Hubble Selection of the Weak Scale

Possibility from QCD quantum phase transition

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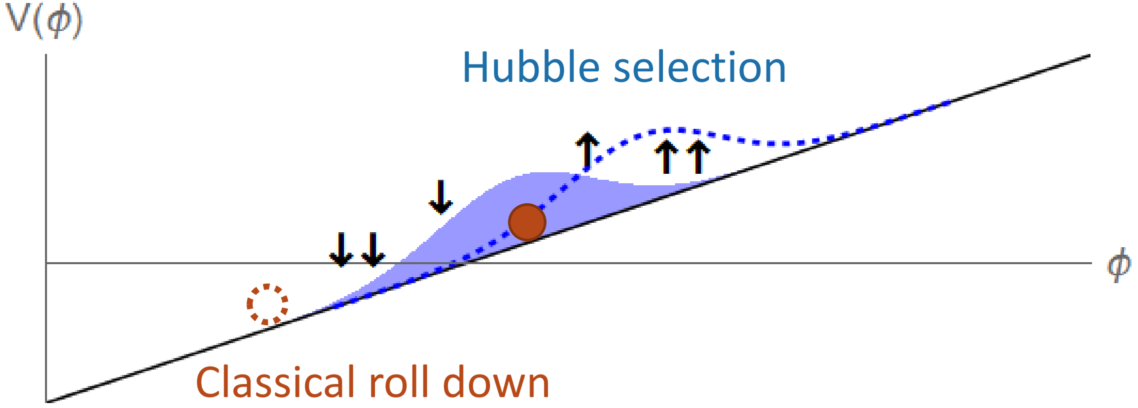
Hubble selection: Concept

- Inflationary fluctuation of quantum field can “kick” the field value toward higher potentials, against the classical rolling.
- Having higher Hubble rate in higher potentials, a global distribution of field values can behave radically different from the field value of each point; “Hubble selection”



Hubble selection: History

- Eternal inflation
  - “Self reproduction of inflating universe”
  - “Every local universe reaches reheating, but the inflation never ends globally”
  - “Dominant volume of the universe is always in inflationary stage”
- Recent developments: stochastic axion, SOL, …

Hubble selection: History

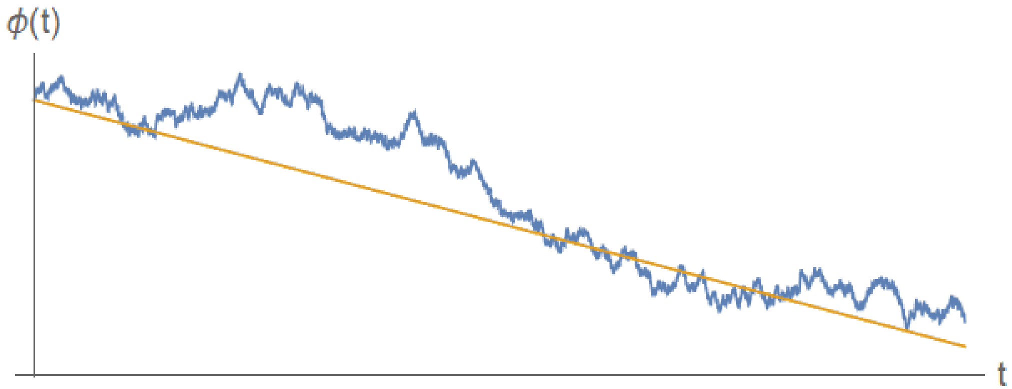
- Self organized localization (SOL) (Giudice et al. 2021)
  - Hubble selection in non-dynamical but equilibrium point of view
  - Driven by Hubble selection (“self organized”), the field value distribution has equilibrium near the boundary of potential (“localization”)
- We give dynamical viewpoint to Hubble selection

Hubble selection: Basic ingredients

- Quantum fluctuation of fields
  - Inflation → de Sitter space time → quantum fluctuations are generated
  - Modes exiting the Hubble horizon (when $\lambda \sim \frac{1}{H}$) “freeze”
  - Spatial field profile: each “Hubble patches” of size $\frac{1}{H}$ acquires its own field value coming from the accumulation of horizon crossed modes
  - Different field values for different patches; we look at patch-by-patch field values

Hubble selection: Basic ingredients

- Stochastic motion of field value of a patch
  - Continuous accumulation of horizon exiting modes
  - "Continuous random kicks" for field value: the Brownian motion
  - $d\phi = -\frac{V'}{3H}\,dt + \sqrt{\frac{H^3}{4\pi^2}}\,dW, \quad \langle dW \rangle = 0, \quad \langle dW^2 \rangle = dt$
    - Classical rolling + stochastic motion



Hubble selection: Basic ingredients

- Distribution of field values among different patches?
  - "Is our universe special?" $\Longleftrightarrow$ "Where in the distribution corresponds to our universe?"
  - Langevin equation $\phi(t)$ $\Longleftrightarrow$ Fokker Planck equation $\rho(\phi, t)$
  - But Hubble expansion comes in…
  - Different $H(\phi)$ for different ϕ's, due to different $V(\phi)$

Hubble selection: FPV equation

- Volume weighted Fokker-Planck (FPV) equation

$$\frac{\partial \rho(\phi, t)}{\partial t} = \frac{\partial}{\partial \phi}\left(\frac{V'}{3H}\rho\right) + \frac{1}{8\pi^2}\frac{\partial^2 (H^3 \rho)}{\partial \phi^2} + 3\Delta H(\phi)\rho$$

Hubble selection: FPV equation

- Volume weighted Fokker-Planck (FPV) equation

$$\overbrace{\frac{\partial\rho(\phi,t)}{\partial t} = \underbrace{\frac{\partial}{\partial\phi}\left(\frac{V'}{3H}\rho\right)}_{\substack{\text{Drift} \\ \text{(classical rolling)}}} + \underbrace{\frac{1}{8\pi^2}\frac{\partial^2(H^3\rho)}{\partial\phi^2}}_{\substack{\text{Diffusion} \\ \text{(quantum fluctuation)}}}}^{\text{Ordinary Fokker Planck eq.}} + 3\Delta H(\phi)\rho$$

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$$V \propto a^3 = e^{3Ht}$$

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Ordinary Fokker Planck eq.

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Drift
(classical rolling)

Diffusion
(quantum fluctuation)

Different Hubble
expansion rate

Greater "proliferation" at
greater Hubble rate

Hubble selection: FPV equation

- Volume weighted Fokker-Planck (FPV) equation

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Greater “proliferation” at greater Hubble rate

Hubble selection

Hubble selection: FPV equation

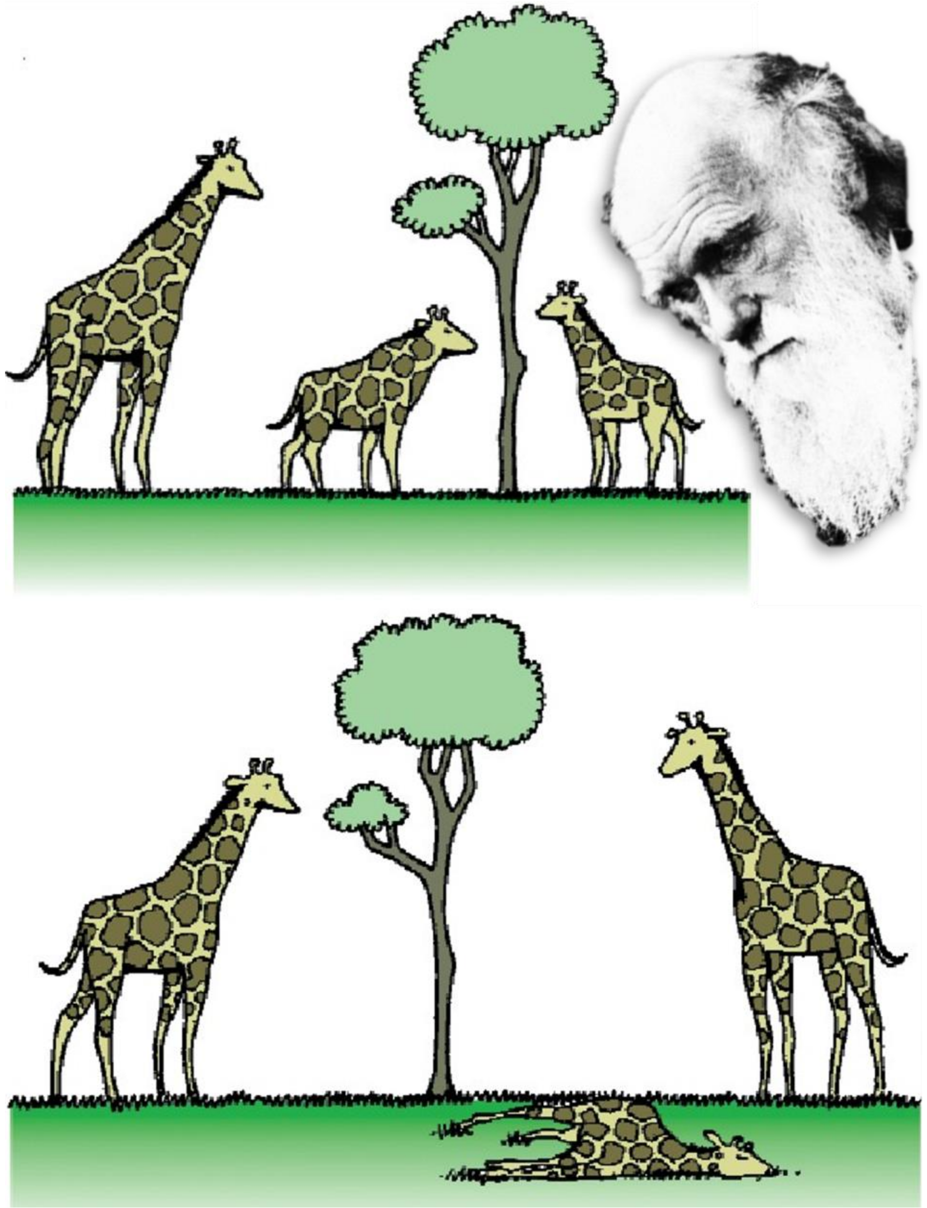
- $V(\phi) = V_0 + \Delta V(\phi) \qquad V_0 \gg \Delta V$
  - Total potential = Sum of inflaton potential (V_0) $+ \phi$ contribution (ΔV)

- $\Delta H(\phi) = \frac{\Delta V(\phi)}{6M_P^2 H_0} \propto \Delta V(\phi) \qquad H_0 = \sqrt{\frac{V_0}{3M_P^2}}$

Hubble selection: Analogy

- Naming comes from “natural selection”

	Natural selection
Applied to	Biological creatures
“Offspring” production	Reproduction
… rate differs by	Adaptation to environment
Diversity comes from	Genetic variation (mutation)
Result	Dominance of genotype with higher reproduction rate



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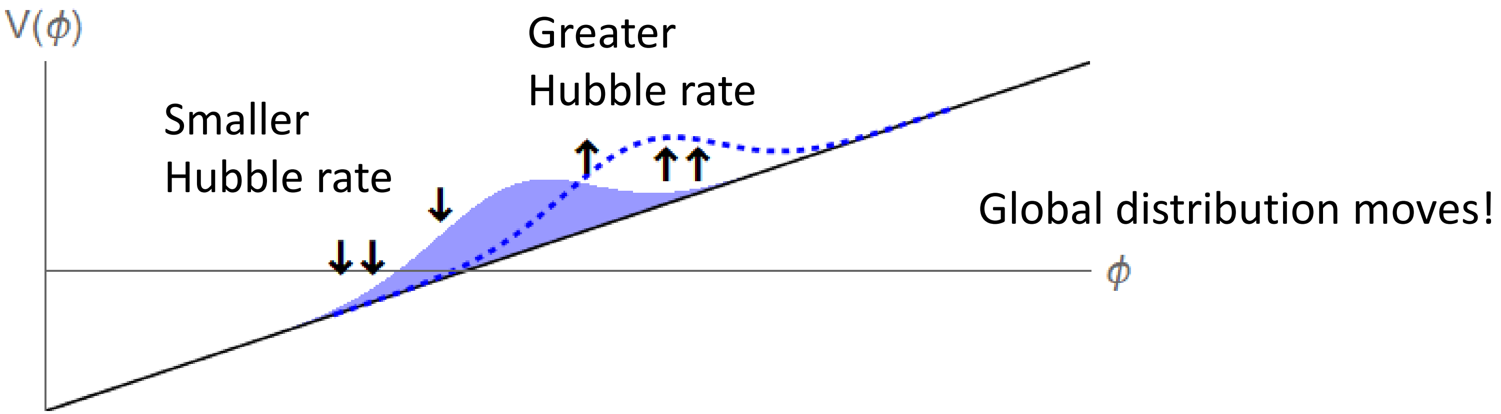
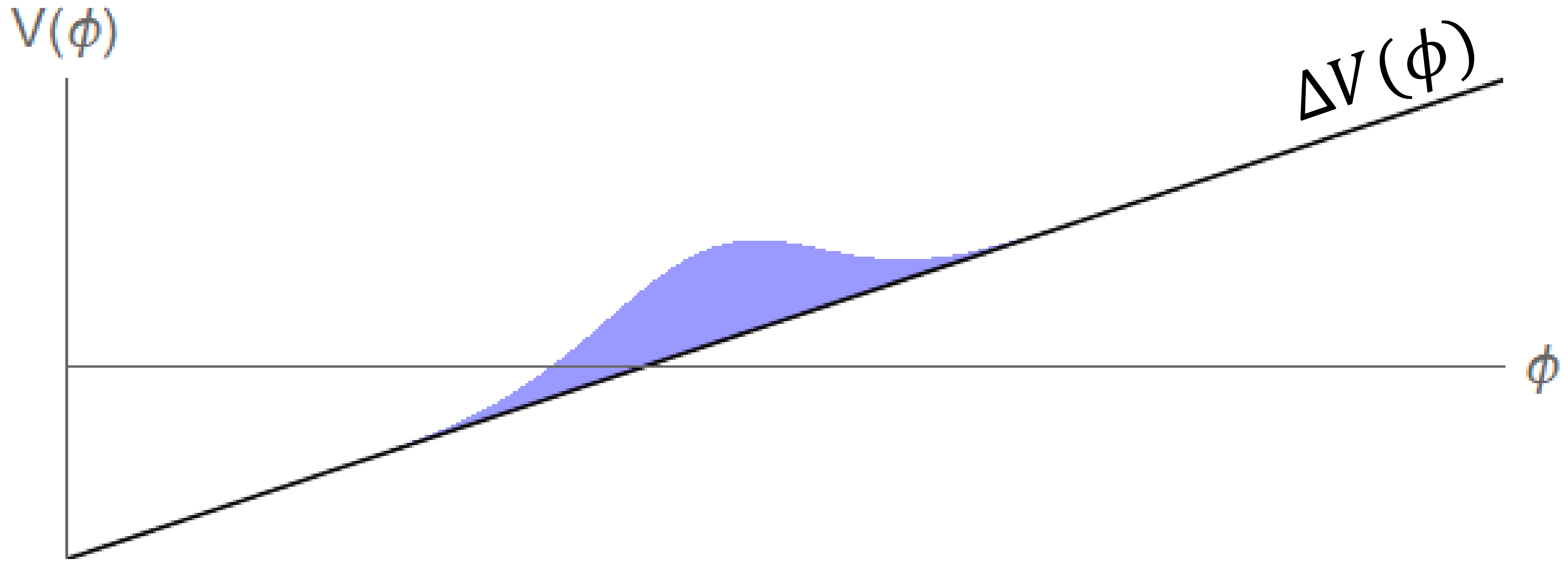
Hubble selection: Analogy

	Natural selection	Hubble selection
Applied to	Biological creatures	Hubble patches
“Offspring” production	Reproduction	Hubble expansion
… rate differs by	Adaptation to environment	$\Delta H(\phi) = \dfrac{\Delta V(\phi)}{6M_P^2 H_0}$
Diversity comes from	Genetic variation (mutation)	Quantum fluctuation
Result	Dominance of genotype with higher reproduction rate	Dominance of field values with higher Hubble rate



Hubble selection: Quantitative analysis

- Quantitative description
  - Classical rolling vs Hubble selection: inevitable competition



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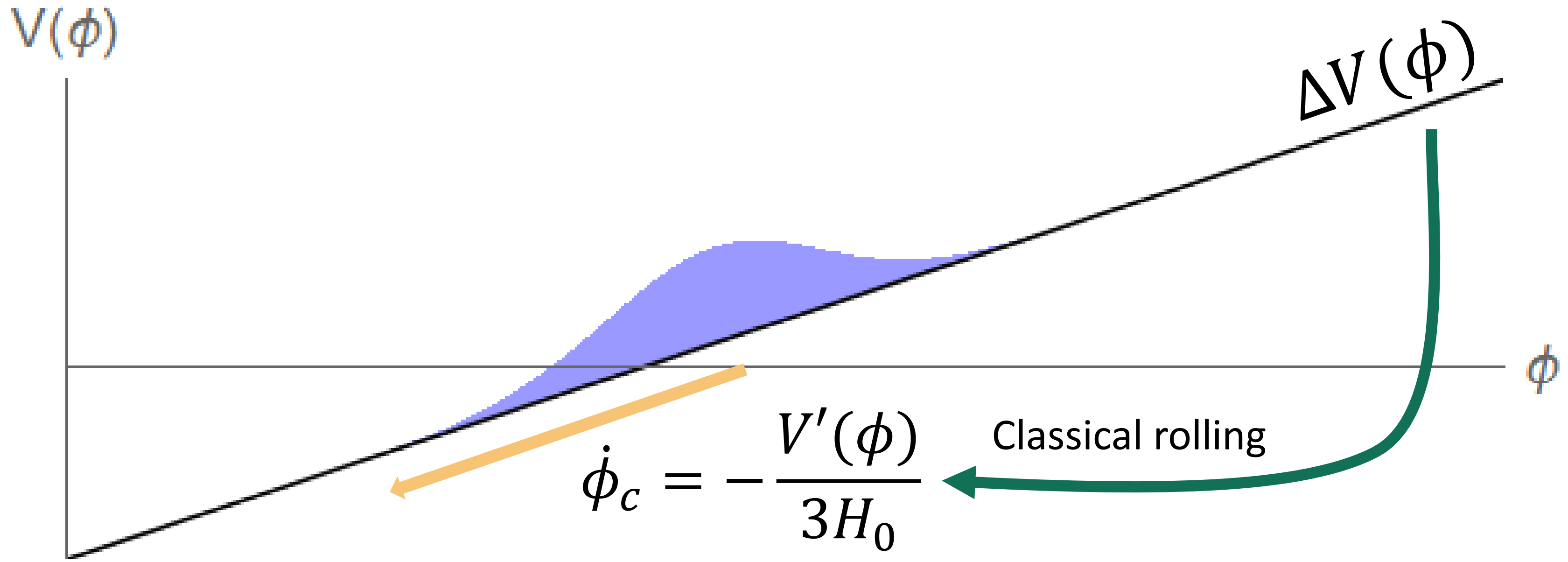

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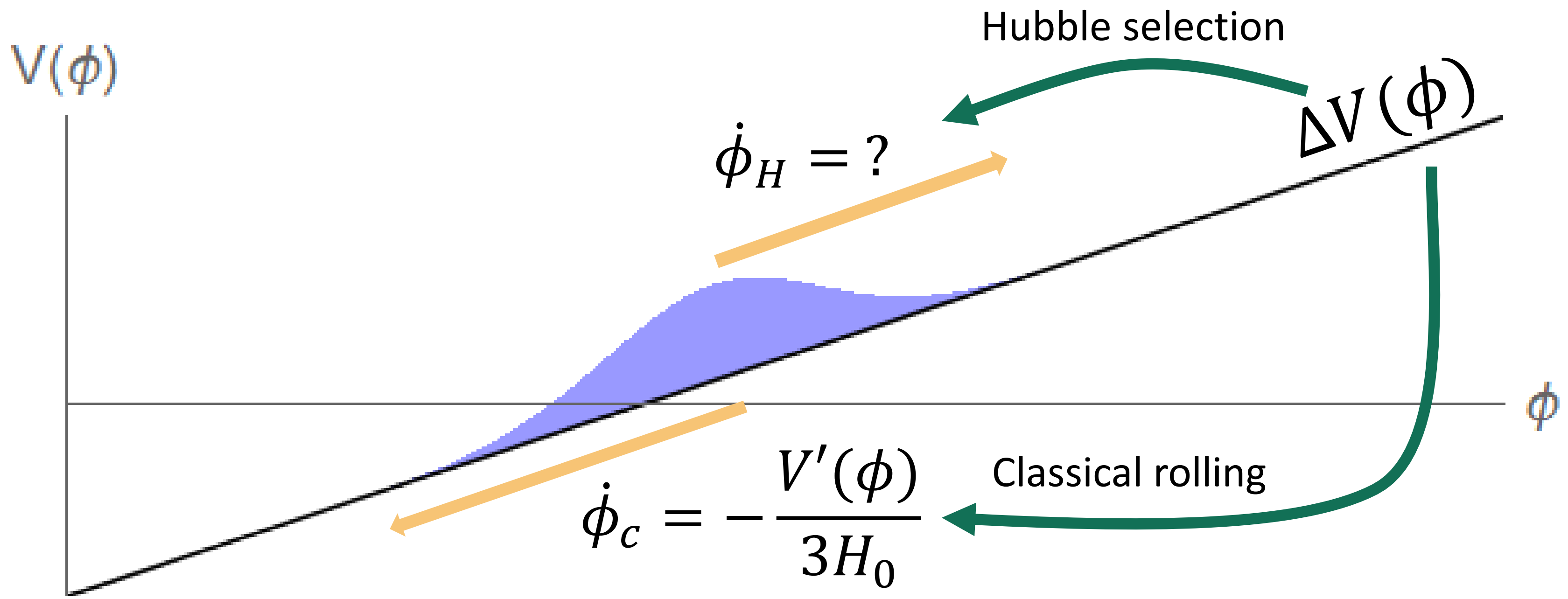

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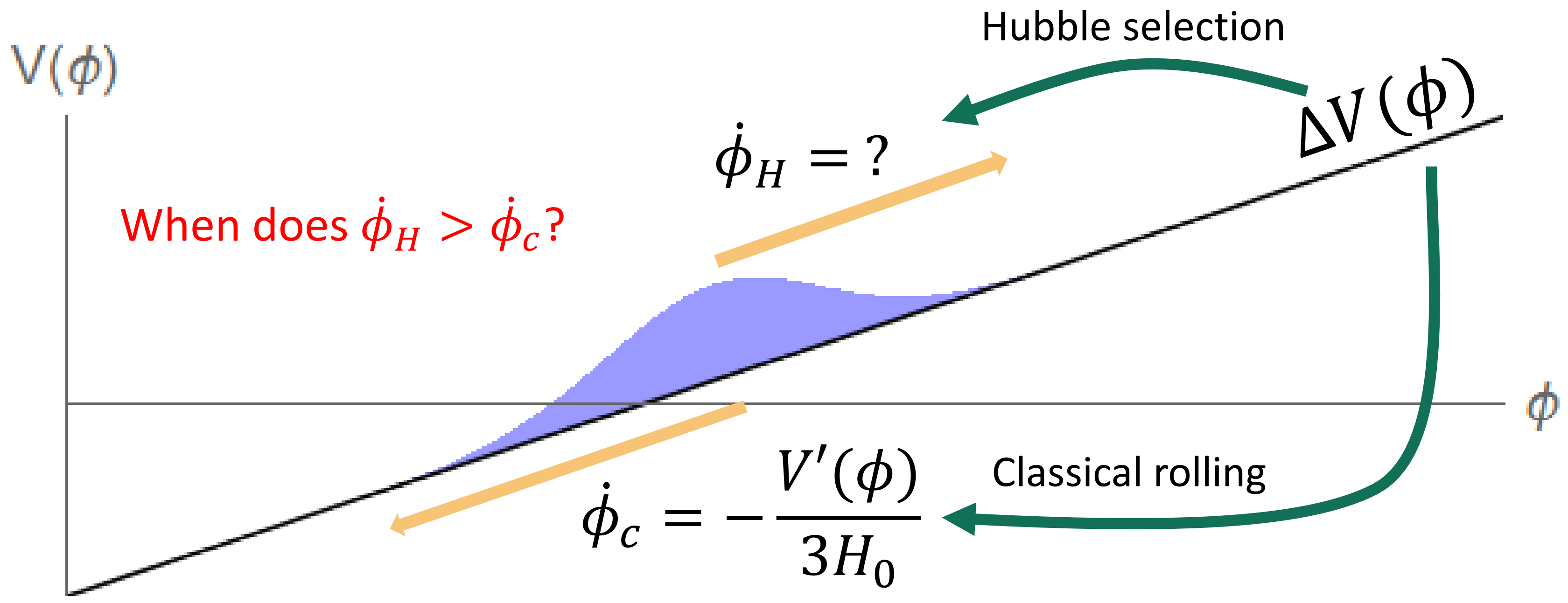


Hubble selection: Quantitative analysis

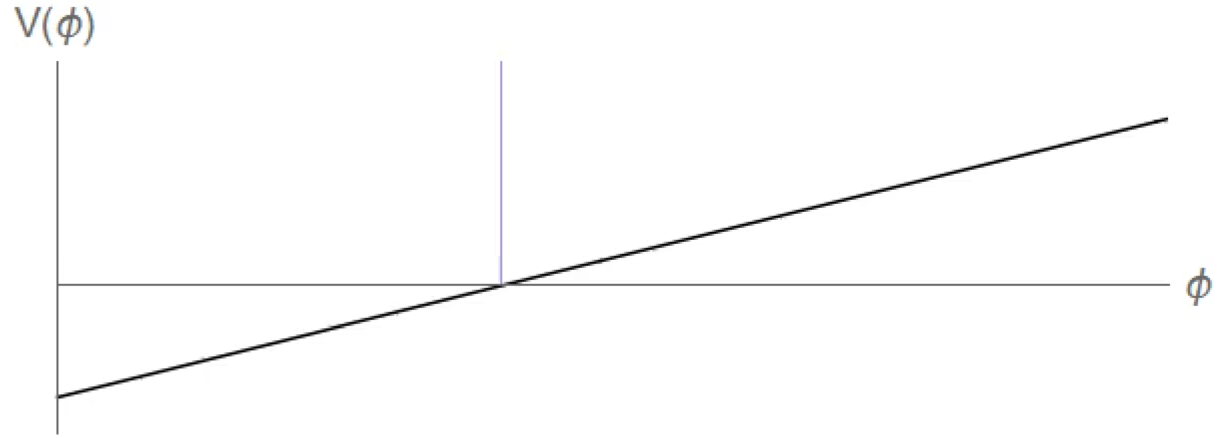
- Recall: necessary ingredient for selection: “diversity”
  - ΔH between the “head” and the “tail” drives the distribution
  - Efficient for broadened distributions
- Linear potential solution to FPV equation

$$\rho(\phi,t) \propto e^{-\frac{1}{\underbrace{2\sigma_\phi^2(t)}_{\text{Peak width}}}\left[\phi-\overbrace{\left(\underbrace{\phi_0}_{\text{Initial value}}+\underbrace{\dot{\phi}_c}_{\text{Motion by classical rolling}}t+\underbrace{\tfrac{3}{2}(\Delta H)'\sigma_\phi^2(t)\,t}_{\text{Motion by Hubble selection}}\right)}^{\text{Peak position}}\right]^2}$$

Hubble selection: Quantitative analysis

- $\sigma_\phi^2(t) = \frac{H^3}{4\pi^2}t$: variance (diffusion by stochastic motion)

- $\dot{\phi}_H = 3(\Delta H)'\sigma_\phi^2 = \frac{V'\sigma_\phi^2}{2M_P^2 H_0}$: motion induced by Hubble selection
  - $\propto \sigma_\phi^2$: "need for diversity", $\propto V'$: "strength of selection"



Hubble selection: Quantitative analysis

Condition to climb up:
Full field range accommodates $\Delta\phi$

- Turning point: $\dot{\phi}_c + \dot{\phi}_H = 0$
  - Planckian width: $\sigma_\phi \sim M_P$
  - Maximum roll down excursion: $\Delta\phi > M_P \sim \sigma_\phi$
  - Eternal inflation: $\Delta N \simeq \frac{8\pi^2 M_P^2}{3\,H^2} > \frac{2\pi^2 M_P^2}{3H^2} =$ "de Sitter entropy bound"
- Super Planckian field range and eternal inflation are necessary
  - No Hubble selection for usual well-known quantum fields

Hubble selection: Equilibrium

- Equilibrium: localization near the boundary
  - out of EFT / potential drop (phase transition) / anything else…
  - Distribution cannot get over and stop near the boundary
- Why equilibrium is important?
  - Eternal inflation: reheating universe is continuously dominated by “young population”
  - After finite time, newly generated (and dominating) young population will follow the equilibrium distribution.

Hubble selection: Equilibrium

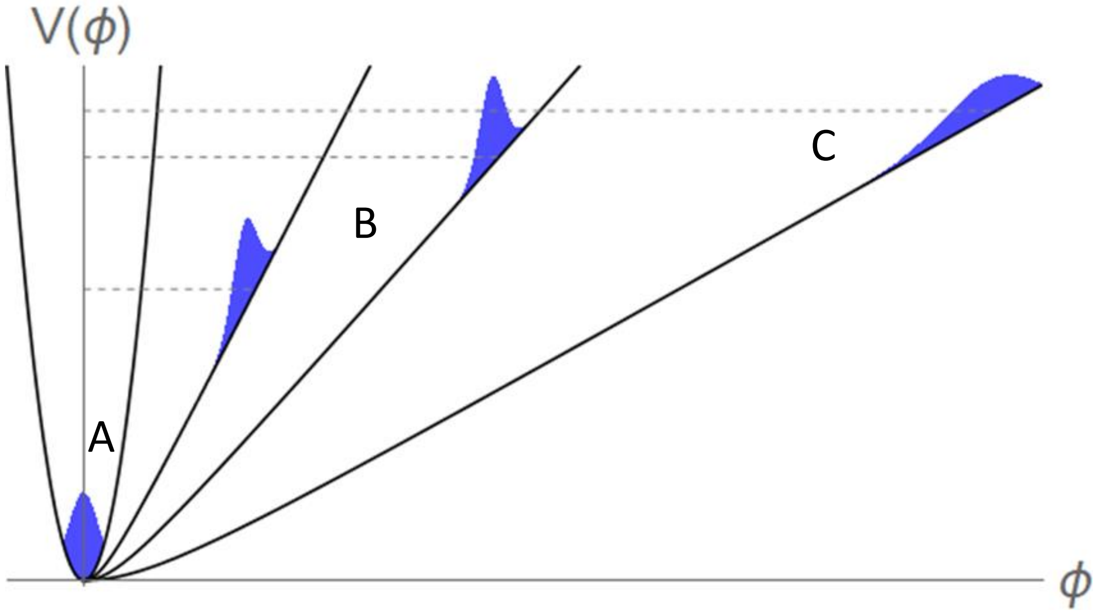
- Width and position of equilibrium distribution
  - How are they determined? We give qualitative description
  - Full quantitative description: Giudice et al. 2021

Hubble selection: Equilibrium

- Rule of thumb:

  less steep potential gives equilibrium

  distribution closer to the upper boundary

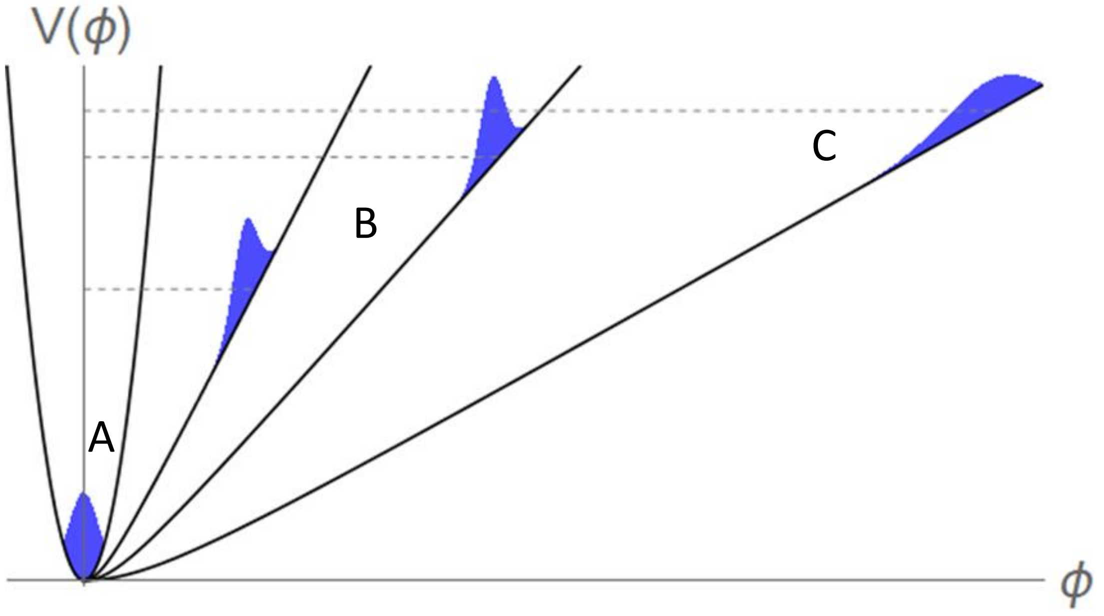


Hubble selection: Equilibrium

- A: Classically dominated
  - $\dot{\phi}_c \gg \dot{\phi}_H$
- B: Quantum dominated
  - $\dot{\phi}_c + \dot{\phi}_H = 0$
- C: Extremely quantum dominated
  - $\dot{\phi}_c + \dot{\phi}_H + \dot{\phi}_b = 0$
- C/QV/Q2V regimes in SOL
  - Quantitatively consistent

More classical → More quantum

Width:	$\sigma_\phi < M_P$	$\sigma_\phi \sim M_P$	$\sigma_\phi > M_P$
Position:	At bottom	Near top	At top



Possibility from QCD

- Can we use Hubble selection to select the weak scale?
  - “Localization near the upper boundary”
- 1^{st} order quantum phase transition gives the boundary
  - Vacuum energy is peaked at the critical point
- QCD chiral phase transition could be 1^{st} order, at QCD scale
  - Not far from the current weak scale
  - Subject to strange quark mass
  - Not firmly established yet… thus a “possibility”

Possibility from QCD

- Need three sectors
  - ϕ : relaxion; scans the Higgs mass (or VEV v_h)
  - h : Higgs; v_h determines quark mass; triggers Σ's phase transition at $v_h^* = v_h(\phi^*)$
  - Σ : meson field; QCD d.o.f. below $\Lambda_{QCD} \simeq 200$ MeV; undergoes 1st order phase transition

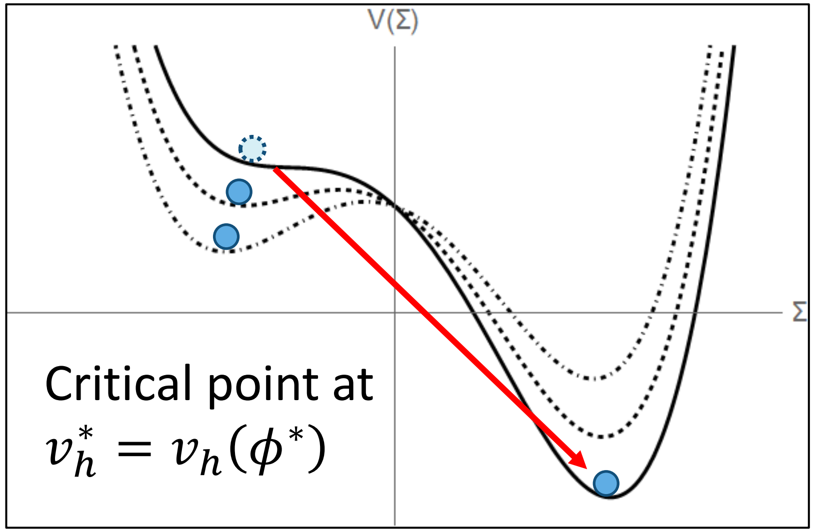
$$\boxed{\phi \text{ relaxion}} \xrightarrow{v_h \text{ scan}} \boxed{h \text{ Higgs}} \xrightarrow{\text{PT at } v_h^*} \boxed{\Sigma \text{ meson field}}$$

- h and Σ always follow their minimum; no Hubble selection for them

Possibility from QCD

- $V = V_\phi(\phi) + V_h(\phi) + V_\Sigma\big(h(\phi)\big)$
  - $V_\phi = \Lambda_\phi^4 \cos\left(\frac{\phi}{f_\phi}\right)$, $\quad V_h = \frac{1}{2}(M^2 - g\phi)h^2 + \frac{\lambda_h}{4}h^4$
  - $V_\Sigma = \mu^2\, \mathrm{Tr}(\Sigma\Sigma^\dagger) + \ \ldots - \mathrm{Tr}\big(\mathcal{H}(\Sigma+\Sigma^\dagger)\big)$

$\mathcal{H}(v_h)$ plays role of external magnetic field in magnetization



Critical point at
$v_h^* = v_h(\phi^*)$

$$\boxed{\phi \text{ relaxion}} \xrightarrow{v_h \text{ scan}} \boxed{h \text{ Higgs}} \xrightarrow{\text{PT at } v_h^*} \boxed{\Sigma \text{ meson field}}$$

$$V_\phi(\phi) \qquad V_h(\phi) \qquad V_\Sigma(\phi)$$

Possibility from QCD

- $V = V_\phi(\phi) + V_h(\phi) + V_\Sigma\big(h(\phi)\big)$
  - $V_\phi = \Lambda_\phi^4 \cos\left(\frac{\phi}{f_\phi}\right)$, $V_h = \frac{1}{2}(M^2 - g\phi)h^2 + \frac{\lambda_h}{4}h^4$
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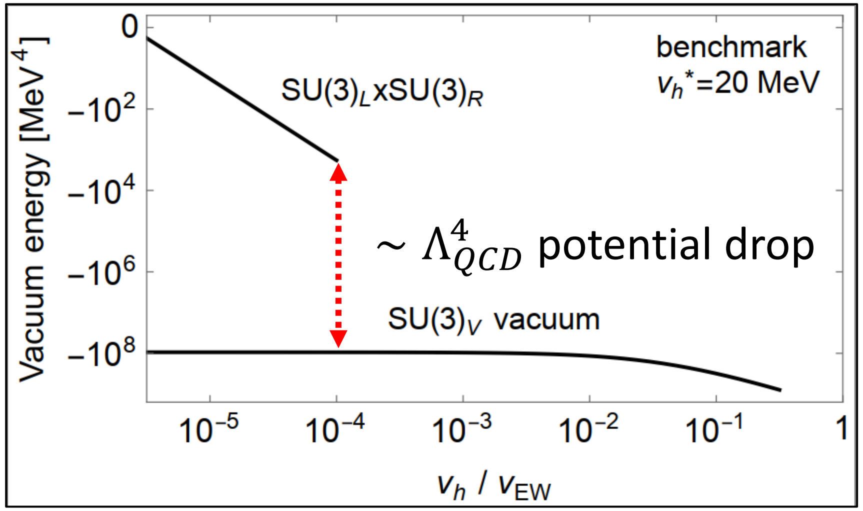
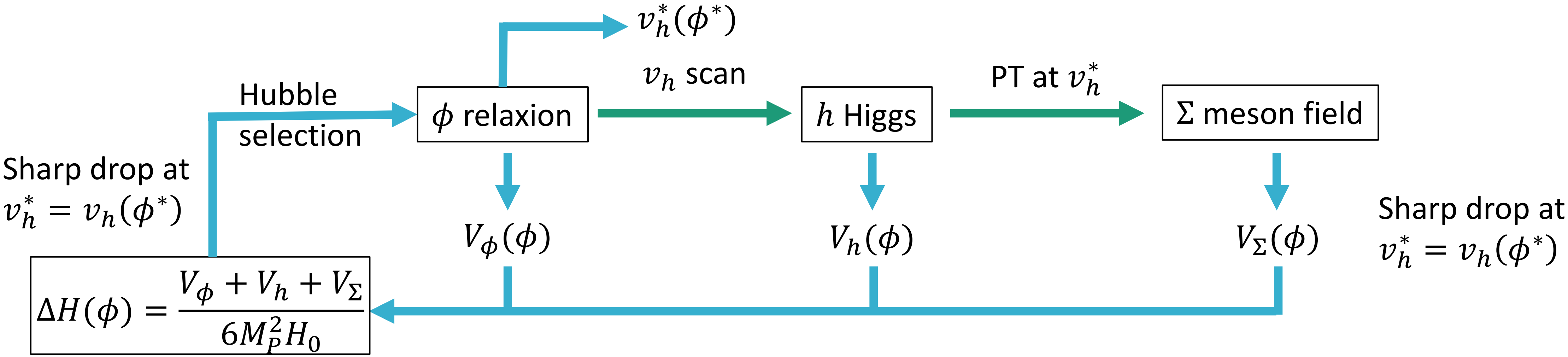
Σ vacuum structure



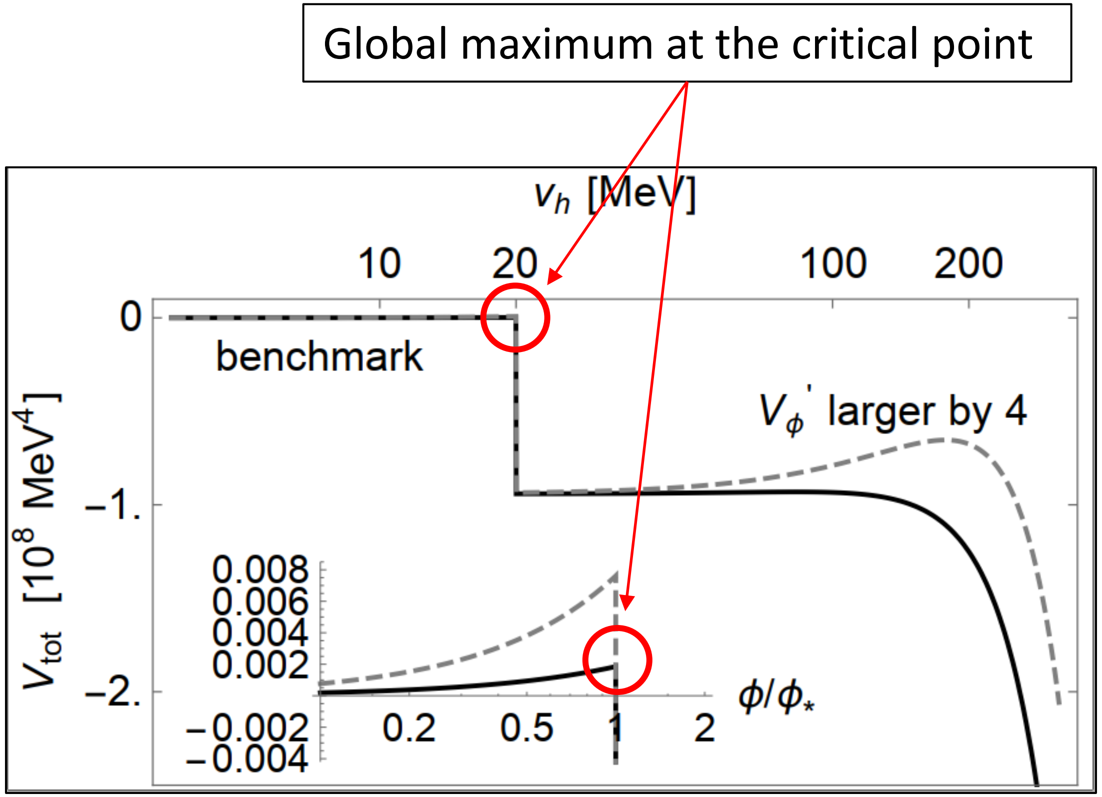


Possibility from QCD

- Required conditions
  - …
  - $V(\phi^*)$ is global maximum (greatest Hubble rate):

    $v_h^* \lesssim \Lambda_\phi^2/M \lesssim \Lambda_{QCD}$

    "Slope of V_ϕ should not be too small nor too large"



- $\Lambda_\phi \ll M$ is need to have high cutoff

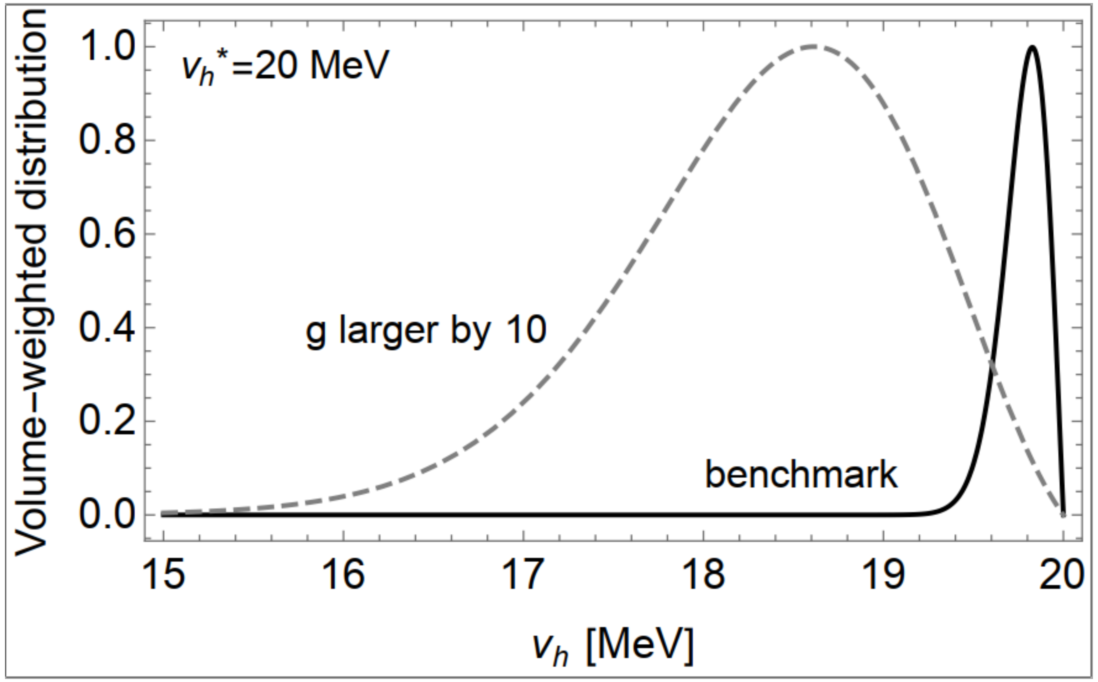
Possibility from QCD

- Successful benchmark
  - $H = v_h^* \simeq 20$ MeV

    $M = 3 \times 10^{-3} M_P$

    $\Lambda_\phi^2 = 10^{-2} H\, M_P$

    $g = 10^{-3} H^2 / M_P$
- Well localized v_h near $v_h^* \simeq 20$ MeV



Possibility from QCD

- High cutoff $M = 3 \times 10^{-3} M_P$, well localized v_h...

  Are we successfully solved the Higgs naturalness problem? Unfortunately, no.

- $\Lambda_\phi \ll M$ is unstable from quantum correction originating from relaxion-Higgs interaction; at least $\Lambda_\phi \sim M$
  - Recall: $V_h = \frac{1}{2}(M^2 - g\phi)h^2 + \frac{\lambda_h}{4}h^4$

Possibility from QCD

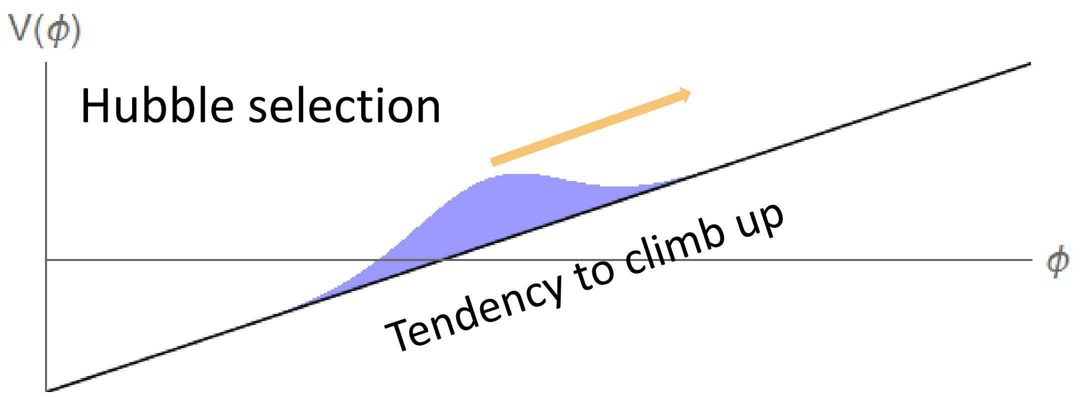
- $\Lambda_\phi \ll M$ = "The tail wagging the dog" is not allowed by naturalness
- Several different translation of original naturalness problems.

  At least some of them are quantum stable fine tunings, but I'd omit here.

- We saw some possibility in QCD, but naïve model building was doomed.
- Then, should Hubble selection be discarded? NO!

Discussion

- One step back… let us see the big picture
- “Climbing up”



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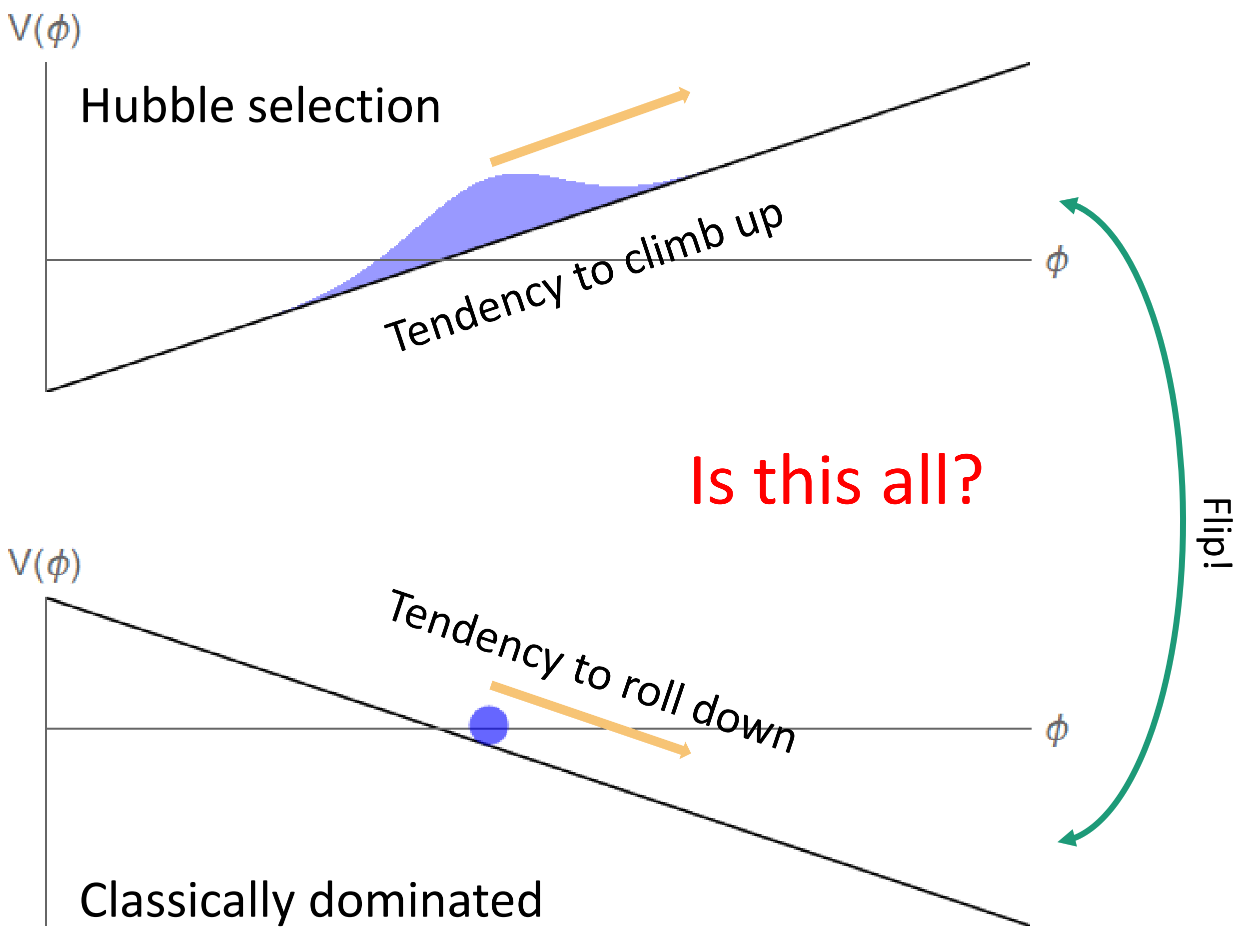
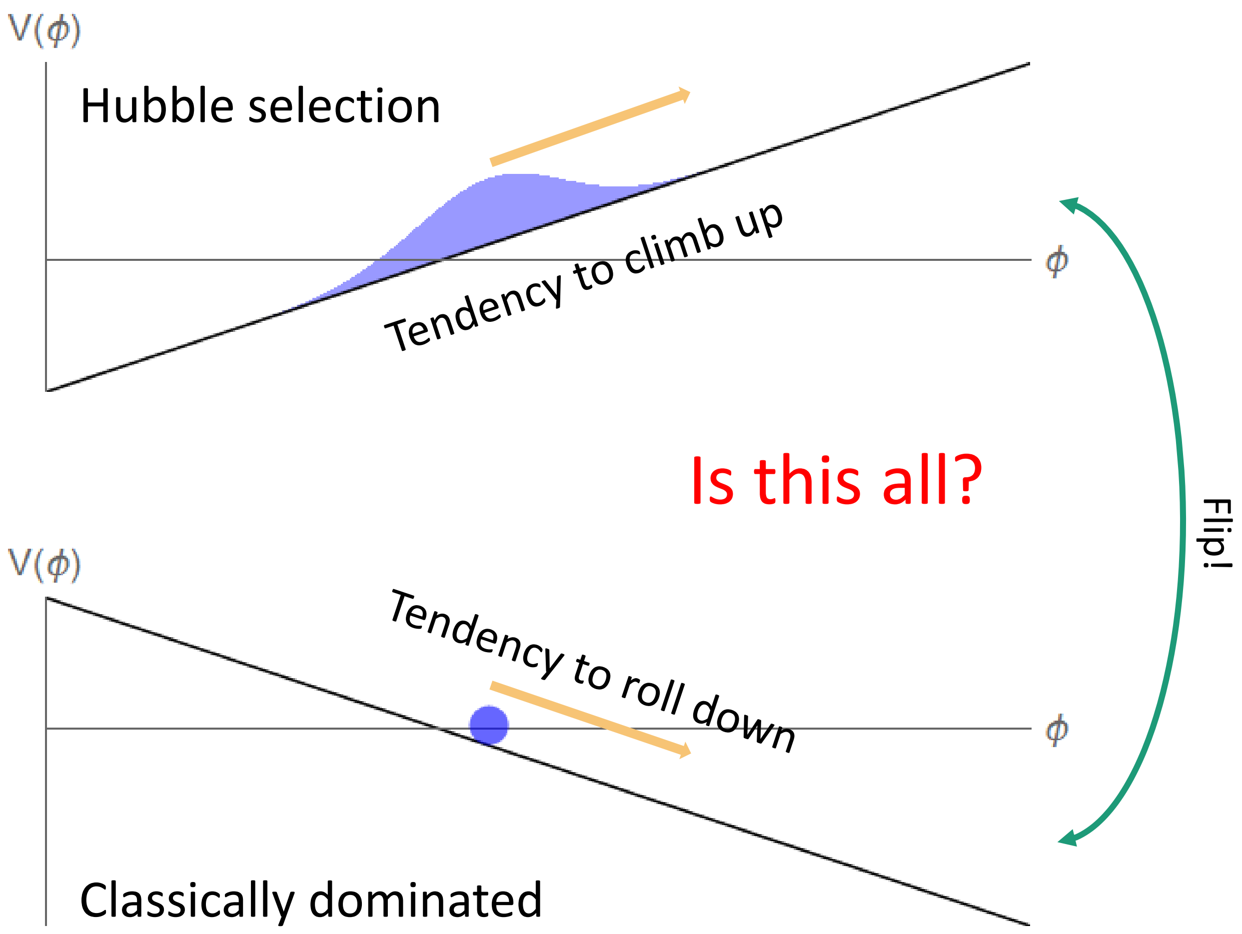
- One step back… let us see the big picture
- “Climbing up”
  - Just different tendency, is this all?



Discussion

- One step back… let us see the big picture
- "Climbing up"
  - Just different tendency, is this all?
  - Why should we keep an eye on Hubble selection?



Discussion

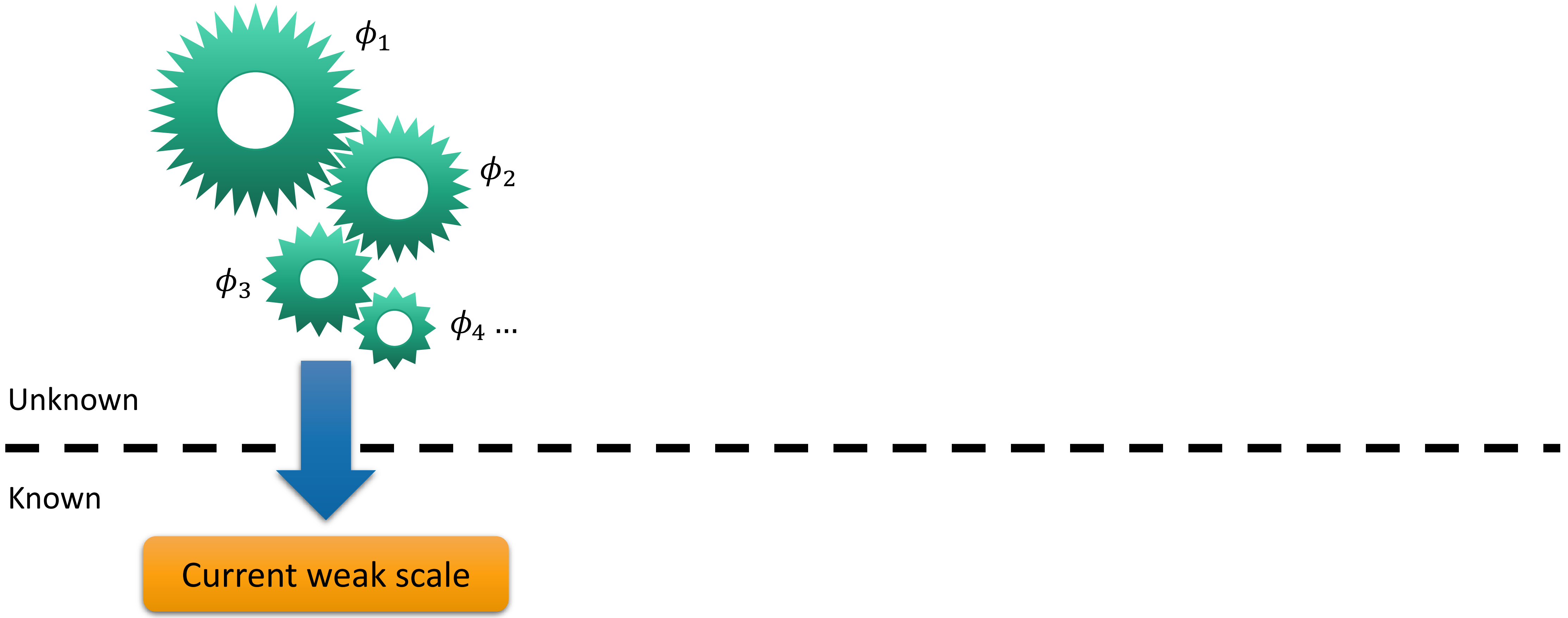
- Answer:

  “Because we now have two competing tendencies”

- Let us ask first…

  “What kinds of model we are looking for?”

Discussion



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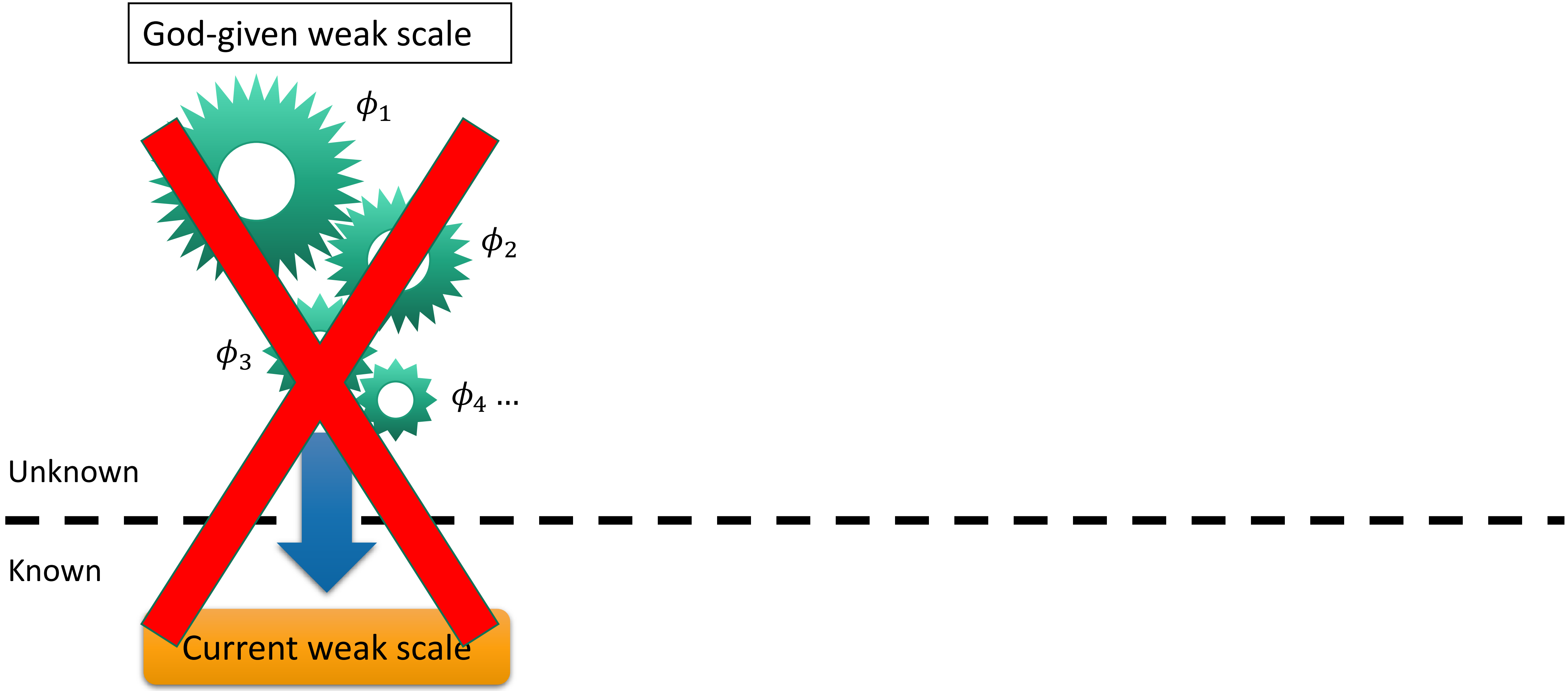
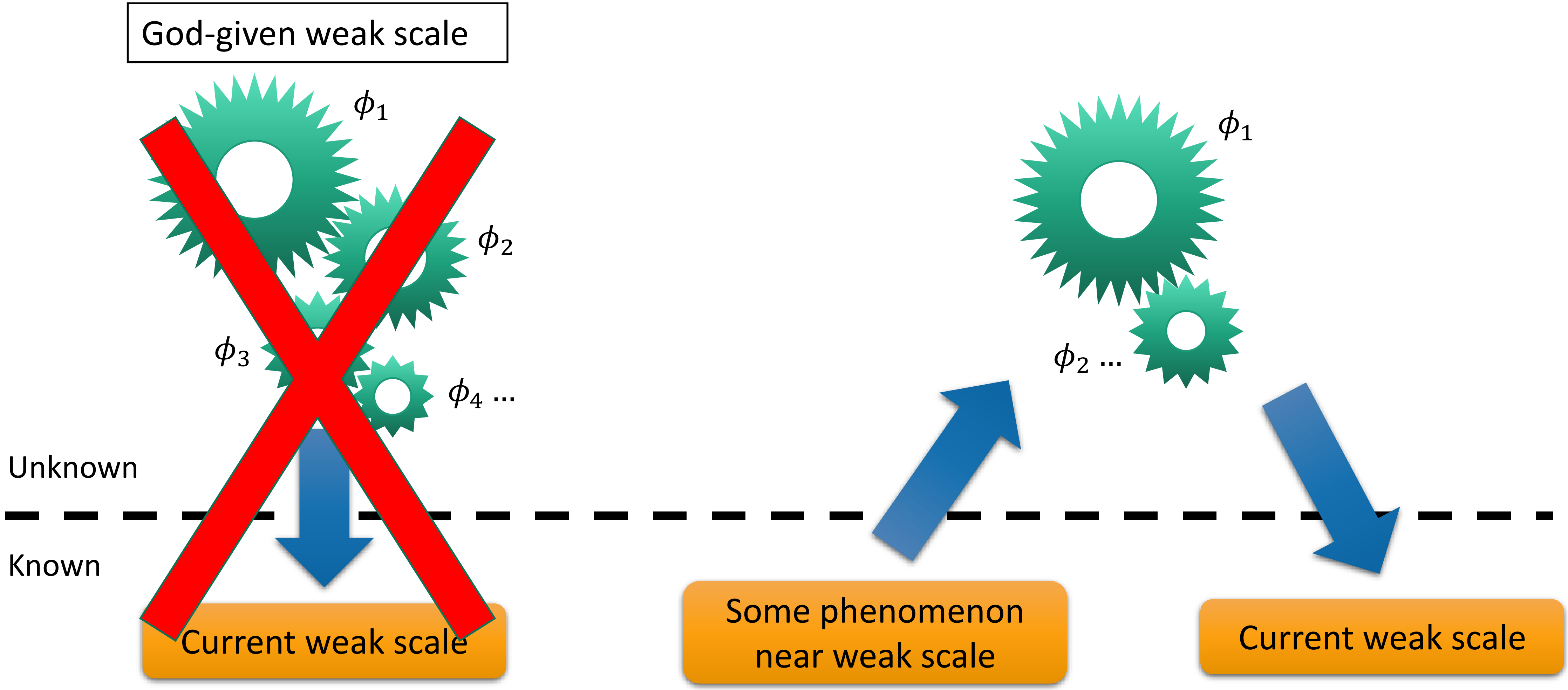


God-given weak scale
ϕ_1
ϕ_2
ϕ_3
ϕ_4 ...
Unknown
Known
Current weak scale

Discussion


God-given weak scale
ϕ_1
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ϕ_3
ϕ_4 …
Unknown
Known
Current weak scale
ϕ_1
ϕ_2 …
Some phenomenon near weak scale
Current weak scale

Discussion

- Known sector → sub Planckian field range
  - No Hubble selection; always rolls down; minimizes potential energy
  - We want this to act as some trigger or brake of unknown sector to select the current weak scale
- But a brake can work only for competing tendency!
  - Rolling down cannot act as a brake for rolling down…
- Thus, Hubble selection is still noteworthy for mechanism in unknown sector

Concluding remarks

- Hubble selection: global field value distribution climbs up the potential
  - Higher potential, higher Hubble rate
  - Super Planckian field range & eternal inflation are required
- Possibility from QCD: weak scale might be selected from QCD phase transition
- Naïve model building was not successful, but Hubble selection is still an attractive mechanism.
  - competing tendency against classical rolling
  - Please keep an eye on it!