close to maximal CPV with CPC being disfavored at 3σ

Leptonic CP violation



$$J_{\rm CP} \equiv {\rm Im}\left[U_{\alpha i} U^*_{\alpha j} U^*_{\beta i} U_{\beta j}\right]$$
$$\equiv J_{\rm CP}^{\rm max} \sin\delta_{\rm CP} = \cos\theta_{12}\sin\theta_{12}\cos\theta_{23}\sin\theta_{23}\cos^2\theta_{13}\sin\theta_{13}\sin\delta_{\rm CP}$$

$$J_{\rm CP}^{\rm max} = 0.0332 \pm 0.0008\,(\pm 0.0019)$$

the best fit value $J_{\rm CP}^{\rm best} = -0.0089$ is only favored over CP conservation $J_{\rm CP} = 0$

Global fit results

without SK atmospheric data

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.7$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.44^{+0.78}_{-0.75}$	$31.27 \to 35.86$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.570^{+0.018}_{-0.024}$	$0.407 \to 0.618$	$0.575^{+0.017}_{-0.021}$	$0.411 \to 0.621$
$\theta_{23}/^\circ$	$49.0^{+1.1}_{-1.4}$	$39.6 \to 51.8$	$49.3^{+1.0}_{-1.2}$	$39.9 \to 52.0$
$\sin^2\theta_{13}$	$0.02221^{+0.00068}_{-0.00062}$	$0.02034 \to 0.02430$	$0.02240^{+0.00062}_{-0.00062}$	$0.02053 \to 0.02436$
$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	$8.20 \to 8.97$	$8.61^{+0.12}_{-0.12}$	$8.24 \to 8.98$
$\delta_{CP}/^\circ$	195^{+51}_{-25}	$107 \to 403$	286^{+27}_{-32}	$192 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.514^{+0.028}_{-0.027}$	$+2.431 \to +2.598$	$-2.497^{+0.028}_{-0.028}$	$-2.583 \to -2.412$

with SK atmospheric data

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 7.1$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.44^{+0.77}_{-0.74}$	$31.27 \to 35.86$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.573^{+0.016}_{-0.020}$	$0.415 \to 0.616$	$0.575^{+0.016}_{-0.019}$	$0.419 \to 0.617$
$\theta_{23}/^\circ$	$49.2^{+0.9}_{-1.2}$	$40.1 \to 51.7$	$49.3^{+0.9}_{-1.1}$	$40.3 \to 51.8$
$\sin^2\theta_{13}$	$0.02219^{+0.00062}_{-0.00063}$	$0.02032 \to 0.02410$	$0.02238^{+0.00063}_{-0.00062}$	$0.02052 \to 0.02428$
$\theta_{13}/^\circ$	$8.57^{+0.12}_{-0.12}$	$8.20 \to 8.93$	$8.60^{+0.12}_{-0.12}$	$8.24 \to 8.96$
$\delta_{CP}/^\circ$	197^{+27}_{-24}	$120 \to 369$	282^{+26}_{-30}	$193 \to 352$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.517^{+0.026}_{-0.028}$	$+2.435 \to +2.598$	$-2.498^{+0.028}_{-0.028}$	$-2.581 \to -2.414$

Implications of global fit:

✓ $\theta_{12} \simeq \pi/4 - \theta_C$ satisfied within 2 σ.

→ quark-lepton complementarity (Raidal, Smirnov, Minakata, SK & CSKim,….'04)

✓ Non-maximal and 2^{nd} octant of θ_{23} is favored at 2 (1.5) σ level for NO (IO) → could be related to $\sqrt{m_2/m_3}$ similar to Gatto-Sartoti-Tonin relation predicting Cabibbo angle. (Roy & Singh PRD91(2015))

✓ Zero θ_{13} is excluded at 10 σ.

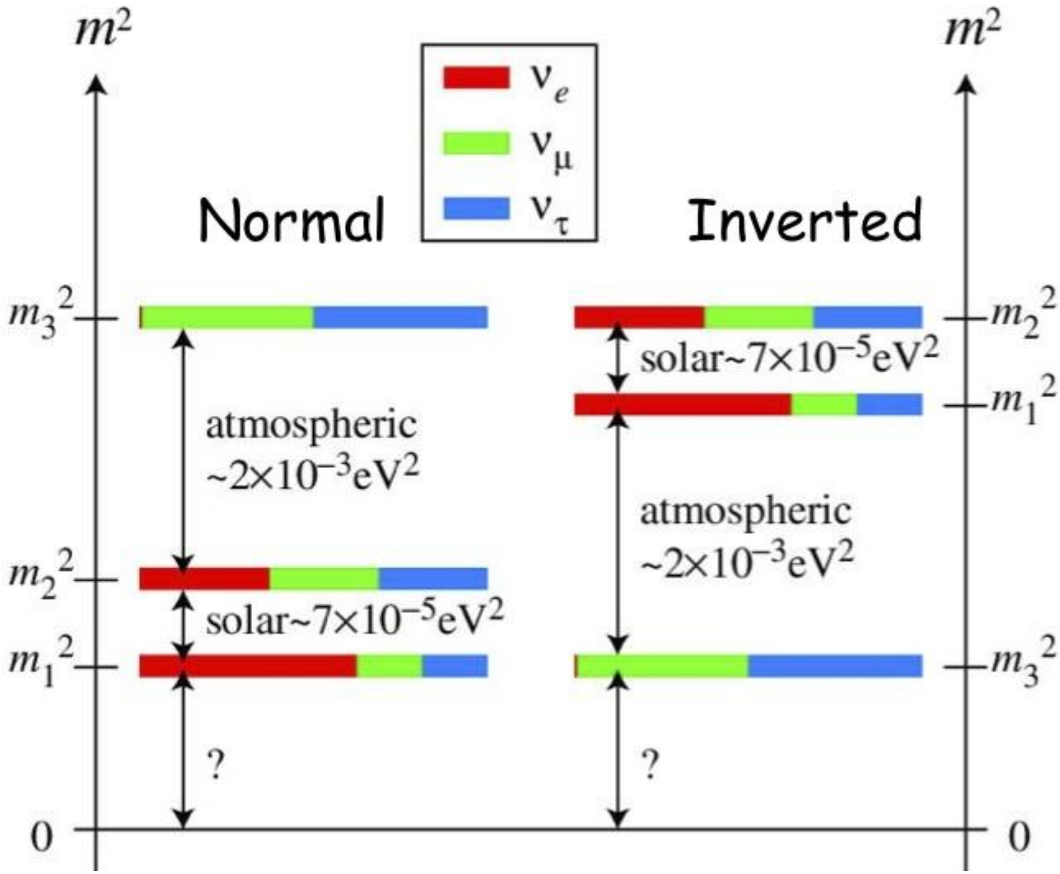
✓ Two large angles → hint for discrete flavor symmetry?

✓ $\delta \simeq 3\pi/2$ is favored by IO case.

→ could be related with mixing angles, flavor symmetries etc. ?

Mass scale of neutrinos

- Atmospheric neutrino; $\Delta m^2_{atm} \approx |\Delta m^2_{31}| \approx |\Delta m^2_{32}| \sim 10^{-3} eV^2$
- Solar neutrino; $\Delta m^2_{sol} \approx \Delta m^2_{21} \sim 10^{-5} eV^2$
- Sum of 3 Δm^2 should be 0; $\Delta m^2_{21} + \Delta m^2_{13} + \Delta m^2_{32} = 0$

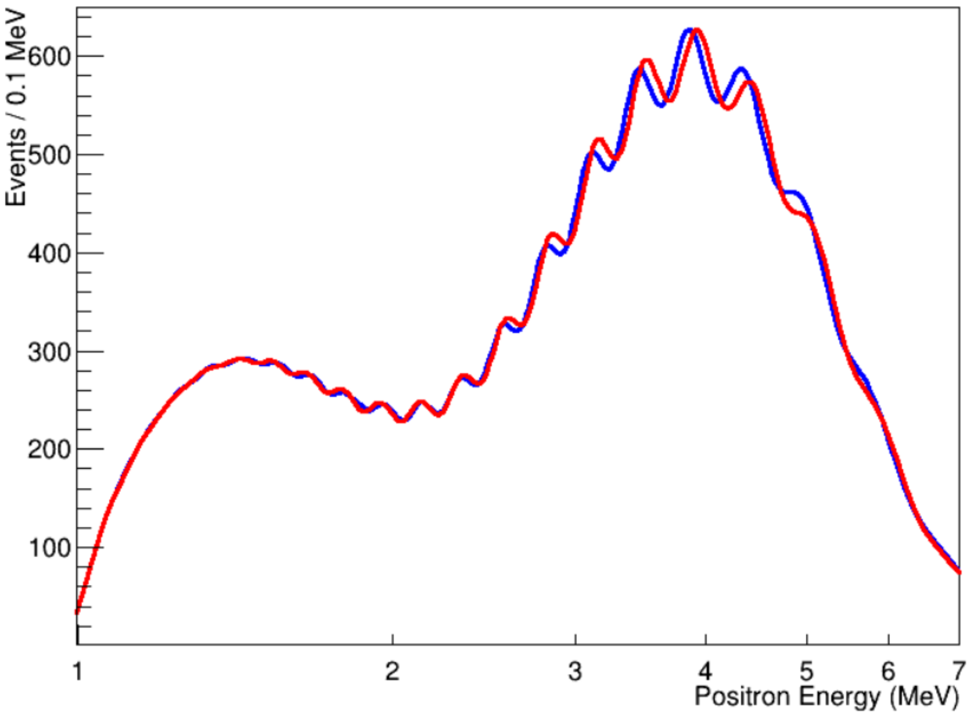


Determination of Mass Hierarchy

With the approximation $\Delta m^2_{32} \approx \Delta m^2_{31}$

$$P_{ee} = 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2\theta_{13}\sin^2|\Delta_{31}|$$
$$- \sin^2\theta_{12}\sin^2 2\theta_{13}\sin^2\Delta_{21}\cos 2|\Delta_{31}|$$
$$\pm \frac{\sin^2\theta_{12}}{2}\sin^2 2\theta_{13}\sin^2 2\Delta_{21}\sin\ 2|\Delta_{31}|$$

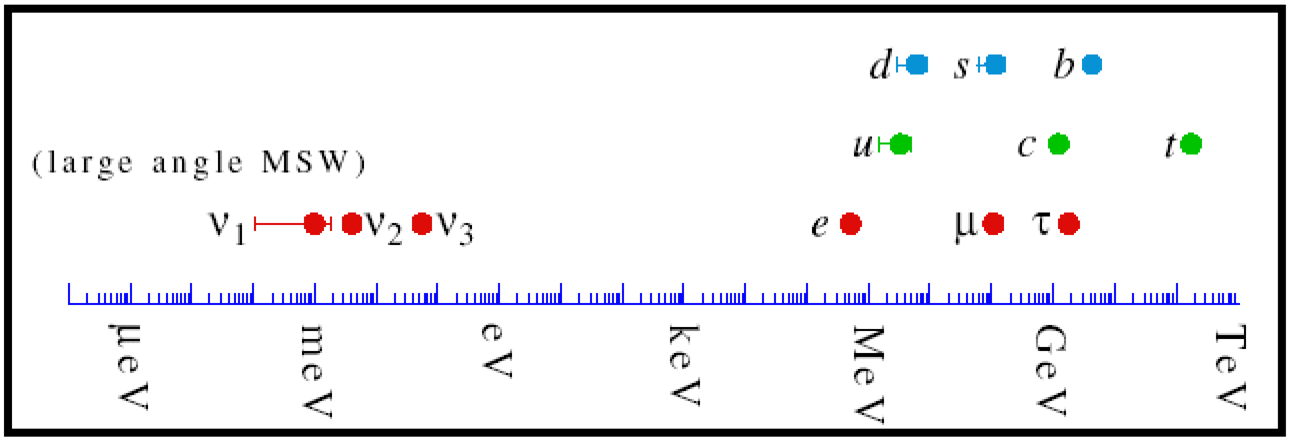
Mass hierarchy difference



- $\bar{\nu}_e$ Energy spectrum at a baseline of 50km
- Assuming $3\%/\sqrt{E}$ detector E resolution

(red: NH; blue : IH)

How small neutrino masses?



- **Cosmological Mass Limit :**

Planck CMB

$$\Omega_\nu h^2 = \sum \frac{m_\nu}{92.5eV} \leq 0.0076$$

$\Rightarrow$ $m_\nu \leq 0.086$ (eV)

Theoretical Issues

- Origins of neutrino mixing & CP violation
  - flavor symmetry
  - predictions
- Origins of tiny neutrino mass
  - seesaw variants
  - radiative generation
- New physics in neutrino oscillation

I. Origin of mixing pattern

- From fit to neutrino data in 3-neutrino paradigm

$$|U|_{3\sigma}^{\text{w/o SK-atm}} = \begin{pmatrix} 0.801 \to 0.845 & 0.513 \to 0.579 & 0.143 \to 0.156 \\ 0.233 \to 0.507 & 0.461 \to 0.694 & 0.631 \to 0.778 \\ 0.261 \to 0.526 & 0.471 \to 0.701 & 0.611 \to 0.761 \end{pmatrix}$$

$$|U|_{3\sigma}^{\text{with SK-atm}} = \begin{pmatrix} 0.801 \to 0.845 & 0.513 \to 0.579 & 0.143 \to 0.155 \\ 0.234 \to 0.500 & 0.471 \to 0.689 & 0.637 \to 0.776 \\ 0.271 \to 0.525 & 0.477 \to 0.694 & 0.613 \to 0.756 \end{pmatrix}$$

Look different from quark mixing matrix !!

$$|V_{CKM}| = \begin{pmatrix} 0.97434 & 0.22506 & 0.00357 \\ 0.22492 & 0.97351 & 0.0414 \\ 0.00875 & 00403 & 0.99915 \end{pmatrix}$$

PDG(2018)

How do we understand ν mixing matrix ?

Before measuring θ_{13} , tri-bimaximal mixing hypothesis :

- $U^{TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

Harrison & Perkins & Scott (2002)

$$\theta_{13} \approx 0; \quad \theta_{23} \approx 45^0; \quad \theta_{12} = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right) \approx 35.3^0$$

- generates specific neutrino mass matrix

$$U M_\nu^D U^T = \begin{pmatrix} m_1 & m_2 & m_2 \\ . & \frac{1}{2}(m_1+m_2+m_3) & \frac{1}{2}(m_1+m_2-m_3) \\ . & . & \frac{1}{2}(m_1+m_2+m_3) \end{pmatrix}$$

$$= \frac{m_1+m_3}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{m_2-m_1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_1-m_3}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

– Integer matrix elements suggest non–Abelian discrete symmetry

(E. Ma, G. Rajasekaran, PRD64(2001))

- TBMixing understanding from discrete symmetries

-setting $U_{PMNS} = (\vec{u}_1, \vec{u}_2, \vec{u}_3)$, we construct group generators:

$$S_1 = \vec{u}_1\overrightarrow{u_1}^{+} - \vec{u}_2\overrightarrow{u_2}^{+} - \vec{u}_3\overrightarrow{u_3}^{+}$$

$$S_2 = -\vec{u}_1\overrightarrow{u_1}^{+} + \vec{u}_2\overrightarrow{u_2}^{+} - \vec{u}_3\overrightarrow{u_3}^{+}$$

(CSLam'06)

$$S_3 = -\vec{u}_1\overrightarrow{u_1}^{+} - \vec{u}_2\overrightarrow{u_2}^{+} + \vec{u}_3\overrightarrow{u_3}^{+}$$

- $S_1 = \frac{1}{3}\begin{pmatrix} 1 & -2 & -2 \\ -2 & -2 & -1 \\ 2 & -1 & -2 \end{pmatrix}$ $S_2 = \frac{1}{3}\begin{pmatrix} -1 & 2 & -2 \\ 2 & -1 & -2 \\ -2 & -2 & -1 \end{pmatrix}$ $S_3 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

  then, $S^T \bar{M}_\nu S = \bar{M}_\nu$

C.S.Lam, PRD98(2008)
arXiv:0809.1185

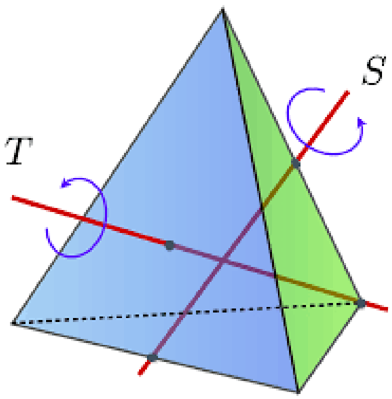
- For charged lepton, $T^+ \bar{M}_e T = \bar{M}_e$ $with$ $\bar{M}_e = {M_e}^+ M_e$ & $T^n = 1$

⇒ $\{S_i, T\}$ forms a discrete group (flavor symmetry)

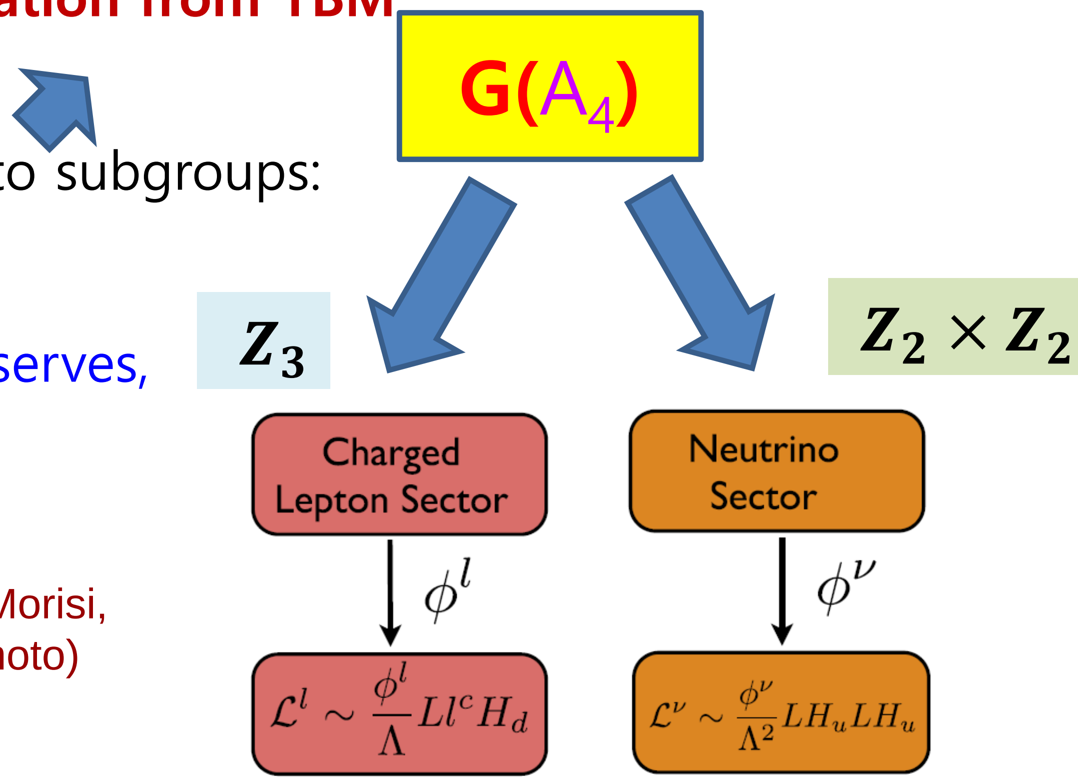
(Mixing matrices diagonalize M_ν and $\bar{M}_e$ also diagonalize S and T)

- Simplest group with a triplet representation: A_4

  A_4 has subgroups: three Z_2, four Z_3, one $Z_2 \times Z_2$



Deviation from TBM



- A_4 is spontaneously broken to subgroups:

  i) Neutrino sector preserves,

  $Z_2 \times Z_2$:

  ii) Charged lepton sector preserves,

  Z_3:

(arXiv: 1402.4271 King, Merle, Morisi, Simizu, Tanimoto)

- Many discrete groups reproducing TBM mixing

Group	d	Irr. Repr.'s	Presentation
$D_3 \sim S_3$	6	$1,\ 1',\ 2$	$A^3 = B^2 = (AB)^2 = 1$
D_4	8	$1_1, ... 1_4, 2$	$A^4 = B^2 = (AB)^2 = 1$
D_7	14	$1,\ 1',\ 2,\ 2',\ 2''$	$A^7 = B^2 = (AB)^2 = 1$
A_4	12	$1,\ 1',\ 1'',\ 3$	$A^3 = B^2 = (AB)^3 = 1$
$A_5 \sim PSL_2(5)$	60	$1,\ 3,\ 3',\ 4,\ 5$	$A^3 = B^2 = (BA)^5 = 1$
T'	24	$1,\ 1',\ 1'',\ 2,\ 2',\ 2'',\ 3$	$A^3 = (AB)^3 = R^2 = 1,\ \ B^2 = R$
S_4	24	$1,\ 1',\ 2,\ 3,\ 3'$	$BM : A^4 = B^2 = (AB)^3 = 1$ $TB : A^3 = B^4 = (BA^2)^2 = 1$
$\Delta(27) \sim Z_3 \rtimes Z_3$	27	$1_1, ... 1_9, 3, \bar{3}$	
$PSL_2(7)$	168	$1, 3, \bar{3}, 6, 7, 8$	$A^3 = B^2 = (BA)^7 = (B^{-1}A^{-1}BA)^4 = 1$
$T_7 \sim Z_7 \rtimes Z_3$	21	$1, 1', \bar{1'}, 3, \bar{3}$	$A^7 = B^3 = 1,\ \ AB = BA^4$

(Altarelli, Feruglio, 1002.0211)

Type	L_i	ℓ_i^c	ν_i^c	Δ
A1	$\underline{3}$	$\underline{1},\ \underline{1}',\ \underline{1}''$	$\cdots$	$\cdots$
A2				$\underline{1},\ \underline{1}',\ \underline{1}'',\ \underline{3}$
B1	$\underline{3}$	$\underline{1},\ \underline{1}',\ \underline{1}''$	$\underline{3}$	$\cdots$
B2				$\underline{1},\ \underline{3}$
C1	$\underline{3}$	$\underline{3}$	$\cdots$	$\cdots$
C2				$\underline{1}$
C3				$\underline{1},\ \underline{3}$
C4				$\underline{1},\ \underline{1}',\ \underline{1}'',\ \underline{3}$
D1	$\underline{3}$	$\underline{3}$	$\underline{3}$	$\cdots$
D2				$\underline{1}$
D3				$\underline{1}'$
D4				$\underline{1}',\ \underline{3}$
E	$\underline{3}$	$\underline{3}$	$\underline{1},\ \underline{1}',\ \underline{1}''$	$\cdots$
F	$\underline{1},\ \underline{1}',\ \underline{1}''$	$\underline{3}$	$\underline{3}$	$\underline{1}$ or $\underline{1}'$
G	$\underline{3}$	$\underline{1},\ \underline{1}',\ \underline{1}''$	$\underline{1},\ \underline{1}',\ \underline{1}''$	$\cdots$
H	$\underline{3}$	$\underline{1},\ \underline{1},\ \underline{1}$	$\cdots$	$\cdots$
I	$\underline{3}$	$\underline{1},\ \underline{1},\ \underline{1}$	$\underline{1},\ \underline{1},\ \underline{1}$	$\cdots$
J	$\underline{3}$	$\underline{1},\ \underline{1},\ \underline{1}$	$\underline{3}$	$\cdots$

Each group has many models!

(Barry, Rodejohann, PRD81(2010))

- Modification of Tri-Bimaximal Mixing

  - Simple possible forms :

$$\begin{cases} U_{TBM}\, U_{ij}(\theta) \\ U_{ij}^{+}(\theta)\, U_{TBM} \end{cases}$$

  - θ possibly gives rise to non-zero θ_{13} and deviation from maximal θ_{23}

(He & Zee, PLB645(2007), SK & CSKim PRD90(2014)
See also, Goswami, Petcov, Ray, Rodejohann,PRD80(2009))

  - Best fit achieved by (SK & CSKim, PRD90(2014))

$$U_{TBM}\cdot U_{23} \sim \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}}\lambda \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}}+\frac{1}{\sqrt{2}}\lambda & \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}\lambda \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}}-\frac{1}{\sqrt{2}}\lambda & \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}\lambda \end{pmatrix} \qquad (c_{23}\sim 1,\ \ s_{23}\sim\lambda)$$

- $$U^{TBM} \cdot U_{12} = \begin{pmatrix} \blacksquare & \blacksquare & 0 \\ \blacksquare & \blacksquare & -\frac{1}{\sqrt{2}} \\ \blacksquare & \blacksquare & \frac{1}{\sqrt{2}} \end{pmatrix}$$ unchanged $$U^{TBM} \cdot U_{13} = \begin{pmatrix} \blacksquare & -\frac{1}{\sqrt{3}} & \blacksquare \\ \blacksquare & \frac{1}{\sqrt{3}} & \blacksquare \\ \blacksquare & \frac{1}{\sqrt{3}} & \blacksquare \end{pmatrix}$$
- $$U^{TBM} \cdot U_{23} = \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \blacksquare & \blacksquare \\ \frac{1}{\sqrt{6}} & \blacksquare & \blacksquare \\ \frac{1}{\sqrt{6}} & \blacksquare & \blacksquare \end{pmatrix}$$

Unchanged columns may reflect the remnants of flavor symmetry → residual symmetry

II. Prediction of CP phase

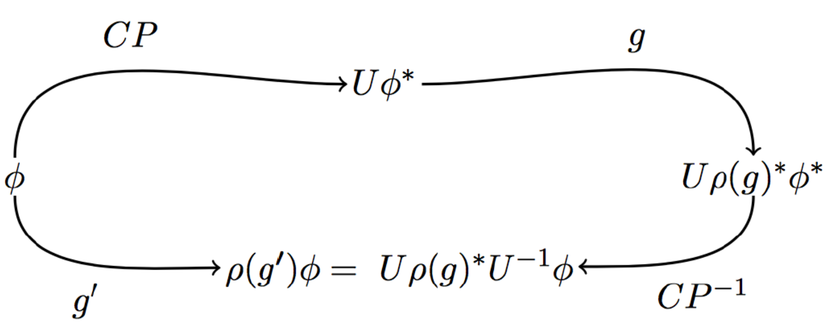
- $\mu - \tau$ reflection symmetry : $\nu_e \leftrightarrow \nu_e^C, \nu_\mu \leftrightarrow \nu_\tau^C$

$$m_\nu = \begin{pmatrix} x & z_1 & z_1^* \\ \cdot & z_2 & y \\ \cdot & \cdot & z_2^* \end{pmatrix} \Rightarrow \delta = \pm\frac{\pi}{2} \;\&\; \theta_{23} = \frac{\pi}{4}$$

(Grimus, Lavoura, PRB578(2004))

- General CP transformation :

  - combining CP with flavor symmetry $\Rightarrow \delta = \pm\frac{\pi}{2}, \pm\pi, 0$



(Grimus; Chen; Feruglio, Hagedorn, Ziegler; Holthausen, Schmidt, Lindner; Ding, King, Stuart; Meroni, Petcov; Branco, King, Varzielas,...)

II. Prediction of CP phase

- Example $G_f = S_4 \times Z_3 \times CP$ broken to $G_\nu = Z_2 \times CP$, $G_l = Z_3$ (Hagedorn, Feruglio, Ziegler, EPJ74)

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \quad T = \frac{1}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \quad X_{3'} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$S m_\nu S = m_\nu$ & $X_{3'} m_\nu X_{3'} = m_\nu^*$

➡ leads to $\delta = \pm\frac{\pi}{2}$

$$\sin^2\theta_{13} = \frac{2}{3}\sin^2\theta$$

$$\sin^2\theta_{12} = \frac{1}{2+\cos 2\theta}$$

$$\sin^2\theta_{23} = \frac{1}{2}$$

$$U_{PMNS} = \frac{1}{\sqrt{6}}\begin{pmatrix} 2\cos\theta & \sqrt{2} & 2\sin\theta \\ -\cos\theta + i\sqrt{3}\sin\theta & \sqrt{2} & -\sin\theta - i\sqrt{3}\cos\theta \\ -\cos\theta - i\sqrt{3}\sin\theta & \sqrt{2} & -\sin\theta + i\sqrt{3}\cos\theta \end{pmatrix} K$$

Modification of Tri-bi-maximal

- Any forms of neutrino mixing matrix should be equivalent to U_{PMNS} presented in the standard parameterization :
- $U_{PMNS} = U_{23}(\theta_{23})U_{13}(\theta_{13},\delta_D)U_{12}(\theta_{12})P_\phi$

$$= \begin{pmatrix} c_{12}c_{13} & -s_{12}c_{13} & s_{13}e^{i\delta_D^*} \\ * & * & -s_{23}c_{13} \\ * & * & c_{23}c_{13} \end{pmatrix} \cdot \begin{pmatrix} e^{i\phi_1} & & \\ & e^{i\phi_2} & \\ & & e^{i\phi_3} \end{pmatrix}$$

$$= P_\alpha \cdot V \cdot P_\beta \qquad , V = U^{TBM} \cdot U_{23(13)}(\theta,\xi)$$

$$\Rightarrow \quad V_{ij}e^{i(\alpha_i+\beta_j)} = (U_{PMNS})_{ij}$$

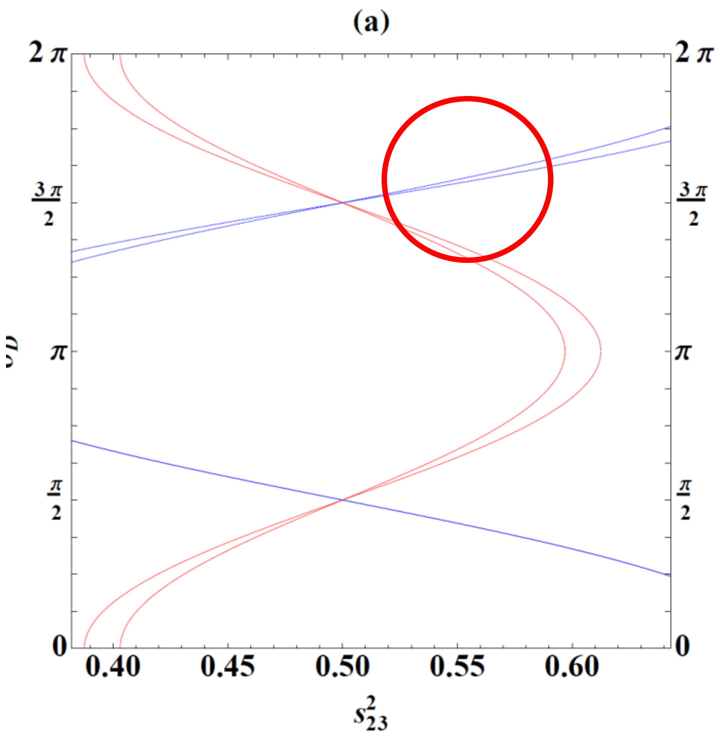
- Predictions : (SK & CSKim, PRD90(2014), SK & Tanimoto, PRD91(2015))

$$s_{12}{}^2 = 1 - \frac{2}{3(1-s_{13}{}^2)}$$

$$s_{12}{}^2 = \frac{1}{3(1-s_{13}{}^2)}$$

$$\cos\delta_D = \frac{1}{2\tan 2\theta_{23}} \cdot \frac{1-5s_{13}{}^2}{s_{13}\sqrt{2-6s_{13}{}^2}}$$

$$\cos\delta_D = \frac{1}{\tan 2\theta_{23}} \cdot \frac{1-2s_{13}{}^2}{s_{13}\sqrt{2-3s_{13}{}^2}}$$



The relations can be obtained from A4 model

(SK & Tanimoto, PRD91 (2015))

Alternatively,

$$\begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{e^{-i\rho}\cos\theta}{\sqrt{3}} & -\frac{ie^{-i\rho}\sin\theta}{\sqrt{3}} \\ -\frac{e^{i\rho}}{\sqrt{6}} & \frac{\cos\theta}{\sqrt{3}} - \frac{ie^{-i\sigma}\sin\theta}{\sqrt{2}} & \frac{e^{-i\sigma}\cos\theta}{\sqrt{2}} - \frac{i\sin\theta}{\sqrt{3}} \\ \frac{e^{i(\rho+\sigma)}}{\sqrt{6}} & -\frac{e^{i\sigma}\cos\theta}{\sqrt{3}} - \frac{i\sin\theta}{\sqrt{2}} & \frac{\cos\theta}{\sqrt{2}} + \frac{ie^{i\sigma}\sin\theta}{\sqrt{3}} \end{bmatrix}$$

$$\sin^2\theta_{12} = \frac{\cos^2\theta}{\cos^2\theta + 2},$$

$$\sin^2\theta_{13} = \frac{\sin^2\theta}{3},$$

$$\sin^2\theta_{23} = \frac{1}{2} + \frac{\sqrt{6}\sin 2\theta \sin\sigma}{2\cos^2\theta + 4}$$

$$\tan\delta_{CP} = \frac{(\cos^2\theta + 2)\cot\sigma}{5\cos^2\theta - 2},$$

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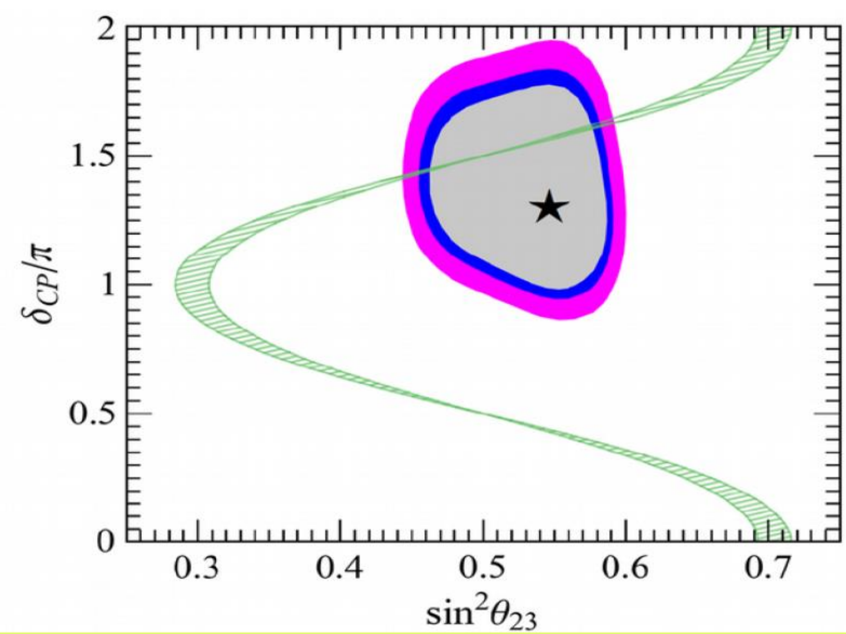
(Cheng, Chulia, Ding, Srivastava, Valle)



How to test Flavor Symmetry

- UV theories giving rise to flavor symmetry in lepton sector contains new scalars→ probe of signal be test of FlaSy.
- Mixing angle sum rules:

Example:

$$\sin^2\theta_{23} = \frac{1}{2}\frac{1}{\cos^2\theta_{13}} \geq \frac{1}{2}\,, \quad \sin^2\theta_{12} \simeq \frac{1}{\sqrt{3}} - \frac{2\sqrt{2}}{3}\sin\theta_{13}\cos\delta_{CP} + \frac{1}{3}\sin^2\theta_{13}\cos 2\delta_{CP}$$

$$\sin^2\theta_{12} = \frac{1}{3}\frac{1}{\cos^2\theta_{13}} \geq \frac{1}{3}\,, \quad \cos\delta_{CP}\tan 2\theta_{23} \simeq \frac{1}{\sqrt{2}\sin\theta_{13}}\left(1 - \frac{5}{4}\sin^2\theta_{13}\right)$$

$$s_{12}{}^2 = 1 - \frac{2}{3(1 - s_{13}{}^2)}$$

- Neutrino mass sum rules in FLaSy ⇔different $0\nu\beta\beta$
- Prediction of CP phase

(Girardi, Petcov, Titov, NPB894(2015))

(e.g.) $\cos\delta_D = \frac{1}{\tan 2\theta_{23}} \cdot \frac{1 - 5s_{13}{}^2}{s_{13}\sqrt{2 - 6s_{13}{}^2}}$

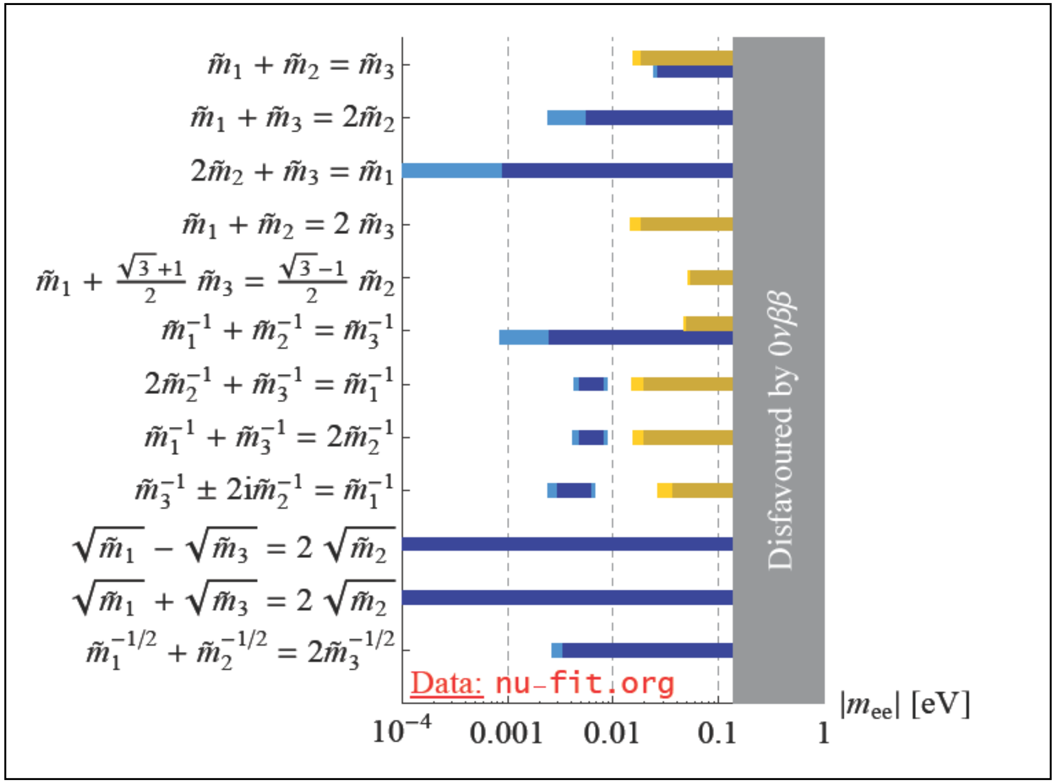
Neutrino mass sum rules

(King, Merle,Morisi, Shimizu, Tanimoto, New J. Phys. 2014)

Sum Rule	Group	Seesaw Type	Matrix
$\tilde{m}_1 + \tilde{m}_2 = \tilde{m}_3$	A_4[167]([175, 178–181]); S_4([182]); A_5[69][a]	Weinberg	m^{ν}_{LL}
$\tilde{m}_1 + \tilde{m}_2 = \tilde{m}_3$	$\Delta(54)$[183]; S_4([163])	Type II	M_L
$\tilde{m}_1 + 2\tilde{m}_2 = \tilde{m}_3$	S_4[120]	Type II	M_L
$2\tilde{m}_2 + \tilde{m}_3 = \tilde{m}_1$	A_4[165, 167] ([36, 37, 188–194, , , , , , , 178–181]) S_4([45, 124])[b]; T'[195, 196] ([46, 134, 197, 198]); T_7([199])	Weinberg	m^{ν}_{LL}
$2\tilde{m}_2 + \tilde{m}_3 = \tilde{m}_1$	A_4([200])	Type II	M_L
$\tilde{m}_1 + \tilde{m}_2 = 2\tilde{m}_3$	S_4[201][c]	Dirac[c]	m^D
$\tilde{m}_1 + \tilde{m}_2 = 2\tilde{m}_3$	$L_e - L_\mu - L_\tau$([202])	Type II	M_L
$\tilde{m}_1 + \frac{\sqrt{3}+1}{2}\tilde{m}_3 = \frac{\sqrt{3}-1}{2}\tilde{m}_2$	A_5'([203])	Weinberg	m^{ν}_{LL}
$\tilde{m}_1^{-1} + \tilde{m}_2^{-1} = \tilde{m}_3^{-1}$	A_4[167]; S_4([163, 175]); A_5[176, 177]	Type I	M_R
$\tilde{m}_1^{-1} + \tilde{m}_2^{-1} = \tilde{m}_3^{-1}$	S_4([163])	Type III	M_Σ
$2\tilde{m}_2^{-1} + \tilde{m}_3^{-1} = \tilde{m}_1^{-1}$	A_4[135, 164, 165, 167, 204] ([37, 137, 145, 205–211]); T'[196]	Type I	M_R
$\tilde{m}_1^{-1} + \tilde{m}_3^{-1} = 2\tilde{m}_2^{-1}$	A_4([212–214]); T'[215]	Type I	M_R
$\tilde{m}_3^{-1} \pm 2i\tilde{m}_2^{-1} = \tilde{m}_1^{-1}$	$\Delta(96)$[66]	Type I	M_R
$\tilde{m}_1^{1/2} - \tilde{m}_3^{1/2} = 2\tilde{m}_2^{1/2}$	A_4([162])	Type I	m^D
$\tilde{m}_1^{1/2} + \tilde{m}_3^{1/2} = 2\tilde{m}_2^{1/2}$	A_4([216])	Scotogenic	h_ν
$\tilde{m}_1^{-1/2} + \tilde{m}_2^{-1/2} = 2\tilde{m}_3^{-1/2}$	S_4[217]	Inverse	M_{RS}

Neutrino mass sum rules

Restrictions on $|m_{ee}|$ by mass sum rules



King, Merle, Stuart, JHEP2013
King, Merle,Morisi, Shimizu, Tanimoto, New J. Phys. 2014

III. Origin of Neutrino Mass

- Why are neutrinos massless in SM ?

  - no right-handed neutrinos
  - only SU(2) doublet Higgs scalars
  - prohibiting non-renormalizable terms

- How can neutrinos have mass ?

  -breaking those restrictions

Non-renormalizable term :

$$\frac{\lambda}{M} LL\Phi\Phi \rightarrow \frac{\lambda}{M}\langle\phi^0\rangle^2$$

Seesaw Origins

Type-I Seesaw

- Introducing right-handed neutrinos

$$Y_\nu \Phi \bar{\nu}_L \nu_R \;\rightarrow\; Y_\nu < \phi^0 > \bar{\nu}_L \nu_R \quad \sim 0.2 \text{ eV}$$

➔ $Y_\nu \sim 10^{-12}$ ： why so small?

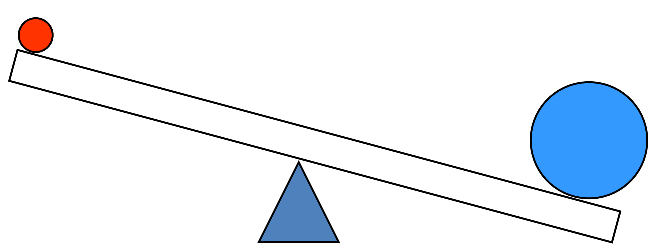
- No principle to prohibit $M_R \overline{\nu_R^C} \nu_R$
- Seesaw mechanism :

-ν_R can have large mass (L-violation: Type-I)

(Minkowski '77 Gellman Ramond Slansky '80 Glashow, Yanagida '79 Mohapatra Senjanovic '80 Lazarides Shafi Weterrich '81 Schechter-Valle '80 & '82)

$$L_{mass} = \frac{1}{2}(\nu_L^T N_R^T) C^{-1} \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} \begin{pmatrix} \nu_L \\ N_R \end{pmatrix} + c.c.$$

$$-\frac{m_D^2}{M}$$

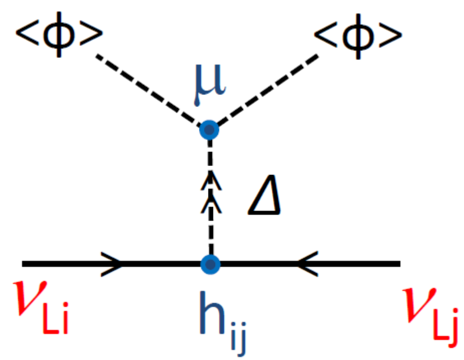


M

Type-II Seesaw

- Introducing SU(2) triplet Higgs (Δ) (type-II):
  $hLL\Delta \leftarrow \langle\Delta\rangle < 8$ GeV from ρ parameter.
  majorana mass

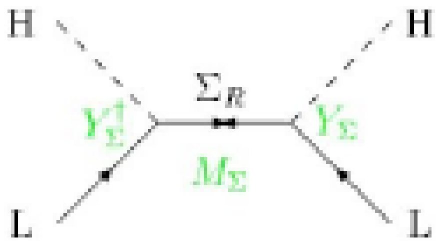
- Due to additional possible terms:
  $\mu\Phi\Delta^{+}\Phi + M_{\Delta}^{2}\mathrm{Tr}[\Delta^{+}\Delta]$
  $\rightarrow \langle\Delta\rangle = \frac{\mu\langle\Phi^{0}\rangle^{2}}{M_{\Delta}^{2}}$



(Magg, Wetterich; Lazarides, Shafi; Mohapatra, Senjanovic; Schechter, Valle)

Type-III Seesaw

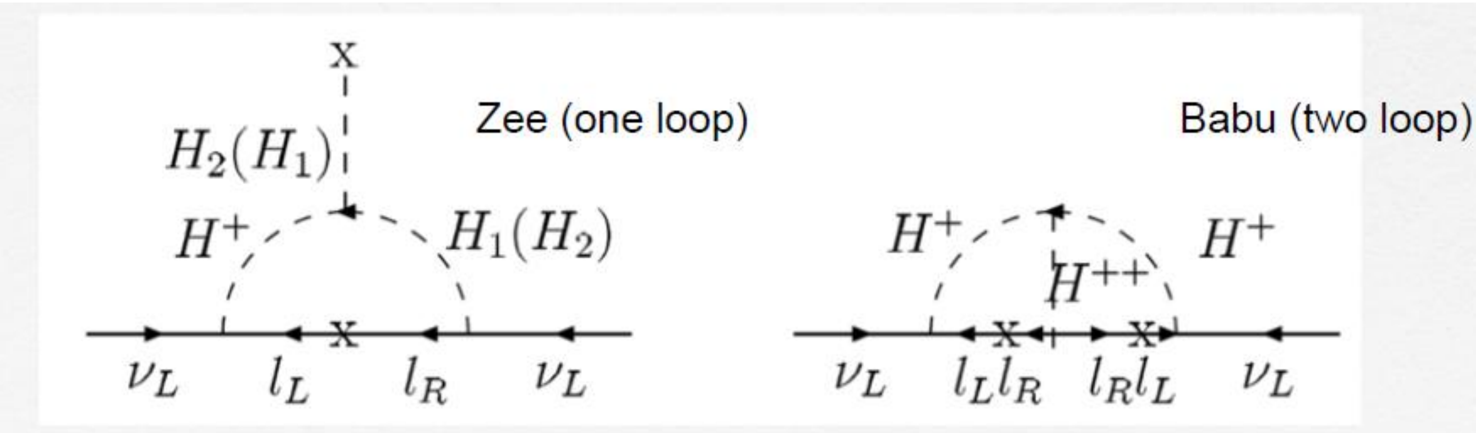
- Introducing SU(2) triplet fermions

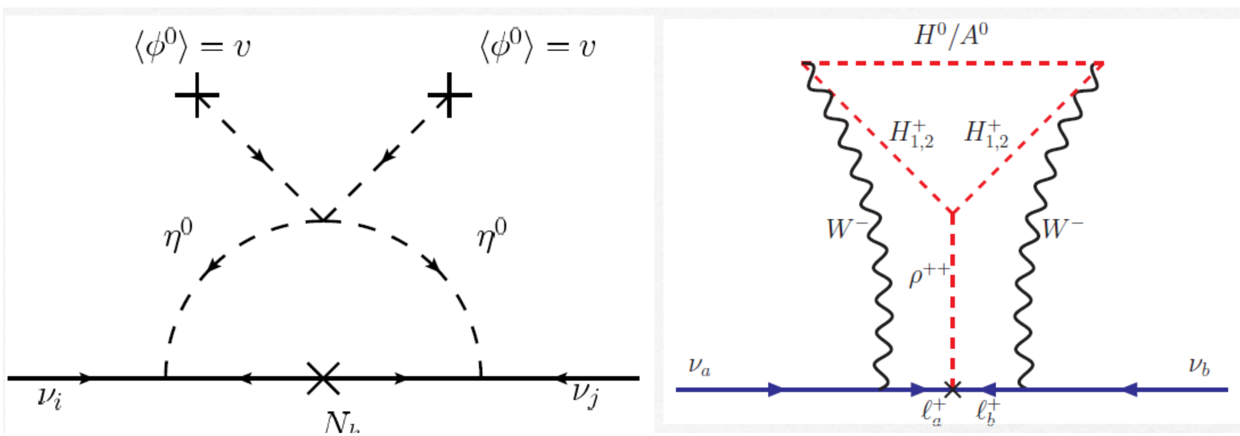


$$m_\nu = Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma v^2$$

Foot, Lew, He, Joshi; Ma; Ma, Roy; T.H., Lin, Notari, Papucci, Strumia; Bajc, Nemevsek, Senjanovic; Dorsner, Fileviez-Perez;....

- Radiative generation of neutrino masses

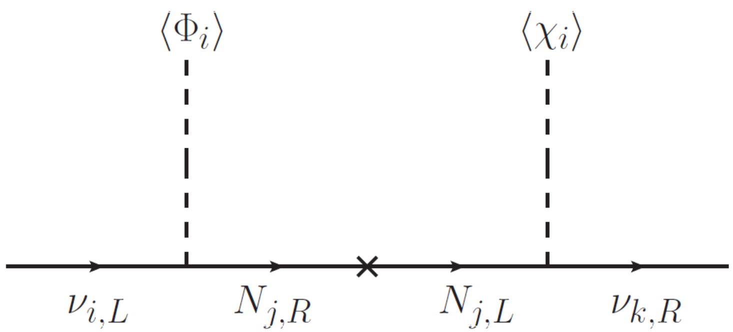




- Scotogenic (Ma) Cocktail (Gustafusson etal.)
- R-parity violating SUSY model

Seesaw for Dirac Neutrino

- Type-I Seesaw



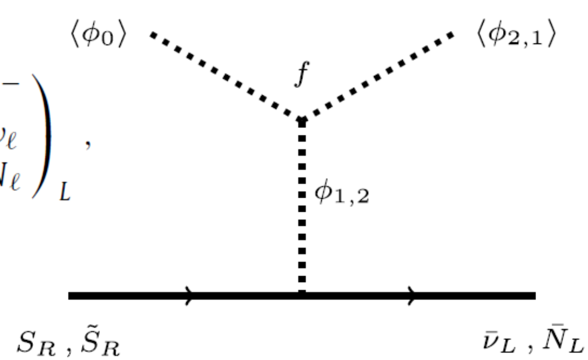
Chulia, Srivastava, Valle, PLB761 (2016),
Chulia, Ma, Srivastava, Valle ,PLB767 (2017)

The Dirac type-I seesaw mechanism. Φ_i and χ_i are triplets under $\Delta(27)$

Type-II Seesaw

(Valle,Vaquera-Araujo, PLB755(2016),
Addazi et al PLB759 (2016))

$$\psi_L^\ell = \begin{pmatrix} \ell^- \\ \nu_\ell \\ N_\ell \end{pmatrix}_L ,$$



Anomaly free $SU(3)_C \times SU(3)_L \times U(1)_x$

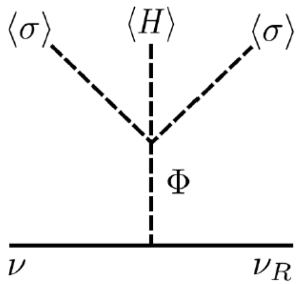
Matter content of the model, where $\hat{u}_R \equiv (u_R, c_R, t_R, U_R)$ and $\hat{d}_R \equiv (d_R, s_R, b_R, D_R, D'_R)$.

	ψ_L^ℓ	ℓ_R	S_R^ℓ, $\tilde{S}_R^\ell$	$Q_L^{1,2}$	Q_L^3	$\hat{u}_R$	$\hat{d}_R$	ϕ_0	ϕ_1	ϕ_2
$SU(3)_C$	**1**	**1**	**1**	**3**	**3**	**3**	**3**	**1**	**1**	**1**
$SU(3)_L$	$\mathbf{3}^*$	**1**	**1**	**3**	$\mathbf{3}^*$	**1**	**1**	$\mathbf{3}^*$	$\mathbf{3}^*$	$\mathbf{3}^*$
$U(1)_X$	$-\frac{1}{3}$	-1	0	0	$+\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$
$\mathcal{L}$	$-\frac{1}{3}$	-1	1	$-\frac{2}{3}$	$+\frac{2}{3}$	0	0	$+\frac{2}{3}$	$-\frac{4}{3}$	$-\frac{4}{3}$
$\mathbb{Z}_3^{\text{aux}}$	ω	ω	ω	ω^2	ω^2	ω^2	ω^2	1	1	1

Seesaw for Dirac Neutrino

- Type-II Seesaw

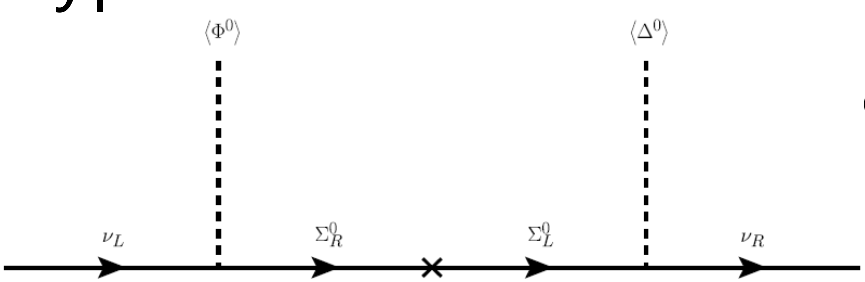
Bonilla, Valle, PLB762(2016)
Reig et al., PRD94(2016)



	$\overline{L}$	ℓ_R	ν_R	H	Φ	σ
$SU(2)_L$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$
$\mathbb{Z}_5$	ω	ω^4	ω	1	ω^3	ω
$\mathbb{Z}_3$	α^2	α	α	1	1	1

Neutrino mass generation in type-II Dirac seesaw mechanism

- Type-III Seesaw

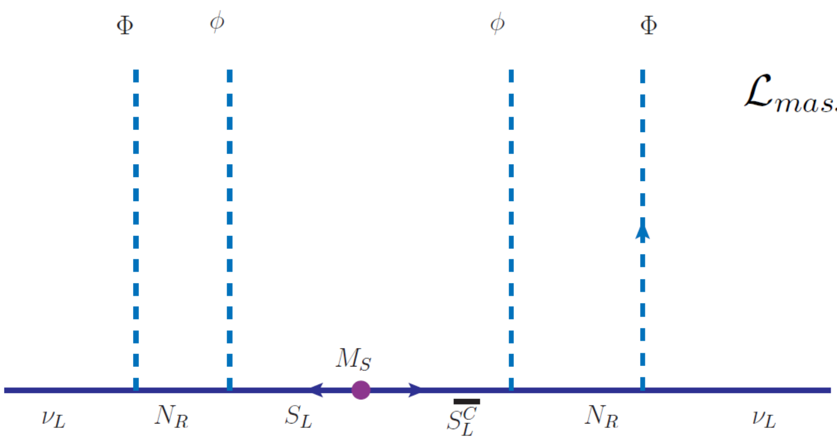


Chulia, Srivastava, Valle, PLB781(2018)

Neutrino mass generation in type-III Dirac seesaw
There can be d=5 op. leading to tiny Dirac mass.

Inverse seesaw

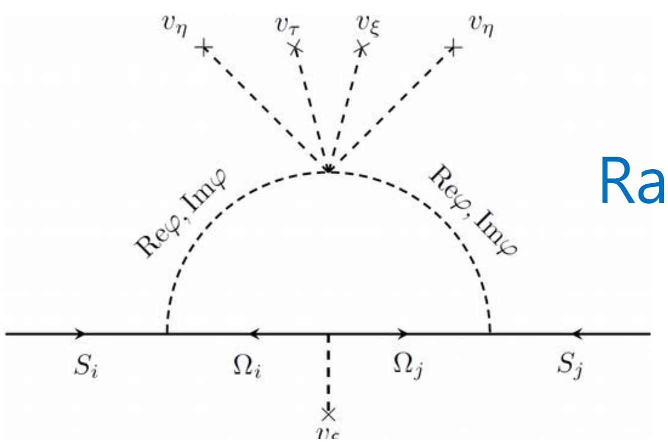
- Inverse seesaw



$$\mathcal{L}_{mass} = \left(\overline{\nu_L^C}\ \overline{N_R}\ \overline{S_L^C}\right) \begin{pmatrix} 0 & m_D^* & 0 \\ m_D^\dagger & 0 & m_{NS} \\ 0 & m_{NS}^T & M_S \end{pmatrix} \begin{pmatrix} \nu_L \\ N_R^C \\ S_L \end{pmatrix}$$

$$m_\nu = (m_D^* m_{NS}^{-1}) M_s (m_D^* m_{NS}^{-1})^T$$

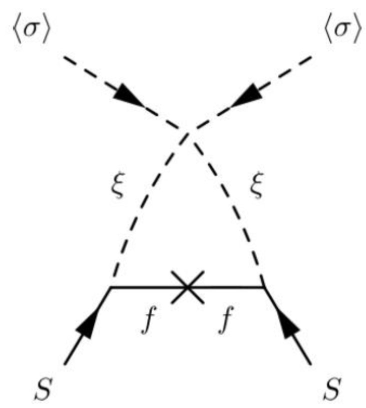
Mohapatra, PRL56(1986)
Mohapatra, Valle, PRD34(1986)



Radiative Inverse seesaw

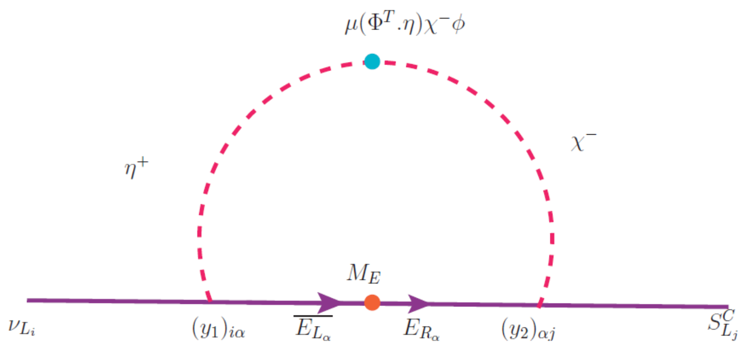
Carcamo Hernandez et al JHEP 1902 (2019)

- Scotogenic inverse seesaw



arXiv:1907.07728

- Inverse seesaw+1-loop (A. Das et al, 1704.02078)



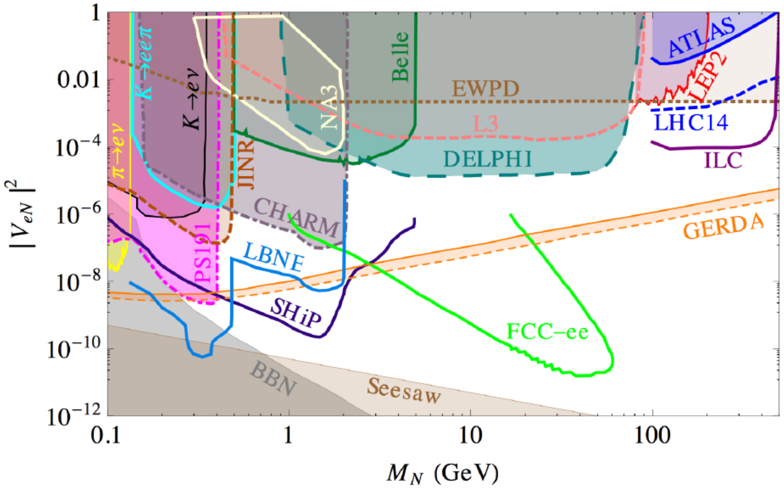
$$m_\nu^{\text{tree+1-loop}} = \begin{pmatrix} 0 & m_D^* & \delta_1^* \\ m_D^\dagger & 0 & m_{NS} \\ \delta_1^\dagger & m_{NS}^T & M_S \end{pmatrix}$$

- Dirac Inverse seesaw (Borah, Karmakar, PLB780(2018))

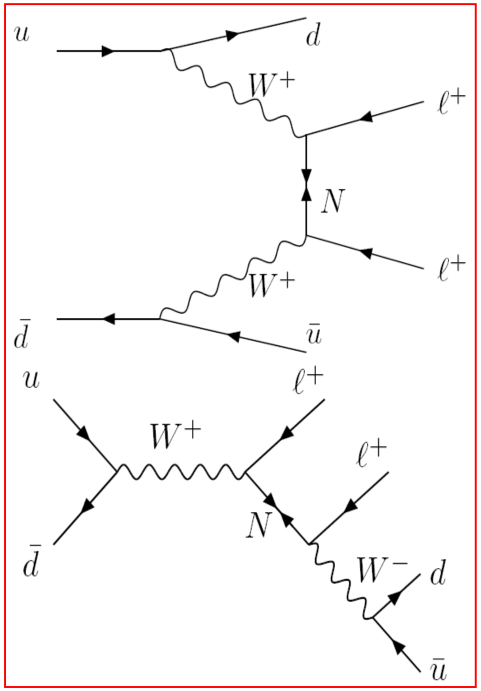
What is the Seesaw scale ?

- For $m_D \sim m_t$, neutrino mass of $m_\nu \leq 1$ eV implies $M_R \sim 10^{14}$ GeV
  - close to the scale of Grand Unification ~ 10^{16} GeV

- For $m_D \sim m_e$, neutrino mass of $m_\nu \leq 1$ eV implies $M_R \sim 1$ TeV.
  -potentially testable at collider



Deppisch, Dev, Pilaftsis, 1502.06541



What is the Seesaw scale ?

- νMSM (Asaka, Blanchet,Shaposhnikov, PLB631(2005)):
  - $M_{R1} \sim$ keV scale warm dark matter
  - $M_{R2(R3)}$ ~few GeV with tiny Yukawa couplings
- Minimal SM accommodating DM, baryogenesis at the price of fine tuning.

IV. New Physics in ν Oscillation

- What causes deviation of oscillations
  - Non-standard Interactions(NSI)
  - Unitarity violation in U_{PMNS}
  - light sterile neutrinos
  - long-range forces
  - Lorentz/CPT violation
  - General neutrino interactions
  - decay etc.

Nonstandard Interactios(NSI)

- Existence of NSI indicates new physics beyond the SM

$$\delta\mathcal{L}_{NSI}^{NC} = -2\sqrt{2}\sum_{f,P}\epsilon_{\alpha\beta}^{fP}(\bar{\nu}_\alpha\gamma^\mu P_L\nu_\beta)(\bar{f}\gamma_\mu P f)$$

$$\varepsilon_{\alpha\beta}^{f} = \varepsilon_{\alpha\beta}^{fL} + \varepsilon_{\alpha\beta}^{fR}$$

- NC-NSI my affect neutrino in propagation, which can be presented through modification of matter potential

$$H_{\rm mat} = \sqrt{2}G_F N_e(x)\begin{pmatrix} 1+\epsilon_{ee}(x) & \epsilon_{e\mu}(x) & \epsilon_{e\tau}(x) \\ \epsilon_{e\mu}^*(x) & \epsilon_{\mu\mu}(x) & \epsilon_{\mu\tau}(x) \\ \epsilon_{e\tau}^*(x) & \epsilon_{\mu\tau}^*(x) & \epsilon_{\tau\tau}(x) \end{pmatrix}$$

If $\varepsilon \propto \frac{c^2}{\Lambda^2}$, TeV scale NP leads to ε~0.01

- Even if no mixing in vacuum, $\nu_\alpha \rightarrow \nu_\beta$ can occur in matter
- Complex phases of off-diag. could be new source of CPV

2 σ Results from fit to oscillation data and COHERENT

OSC			+COHERENT		
	LMA	LMA ⊕ LMA-D		LMA	LMA ⊕ LMA-D
$\varepsilon^u_{ee} - \varepsilon^u_{\mu\mu}$	[−0.020, +0.456]	⊕[−1.192, −0.802]	ε^u_{ee}	[−0.008, +0.618]	[−0.008, +0.618]
$\varepsilon^u_{\tau\tau} - \varepsilon^u_{\mu\mu}$	[−0.005, +0.130]	[−0.152, +0.130]	$\varepsilon^u_{\mu\mu}$	[−0.111, +0.402]	[−0.111, +0.402]
			$\varepsilon^u_{\tau\tau}$	[−0.110, +0.404]	[−0.110, +0.404]
$\varepsilon^u_{e\mu}$	[−0.060, +0.049]	[−0.060, +0.067]	$\varepsilon^u_{e\mu}$	[−0.060, +0.049]	[−0.060, +0.049]
$\varepsilon^u_{e\tau}$	[−0.292, +0.119]	[−0.292, +0.336]	$\varepsilon^u_{e\tau}$	[−0.248, +0.116]	[−0.248, +0.116]
$\varepsilon^u_{\mu\tau}$	[−0.013, +0.010]	[−0.013, +0.014]	$\varepsilon^u_{\mu\tau}$	[−0.012, +0.009]	[−0.012, +0.009]
$\varepsilon^d_{ee} - \varepsilon^d_{\mu\mu}$	[−0.027, +0.474]	⊕[−1.232, −1.111]	ε^d_{ee}	[−0.012, +0.565]	[−0.012, +0.565]
$\varepsilon^d_{\tau\tau} - \varepsilon^d_{\mu\mu}$	[−0.005, +0.095]	[−0.013, +0.095]	$\varepsilon^d_{\mu\mu}$	[−0.103, +0.361]	[−0.103, +0.361]
			$\varepsilon^d_{\tau\tau}$	[−0.102, +0.361]	[−0.102, +0.361]
$\varepsilon^d_{e\mu}$	[−0.061, +0.049]	[−0.061, +0.073]	$\varepsilon^d_{e\mu}$	[−0.058, +0.049]	[−0.058, +0.049]
$\varepsilon^d_{e\tau}$	[−0.247, +0.119]	[−0.247, +0.119]	$\varepsilon^d_{e\tau}$	[−0.206, +0.110]	[−0.206, +0.110]
$\varepsilon^d_{\mu\tau}$	[−0.012, +0.009]	[−0.012, +0.009]	$\varepsilon^d_{\mu\tau}$	[−0.011, +0.009]	[−0.011, +0.009]
$\varepsilon^p_{ee} - \varepsilon^p_{\mu\mu}$	[−0.041, +1.312]	⊕[−3.327, −1.958]	ε^p_{ee}	[−0.010, +2.039]	[−0.010, +2.039]
$\varepsilon^p_{\tau\tau} - \varepsilon^p_{\mu\mu}$	[−0.015, +0.426]	[−0.424, +0.426]	$\varepsilon^p_{\mu\mu}$	[−0.364, +1.387]	[−0.364, +1.387]
			$\varepsilon^p_{\tau\tau}$	[−0.350, +1.400]	[−0.350, +1.400]
$\varepsilon^p_{e\mu}$	[−0.178, +0.147]	[−0.178, +0.178]	$\varepsilon^p_{e\mu}$	[−0.179, +0.146]	[−0.179, +0.146]
$\varepsilon^p_{e\tau}$	[−0.954, +0.356]	[−0.954, +0.949]	$\varepsilon^p_{e\tau}$	[−0.860, +0.350]	[−0.860, +0.350]
$\varepsilon^p_{\mu\tau}$	[−0.035, +0.027]	[−0.035, +0.035]	$\varepsilon^p_{\mu\tau}$	[−0.035, +0.028]	[−0.035, +0.028]

(Esteban et al., 1805.04530)

- NSI may affect neutrinos at the production point as well as detection point.
- To see those effects, we use

$$\delta\mathcal{L}_{NSI}^{CC} = -2\sqrt{2}\sum_{f,P}\epsilon_{\alpha\beta}^{fP}\left(\bar{\nu}_\alpha\gamma^\mu P_L l_\beta\right)\left(\bar{f}\gamma_\mu P f'\right)$$

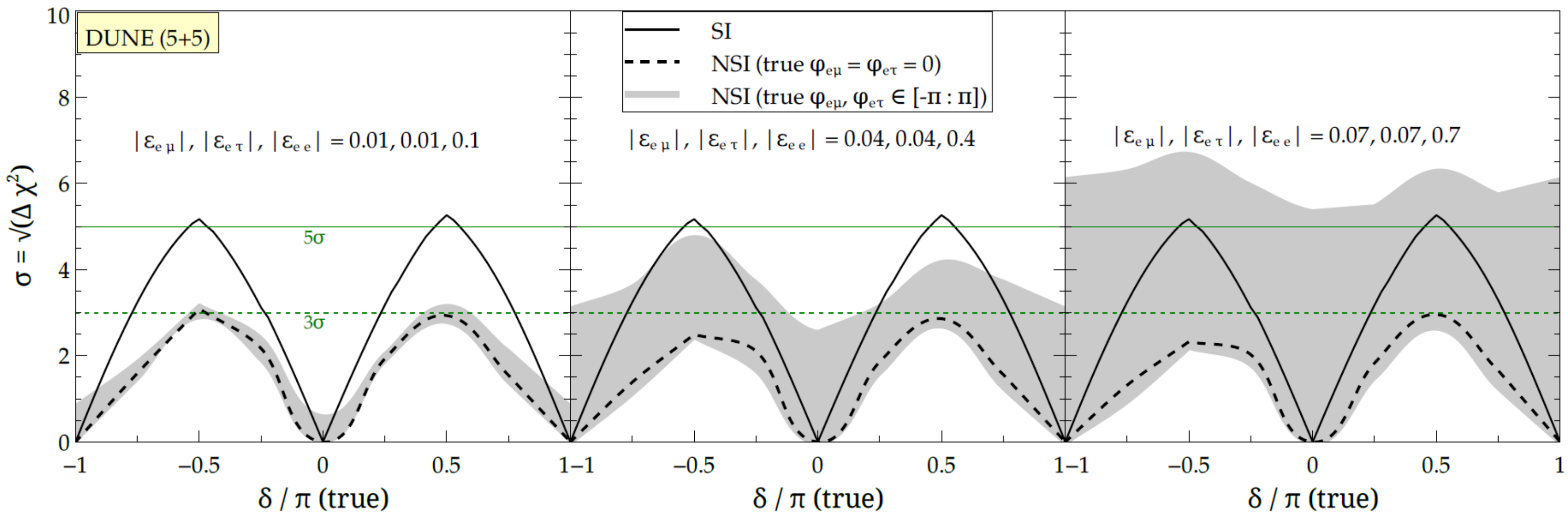
	90% C.L. range	Origin	References
SEMILEPTONIC NSI			
ϵ_{ee}^{udP}	$[-0.015, 0.015]$	Daya Bay	[13]
$\epsilon_{e\mu}^{udL}$	$[-0.026, 0.026]$	NOMAD	[33]
$\epsilon_{e\mu}^{udR}$	$[-0.037, 0.037]$	NOMAD	[33]
$\epsilon_{\tau e}^{udL}$	$[-0.087, 0.087]$	NOMAD	[33]
$\epsilon_{\tau e}^{udR}$	$[-0.12, 0.12]$	NOMAD	[33]
$\epsilon_{\tau\mu}^{udL}$	$[-0.013, 0.013]$	NOMAD	[33]
$\epsilon_{\tau\mu}^{udR}$	$[-0.018, 0.018]$	NOMAD	[33]
PURELY LEPTONIC NSI			
$\epsilon_{\alpha e}^{\mu eL}$, $\epsilon_{\alpha e}^{\mu eR}$	$[-0.025, 0.025]$	KARMEN	[33]
$\epsilon_{\alpha\beta}^{\mu eL}$, $\epsilon_{\alpha\beta}^{\mu eR}$	$[-0.030, 0.030]$	kinematic G_F	[33]

(Farzan, Torotla, *Front.in Phys.* 6 (2018))

NSI & CPV

$$\epsilon_{\alpha\beta} = |\epsilon_{\alpha\beta}| e^{i\phi_{\alpha\beta}}$$

another source of CPV



- NSI can prevent determination of CP violation

Masud, Mehta, PRD94(2016)

NSI & CPV

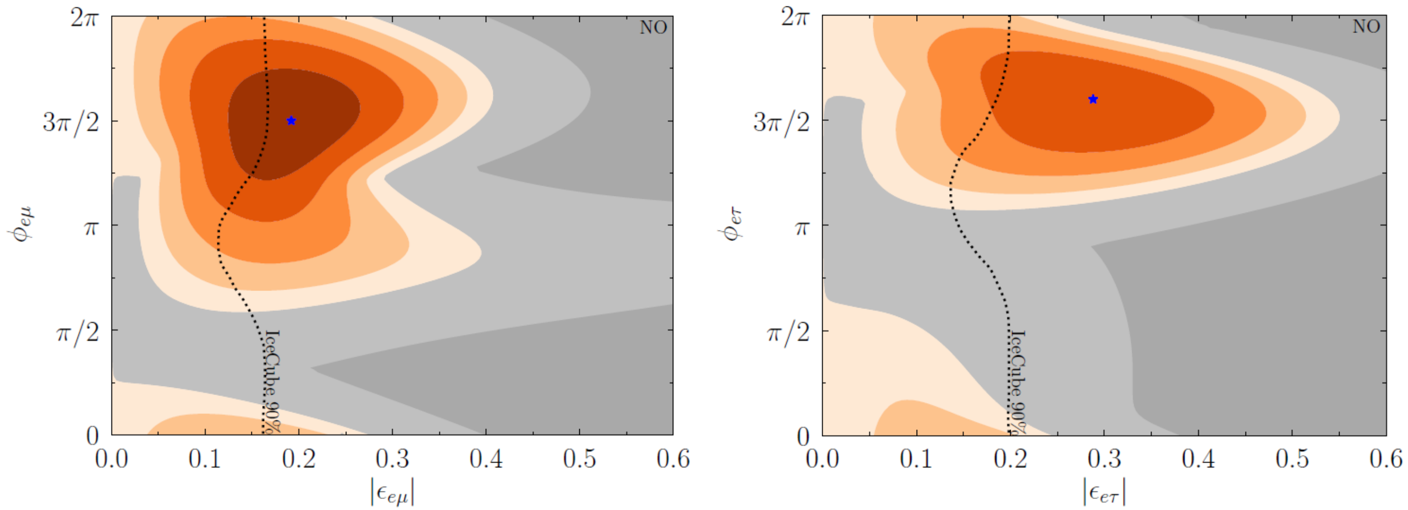


FIG. 2. The preferred parameter regions for $\epsilon_{e\mu}$ and $\epsilon_{e\tau}$ using the newest appearance and disappearance data from NOvA and T2K and assuming the NO. The gray region is disfavored compared to the SM, and the dark gray region is ruled out by NOvA and T2K data at $\Delta\chi^2 = -4.61$. The blue stars show the best fit points. Each of the orange contours are drawn at integer values of $\Delta\chi^2$. See table I for the best parameters. IceCube disfavors the region to the right of the black dotted curve at 90%

MO	NSI	$\lvert\epsilon_{\alpha\beta}\rvert$	$\phi_{\alpha\beta}/\pi$	δ/π	$\Delta\chi^2$
NO	$\epsilon_{e\mu}$	0.19	1.50	1.46	4.68
	$\epsilon_{e\tau}$	0.29	1.60	1.46	3.99
	$\epsilon_{\mu\tau}$	0.38	0.60	1.16	1.03
IO	$\epsilon_{e\mu}$	0.05	1.24	1.52	0.30
	$\epsilon_{e\tau}$	0.07	1.70	1.47	0.28
	$\epsilon_{\mu\tau}$	0.31	0.12	1.51	2.53

Denton, Gehrlein, Pestes, arXiv:2008.01110

Origin of NSI

- ε from integrating out scalar of type II seesaw: $\varepsilon^{e}_{\alpha\beta} \propto (m_\nu)_{\alpha\beta}$ (Malinsky, Ohlsson, Zhang, 0811.3346)
- ε from integrating out leptoquarks (Wise, Zhang, 1404.4663)
- ε from integrating out charge +1 scalar singlet:
- ε from loop effects, including secret neutrino interactions (Bischer, Rodejohann, Xu, 1807.08102)
- ε from higher dimensional operators (Gavela et al., 0809.3451); within flavor symmetry models have information on flavor symmetry (Wang, Zhou, 1801.05656)
- ε from integrating out Z′ (Heeck, Lindner, Rodejohann, Vogl, 1812.04067)

Non-unitarity

- Source of non-unitary : sterile neutrino, NSI,…(minimal unitarity violation: Antusch et al, 2006)
- Parametrization (Z. Xing, PLB 2008, Escrihuela et al. PRD92(2015))

$$N = N^{NP} U = \begin{pmatrix} \alpha_{11} & 0 & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} U$$

- Constraints from experimental data: ν oscillations, *W* & *Z* decays, rare lepton-flavor-violating decays, lepton universality tests, ……

- Sensitivity to CPV in LBL exps. can be affected by the presence of non-unitarity.

	One parameter (1 d.o.f.)		All parameters (6 d.o.f.)	
	90% C.L.	3σ	90% C.L.	3σ
Neutrinos + charged leptons				
$\alpha_{11} >$	0.9974	0.9963	0.9961	0.9952
$\alpha_{22} >$	0.9994	0.9991	0.9990	0.9987
$\alpha_{33} >$	0.9988	0.9976	0.9973	0.9961
$\|\alpha_{21}\| <$	1.7×10^{-3}	2.5×10^{-3}	2.6×10^{-3}	4.0×10^{-3}
$\|\alpha_{31}\| <$	2.0×10^{-3}	4.4×10^{-3}	5.0×10^{-3}	7.0×10^{-3}
$\|\alpha_{32}\| <$	1.1×10^{-3}	2.0×10^{-3}	2.4×10^{-3}	3.4×10^{-3}
Neutrinos only				
$\alpha_{11} >$	0.98	0.95	0.96	0.93
$\alpha_{22} >$	0.99	0.96	0.97	0.95
$\alpha_{33} >$	0.93	0.76	0.79	0.61
$\|\alpha_{21}\| <$	1.0×10^{-2}	2.6×10^{-2}	2.4×10^{-2}	3.6×10^{-2}
$\|\alpha_{31}\| <$	4.2×10^{-2}	9.8×10^{-2}	9.0×10^{-2}	1.3×10^{-1}
$\|\alpha_{32}\| <$	9.8×10^{-3}	1.7×10^{-2}	1.6×10^{-2}	2.1×10^{-2}

Escrihuela, Forero, Miranda, Tortola, Valle, New.J.Phys.19(2017)

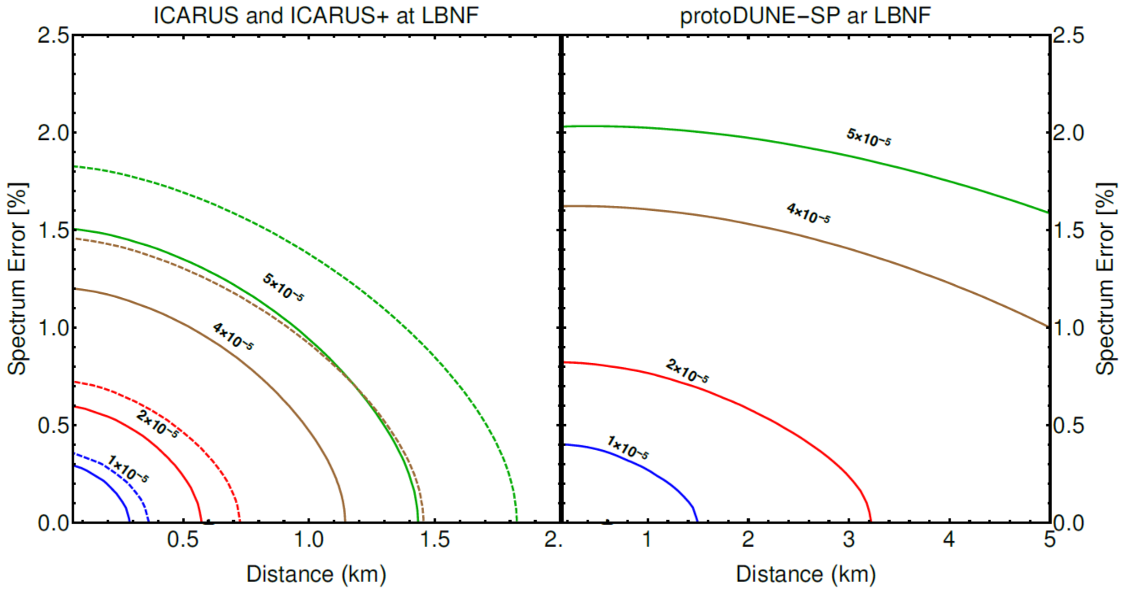
- Non-unitarity predicts "zero-distance effect"

$$P(\nu_\mu \to \nu_e) = \alpha_{11}|\alpha_{21}|^2$$

- Thus, at very short distances from the neutrino source, # of detected electron neutrinos, N_e, is given by

$$N_e = \phi^0_{\nu_e} + |\alpha_{21}|^2 \phi^0_{\nu_\mu}$$

- Capabilities of SBL as well as LBL as a probe of the unitarity of lepton mixing :(Miranda et al. PRD97(2018); Escrihuela et al ,New.J.Phys.19(2017)



Conclusion

- Lots of progress in neutrino physics in the past years

  → PMNS parameters approach CKM precision

- Still lots to learn about neutrino

  → mass ordering, CP violation, Majorana or Dirac etc.

- Lots of theoretical idea proposed to understand our universe via neutrino

  → More idea will emerge in future

- Lots of experimental programs and proposals exist

  → New era of neutrino physics

Recent development of neutrino mass models

- Seesaw a la Dirac neutrino
- Inverse seesaw
- Linear seesaw
- Low scale seesaw
- Radiative generation -talk by Volkas
- Neutrinoless double beta decay
  - absolute mass and Majorana phases
- Testability

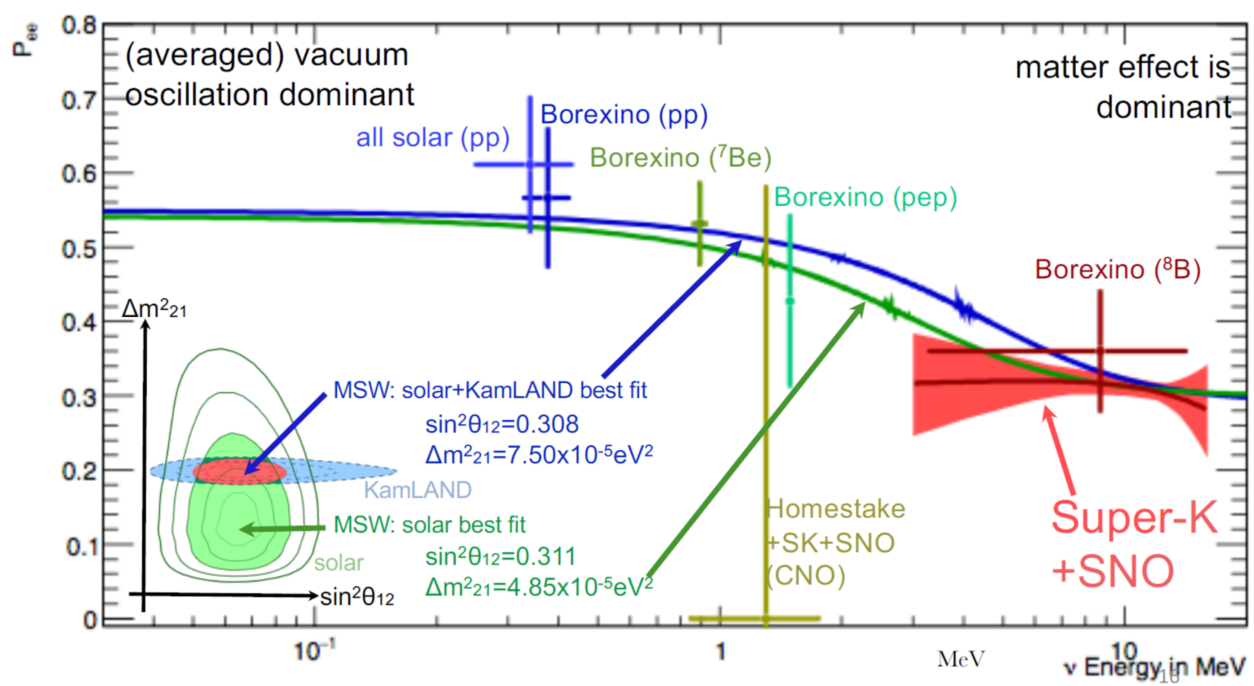
• MSW matter effect

$$\tan 2\theta_m = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - \xi}$$

$$\xi \equiv 2EV \approx 1.526 \times 10^{-7} \frac{Z}{A} \rho E \quad [\mathrm{eV}^2]$$

$$V = \pm\sqrt{2} G_F N_e \quad (\rho \text{ in g/cm}^3, E \text{ in MeV})$$

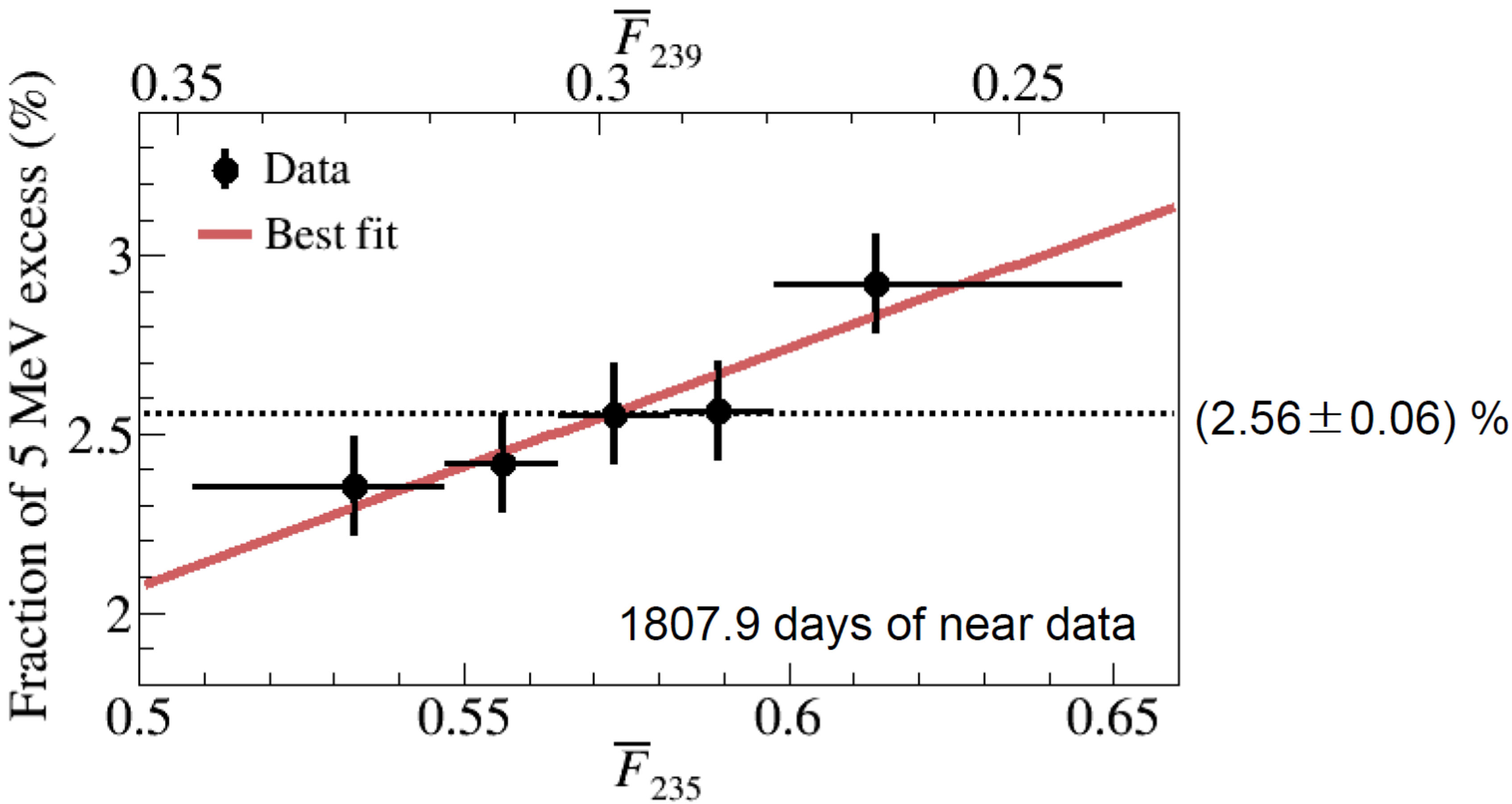
$$P_{ee} = 1 - P_{e\mu} = 1 - sin^2\theta cos^2\theta_m - cos^2\theta sin^2\theta_m$$



BOREXINO (*Barbara Caccianiga* 2019)

Correlation of 5 MeV excess with fuel ^{235}U

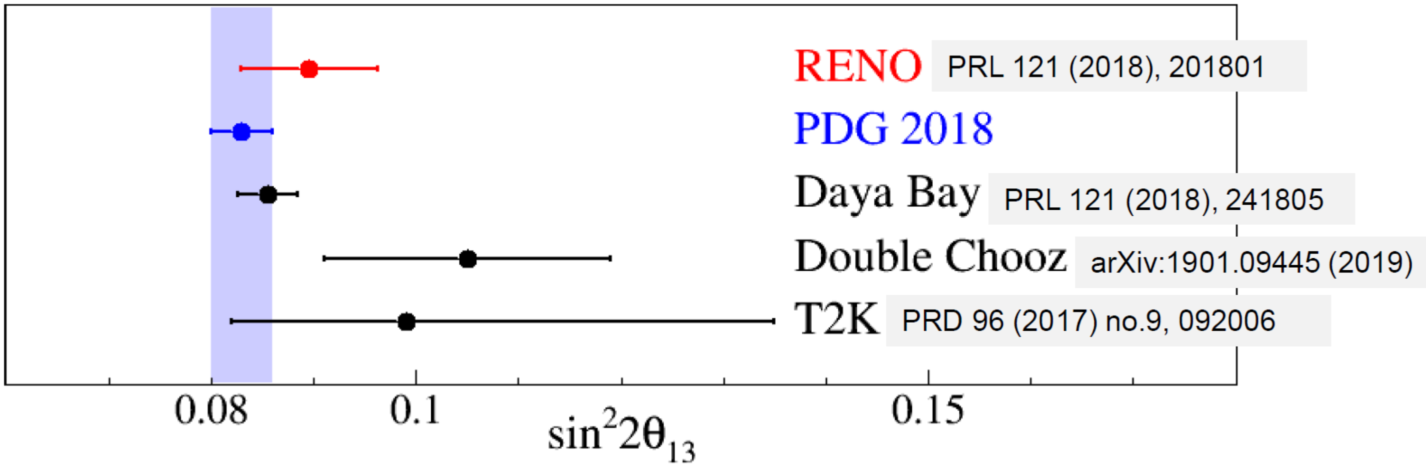
2.9σ indication of 5 MeV excess coming from ^{235}U fuel isotope fission !!

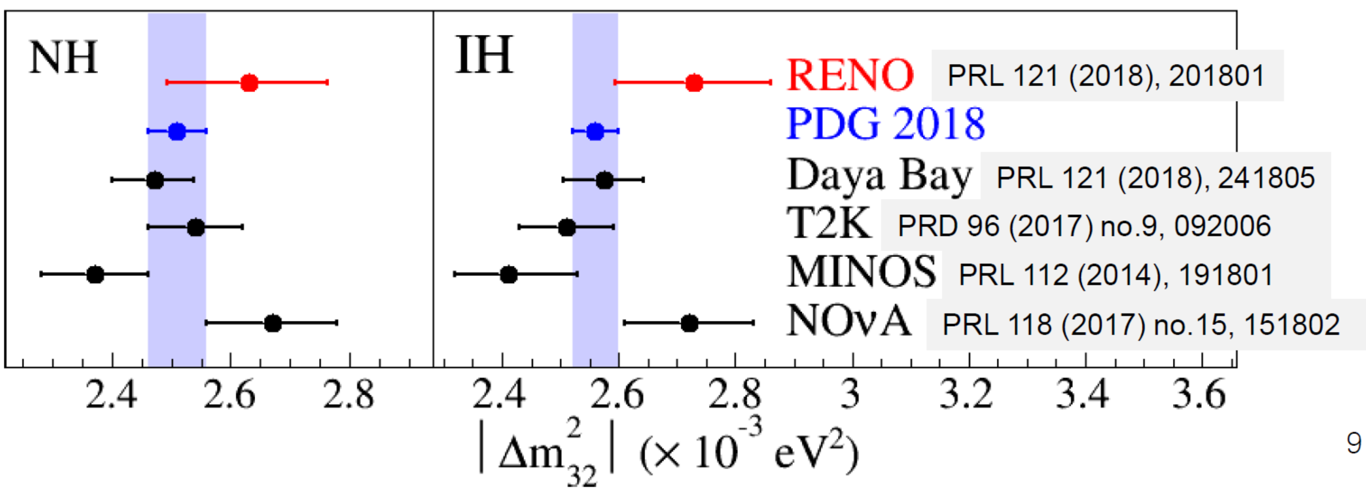


(End of reactor cycle)

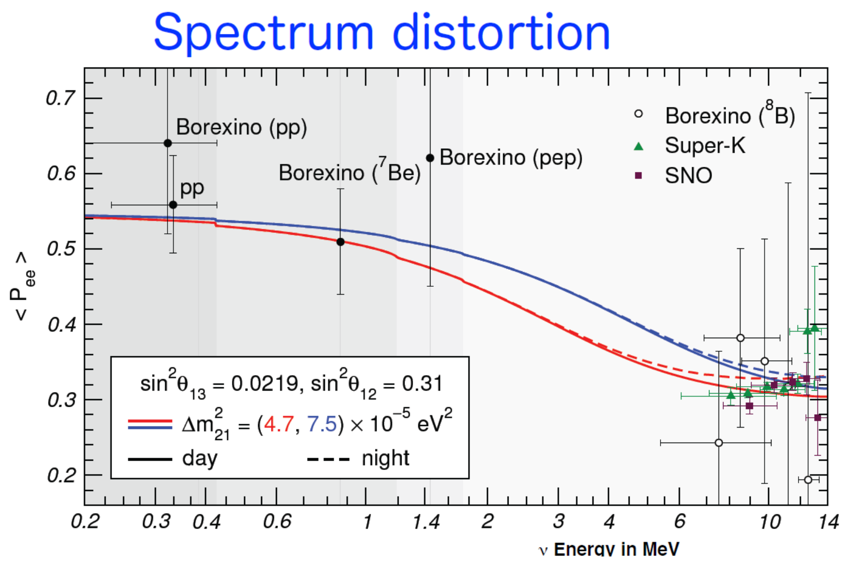
(Beginning of reactor cycle)

Comparison of θ_{13} and $|\Delta m^2_{ee}|$

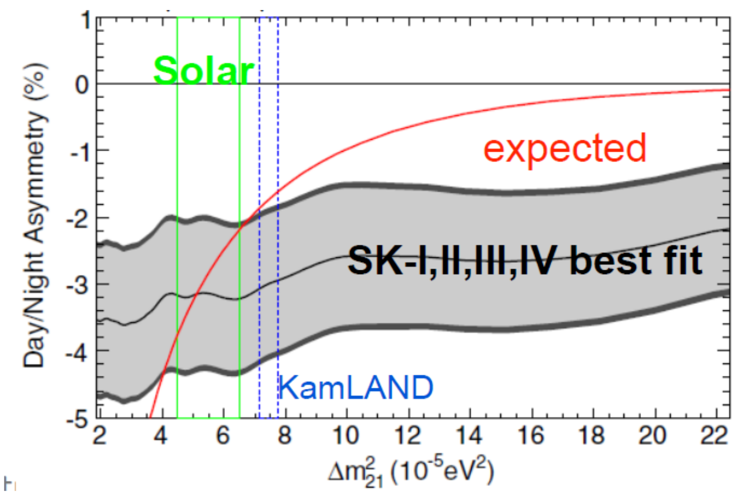




- Δm_{21}^2 preferred by KamLAND predicts steeper upturn at solar spectrum and smaller A(D/N)

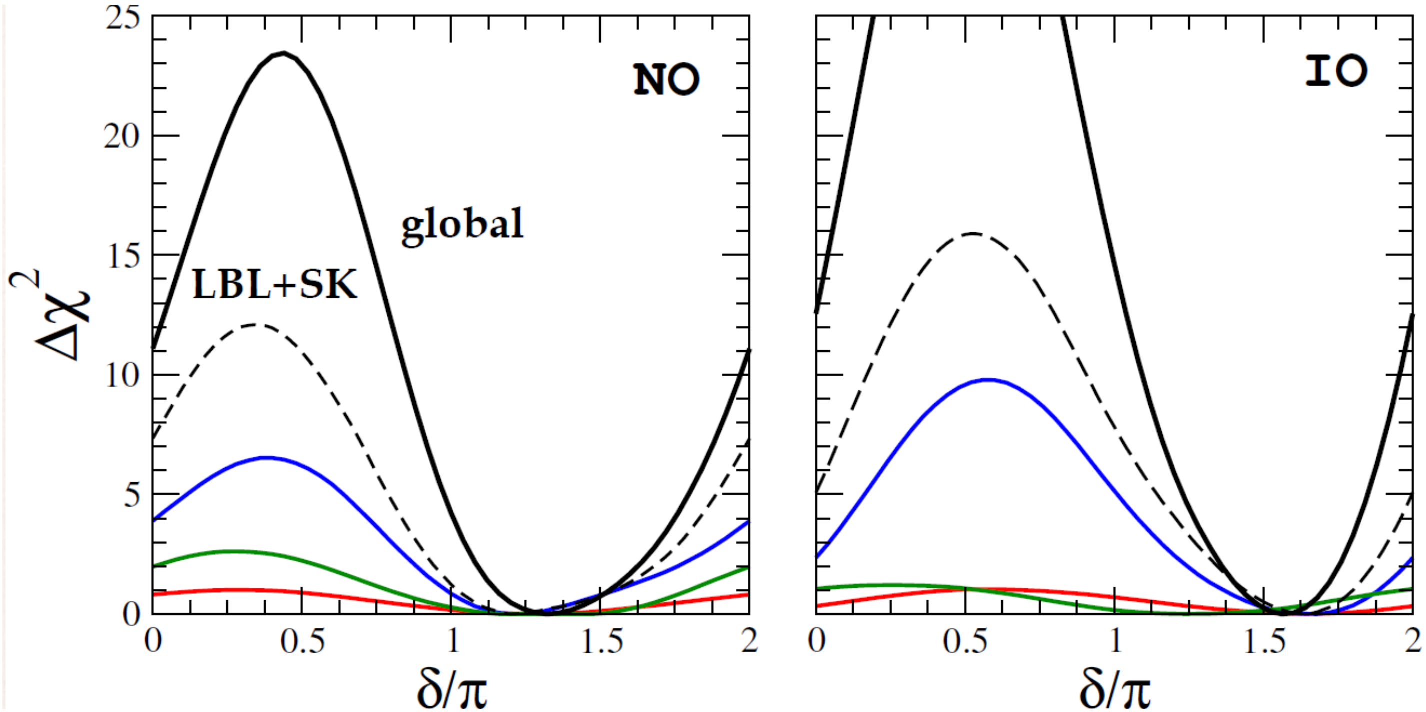


Maltoni, Smirnov EJP A52(2016)



Koshino (SK, 2019)

CP violation

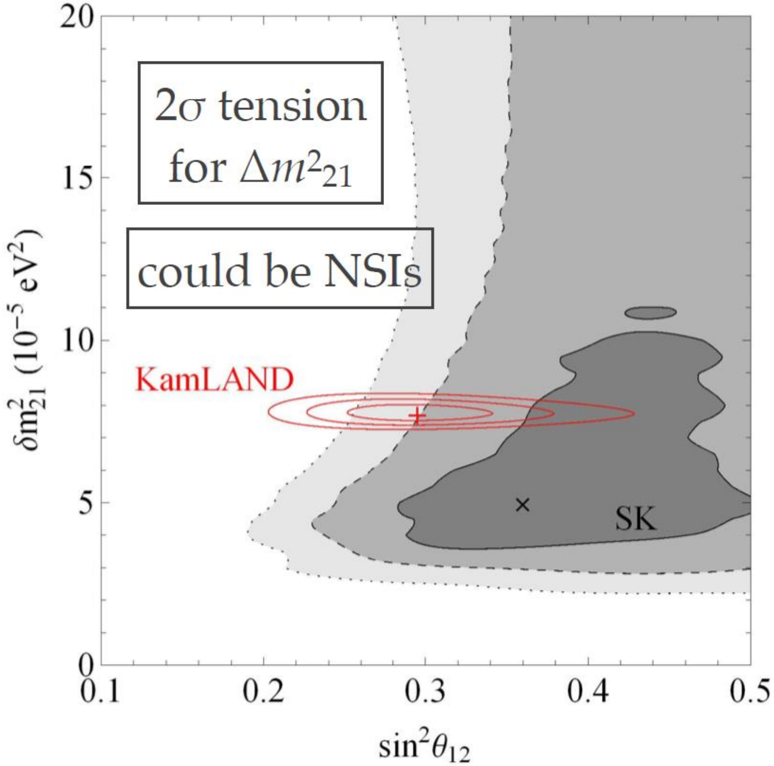


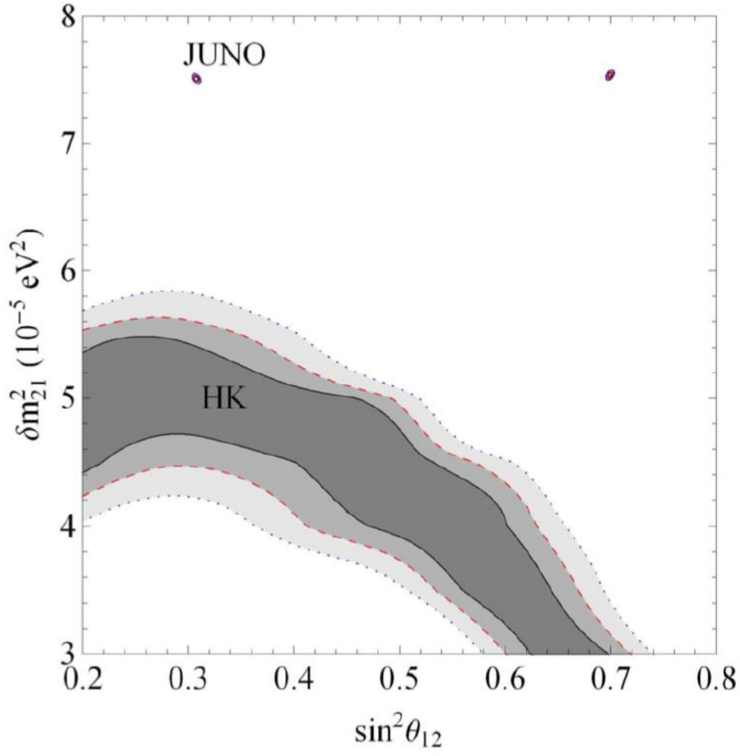
- NOvA , Super-K and T2K prefer $\pi < \delta < 2\pi$ (as well as NO)
- The combination of LBL and Super-K enhances rejection against $\delta = \pi/2$
- From the global analysis, $\delta = \pi/2$ is disfavoured at 4.8σ (6.1σ) for NO (IO)

NSI

- 2σ tension for Δm^2_{21} could be due to NSI

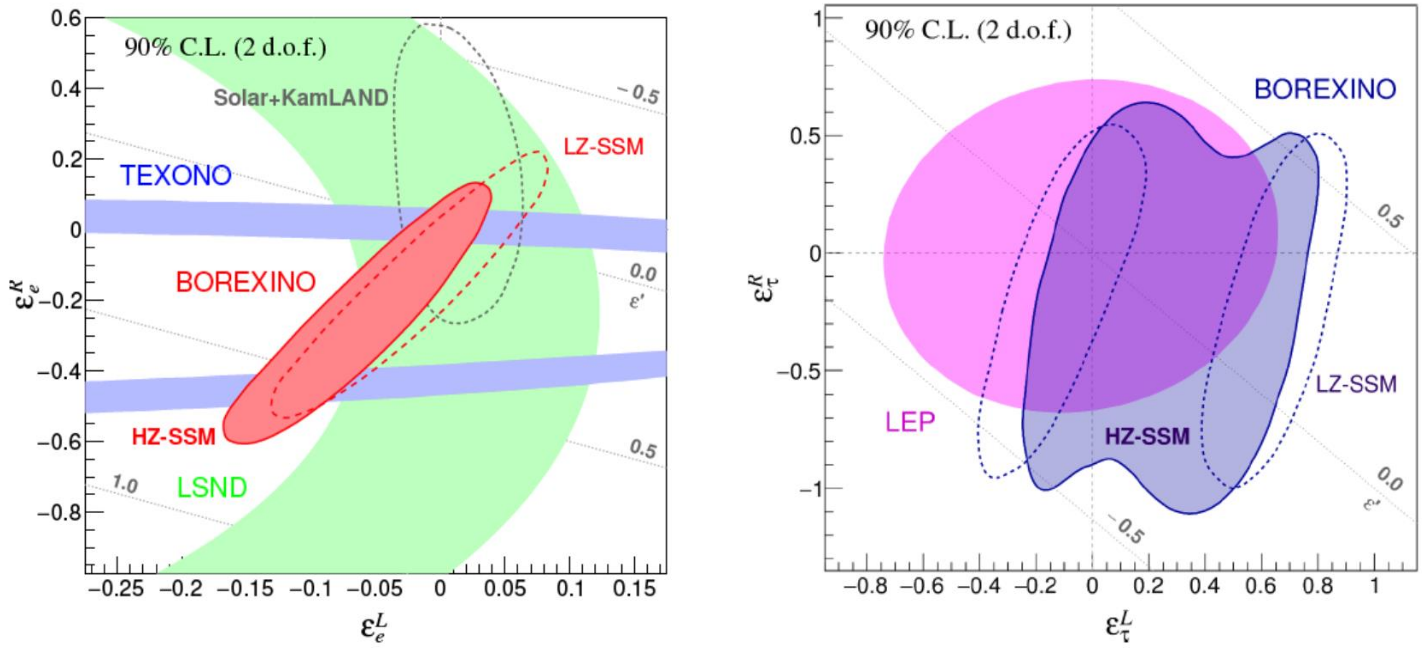
Liao, Marfatia, Whisnant, 1704.04711





(JUNO and HyperK would reject no NSI-solution by 7σ)

New constraints on NC NSIs from Borexion phase-II

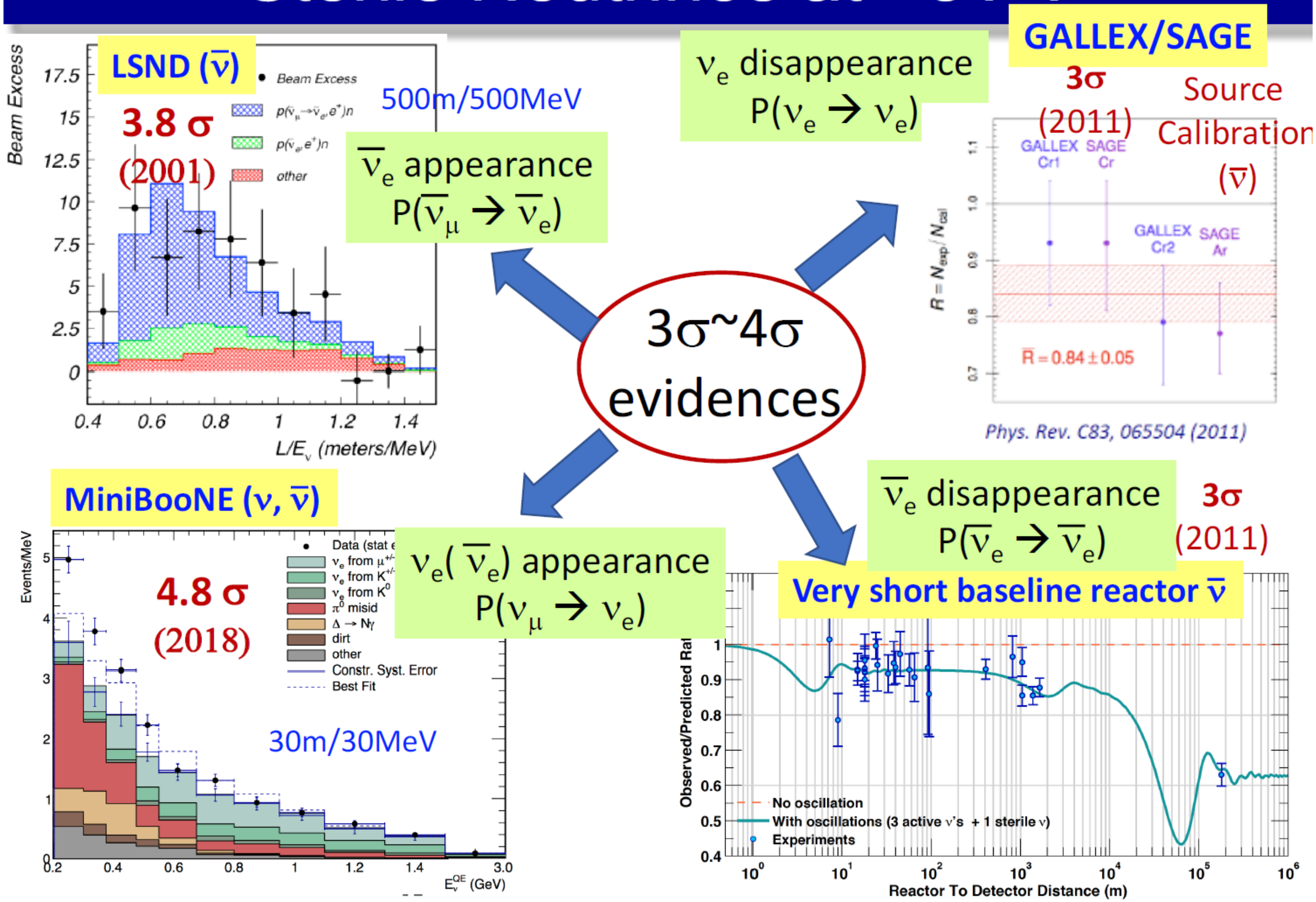


	HZ-SSM	LZ-SSM
ε_e^R	$[-0.15, +0.11\ ]$	$[-0.20, +0.03\ ]$
ε_e^L	$[-0.035, +0.032\ ]$	$[-0.013, +0.052\ ]$
ε_τ^R	$[-0.83, +0.36\ ]$	$[-0.42, +0.43\ ]$
ε_τ^L	$[-0.11, +0.67\ ]$	$[-0.19, +0.79\ ]$

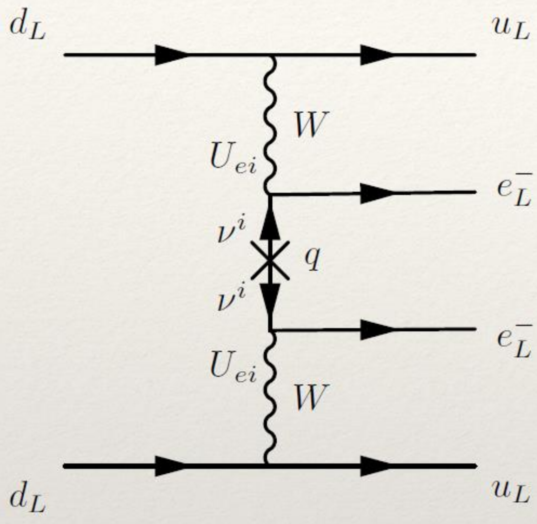
90% C.L. (1 d.o.f.)

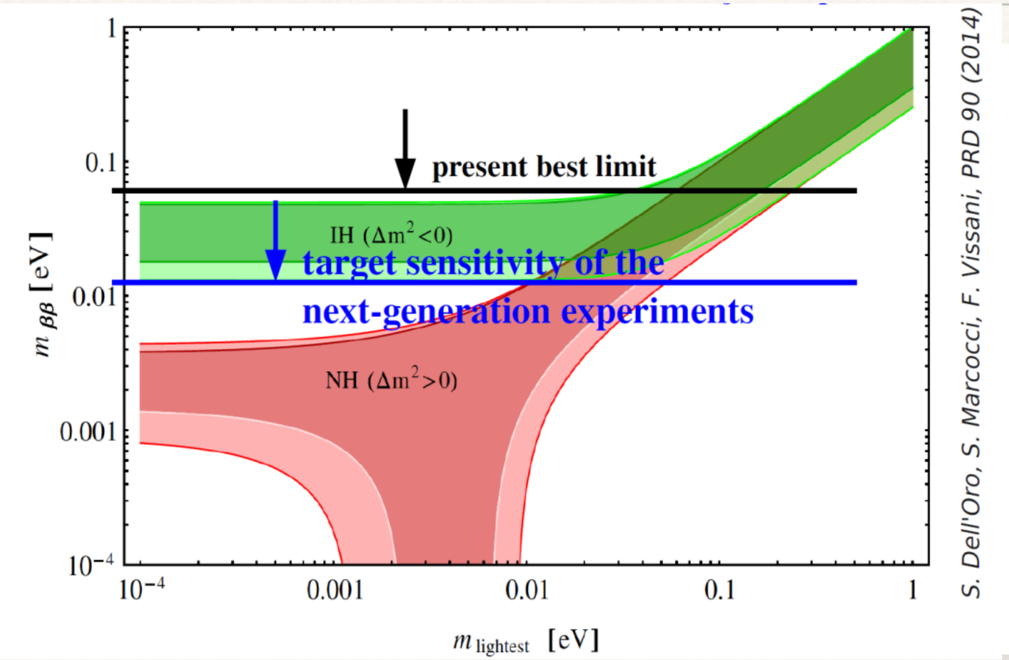
Agarwalla et al. JHEP02(2020) 038

Sterile Neutrinos at ~eV ?



Double Beta Decay





S. Dell'Oro, S. Marcocci, F. Vissani, PRD 90 (2014)

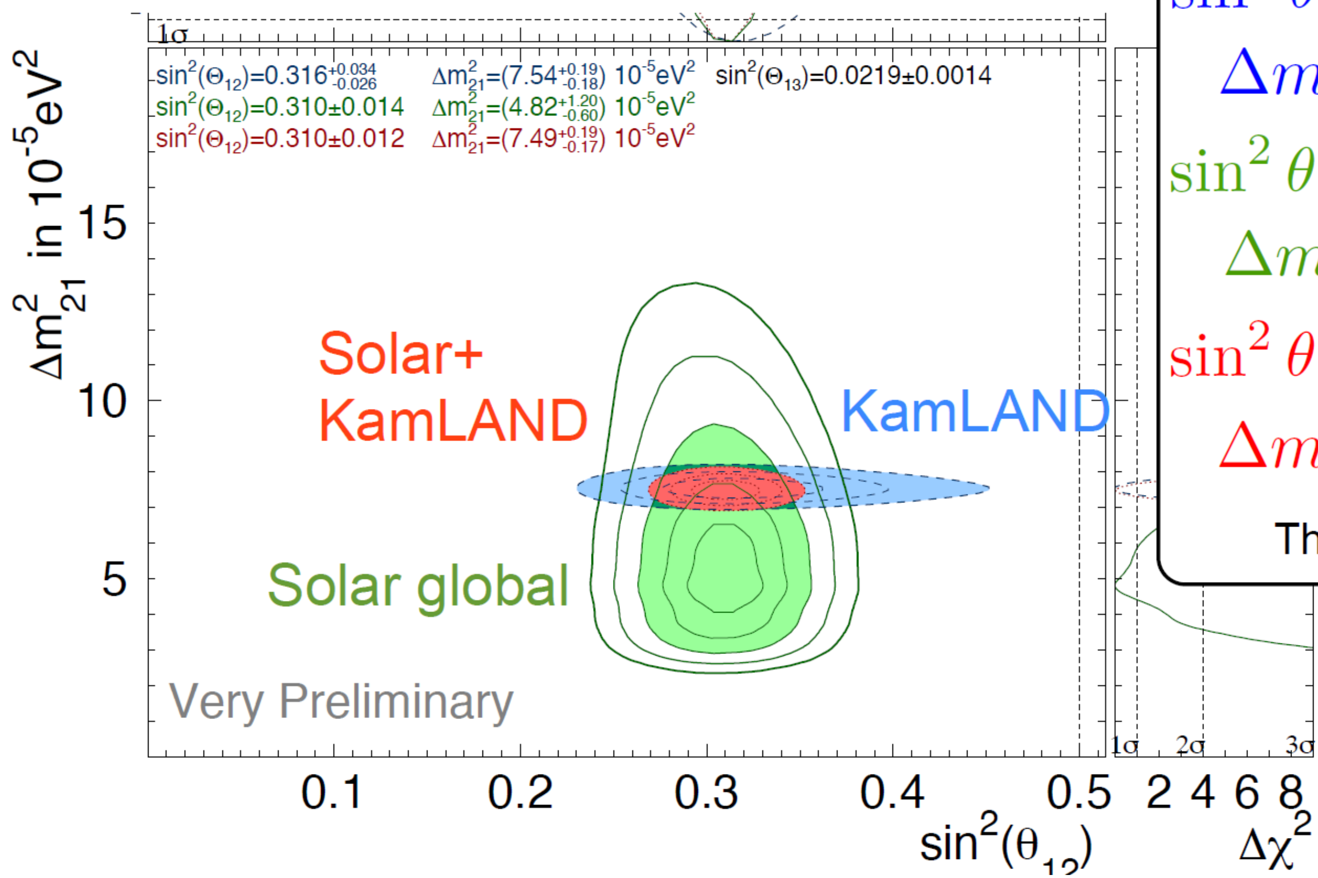
$$|m_{ee}| = |\sum U_{ei}^2 m_i| = |U_{e1}^2 m_1 + U_{e2}^2 m_2 e^{i\alpha} + U_{e3}^2 m_3 e^{i\beta}|$$
$$= f\left(\theta_{12}, |U_{e3}|, m_i, \mathrm{sgn}(\Delta m_A^2), \alpha, \beta\right)$$

known   limits   unknown

θ_{12} & Δm^2_{21}

In 2018

~2σ tension between solar global and KamLAND in Δm^2_{21}



$$\sin^2\theta_{12} = 0.316^{+0.034}_{-0.026}$$
$$\Delta m^2_{21} = 7.54^{+0.19}_{-0.18}$$
$$\sin^2\theta_{12} = 0.310 \pm 0.014$$
$$\Delta m^2_{21} = 4.82^{+1.20}_{-0.60}$$
$$\sin^2\theta_{12} = 0.310 \pm 0.012$$
$$\Delta m^2_{21} = 7.49^{+0.19}_{-0.17}$$

The unit of Δm^2_{21} is 10^{-5} eV^2

$\sin^2\theta_{13}=0.0219\pm0.0014$

M. Ikeda, Neutrino 2018

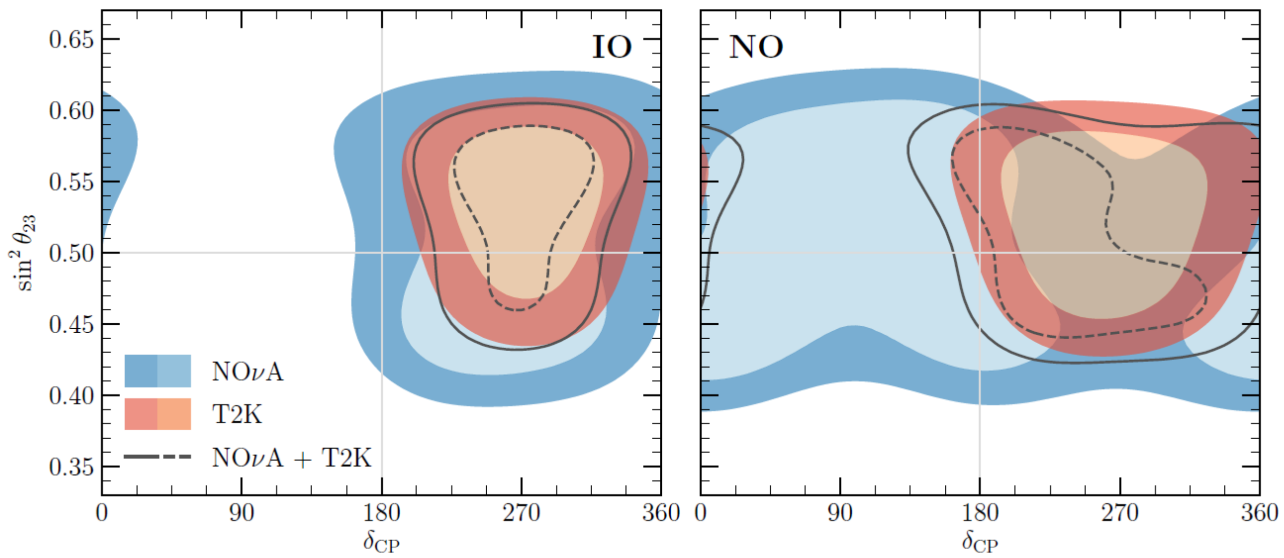
DOI: 10.5281/zenodo.1286857

Characteristic values of E and L for experiments performed using various neutrino soures and corresponding range of $|\Delta m^2|$

Experiment		L (m)	E (MeV)	$\|\Delta m^2\|$ (eV^2)
Solar		10^{10}	1	10^{-10}
Atmospheric		$10^4 - 10^7$	10^2–10^5	$10^{-1} - 10^{-4}$
Reactor	SBL	$10^2 - 10^3$	1	$10^{-2} - 10^{-3}$
	LBL	$10^4 - 10^5$		$10^{-4} - 10^{-5}$
Accelerator	SBL	10^2	10^3–10^4	> 0.1
	LBL	$10^5 - 10^6$	$10^3 - 10^4$	$10^{-2} - 10^{-3}$

Leptonic CP violation

$\sin^2\theta_{13} = 0.0224,\ \sin^2\theta_{12} = 0.310,\ \Delta m^2_{21} = 7.40\times10^{-5}\ \mathrm{eV}^2$



-For IO, Both NovA & T2K are fitted best at δ~3π/2

-For NO, T2K is fitted best at δ~3π/2 while NovA is fitted bets at δ~0.8π