Does existence of dark matter imply grand unification?

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Based on the collaboration with

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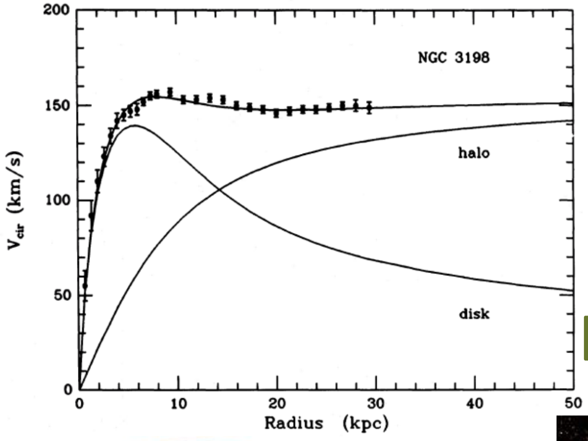
arXiv:2104.13523 [hep-ph] to be published in PRD

Dark matter

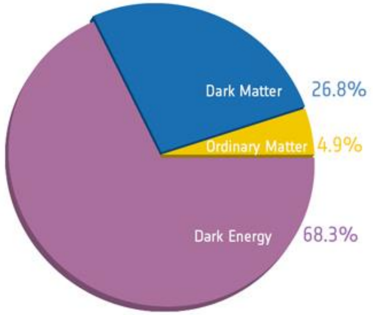
- Dark matter (DM)
  - The existence of dark matter is inferred from various observations.
  - The nature of dark matter is still unknown.
  - Identification of dark matter ⇒ big probe for BSM

$$\boxed{\Omega_{\mathrm{DM}} h^2 = 0.120 \pm 0.001}$$

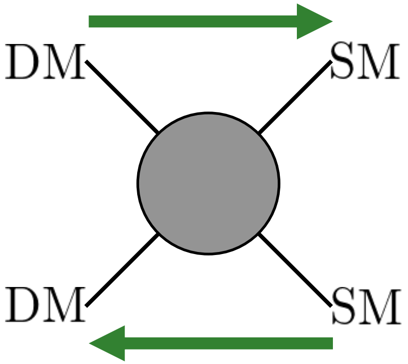
[PLANCK collaboration arXiv:1807.06209]

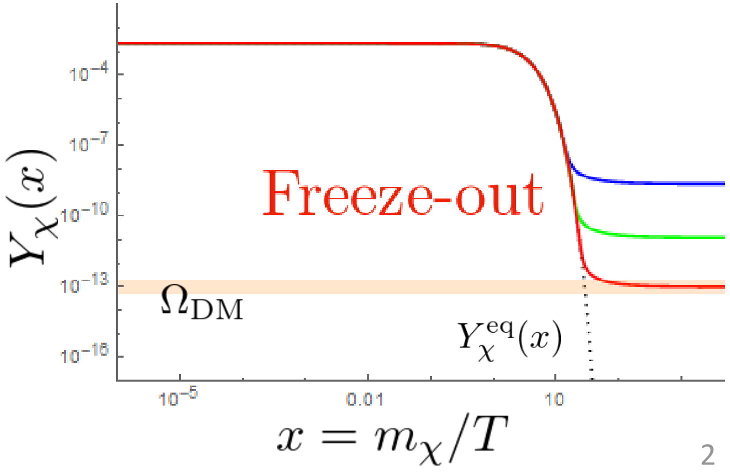


[Albada *et al.* ’85]



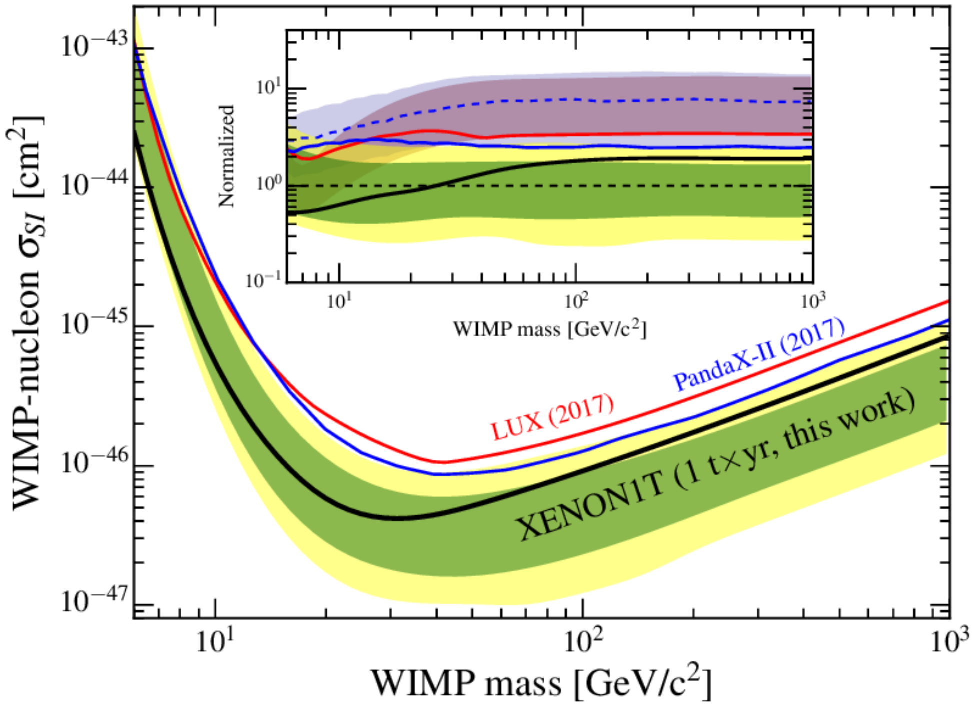
- WIMP DM
  - Dark matter relic abundance is realized as the thermal relic



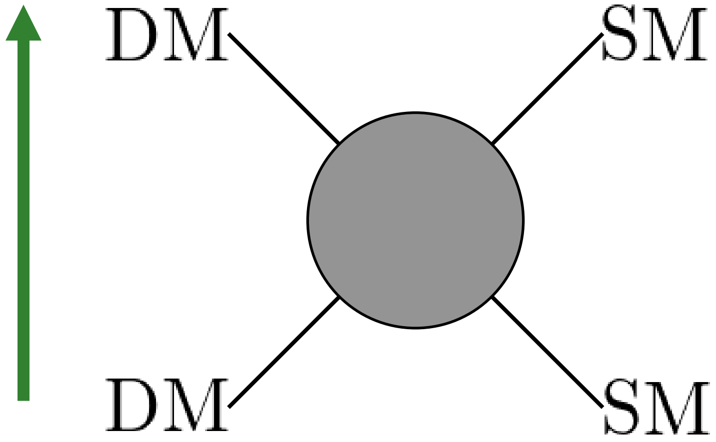


Constraints from direct detection

- Direct detection experiments
  LUX, PandaX-II, XENON
  ⇒ **Severe constraints** on the WIMP nucleon cross section



[XENON collaboration '18]



Nuclear recoil by dark matter

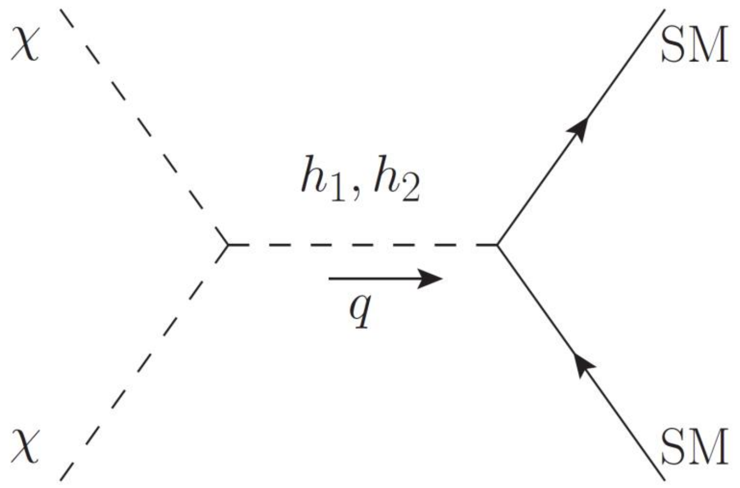
pNGB DM

- Pseudo-Nambu-Goldstone boson (pNGB) DM [Gross-Lebedev-Toma '17]
  - SM + SM singlet scalar S w/ global $U(1)_S$ softly broken to $\mathbb{Z}_2$ by $\boldsymbol{m}^2$

$$V(H,S) = -\frac{\mu_H^2}{2}|H|^2 - \frac{\mu_S^2}{2}|S|^2 + \frac{\lambda_H}{2}|H|^4 + \frac{\lambda_S}{2}|S|^4 + \lambda_{HS}|H|^2|S|^2 - \frac{\boldsymbol{m^2}}{4}(S^2 + S^{*2})$$

  - Breaking of $U(1)_S \Rightarrow$ Nambu-Goldstone boson arises $\mathrm{Im}\, S$
  - This is an attractive WIMP candidate escaping the direct detection constraint naturally

- Scattering between pNGB DM and SM fermions



$$\boxed{i\mathcal{M}_{\chi \mathrm{SM} \to \chi \mathrm{SM}} \sim \frac{\sin\theta\cos\theta(m_{h_1}^2 - m_{h_2}^2)}{v_s m_{h_1}^2 m_{h_2}^2} q^2 \xrightarrow[q^2 \to 0]{} 0}$$

Cf. soft pion theorem

What is origin of pNGB DM?

- What is the UV completion of the simple pNGB DM model?
- Where does the soft breaking term come?

$$V_{\text{soft}}(S) = -\frac{m^2}{4}(S^2 + S^{*2})$$

- Our proposal: pNGB DM from *gauged* $U(1)_{B-L}$ model

[YA-Toma-Tsumura '20, Okada *et al.* '20]

The symmetry of the UV physics (consistent w/ QG) maybe *gauge symmetry*
(discrete symmetry should be ***gauged***)

[Bansk-Seiberg '10, Vafa '05]

DM

	Q_L	u_R^c	d_R^c	L	e_R^c	H	ν_R^c	S	Φ
$SU(3)_C$	**3**	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	**1**	**1**	**1**	**1**	**1**	**1**
$SU(2)_L$	**2**	**1**	**1**	**2**	**1**	**2**	**1**	**1**	**1**
$U(1)_Y$	$+1/6$	$-2/3$	$+1/3$	$-1/2$	$+1$	$+1/2$	0	0	0
$U(1)_{B-L}$	$+1/3$	$-1/3$	$-1/3$	-1	$+1$	0	$+1$	$+1$	$+2$

$$V(H,S,\Phi) \supset -\frac{\mu_c}{\sqrt{2}}(\Phi^* S^2 + \text{h.c.}) \to -\underbrace{\frac{\mu_c}{\sqrt{2}}(\langle\Phi^*\rangle}_{\sim m^2} S^2 + \text{h.c.})$$

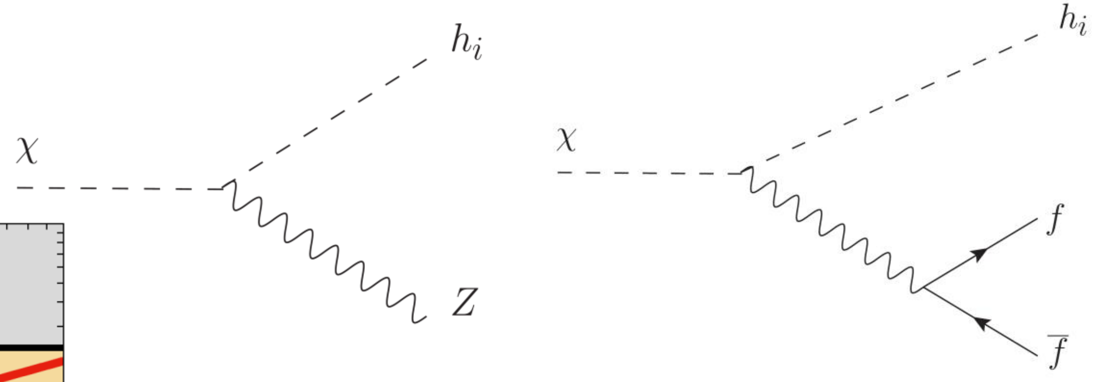
$$\langle\Phi\rangle \sim 10^{13}\ \text{GeV}$$

- Simple pNGB DM is realized as the low energy effective field theory

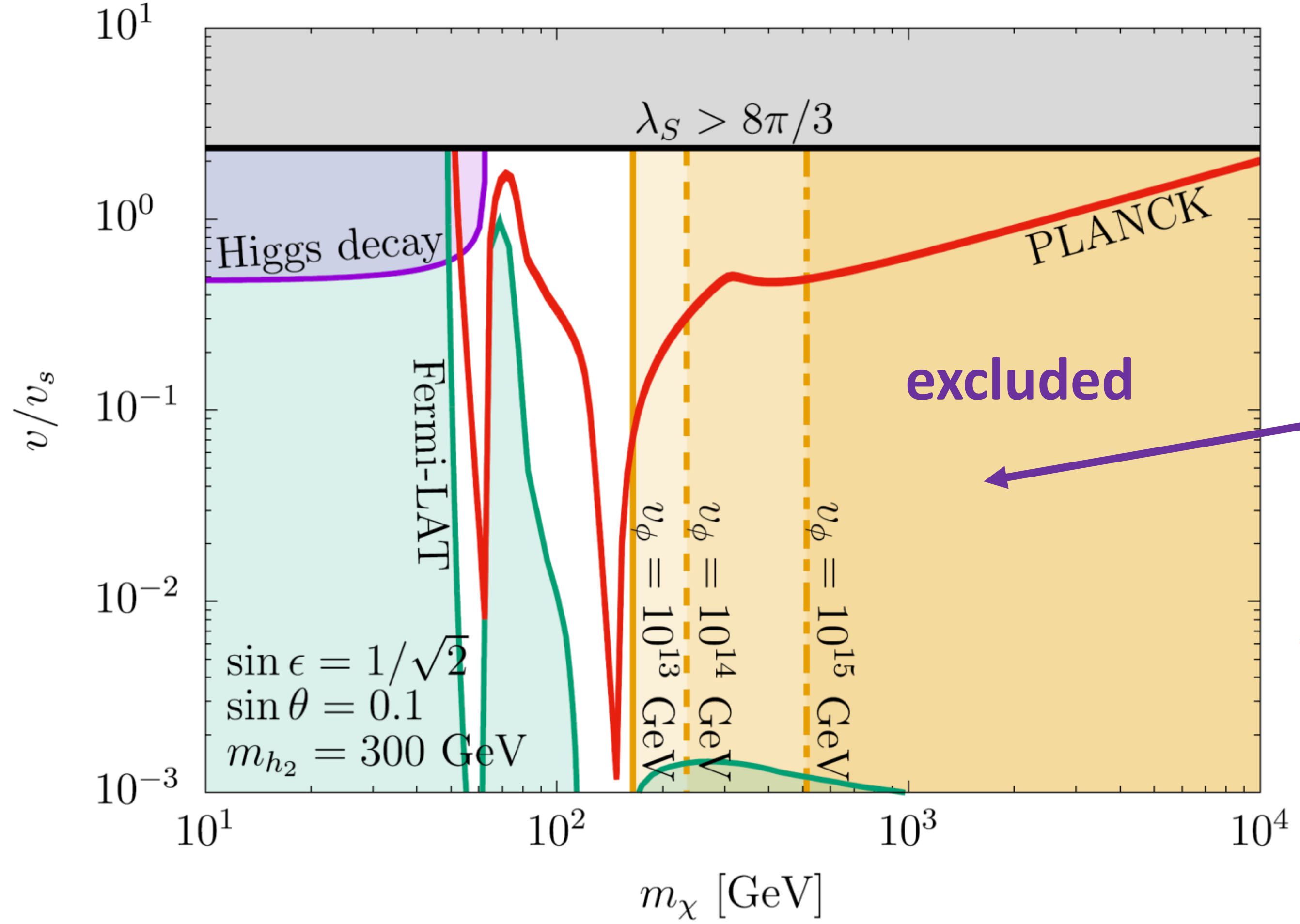
Gauged $U(1)_{B-L}$ model

[YA-Toma-Tsumura '20]

- The new interactions and scalar mixing give the following decay processes



- Allowed region for DM w/ TeV scale mass



Constraint on the DM lifetime

$$\tau_{\rm DM} \gtrsim 10^{27}\ {\rm s}$$

[Baring-Ghosh-Queiroz-Sinha '16]

$$\langle \Phi \rangle \sim 10^{13}\ {\rm GeV}$$

pNGB DM inspired by grand unification

[YA-Toma-Tsumura-Yamatsu ’21]

$SO(10)$ pNGB DM model

- What is the further UV completion of the gauged $U(1)_{B-L}$ pNGB DM model?

[YA-Toma-Tsumura '20, Okada *et al.* '20]

$$\langle \Phi \rangle \sim 10^{13}\ \mathrm{GeV}$$

- Charge quantization of $U(1)_Y$, anomaly cancellation in SM

[Gerogi-Glashow '74]

$\Rightarrow$ Grand unified theory (GUT) $\quad M_U \sim 10^{15-17}\ \mathrm{GeV}$

- The symmetry $G_{\mathrm{SM}} \times U(1)_{B-L}$ can appear in the GUT symmetry breaking pattern

Motivation

Can the pNGB DM model be embedded to GUT?

Does the existence of DM (+ QG assumption?) imply the grand unification?

- In this work, we focus on the Pati-Salam gauge group G_{PS} [Pati-Salam '74]

$$G_{\mathrm{PS}} = SU(4)_C \times SU(2)_L \times SU(2)_R \supset G_{\mathrm{SM}} \times U(1)_{B-L}$$

$SO(10)$ pNGB DM model

[YA-Toma-Tsumura-Yamatsu ’21]

- Particle contents
  and symmetry breaking pattern

$$G_{\rm PS} \supset G_{\rm SM} \times U(1)_{B-L}$$

$$\xrightarrow{\langle \Phi_{\overline{\mathbf{126}}} \rangle \neq 0} G_{\rm SM}$$

$$\xrightarrow{\langle \Phi_{\mathbf{10}} \rangle \neq 0} SU(3)_C \times U(1)_{\rm EM}$$

	$\Psi_{\mathbf{16}}$						$\Phi_{\mathbf{10}}$	$\Phi_{\mathbf{16}}$	$\Phi_{\overline{\mathbf{126}}}$
$SO(10)$	$\mathbf{16}$						$\mathbf{10}$	$\mathbf{16}$	$\overline{\mathbf{126}}$
	$\psi_{(\mathbf{4},\mathbf{2},\mathbf{1})}$		$\psi_{(\overline{\mathbf{4}},\mathbf{1},\mathbf{2})}$				$\phi_{(\mathbf{1},\mathbf{2},\mathbf{2})}$	$\phi_{(\overline{\mathbf{4}},\mathbf{1},\mathbf{2})}$	$\phi_{(\overline{\mathbf{10}},\mathbf{1},\mathbf{3})}$
$G_{\rm PS}$	$(\mathbf{4},\mathbf{2},\mathbf{1})$		$(\overline{\mathbf{4}},\mathbf{1},\mathbf{2})$				$(\mathbf{1},\mathbf{2},\mathbf{2})$	$(\overline{\mathbf{4}},\mathbf{1},\mathbf{2})$	$(\overline{\mathbf{10}},\mathbf{1},\mathbf{3})$
	Q_L	L	u_R^c	d_R^c	e_R^c	ν_R^c	H	S	Φ
$SU(3)_C$	$\mathbf{3}$	$\mathbf{1}$	$\overline{\mathbf{3}}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$SU(2)_L$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$
$U(1)_Y$	$+1/6$	$-1/2$	$-2/3$	$+1/3$	$+1$	0	$+1/2$	0	0
$U(1)_{B-L}$	$+1/3$	-1	$-1/3$	$-1/3$	$+1$	$+1$	0	$+1$	$+2$

- Grand unification determines some free parameters in gauged $U(1)_{B-L}$ model

$$\mathcal{L} = |D_\mu S|^2 + \boxed{|D_\mu \Phi|^2} + i\overline{\nu_{Ri}} \not{D} \nu_{Ri} - \frac{1}{4} X_{\mu\nu}^2 \boxed{- \frac{\sin\epsilon}{2} X_{\mu\nu} B^{\mu\nu}} - y_{ij}^{\nu} \tilde{H}^\dagger \overline{\nu_{Ri}} L_j - \frac{y_{ij}^{\Phi}}{2} \Phi \overline{\nu_{Ri}^c} \nu_{Rj} + \text{h.c.} + \mathcal{L}_{\rm SM} - \boxed{V(H,S,\Phi)}$$

✓ Gauge kinetic mixing
✓ Gauge couplings
✓ $\langle \Phi \rangle$: $U(1)_{B-L}$ breaking scale

$SO(10)$ pNGB DM model

- Gauge kinetic mixing ~ mixing angle (cf. Weinberg angle in the SM)

$$G_{\rm PS} = SU(4)_C \times SU(2)_L \times SU(2)_R \xrightarrow{\langle \Phi_{\overline{\mathbf{126}}} \rangle \neq 0} SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$\cup \qquad\qquad\qquad \cup$$

$$U(1)_{B-L} \quad \times \quad U(1)_R$$

Gauge fields
$$\begin{array}{c} U(1)_Y \\ U(1)_{B-L} \end{array} : \begin{pmatrix} B_\mu \\ X_\mu \end{pmatrix} = \begin{pmatrix} 1 & -\tan\epsilon \\ 0 & 1/\cos\epsilon \end{pmatrix} \begin{pmatrix} B'_\mu \\ C'_\mu \end{pmatrix} : \begin{array}{c} U(1)_R \\ U(1)_{B-L} \end{array}$$

U(1) generator (in GUT)
$$I_Y = \sqrt{\frac{3}{5}} I_{3R} + \sqrt{\frac{2}{5}} I_{B-L}$$

- The gauge kinetic mixing angle is determined as

$$\boxed{\epsilon = -\arctan\sqrt{\frac{2}{3}}}$$

$$\mathcal{L} \supset -\frac{1}{4} B_{\mu\nu}^2 - \frac{1}{4} X_{\mu\nu}^2 - \frac{\sin\epsilon}{2} X_{\mu\nu} B^{\mu\nu}$$

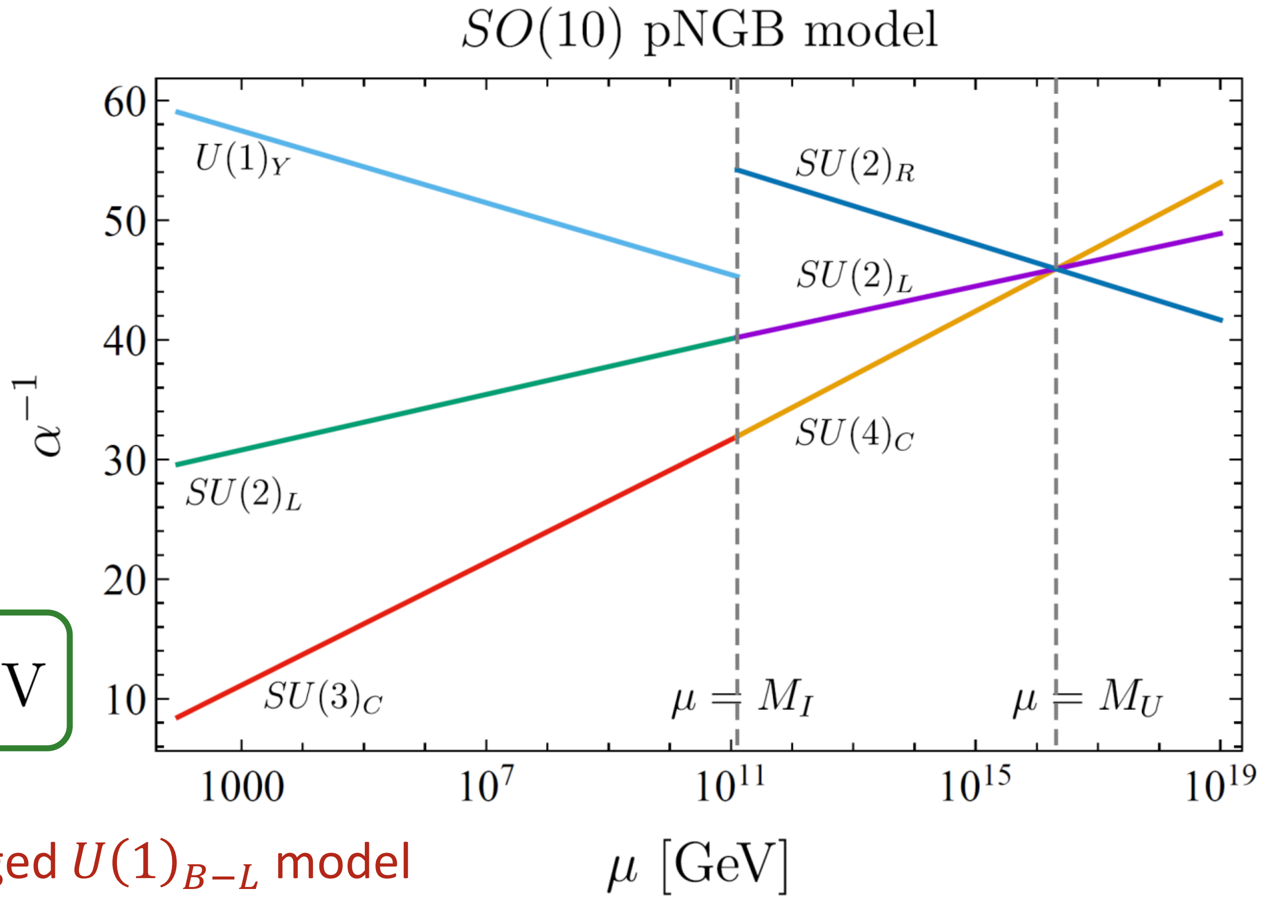
Gauge coupling unification

$$\frac{d}{d\log\mu}\alpha_i^{-1}(\mu) = -\frac{b_i}{2\pi}$$

$\mu \le M_I$

$$\begin{pmatrix} b_{3C} \\ b_{2L} \\ b_{1Y} \end{pmatrix} = \begin{pmatrix} -7 \\ -19/6 \\ +41/10 \end{pmatrix}$$

$$\boxed{M_I = 1.26 \times 10^{11}\ \mathrm{GeV}}$$



$M_I \le \mu \le M_U$

$$\begin{pmatrix} b_{4C} \\ b'_{2L} \\ b_{2R} \end{pmatrix} = \begin{pmatrix} -22/3 \\ -3 \\ +13/3 \end{pmatrix}$$

$$\boxed{M_U = 2.06 \times 10^{16}\ \mathrm{GeV}}$$

$$\boxed{\alpha_U^{-1} = 45.92}$$

Smaller than that in gauged $U(1)_{B-L}$ model
→ DM tends to be short-lived.

Cf. proton decay
[Super-Kamiokande Collaboration '10]

Long-lived DM in $SO(10)$ pNGB DM model

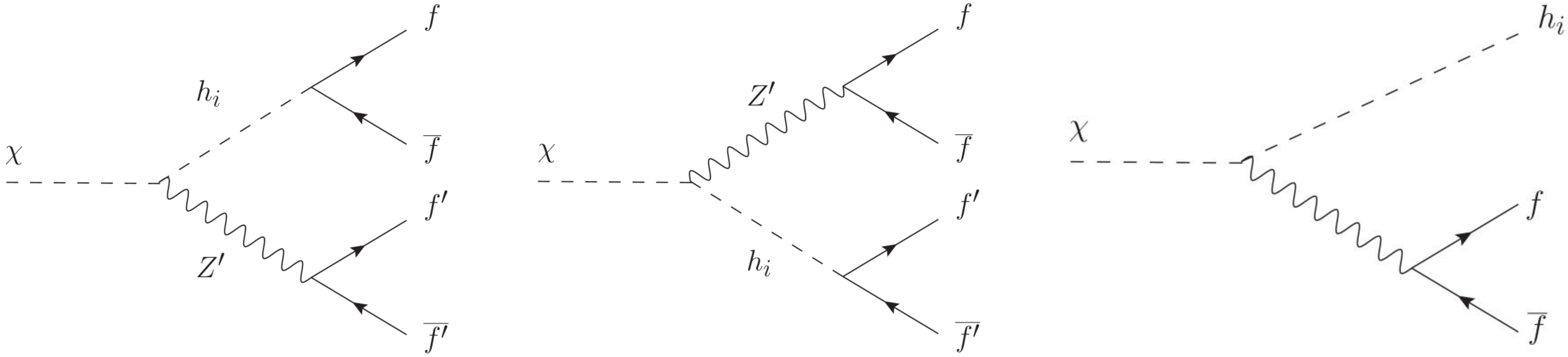
- Long-lived DM in $SO(10)$ pNGB DM model

$$M_I = 1.26 \times 10^{11}\ \text{GeV}$$

DM should be lighter than that in the previous work.

$$\langle \Phi \rangle \sim 10^{13}\ \text{GeV}$$

- Four-body decays and three-body decay of pNGB DM



- DM lifetime constraint

$$\boxed{\tau_{\text{DM}} \gtrsim 10^{27}\ \text{s}}$$

[Baring-Ghosh-Queiroz-Sinha '16]

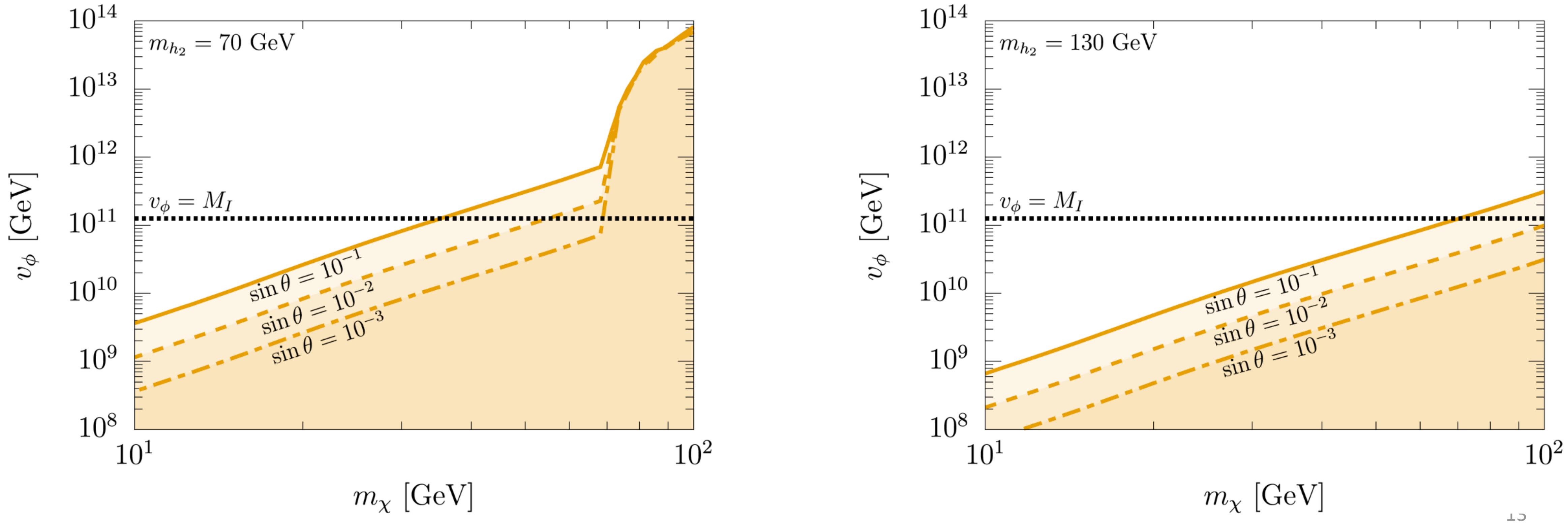
Long-lived DM in $SO(10)$ pNGB DM model

- The typical DM mass $\mathcal{O}(10^{1-2})$ GeV

$$\begin{pmatrix} h \\ s \end{pmatrix} \approx \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

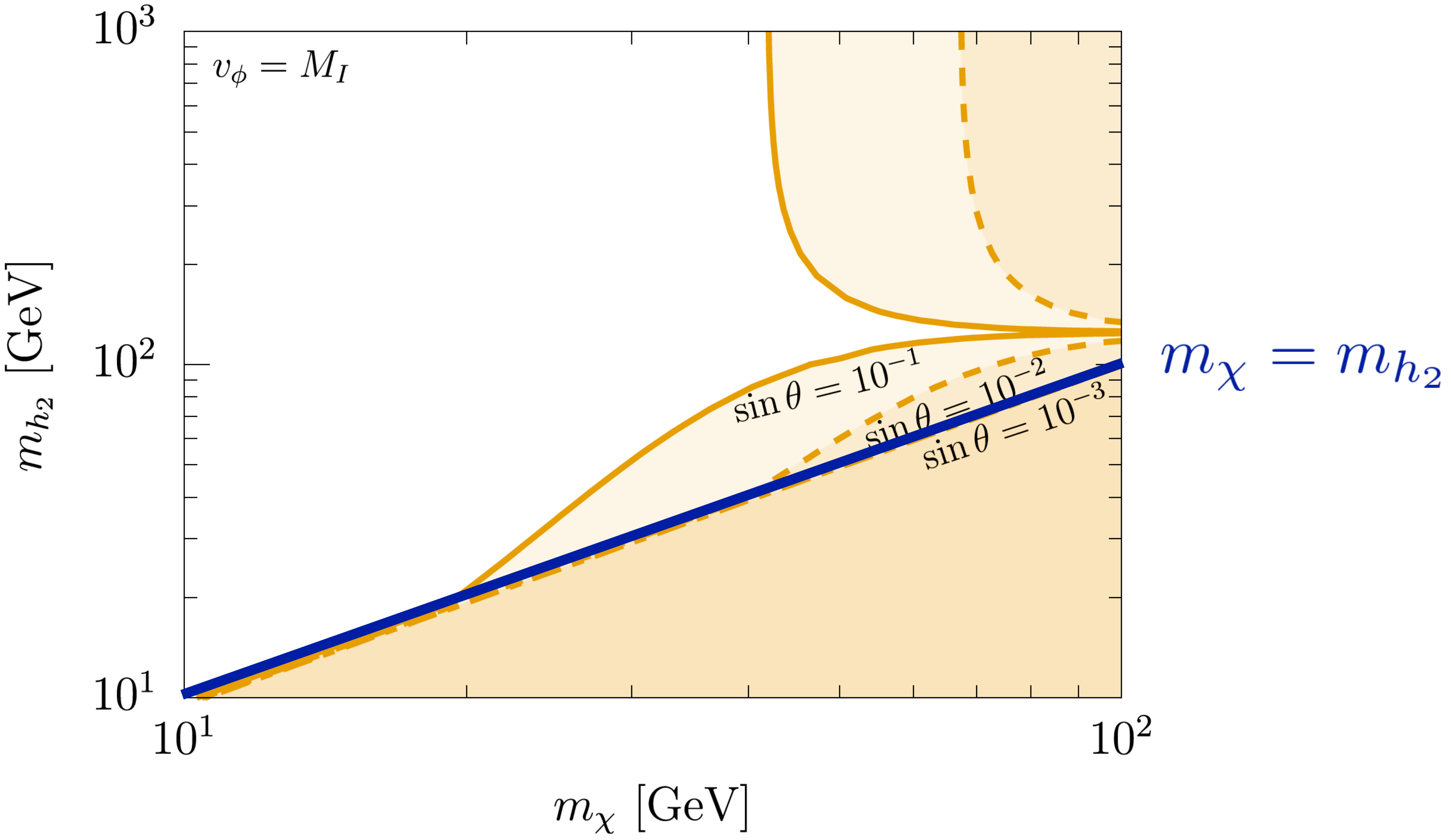
θ: mixing angle of CP-even scalars

- Parameter space in (m_χ, v_ϕ)-plane



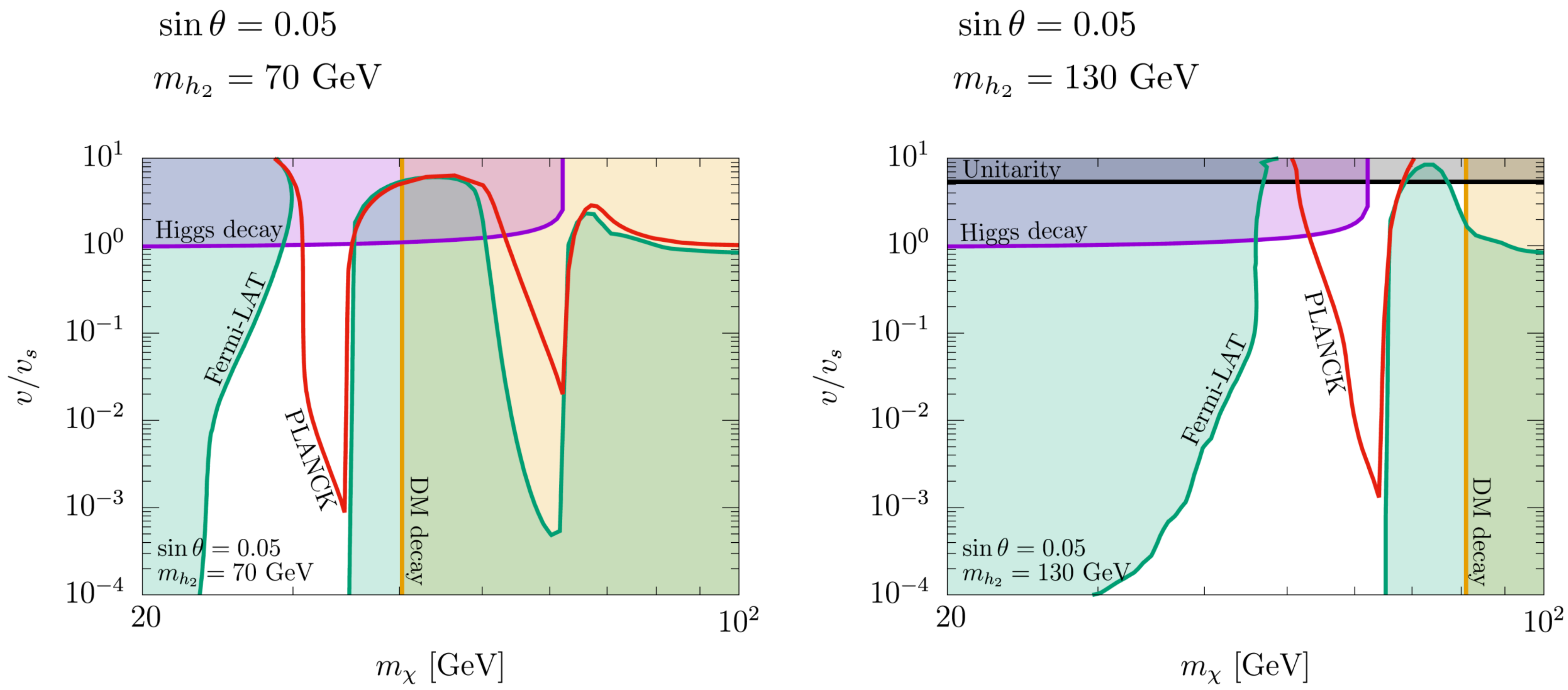
Long-lived DM in $SO(10)$ pNGB DM model

- (m_χ, m_{h_2})-plane



Parameter space of $SO(10)$ pNGB DM model

- Allowed region in $(m_\chi, v/v_s)$ plane realizing the DM relic abundance



Summary

- We proposed an $SO(10)$ pNGB DM model in the framework of GUTs

- The gauged $U(1)_{B-L}$ pNGB DM model can be embedded to $SO(10)$ GUT

$$H \subset \Phi_{\mathbf{10}}, \quad S \subset \Phi_{\mathbf{16}}, \quad \Phi \subset \Phi_{\overline{\mathbf{126}}}$$

- The grand unification condition requires the following relations

$$\sin\epsilon = -\sqrt{2/5}, \quad v_\phi \approx 10^{11}\ \mathrm{GeV}, \quad m_\chi \lesssim \mathcal{O}(100)\ \mathrm{GeV}$$

- We find that the thermal relic abundance can be consistent with all the constraints when the DM mass is rather close to the resonances

Backup

Gauged $U(1)_{B-L}$ model

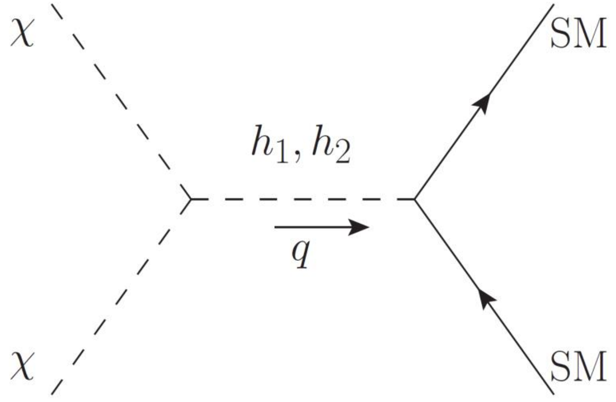
[YA-Toma-Tsumura '20]

- Lagrangian

$$\mathcal{L} = |D_\mu S|^2 + |D_\mu \Phi|^2 + i\overline{\nu_{Ri}} \not{D} \nu_{Ri} - \frac{1}{4} X_{\mu\nu}^2 \boxed{- \frac{\sin\epsilon}{2} X_{\mu\nu} B^{\mu\nu}} - y_{ij}^{\nu} \tilde{H}^\dagger \overline{\nu_{Ri}} L_j - \frac{y_{ij}^{\Phi}}{2} \Phi \overline{\nu_{Ri}^c} \nu_{Rj} + \text{h.c.} + \mathcal{L}_{\text{SM}} - V(H, S, \Phi)$$

Gauge kinetic mixing of $U(1)_Y$ and $U(1)_{B-L}$

- Intuitive story of our gauged $U(1)_{B-L}$ model



Energy scale

$|d\Phi + 2ig_{B-L}X|^2$

$\langle\Phi\rangle = v_\phi \sim 10^{13}$ GeV: $U(1)_{B-L}$ is broken by v_ϕ

$\tilde{\chi}$ is eaten by $X_\mu \Rightarrow X_\mu$ becomes massive $m_X \sim v_\phi$

Large VEV hierarchy ~ heavy particles decouple X_μ, ϕ

$$\boxed{i\mathcal{M} \propto -\frac{\sin\theta\cos\theta(m_{h_1}^2 - m_{h_2}^2)}{v_s m_{h_1}^2 m_{h_2}^2} q^2 + \mathcal{O}(1/v_\phi)}$$

$v_s \sim$ TeV   SM + singlet scalar S with S^2 term

$\Rightarrow$ pNGB dark matter χ + second Higgs h_2 = simple pNGB DM model

$v \sim 246$ GeV

Mass spectrum of scalar fields in gauged $U(1)_{B-L}$ model

- Mass eigenstates

Mixing angle $\tan 2\theta \approx \frac{2vv_s(\lambda_{HS}\lambda_\Phi - \lambda_{H\Phi}\lambda_{S\Phi})}{v^2(\lambda_{H\Phi}^2 - \lambda_H\lambda_\Phi) - v_s^2(\lambda_{S\Phi}^2 - \lambda_S\lambda_\Phi)}$

  - CP-even scalars

$$\begin{pmatrix} h \\ s \\ \phi \end{pmatrix} \approx \begin{pmatrix} 1 & 0 & \frac{\lambda_{H\Phi}v}{\lambda_\Phi v_\phi} \\ 0 & 1 & \frac{\lambda_{S\Phi}v_s}{\lambda_\Phi v_\phi} \\ -\frac{\lambda_{H\Phi}v}{\lambda_\Phi v_\phi} & -\frac{\lambda_{S\Phi}v_s}{\lambda_\Phi v_\phi} & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}$$

h_1: SM-like Higgs boson

$$m_{h_1}^2 \approx \lambda_H v^2 - \frac{\lambda_{H\Phi}^2\lambda_S - 2\lambda_{HS}\lambda_{H\phi}\lambda_{S\Phi} + \lambda_\Phi\lambda_{HS}^2}{\lambda_S\lambda_\Phi - \lambda_{S\Phi}^2} v^2 = 125\ \mathrm{GeV},$$

$$m_{h_2}^2 \approx \frac{\lambda_S\lambda_\Phi - \lambda_{S\Phi}^2}{\lambda_\Phi} v_s^2 + \frac{(\lambda_\Phi\lambda_{HS} - \lambda_{H\Phi}\lambda_{S\Phi})^2}{\lambda_\Phi(\lambda_S\lambda_\Phi - \lambda_{S\Phi}^2)} v^2, \quad m_{h_3}^2 \approx \lambda_\Phi v_\phi^2$$

  - CP-odd scalars

$$\begin{pmatrix} \eta_s \\ \eta_\phi \end{pmatrix} = \frac{1}{(v_s^2 + 4v_\phi^2)^{1/2}} \begin{pmatrix} 2v_\phi & v_s \\ -v_s & 2v_\phi \end{pmatrix} \begin{pmatrix} \chi \\ \tilde{\chi} \end{pmatrix}$$

χ: pNGB

$\tilde{\chi}$ is NGB eaten by the new gauge boson

$$m_\chi^2 = \frac{\mu_c(v_s^2 + 4v_\phi^2)}{4v_\phi}$$

If $\mu_c \to 0$, the pNGB becomes massless

Gauge kinetic mixing

- Gauge groups $$SU(2)_L \times U(1)_Y \times U(1)_{B-L}$$

- Mixing $$\tilde{V}_{\rm GK} = \begin{pmatrix} 1 & 0 & -\tan\epsilon \\ 0 & 1 & 0 \\ 0 & 0 & 1/\cos\epsilon \end{pmatrix}, \quad \tan 2\zeta = \frac{-m_{\tilde{Z}}^2 \sin\theta_W \sin 2\epsilon}{m_X^2 - m_{\tilde{Z}}^2(\cos^2\epsilon - \sin^2\theta_W \sin^2\epsilon)}$$

$$U_G = \begin{pmatrix} \cos\theta_W & -\sin\theta_W & 0 \\ \sin\theta_W & \cos\theta_W & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\zeta & -\sin\zeta \\ 0 & \sin\zeta & \cos\zeta \end{pmatrix}$$

- Mass eigenstate

$$\begin{pmatrix} B_\mu \\ W_\mu^3 \\ X_\mu \end{pmatrix} = \tilde{V}_{\rm GK} U_G \begin{pmatrix} A_\mu \\ Z_\mu \\ Z'_\mu \end{pmatrix}$$

- Mass eigenvalues

$$m_Z^2 = \frac{1}{2}\left[\overline{M}^2 - \sqrt{\overline{M}^4 - \frac{4 m_{\tilde{Z}}^2 m_X^2}{\cos^2\epsilon}}\right], \quad m_{Z'}^2 = \frac{1}{2}\left[\overline{M}^2 - \sqrt{\overline{M}^4 + \frac{4 m_{\tilde{Z}}^2 m_X^2}{\cos^2\epsilon}}\right],$$

$$\overline{M}^2 := m_{\tilde{Z}}^2(1 + \sin^2\theta_W \tan^2\epsilon) + m_X^2/\cos^2\epsilon$$

Path of $SO(10) \to SU(3)_C \times U(1)_{\mathrm{em}}$

$$
\begin{tikzcd}
SO(10) \arrow[d, "\mathbf{54}, \mathbf{210}"] \arrow[dr, "\mathbf{45}, \mathbf{210}"] & \\
SU(4)_C \times SU(2)_L \times SU(2)_R \arrow[r, "\mathbf{45}, \mathbf{210}"] \arrow[d, "\mathbf{45}, \mathbf{210}"] & SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \arrow[dl, "\mathbf{45}, \mathbf{210}"] \\
SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L} \arrow[d, "\mathbf{16}, \mathbf{126} \Leftrightarrow \Phi_{\mathbf{16}}(\ni S), \Phi_{\overline{\mathbf{126}}}(\ni \Phi)"] & \\
SU(3)_C \times SU(2)_L \times U(1)_Y \arrow[d, "\mathbf{10} \Leftrightarrow \Phi_{\mathbf{10}}(\ni H)"] & \\
SU(3)_C \times U(1)_{\mathrm{em}} &
\end{tikzcd}
$$

Our model

[YA-Toma-Tsumura-Yamatsu '21]

$$
\begin{tikzcd}
SO(10) \arrow[d, "\mathbf{210}"] \\
G_I = G_{\mathrm{PS}} = SU(4)_C \times SU(2)_L \times SU(2)_R \arrow[d, "\overline{\mathbf{126}}, \Phi_{\overline{\mathbf{126}}} \ni \Phi"] \\
G_{\mathrm{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y \arrow[d, "\mathbf{10}, \Phi_{\mathbf{10}} \ni H"] \\
SU(3)_C \times U(1)_{\mathrm{em}}
\end{tikzcd}
$$

$SO(10)$ pNGB DM model

- Particle contents of our model

[YA-Toma-Tsumura-Yamatsu '21]

	Ψ_{16}						Φ_{10}	Φ_{16}	$\Phi_{\overline{126}}$
$SO(10)$	$\mathbf{16}$						$\mathbf{10}$	$\mathbf{16}$	$\overline{\mathbf{126}}$
	$\psi_{(\mathbf{4},\mathbf{2},\mathbf{1})}$		$\psi_{(\overline{\mathbf{4}},\mathbf{1},\mathbf{2})}$				$\phi_{(\mathbf{1},\mathbf{2},\mathbf{2})}$	$\phi_{(\overline{\mathbf{4}},\mathbf{1},\mathbf{2})}$	$\phi_{(\overline{\mathbf{10}},\mathbf{1},\mathbf{3})}$
G_{PS}	$(\mathbf{4},\mathbf{2},\mathbf{1})$		$(\overline{\mathbf{4}},\mathbf{1},\mathbf{2})$				$(\mathbf{1},\mathbf{2},\mathbf{2})$	$(\overline{\mathbf{4}},\mathbf{1},\mathbf{2})$	$(\overline{\mathbf{10}},\mathbf{1},\mathbf{3})$
	Q_L	L	u_R^c	d_R^c	e_R^c	ν_R^c	H	S	Φ
$SU(3)_C$	$\mathbf{3}$	$\mathbf{1}$	$\overline{\mathbf{3}}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$SU(2)_L$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$
$U(1)_Y$	$+1/6$	$-1/2$	$-2/3$	$+1/3$	$+1$	0	$+1/2$	0	0
$U(1)_{B-L}$	$+1/3$	-1	$-1/3$	$-1/3$	$+1$	$+1$	0	$+1$	$+2$

- Symmetry breaking pattern

$$SO(10) \xrightarrow{\langle\Phi_{\mathbf{210}}\rangle\neq 0} \boxed{G_{\mathrm{PS}}(\supset G_{\mathrm{SM}}\times U(1)_{B-L}) \xrightarrow{\langle\Phi_{\overline{\mathbf{126}}}\rangle\neq 0} G_{\mathrm{SM}} \xrightarrow{\langle\Phi_{\mathbf{10}}\rangle\neq 0} SU(3)_C\times U(1)_{\mathrm{EM}}}$$

$$G_{\mathrm{SM}}\times U(1)_{B-L} \xrightarrow{\langle\Phi\rangle\neq 0} G_{\mathrm{SM}} \xrightarrow{\langle H\rangle\neq 0} SU(3)_C\times U(1)_{\mathrm{EM}}$$

[YA-Toma-Tsumura '20]

Gauge kinetic mixing

- E_μ: gauge field of $U(1) \subset SU(4)_C$, W'^3_μ: gauge filed of $U(1) \subset SU(2)_R$
- Lagrangian

$$\mathcal{L} \supset -\frac{1}{4}(E_{\mu\nu})^2 - \frac{1}{4}(W'^3_{\mu\nu})^2 + \frac{1}{2}\left(\frac{v_s^2}{4} + v_\phi^2\right)\left(2g_{B-L}E_\mu - g_R W'^3_\mu\right)^2$$

$$= -\frac{1}{4}(B'_{\mu\nu})^2 - \frac{1}{4}(C'_{\mu\nu})^2 + \frac{1}{2}M_{C'}^2 C'^2_\mu$$

$$= -\frac{1}{4}(B_{\mu\nu})^2 - \frac{1}{4}(C_{\mu\nu})^2 - \frac{\sin\epsilon}{2}B_{\mu\nu}C^{\mu\nu} + \frac{1}{2}M_C^2 C_\mu^2$$

- Relations of gauge fields

$$\begin{pmatrix} B'_\mu \\ C'_\mu \end{pmatrix} = \begin{pmatrix} \cos\epsilon & \sin\epsilon \\ -\sin\epsilon & \cos\epsilon \end{pmatrix}\begin{pmatrix} W'^3_\mu \\ E_\mu \end{pmatrix}, \quad \begin{pmatrix} B_\mu \\ C_\mu \end{pmatrix} = \begin{pmatrix} 1 & -\tan\epsilon \\ 0 & 1/\cos\epsilon \end{pmatrix}\begin{pmatrix} B'_\mu \\ C'_\mu \end{pmatrix}, \quad \begin{pmatrix} W'^3_\mu \\ E_\mu \end{pmatrix} = \begin{pmatrix} \cos\epsilon & 0 \\ \sin\epsilon & 1 \end{pmatrix}\begin{pmatrix} B_\mu \\ C_\mu \end{pmatrix}$$

- Covariant derivative

$$D_\mu \supset \mathrm{i}g_{B-L}E_\mu Q_{B-L} + \mathrm{i}g_R W'^3_\mu I_3^{SU(2)_R} = \mathrm{i}g_{B-L}C_\mu Q_{B-L} + \mathrm{i}g_1 B_\mu Q_Y$$

Beta function coefficient

- Beta function coefficient for vector, Weyl fermion, real, complex scalar fields

$$b_i = -\frac{11}{3}\sum_{\text{vector}} T(R_V) + \frac{2}{3}\sum_{\text{Weyl}} T(R_F) + \frac{1}{6}\sum_{\text{Real}} T(R_{RS}) + \frac{1}{3}\sum_{\text{Complex}} T(R_{CS})$$

- Vector fields => gauge fields $\quad T(R_V) = C_2(G)$
  - $C_2(G)$: Quadratic Casimir invariant of the adjoint representation of G
- $T(R_i)$ is a Dynkin index of the irreducible representation R_i

$\mathfrak{su}_n$ irrep.	$d(R)$	$C_2(R)$	$T(R)$	$A(R)$	$C_c(R)$	C/R/PR
$(0000\cdots0000)$	1	0	0	0	0	R
$(1000\cdots0000)$	n	$\frac{n^2-1}{2n}$	$\frac{1}{2}$	$+1$	1	$\mathrm{C/PR}_{n=2}$
$(0000\cdots0001)$	n	$\frac{n^2-1}{2n}$	$\frac{1}{2}$	-1	$n-1$	$\mathrm{C/PR}_{n=2}$
$(0100\cdots0000)$	$\frac{n(n-1)}{2}$	$\frac{(n+1)(n-2)}{n}$	$\frac{n-2}{2}$	$n-4$	2	$\mathrm{C/R}_{n=4}$
$(0000\cdots0010)$	$\frac{n(n-1)}{2}$	$\frac{(n+1)(n-2)}{n}$	$\frac{n-2}{2}$	$-n+4$	$n-2$	$\mathrm{C/R}_{n=4}$
$(2000\cdots0000)$	$\frac{n(n+1)}{2}$	$\frac{(n-1)(n+2)}{n}$	$\frac{n+2}{2}$	$n+4$	2	$\mathrm{C/R}_{n=2}$
$(0000\cdots0002)$	$\frac{n(n+1)}{2}$	$\frac{(n-1)(n+2)}{n}$	$\frac{n+2}{2}$	$-n-4$	$n-2$	$\mathrm{C/R}_{n=2}$
$(1000\cdots0001)$	n^2-1	n	n	0	0	R
$(0010\cdots0000)$	$\frac{n(n-1)(n-2)}{6}$	$\frac{(n-2)(n-3)(n-4)}{12}$	$\frac{(n-3)(n-2)}{4}$	$\frac{(n-3)(n-6)}{2}$	3	$\mathrm{C/PR}_{n=6}$
$(0000\cdots0100)$	$\frac{n(n-1)(n-2)}{6}$	$\frac{(n-2)(n-3)(n-4)}{12}$	$\frac{(n-3)(n-2)}{4}$	$-\frac{(n-3)(n-6)}{2}$	$n-3$	$\mathrm{C/PR}_{n=6}$
$(1100\cdots0000)$	$\frac{n(n-1)(n+1)}{3}$	$\frac{3(n^2-3)}{2n}$	$\frac{n^2-3}{2}$	n^2-9	3	$\mathrm{C/R}_{n=3}$
$(0000\cdots0011)$	$\frac{n(n-1)(n+1)}{3}$	$\frac{3(n^2-3)}{2n}$	$\frac{n^2-3}{2}$	$-n^2+9$	$n-3$	$\mathrm{C/R}_{n=3}$
$(3000\cdots0000)$	$\frac{n(n+1)(n+2)}{6}$	$\frac{(n+2)(n+3)(n+4)}{12}$	$\frac{(n+2)(n+3)}{4}$	$\frac{(n+3)(n+6)}{2}$	3	$\mathrm{C/PR}_{n=2}$
$(0000\cdots0003)$	$\frac{n(n+1)(n+2)}{6}$	$\frac{(n+2)(n+3)(n+4)}{12}$	$\frac{(n+2)(n+3)}{4}$	$-\frac{(n+3)(n+6)}{2}$	$n-3$	$\mathrm{C/PR}_{n=2}$
$(0200\cdots0000)$	$\frac{n^2(n+1)(n-1)}{12}$	$\frac{n(n^2-16)}{3}$	$\frac{n(n-2)(n+2)}{6}$	$\frac{n(n-4)(n+4)}{3}$	4	$\mathrm{C/R}_{n=4}$
$(0000\cdots0020)$	$\frac{n^2(n+1)(n-1)}{12}$	$\frac{n(n^2-16)}{3}$	$\frac{n(n-2)(n+2)}{6}$	$-\frac{n(n-4)(n+4)}{3}$	$n-4$	$\mathrm{C/R}_{n=4}$

[Yamatsu '15]

Pati-Salam or Left-Right

- Summary of the intermediate and unification scales and unified coupling

Group G_I	Scalars at $\mu = M_I$	b_j	$\log_{10}(M/1[\mathrm{GeV}])$ M_I	$\log_{10}(M/1[\mathrm{GeV}])$ M_U	α_U^{-1}
G_{PS}	$(\mathbf{1},\mathbf{2},\mathbf{2})_{\mathbf{10}}$ $(\overline{\mathbf{4}},\mathbf{1},\mathbf{2})_{\mathbf{16}}$ $(\overline{\mathbf{10}},\mathbf{1},\mathbf{3})_{\overline{\mathbf{126}}}$	$\begin{pmatrix} b_{4C} \\ b'_{2L} \\ b_{2R} \end{pmatrix} = \begin{pmatrix} -\frac{22}{3} \\ -3 \\ +\frac{13}{3} \end{pmatrix}$	11.10 ± 0.08	16.31 ± 0.15	45.92 ± 0.50
$G_{\mathrm{PS}} \times D$	$(\mathbf{1},\mathbf{2},\mathbf{2})_{\mathbf{10}}$ $(\mathbf{4},\mathbf{2},\mathbf{1})_{\mathbf{16}}$ $(\overline{\mathbf{4}},\mathbf{1},\mathbf{2})_{\mathbf{16}}$ $(\overline{\mathbf{10}},\mathbf{1},\mathbf{3})_{\overline{\mathbf{126}}}$ $(\mathbf{10},\mathbf{3},\mathbf{1})_{\overline{\mathbf{126}}}$	$\begin{pmatrix} b_{4C} \\ b'_{2L} \\ b_{2R} \end{pmatrix} = \begin{pmatrix} -4 \\ +\frac{13}{3} \\ +\frac{13}{3} \end{pmatrix}$	13.71 ± 0.03	15.22 ± 0.04	40.82 ± 0.13
G_{LR}	$(\mathbf{1},\mathbf{2},\mathbf{2},0)_{\mathbf{10}}$ $(\mathbf{1},\mathbf{1},\mathbf{2},1)_{\mathbf{16}}$ $(\mathbf{1},\mathbf{1},\mathbf{3},2)_{\overline{\mathbf{126}}}$	$\begin{pmatrix} b'_{3C} \\ b'_{2L} \\ b_{2R} \\ b_{B-L} \end{pmatrix} = \begin{pmatrix} -7 \\ -3 \\ -\frac{13}{6} \\ +\frac{23}{4} \end{pmatrix}$	8.57 ± 0.06	16.64 ± 0.13	46.13 ± 0.41
$G_{\mathrm{LR}} \times D$	$(\mathbf{1},\mathbf{2},\mathbf{2},0)_{\mathbf{10}}$ $(\mathbf{1},\mathbf{1},\mathbf{2},1)_{\mathbf{16}}$ $(\mathbf{1},\mathbf{2},\mathbf{1},1)_{\mathbf{16}}$ $(\mathbf{1},\mathbf{1},\mathbf{3},2)_{\overline{\mathbf{126}}}$ $(\mathbf{1},\mathbf{3},\mathbf{1},-2)_{\overline{\mathbf{126}}}$	$\begin{pmatrix} b'_{3C} \\ b'_{2L} \\ b_{2R} \\ b_{B-L} \end{pmatrix} = \begin{pmatrix} -7 \\ -\frac{13}{6} \\ -\frac{13}{6} \\ +\frac{15}{2} \end{pmatrix}$	10.11 ± 0.04	15.57 ± 0.09	43.38 ± 0.30

Complex **10** rep.