Sum Rules for Neutrino Parameters

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Mostly based on collaborations with
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Outline

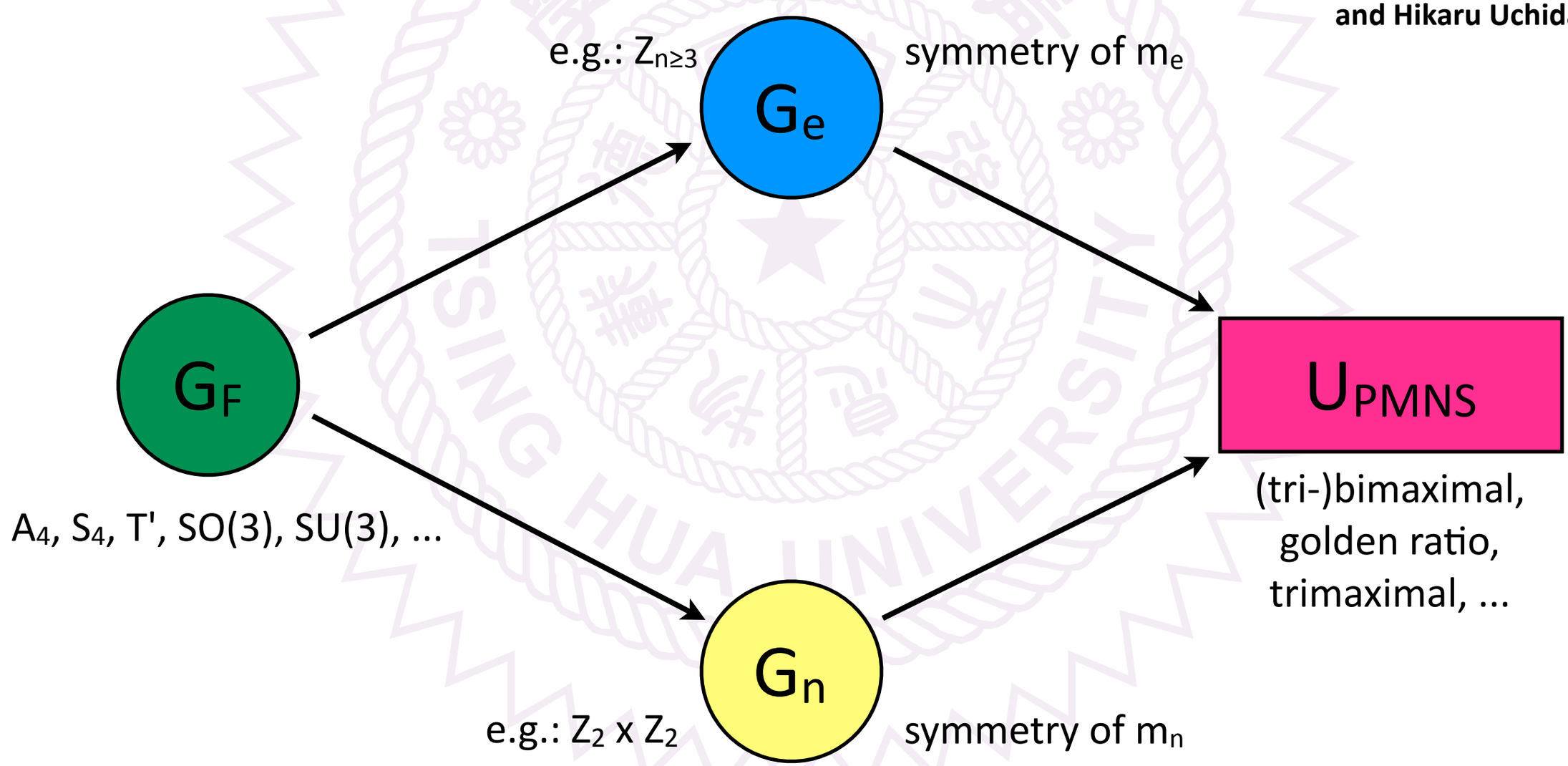
- **Introduction**
- Sum Rules for Lepton Parameters
- Summary and Conclusions

The Flavour Puzzle

- Why do we have three generations?
- Why are the SM fermion masses so vastly different?
- What is the origin of CP violation?
- Why are the quark mixing angles rather small and the leptonic mixing angles rather large?

Non-Abelian, discrete, modular family symmetries

[see also talks on Thursday by Kei Yamamoto, Hajime Otsuka and Hikaru Uchida]



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Sum Rules in the Lepton Sector

- Mixing sum rules, e.g.,

$$\sin^2\theta_{12} \approx \frac{1}{3} + \frac{2\sqrt{2}}{3}\sin\theta_{13}\cos\delta$$

[Antusch, Aranda, Ballett, Boudjemaa, de Medeiros-Varzielas, Ding, Gehrlein, Girardi, Hagedorn, He, Hochmuth, King, Luhn, Malinsky, Marzocca, Masina, Merle, Pascoli, Petcov, Rodejohann, Romanino, Schmidt, Stuart, Tanimoto, Titov, Valle, Volkas, ….?]

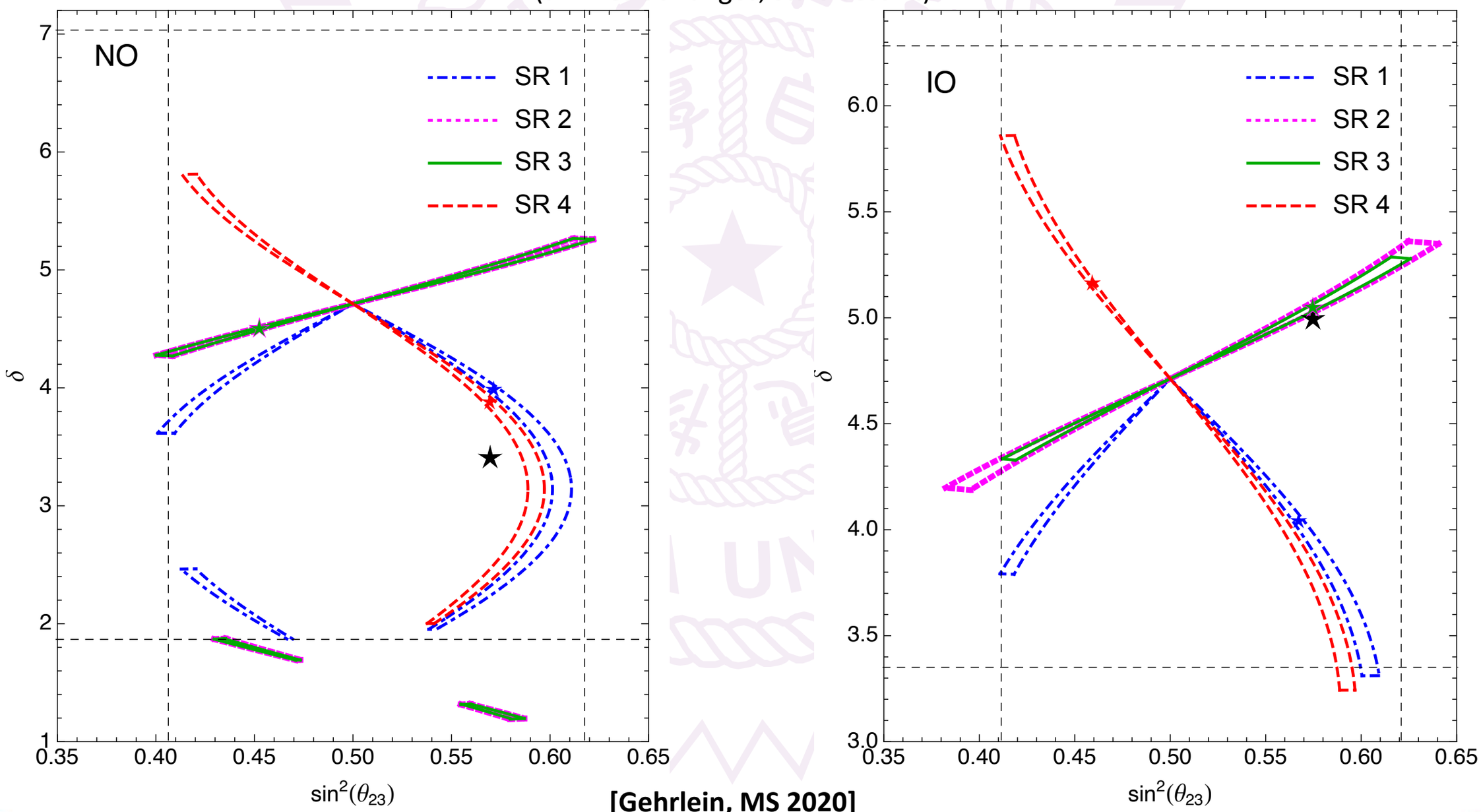
- Mass sum rules, e.g.,

$$m_1 \mathrm{e}^{-\mathrm{i}\,\phi_1} + m_2 \mathrm{e}^{-\mathrm{i}\,\phi_2} - m_3 = 0$$

[Antusch, Aranda, Ballett, Boudjemaa, de Medeiros-Varzielas, Ding, Gehrlein, Girardi, Hagedorn, He, Hochmuth, King, Luhn, Malinsky, Marzocca, Masina, Merle, Pascoli, Petcov, Rodejohann, Romanino, Schmidt, Stuart, Tanimoto, Titov, Valle, Volkas, ….?]

Consequences of Mixing Sum Rules

(all lines 3σ ranges, stars best-fit)



[Gehrlein, MS 2020]

Mass Sum Rules: General Remarks

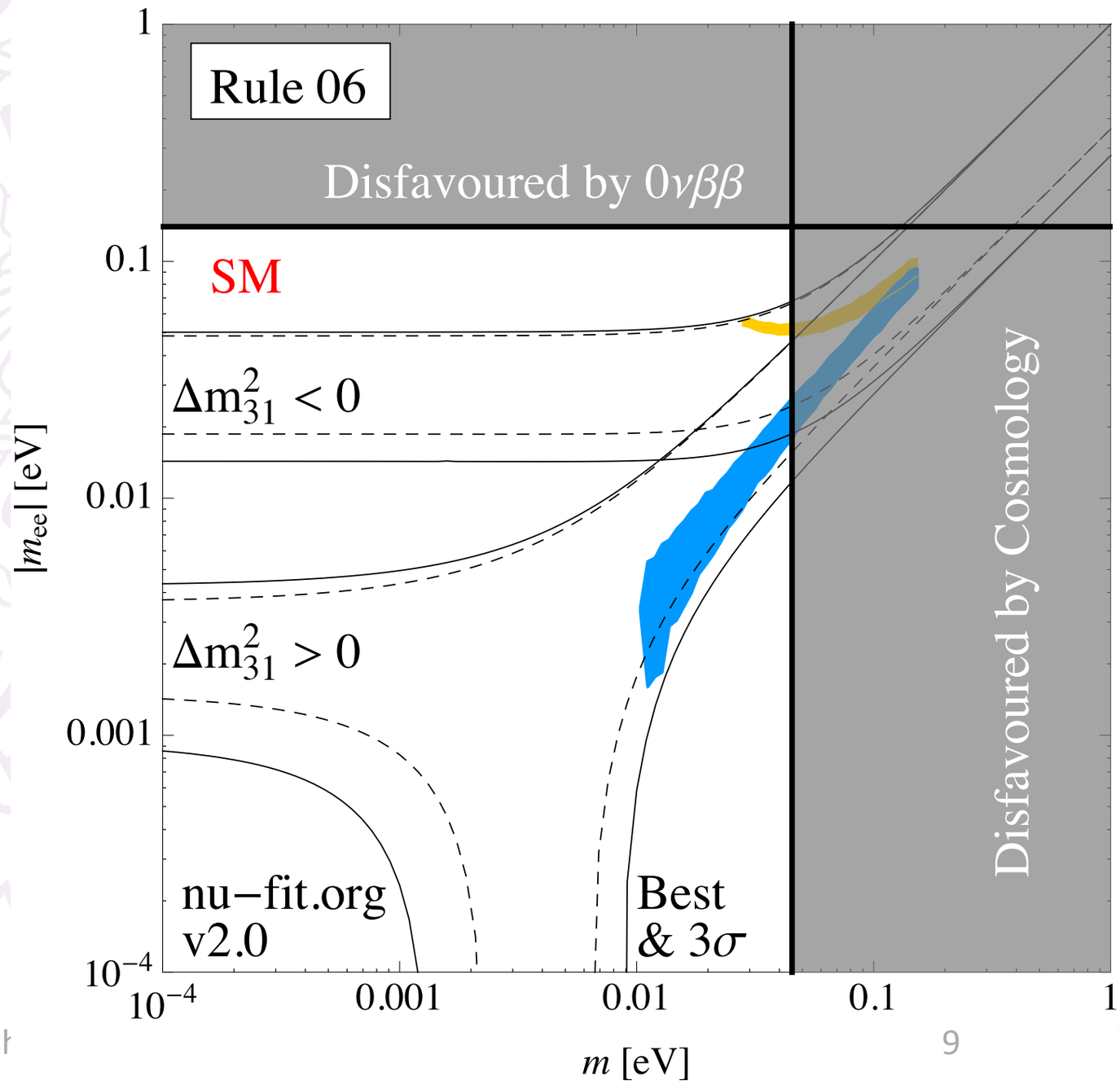
[Gehrlein, MS 2017]

- Not specific to a special family symmetry
- Not related to any residual symmetry
- Not specific to any mass mechanism
- Only in common: 3 complex neutrino masses depend on 2 complex parameters
  ➜ 2 Relations
- Only 12 known for ordinary, non-modular symmetries (about 60 models)

Phenomenological Implications

[Gehrlein, Merle, MS 2015]

$$\frac{e^{i\,\alpha_1}}{m_1} + \frac{e^{i\,\alpha_2}}{m_2} = \frac{1}{m_3}$$

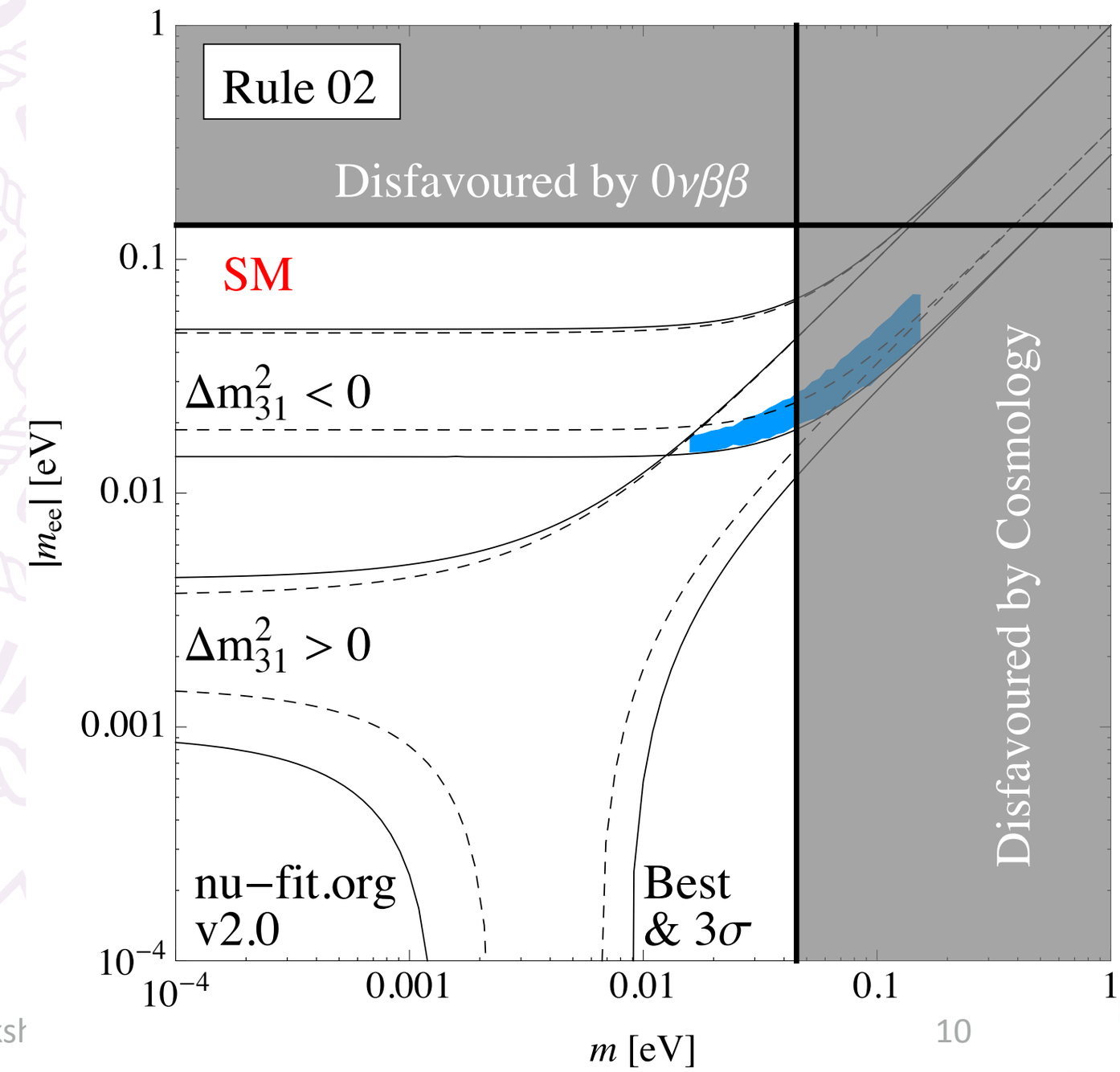


	Sum Rule & IO
	Sum Rule & NO
- - - - -	BF w/o Sum Rule
———	3σ w/o Sum Rule

Phenomenological Implications

[Gehrlein, Merle, MS 2015]

$$m_1 e^{-i\phi_1} + 2m_2 e^{-i\phi_2} = m_3$$



	Sum Rule & IO
	Sum Rule & NO
- - - - -	BF w/o Sum Rule
———	3σ w/o Sum Rule

Modular Family Symmetries

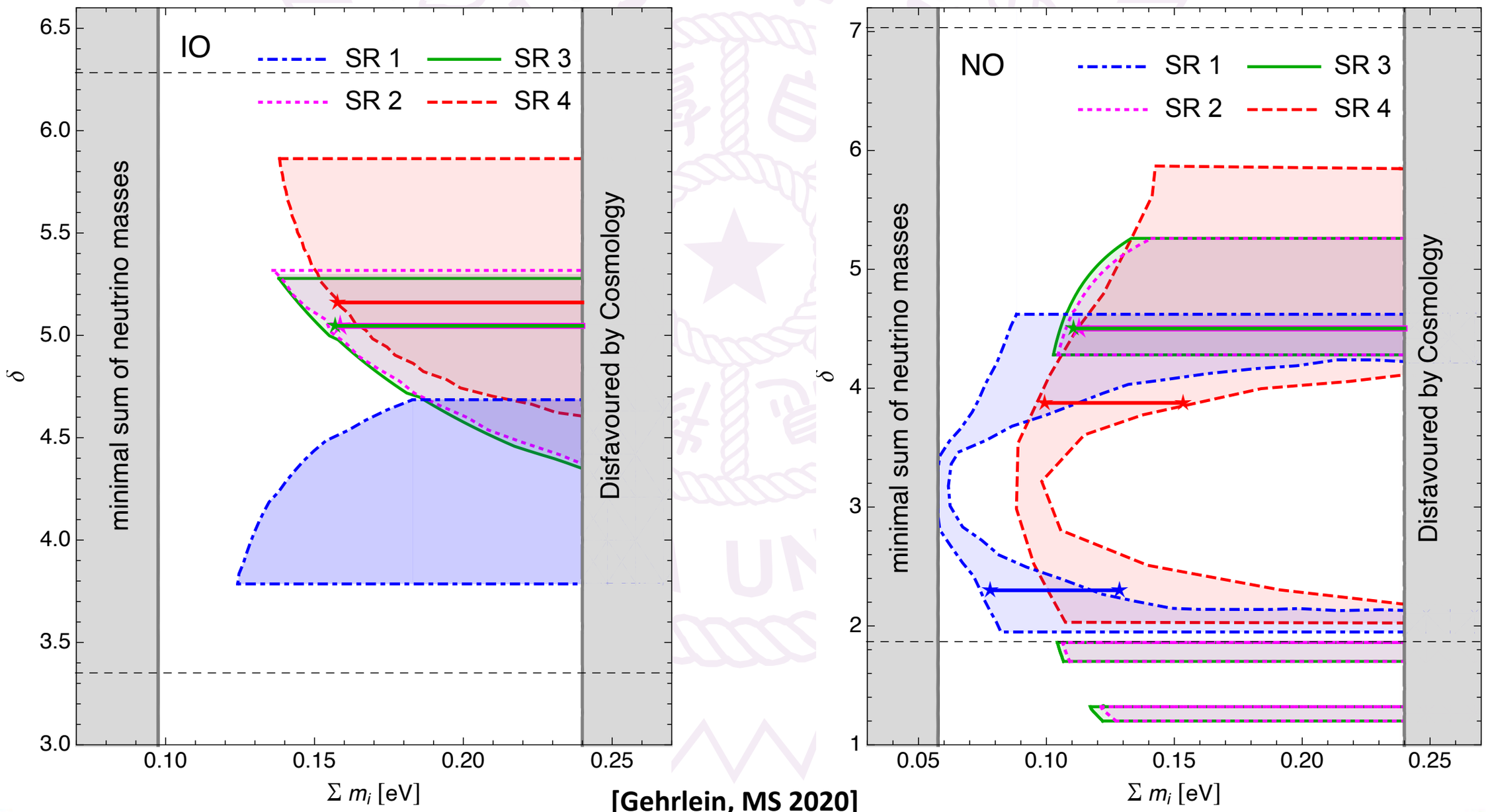
[Gehrlein, MS 2020]

- These models often contain both kinds of sum rules
- The mass sum rules gets mixing dependent coefficients

$$f_1(\theta_{12},\theta_{13},\theta_{23},\delta)(m_1\,\mathrm{e}^{-\mathrm{i}\,\phi_1})^d + f_2(\theta_{12},\theta_{13},\theta_{23},\delta)(m_2\,\mathrm{e}^{-\mathrm{i}\,\phi_2})^d + m_3^d \stackrel{!}{=} 0$$

Consequences of Interplay

(regions 3σ ranges, lines with stars best-fit angles and CP)



[Gehrlein, MS 2020]

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Summary and Conclusions

- Flavour Models predict (relations between) lepton parameters
- In conventional models, you can get
  - Mixing Sum Rules
  - Mass Sum Rules
- In models with modular symmetries
  - Non-trivial combination of both