The domain of thermal dark matter candidates

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Based on arXiv: 2105.01263 with Thomas Hambye, Michel Tytgat and Laurent Vanderheyden

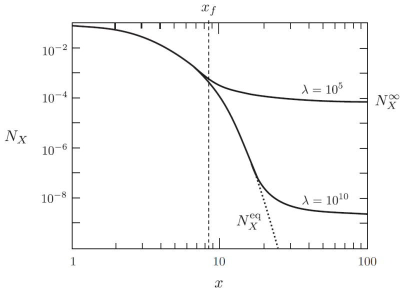
Outline

Introduction

- Understanding the nature of dark matter is an important open question
- Part of the challenge is the vast parameter space, for instance the allowed DM mass spans dozens of orders of magnitude
- One way to refine this parameter space is to consider the implications of the production mechanism

Thermal dark matter

- DM may be produced in many ways, one of the most appealing is freeze-out
- DM thermalises at some point ($\Gamma > H$), then at a later time the interactions drop out of equilibrium ($\Gamma < H$) and the DM yield, $Y_{\rm dm} \equiv n_{\rm dm}/s$, is essentially fixed
- Stronger interactions $\Rightarrow$ later freeze-out $\Rightarrow$ smaller yield (cf. figure from Baumann/Dodelson)



Thermal dark matter equilibrated with the SM

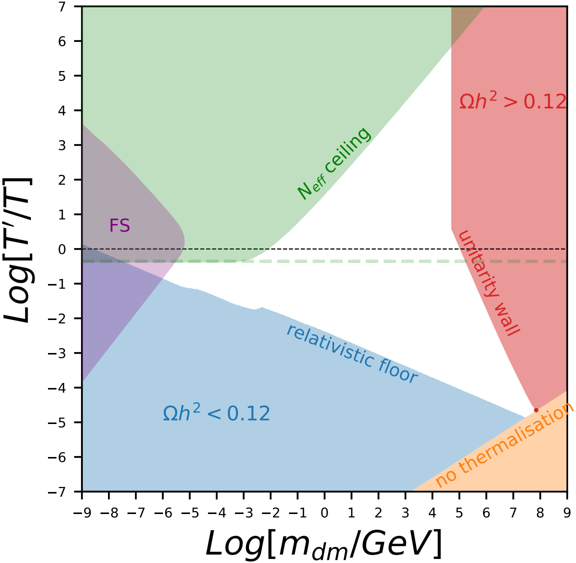
- For DM which entered into equilibrium with the SM, we have:
  - $m_{\rm dm} \gtrsim 100$ eV, since for smaller masses it is underabundant—the Cowsik-McClelland bound [Cowsik+McClelland, PRL **29**, 669 (1972)]
  - $m_{\rm dm} \gtrsim 5.3$ keV from structure formation data, specifically from the Lyman-α forest [Irsic+, PRD **96**, 023522 (2017)]
  - $m_{\rm dm} \gtrsim 3$ MeV from the constraint on $N_{\rm eff}$ [Planck, Astron. Astrophys. **641**, A6 (2020); Fields+, JCAP **03**, 010 (2020)]
  - $m_{\rm dm} \lesssim 300$ TeV from the unitarity limit—the Griest-Kamionkowski bound [Griest+Kamionkowski, PRL **64**, 615 (1990)]
- Thus, the DM mass is limited to the range $[3\ \text{MeV}, 300\ \text{TeV}]$, about 8 orders of magnitude

Thermal dark matter with general T'

- Dark matter may thermalise, but not with the SM
- This scenario broadly needs:
  - Some way of producing some dark sector particle, e.g. through reheating or freeze-in
  - Sufficiently strong dark sector interactions that it thermalises with temperature T', independent of the SM temperature, T
  - Sufficiently feeble dark sector interactions with the SM that the two sectors don't thermalise
- **In this more general scenario, what is the allowed range of DM mass as a function of T'/T?**

Outline

The domain



$\Omega h^2 > 0.12$
N_{eff} ceiling
FS
unitarity wall
relativistic floor
$\Omega h^2 < 0.12$
no thermalisation
$Log[T'/T]$
$Log[m_{dm}/GeV]$

The domain

The domain has five components:

1. The relativistic floor (blue)
2. The unitarity wall (red)
3. The thermalisation ‘cellar’ (orange)
4. The N_{eff} ceiling (green)
5. The free-streaming ‘door’ (purple)

1. Relativistic floor

- The relic DM abundance is proportional to

$$\Omega_{\rm dm} \propto m_{\rm dm} Y_{\rm dm,FO} \qquad (1)$$

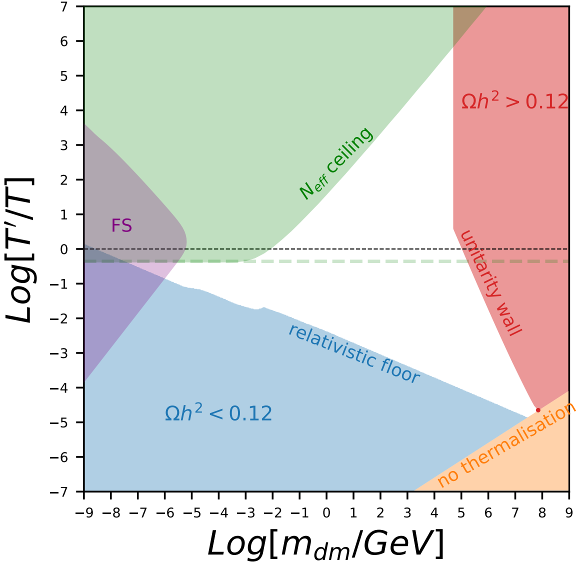
since $Y_{\rm dm,FO} \simeq Y_{\rm dm,0}$

- Given some $\xi \equiv T'/T$, the smallest $m_{\rm dm}$ is obtained for the largest $Y_{\rm dm}$
- Largest thermal number density is relativistic one since there's no Boltzmann suppression, i.e. $n_{\rm dm} \sim T'^3 \Rightarrow Y_{dm,\rm FO} \sim \xi^3$
- In this case, the DM abundance is

$$\Omega_{\rm dm} h^2 = 0.12 \left(\frac{g' m_{\rm dm}}{6 \text{ eV}} \right) \left(\frac{g_{*s,0}}{g_{*s,\rm FO}} \right) \left(\frac{g'_{*s,\rm in}}{g'_{*s,\rm FO}} \right) \xi_{\rm in}^3 \,, \qquad (2)$$

so the floor scales as $m_{\rm dm,min} \propto \xi^{-3}$, cf. [Hambye, Lucca and Vanderheyden, PLB **807**, 135553 (2020)]

The domain



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Log[m_dm/GeV]

2. The unitarity wall

- Since $\Omega_{\rm dm} \propto m_{\rm dm} Y_{\rm dm,FO}$, the largest $m_{\rm dm}$ is obtained for the smallest possible $Y_{\rm dm}$ $\Rightarrow$ annihilate away as much as possible

- What is the largest annihilation cross-section? Unitarity limit,

$$\langle \sigma v \rangle = \frac{\pi}{T'^2} \mathcal{I}_\epsilon(m_{\rm dm}/T') \,, \tag{3}$$

for $J = 0$, where $\mathcal{I}_\epsilon$ is the thermal-averaging integral

- When $m_{\rm dm}/T' \gtrsim 3$, find $\langle \sigma v \rangle \sim \pi / m_{\rm dm}^2$

The unitarity wall for $T' \ll T$

- Taking the instantaneous freeze-out approximation, i.e. a) freeze-out when $\Gamma = H$, and b) $Y_{\rm dm,FO} = Y_{\rm dm,0}$, one finds

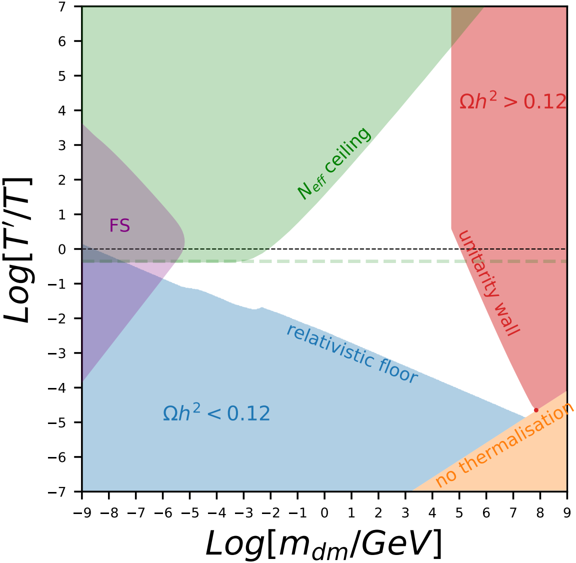
$$\Omega_{\rm dm} h^2 = 4.7 \times 10^8 \frac{g_{\rm dm} g_{*,\rm FO}^{1/2} x'_{\rm FO} \xi}{g_{*s,\rm FO} m_{Pl} \langle \sigma v \rangle \text{GeV}} \tag{4}$$

- Expression agrees well with a full Boltzmann equation when DM undergoes non-relativistic freeze-out, $x'_{\rm FO} \equiv m_{\rm dm}/T'_{\rm FO} \gtrsim 3$
- Heavy DM freezes-out while non-relativistic, so $x'_{\rm FO} \sim 20$ and the unitarity cross-section $\langle \sigma v \rangle \sim \pi / m_{\rm dm}^2$ gives $\Omega_{\rm dm} h^2 \propto \xi m_{\rm dm}^2$
- Thus $m_{\rm dm}$ larger than GK bound is permitted if $\xi < 1$

The unitarity wall for $T' \gg T$

- It's a different story for $T' \gg T$ (or if $g'_* \gg g_*$)
- Key point: for $T' \gtrsim 3T$, the Hubble rate is dominated by the hidden sector, so FO is no longer at $n_{\rm dm}\langle \sigma v \rangle \simeq T^2/m_{Pl}$, instead it's at $n_{\rm dm}\langle \sigma v \rangle \simeq T'^2/m_{Pl}$
- Consequently, the unitarity upper bound is independent of T, hence of ξ: a vertical line in the domain
- The dark sector must decay/annihilate away: how it does this modifies the result

The domain



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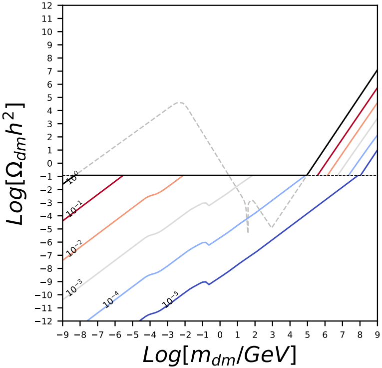
An aside: more on $m_{\rm dm}$

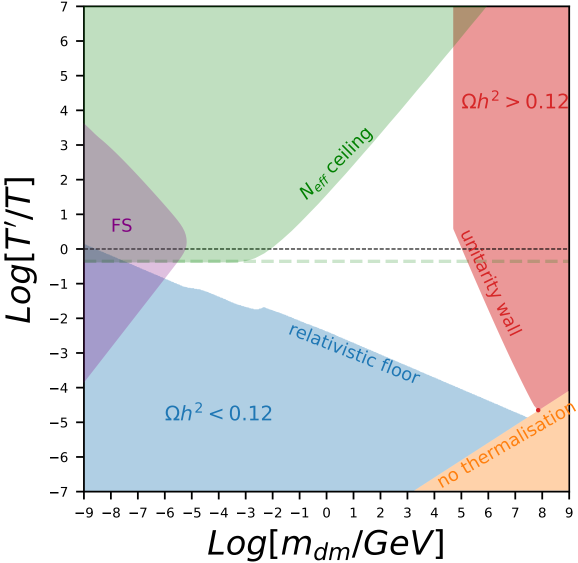
- As $m_{\rm dm} \to 0$ (relativistic FO), we have $\Omega_{\rm dm} \propto m_{\rm dm}$
- As $m_{\rm dm} \to \infty$ (non-relativistic FO), we have $\Omega_{\rm dm} \propto m_{\rm dm}^2$
- Assuming continuity of $\Omega_{\rm dm}(m_{\rm dm})$, must be at least one $m_{\rm dm}$ giving the correct abundance, in general an odd # of solns.
- The allowed range of $m_{\rm dm}$ decreases with ξ



3. The thermalisation cellar

- Assumed the DM sector thermalises—need to check this is true
- DM barely equilibrates if $T'_{\rm eq} \simeq T'_{\rm FO} (\simeq m_{\rm dm})$
- Solved $T'_{\rm eq} = T'_{\rm FO}$ for the unitarity cross-section
- Basic behaviour of this bound is $\xi \propto \sqrt{m_{\rm dm}}$

The domain



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Observational bounds on the domain

- What model-independent constraints can we impose?

- Can't say anything about direct or indirect detection because make no assumption about dark sector interactions with SM, nor about DM self-interactions

- Look towards cosmology, where we may see effects which depend only on DM mass, temperature, etc.

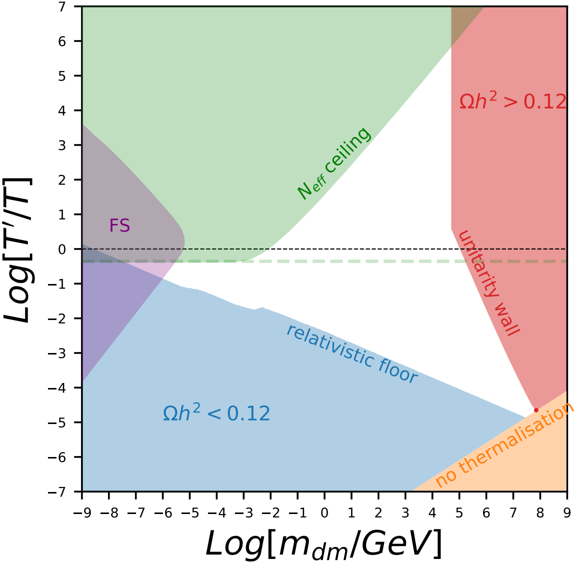
4. The N_{eff} ceiling

- One well-known bound is $\Delta N_{\text{eff}} \lesssim 0.3$ at BBN
- The DM contribution is

$$\Delta N_{\text{eff}} = \frac{60 g_{\rm dm} \xi^4}{7\pi^4} \left(\frac{11}{4}\right)^{4/3} \int_{m_{\rm dm}/T'_{\rm BBN}}^{\infty} dz \frac{z^2 \sqrt{z^2 - m_{\rm dm}^2/T'^2_{\rm BBN}}}{e^z \pm 1} \tag{5}$$

- For $m_{\rm dm} \ll T'_{\text{BBN}} = \xi T_{\text{BBN}}$, have $\Delta N_{\text{eff}} \sim \xi^4$, so require $\xi \lesssim 0.5$ independently of the DM mass
- Larger ξ is possible as long as the DM is non-relativistic, i.e. $m_{\rm dm} \gtrsim T'_{\rm BBN}$

The domain



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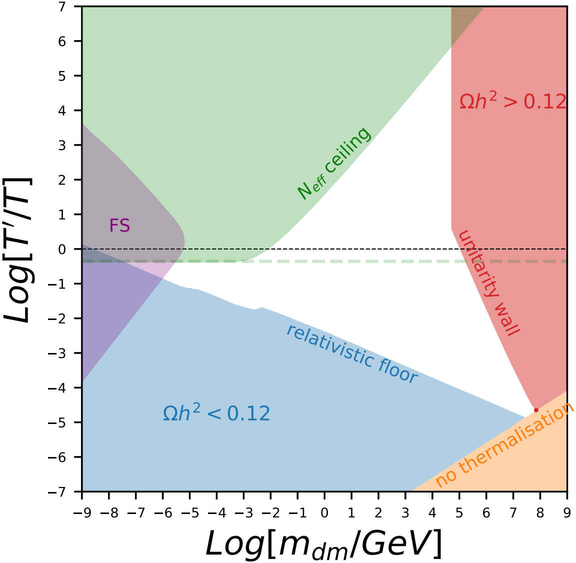
5. The free-streaming door

- Consider DM's role in structure formation, saw that Lyman-α forest data gave $m_{\rm dm} \gtrsim 5.3$ keV for DM thermalised with SM
- Apply this constraint to different ξ via free-streaming horizon: the distance travelled by DM assuming it's collisionless
- Fermionic DM thermalised with SM with mass $m_{\rm dm} \gtrsim 5.3$ keV corresponds to $\lambda_{FS} \lesssim 0.066$ Mpc
- For $\xi \ll 1$, have roughly $\lambda_{FS} \propto \xi/m_{\rm dm}$
- For $\xi \gg 1$, have roughly $\lambda_{FS} \propto 1/(\xi m_{\rm dm})$ due to entropy dilution

The domain



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Summary

- For thermal DM that is a Dirac fermion, find mass range

$$m_{\rm dm} \in [1.0\ \text{keV}, 52\ \text{PeV}]\,, \tag{6}$$

with extrema reached at $\xi \simeq 0.16, 6.9 \times 10^{-5}$

- The temperature ratio range is

$$\xi \in [1.4 \times 10^{-5}, 6.9 \times 10^{5}]\,, \tag{7}$$

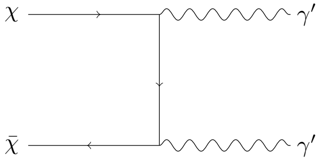
with extrema at $m_{\rm dm} \simeq 50$ TeV, 30 PeV

Outline

1. Introduction
2. Finding the domain
3. A simplified example

t-channel scenario

- Want to see how an explicit simplified model fits into the domain
- Consider scenario with Dirac fermion DM, χ, and massive dark photon, γ': dark QED with DM abundance driven by t-channel annihilation $\chi\bar{\chi} \to \gamma'\gamma'$



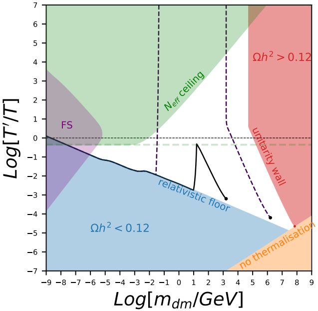
- What values of ξ and $m_{\rm dm}$ give the correct abundance? Can it fill part of the domain, or all of the domain, or go beyond the domain?

t-channel scenario

- Very light $\Rightarrow$ rel. floor; very heavy $\Rightarrow$ parallel to unitarity wall
- Larger α' $\Rightarrow$ closer to unitarity wall
- Don't decouple relativistically when $m_{\rm dm} \gtrsim m_{\gamma'}$, increases of ξ compensates Boltzmann suppression of DM abundance
- For dashed scenario, ξ must increase to $\xi \gg 1$, there is a unique solution in this regime

Solid:
$\alpha' = 3 \times 10^{-4}$, $m_{\gamma'} = 10$ GeV

Dashed:
$\alpha' = 0.1$, $m_{\gamma'} = 30$ MeV



Conclusions

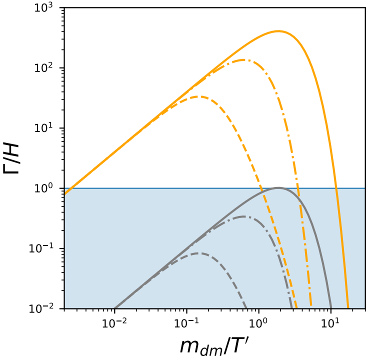
- Considered DM freeze-out for dark sector with temperature T'
- Generalising well-known theoretical and observational bounds, found the allowed DM domain as a function of $m_{\rm dm}$ and T'/T, with $m_{\rm dm} \in [1.0\ \text{keV}, 52\ \text{PeV}]$ and $\xi \in [1.4 \times 10^{-5}, 6.9 \times 10^{5}]$
- Due mainly to relativistic floor and N_{eff} ceiling, allowed mass range shrinks both for small ξ and large ξ
- Useful framework for studying specific scenarios
- Domain can be further shrunk by model-dependent constraints, are there other general bounds?
- Unclear how viable cases with $T' >$ (or $\gg$) T are

Back-up slides

Back-up slides

Thermalisation

- Solid: the DM particles become non-relativistic
- Dot-dashed: the DM annihilates into heavier particles of mass m', here chosen to be $m' = 3m_{\rm dm}$
- Dashed: the DM interacts through some heavy mediator and decouples while still relativistic



A slide on g_*

- Lots of different g_* and g'_* factors throughout this paper, qualitatively not that important
- Note that DM reheating may occur
  1. If DM scattering freezes out relativistically because its annihilation is mediated by some particle with $m_{\rm med} \gg m_{\rm dm}$, then nothing happens to g'_* (this is like SM neutrinos)
  2. If DM freezes out because it scatters into particles that become non-relativistic, then annihilations of this heavier particle reheats DM (like e^+e^- annihilations reheating photons)
- Consequently, we get

$$\xi_{\rm FO} = \left(\frac{g'_{*s,\rm in}}{g'_{*s,\rm FO}} \right)^{1/3} \xi_{\rm in} \tag{8}$$

The unitarity wall

- Since $\Omega_{\rm dm} \propto m_{\rm dm} Y_{\rm dm,FO}$, the largest $m_{\rm dm}$ is obtained for the smallest possible $Y_{\rm dm}$ $\Rightarrow$ annihilate away as much as possible
- What is the largest annihilation cross-section? Unitarity limit,

$$\langle \sigma v \rangle = \frac{\pi}{T'^2} \mathcal{I}_\epsilon(m_{\rm dm}/T')\,, \tag{9}$$

for $J = 0$, where the thermal-averaging integral gives

$$\mathcal{I}_\epsilon(x') = \frac{1}{N_\epsilon^2} \int_{4x'^2}^{\infty} dw \int_{\sqrt{w}}^{\infty} dk_+ \int_{-k_{-,\rm max}}^{k_{-,\rm max}} dk_- \times \frac{\sqrt{w/(w-4x'^2)}}{(e^{(k_+ + k_-)/2} + \epsilon)(e^{(k_+ - k_-)/2} + \epsilon)}\,, \tag{10}$$

and $N_\epsilon = \int_{x'}^{\infty} dk\, k \sqrt{k^2 - x'^2}/(e^k + \epsilon)$ and $k_{-,\rm max} = \sqrt{(1 - 4x'^2/w)(k_+^2 - w)}$

The unitarity wall for $T' \gg T$

- If the hidden sector decays/annihilates to the visible sector via equilibrium processes, entropy is conserved and one can use the general formula

$$m_{\rm dm} \lesssim 35 \text{ TeV } g_{\rm dm}^{1/4} (x'_{\rm FO} \mathcal{I}(x'_{\rm FO}))^{1/2} \tag{11}$$

- If hidden sector $\rightarrow$ visible sector is out-of-equilibrium, energy is conserved but entropy created, in which case the upper bound on the mass increases by a factor,

$$f \simeq \left(\frac{g_{*s}^4}{g'_* g_*^3} \right)^{1/8} , \tag{12}$$

where $g_*^{(\prime)}$ is relativisitic d.o.f. while $g_{*s}^{(\prime)}$ is d.o.f. in entropy

- If $g'_* \sim \mathcal{O}(1)$, then $1 \lesssim f \lesssim 1.6$

Beyond canonical freeze-out

- We have in mind the simplest kind of FO, $\chi\overline{\chi} \to \psi\overline{\psi}$, but what else could happen?
  - Co-annihilation, $\chi\phi \to \psi\eta$, then unitarity limit will be on the heavier of χ, ϕ
  - DM is asymmetric, in which case a larger cross-section is required, and the unitarity bound on the DM mass is stronger
  - DM may be composite, conceivably with $\sigma \sim r_{\rm dm}^2 \gg 1/m_{\rm dm}^2$, see e.g. [Harigaya, Ibe, Kaneta, Nakano and Suzuki, JHEP **08**, 151 (2016)]

- The first two cases are subject to the general bounds we impose, the third is beyond our scope and will not be discussed

The thermalisation cellar

- Assumed the DM sector thermalises—need to check this is true
- At some very high temperature above all relevant scales of the DM interaction, $\Gamma \sim \alpha'^2 T'$, and this entered into equilibrium at

$$T'_{\rm eq} \simeq \alpha'^2 \xi^2 \frac{M_{Pl}}{\sqrt{g_*}} \tag{13}$$

- The value of Γ/H typically peaks around $T' \sim m_{\rm dm}$: for larger T', $H \propto T^2$ grows faster than Γ, while for smaller T' the DM abundance is Boltzmann-suppressed
- DM barely equilibrates if $T'_{\rm eq} \simeq T'_{\rm FO} \sim m_{\rm dm}$

The thermalisation cellar

- Solving for $T'_{\rm eq} = m_{\rm dm}$ gives

$$\xi \simeq \sqrt{\frac{m_{\rm dm} g_*^{1/2}}{\alpha'^2 M_{Pl}}}\,. \tag{14}$$

- For smaller ξ, the DM does not equilibrate
- If there is some heavier mass involved, e.g. some other dark sector mass, m', then the bound is increased by $m_{\rm dm} \to m'$
- Doing this more carefully by solving $T'_{\rm eq} = T'_{\rm FO}$ for the unitarity cross-section gives a similar result

Other dark sector contributions to N_{eff}

- Considering only the DM contribution to N_{eff} gives a conservative limit

- If hidden sector particles decouple from the dark matter and then decay/annihilate back into the SM, there is no problem

- If not, and they are still relativisitic at BBN, we have

$$\xi \lesssim 0.6\, g_{\mathrm{dark,light}}^{-1/4} \,. \tag{15}$$

- For heavier dark sector particles, have an equivalent, perhaps stronger diagonal line, $m_{\mathrm{partner}} \gtrsim T'_{\text{BBN}}$

The free-streaming door

- Formally, free-streaming horizon given by

$$\lambda_{FS} = \int_{t_{\rm FO}}^{t_0} dt \frac{\langle v(t) \rangle}{a(t)} \tag{16}$$

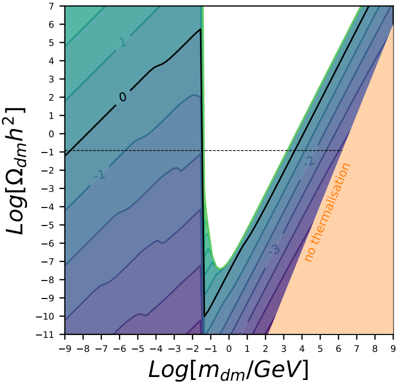
- If time when DM becomes non-relativistic, t_{nr}, is before the time of matter-radiation equality, $t_{eq} = 1.9 \times 10^{11}$s, have

$$\lambda_{FS} = \frac{\sqrt{t_{nr} t_{eq}}}{a_{eq}} \left(5 + \log \frac{t_{nr}}{t_{eq}}\right) \left(\frac{g_{*s,0} + \xi^3 g'_{*s,0}}{g_{*s,\rm FO} + \xi^3 g'_{*s,\rm FO}}\right)^{1/3} \tag{17}$$

- For $\xi \ll 1$, have roughly $\lambda_{FS} \propto \xi / m_{\rm dm}$
- For $\xi \gg 1$, have $t_{nr} \propto 1/H(T'_{\rm nr}) \propto 1/m^2_{\rm dm}$ hence $\lambda_{FS} \propto 1/(\xi m_{\rm dm})$, neglecting the log

t-channel scenario

- Can see how the abundance changes as a function of $m_{\rm dm}$, explicit example of the plot we saw before
- See that there are an odd number of DM candidates in general
- Figure for $\alpha' = 0.1$, $m_{\gamma'} = 30$ MeV



s-channel model

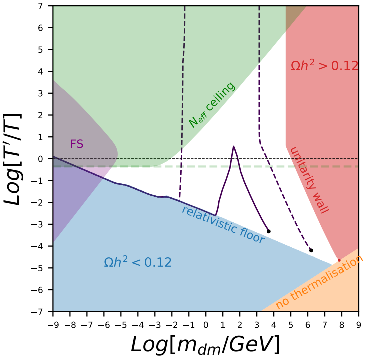
- Introduce Dirac fermion DM, χ, as well as another Dirac fermion, ψ, and scalar, ϕ:

$$\begin{aligned} \mathcal{L}_{\rm HS} &= i\bar{\chi}\partial\!\!\!/\chi - m_{\rm dm}\bar{\chi}\chi + i\bar{\psi}\partial\!\!\!/\psi - m_\psi\bar{\psi}\psi \\ &+ \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}m_\phi^2\phi^2 - y_\chi\phi\bar{\chi}\chi - y_\psi\phi\bar{\psi}\psi \end{aligned} \tag{18}$$

- If $m_\psi \gg m_\chi, m_\phi$, integrate it out and have a similar situation to t-channel, with $\chi\bar{\chi} \to \phi\phi$
- Otherwise, have *s*-channel annihilation, $\chi\bar{\chi} \to \phi \to \psi\psi$, with $\sigma \propto \alpha_x^2/(s - m_\phi^2)^2$, where $\alpha_x \equiv y_\chi y_\psi/(4\pi)$
- This annihilation can have a resonance at $m_{\rm dm} \sim m_\phi$

s-channel model

- As expected, similar behaviour in the large $m_{\rm dm}$ and small $m_{\rm dm}$ limits
- Solid line: $\alpha_x = 0.001$, $m_\chi = 5$ GeV, $m_\phi = 100$ GeV
  Dashed line: $\alpha_x = 0.1$, $m_\psi = 30$ MeV, $m_\phi = 1$



Contact interaction

- Potential problem in previous model (and many models!): what happens to the γ'?
- In some instances its abundance may be comparable to or larger than the DM
- Can it decay away before BBN, but yet have sufficiently weak interactions with the SM that $T' \neq T$? Not sure
- Now consider scenario with scalar DM, Φ, and scalar partner, η,

$$\mathcal{L} \supset -\frac{1}{2}m_{\rm dm}^2\Phi^2 - \frac{\lambda_\Phi}{4!}\Phi^4 - \frac{1}{2}m_\eta^2\eta^2 - \frac{\lambda_\eta}{4!}\eta^4 - \frac{\lambda_{\Phi\eta}}{4}\Phi^2\eta^2\,, \quad (19)$$

  neglecting other quartic terms and imposing a $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry to forbid cubic terms
- DM abundance set by $\Phi\Phi \to \eta\eta$

$2 \leftrightarrow 4$ interactions

- If λ_Φ is large enough, the DM abundance modified by number-violating $2 \leftrightarrow 4$ interactions, see e.g. [Arcadi, Lebedev, Pokorski and Toma, JHEP **08** 050 (2019)] , then concerned with unitarity of this process
- If $m_\eta > m_\Phi$, its abundance is efficiently suppressed by equilibration of $\eta\eta \leftrightarrow \Phi\Phi$ alone
- However, if $m_\eta < m_\Phi$, the η is not Boltzmann suppressed, hence may be too abundant
- Here the $2 \leftrightarrow 4$ process in the η sector may efficiently deplete it

Contact interaction

- Similar behaviour to t-channel model at small and large $m_{\rm dm}$
- Can we deplete the η? Consider $m_\eta = 10$ MeV for concreteness
  - Taking $\lambda_\eta = 10$, find $\Omega_\eta < \Omega_{\rm dm}$ for $\xi \lesssim 0.1$
  - Taking $\lambda_\eta = 10^{-3}$ (η fails to thermalise if λ_η is much smaller), find $\Omega_\eta < \Omega_{\rm dm}$ for $\xi \lesssim 0.03$

Solid:
$\lambda_{\Phi\eta} = 0.01$, $m_\eta = 100$ GeV

Dashed:
$\lambda_{\Phi\eta} = 1$, $m_\eta = 1$ GeV