

Black Hole Quasinormal Modes in General Relativity and Beyond

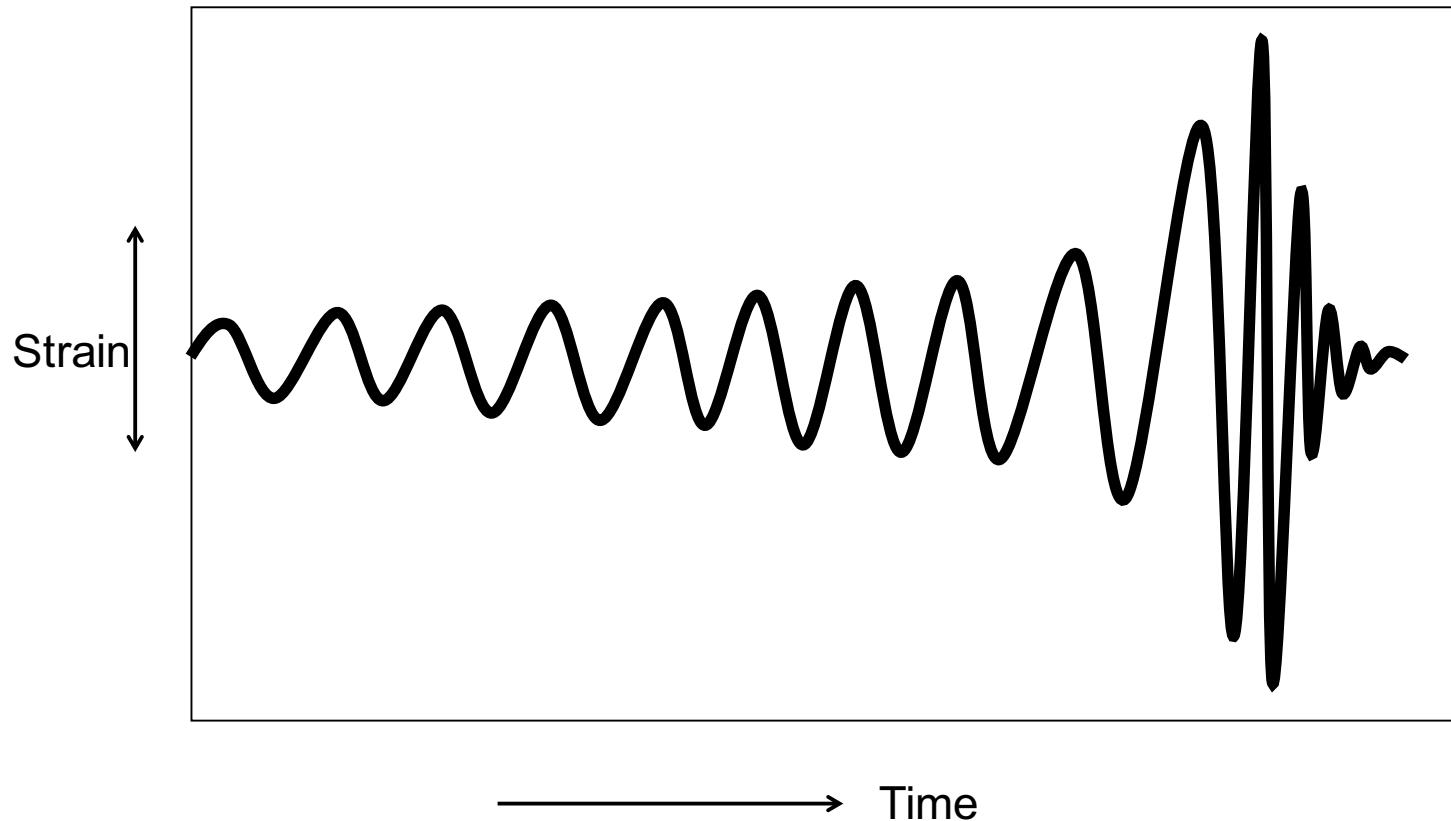
Che-Yu Chen

Institute of Physics, Academia Sinica, Taiwan

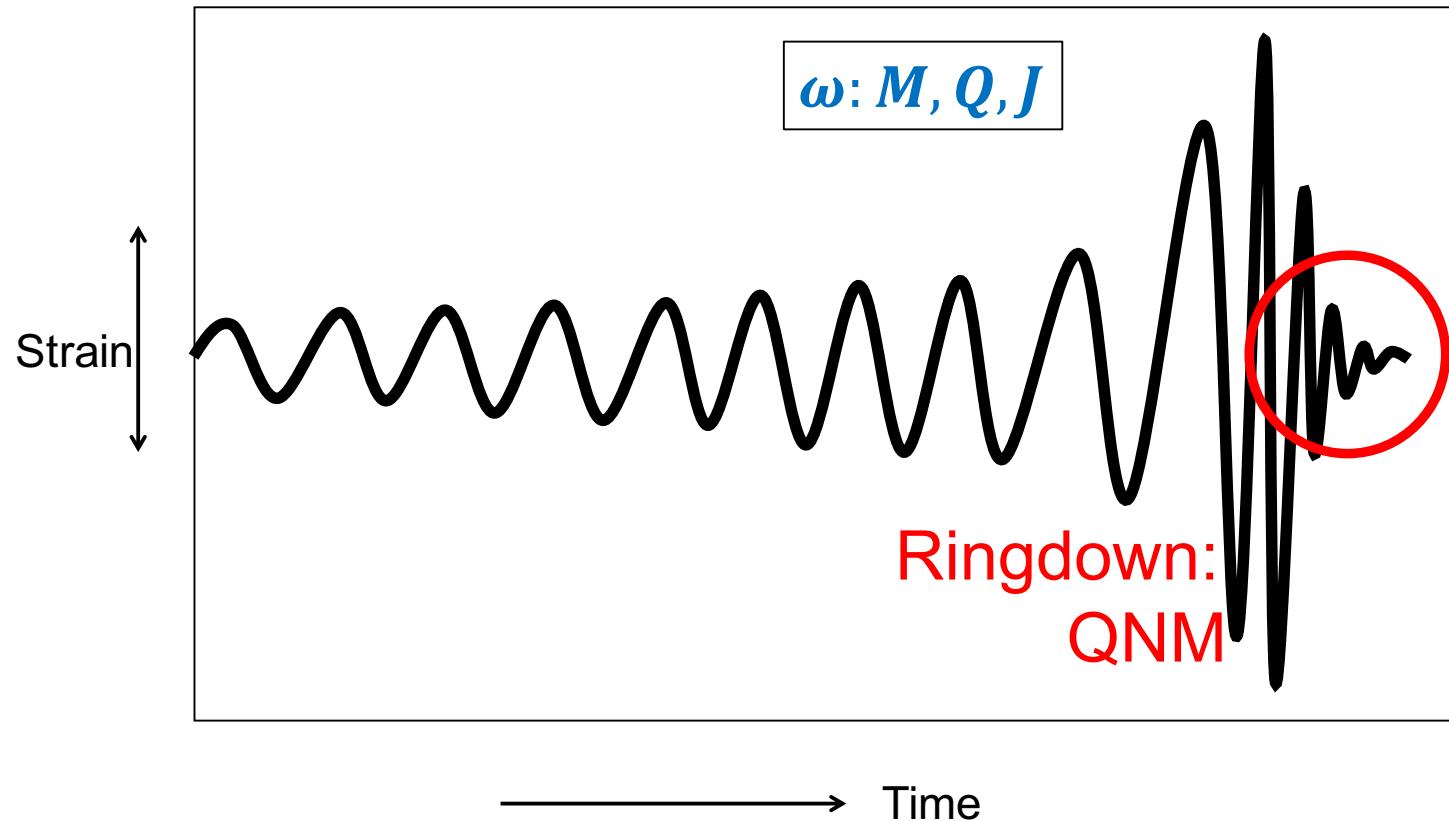
- **CYC, MBL, PC**, EPJP (2021) 136:253



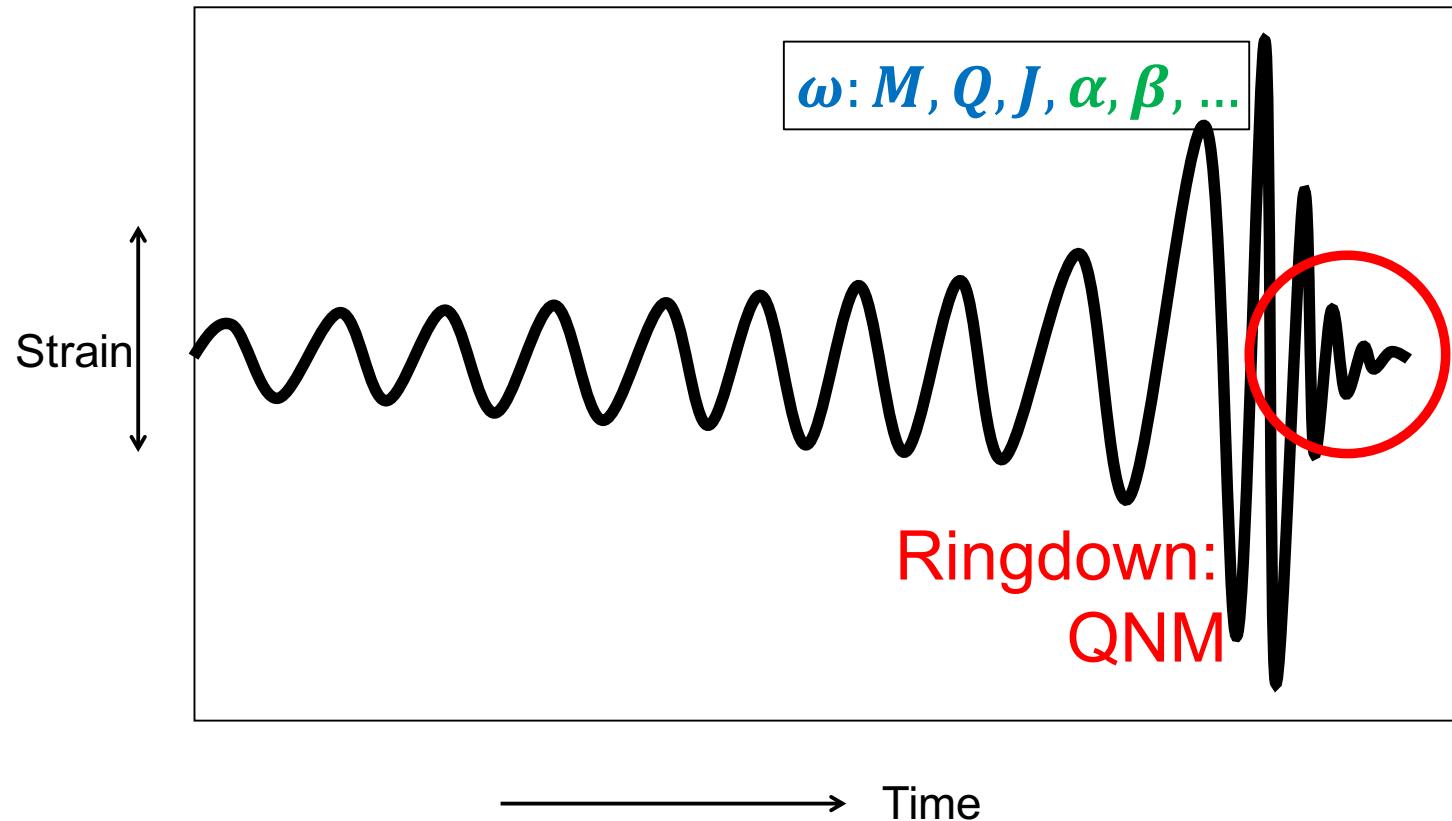
Black Hole Merger



Black Hole Merger: Ringdown and QNMs



Black Hole Merger: Ringdown and QNMs



Black hole perturbations: Master equations

- $g_{\mu\nu} = g_{0\mu\nu} + h_{\mu\nu}$ in Regge-Wheeler gauge and after Fourier decom.:

Odd parity (axial) $\tilde{h}_{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & h_0(r) \\ 0 & 0 & 0 & h_1(r) \\ 0 & 0 & 0 & 0 \\ h_0(r) & h_1(r) & 0 & 0 \end{bmatrix} \left(\sin \theta \frac{\partial}{\partial \theta} \right) Y_{l0}(\theta),$

Even parity (polar) $\tilde{h}_{\mu\nu} = \begin{bmatrix} H_0(r)f & H_1(r) & 0 & 0 \\ H_1(r) & H_2(r)/f & 0 & 0 \\ 0 & 0 & r^2 K(r) & 0 \\ 0 & 0 & 0 & r^2 K(r) \sin^2 \theta \end{bmatrix} Y_{l0}(\theta).$

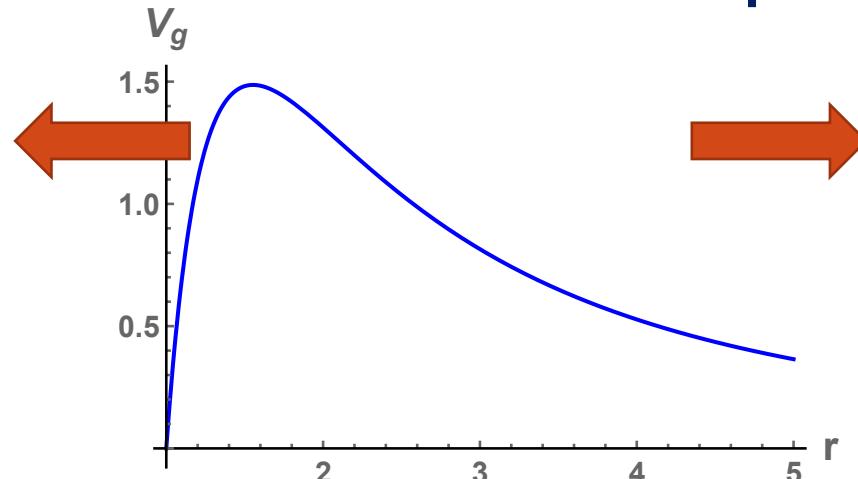
Black hole perturbations: Master equations

$$\left(\frac{d^2}{dr_*^2} + \omega^2 \right) \Psi = V_{a/p} \Psi$$

■ Schwarzschild black hole:

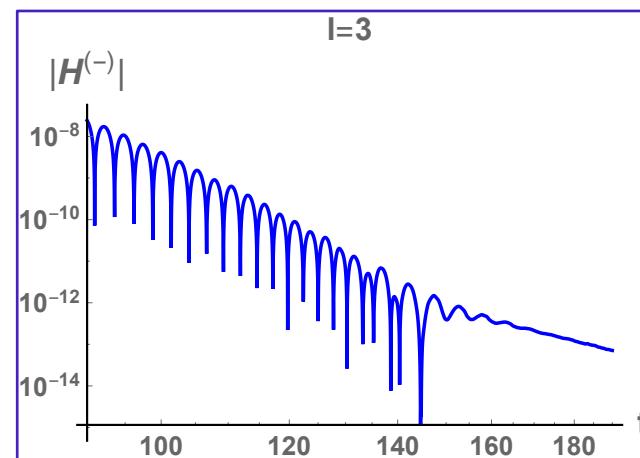
- Odd parity (axial): **Regge-Wheeler equation**

$$V_a = \left(1 - \frac{2M}{r}\right) \left[\frac{l(l+1)}{r^2} - \frac{6M}{r^3} \right]$$



- Even parity (polar): **Zerilli equation**

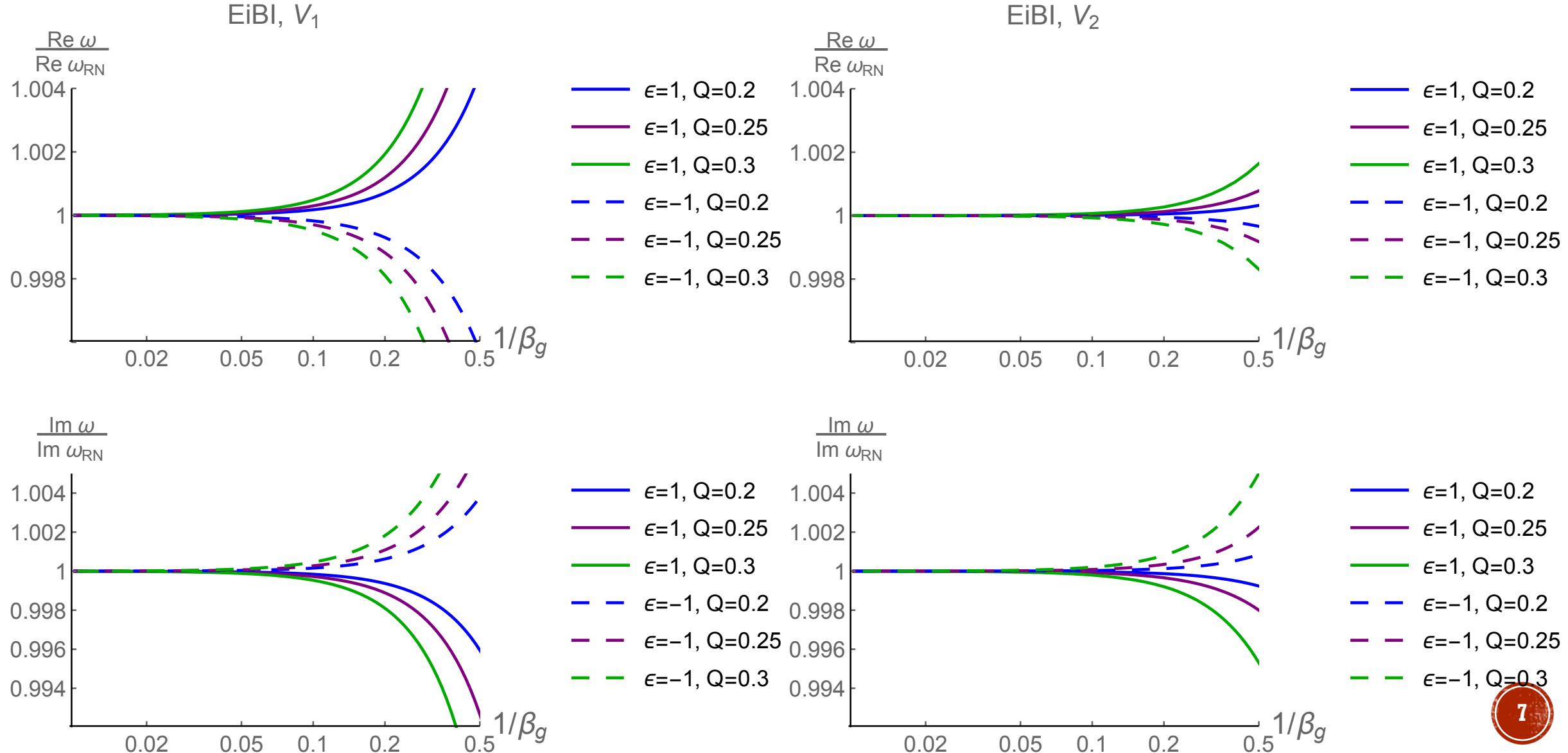
$$V_p = \frac{2\left(1-\frac{2M}{r}\right)[\lambda^2(\lambda+1)r^3 + 3M\lambda^2r^2 + 9M^2\lambda r + 9M^3]}{r^3(\lambda r + 3M)^2}$$
$$\lambda = (l+2)(l-1)/2$$



- Beyond GR?

- **QNM spectrum**
- Eikonal QNM and photon orbits
- Isospectrality

QNMs of EiBI black holes: Axial pert.



- Beyond GR?

- QNM spectrum
- Eikonal QNM and photon orbits
- Isospectrality

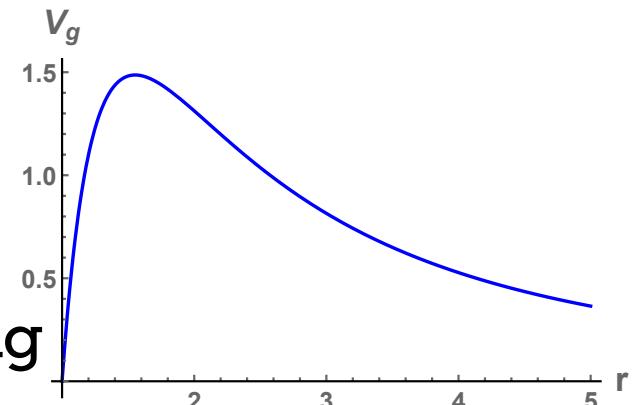
Eikonal QNMs

- The eikonal QNMs ($l \rightarrow \infty$) and the photon ring
 $\omega \approx \Omega_c l - i(n + 1/2)|\lambda_c|$
- $Re(\omega)$: $\leftrightarrow \Omega_c$ (orbital frequency of the photon ring)
- $Im(\omega)$: $\leftrightarrow \lambda_c$ (Lyapunov exponent)

Cardoso, Miranda, Berti, Witek, Zanchin (2009)

$$ds^2 = -\psi^2 \bar{f} dt^2 + \frac{dr^2}{\bar{f}} + r^2 d\Omega^2$$

- In GR, we have $V \approx \psi^2 \bar{f} \left(\frac{l^2}{r^2} \right)$
- The peak of the potential coincides with the photon ring
- This may not be true for modified gravity



Eikonal QNMs

- For examples: In theories with non-minimal matter couplings:

$$V \approx \alpha(r) \psi^2 \bar{f} \left(\frac{l^2}{r^2} \right)$$

cf. $V \approx \psi^2 \bar{f} \left(\frac{l^2}{r^2} \right)$ in GR

- Charged black holes in gEMSG $f(R, T^2)$:

$$V_1 = \frac{\psi^2 \bar{f}}{r^2} \left(\frac{1 - \frac{f_{T^2}}{16\pi^2} F_{(t)(r)}^2}{1 + \frac{f_{T^2}}{16\pi^2} F_{(t)(r)}^2} \right) l^2$$

$$V_2 = \frac{\psi^2 \bar{f}}{r^2} l^2$$

Chen, Chen (2020)

- Charged black holes in EiBI gravity:

$$V_1 = \frac{\psi^2 \bar{f}}{r^2} \left(\frac{\beta_g^2 r^4 + Q^2}{\beta_g^2 r^4 - Q^2} \right) l^2$$

$$V_2 = \frac{\psi^2 \bar{f}}{r^2} \left(\frac{\beta_g^2 r^4 - Q^2}{\beta_g^2 r^4 + Q^2} \right) l^2$$

Chen, Bouhmadi-López, Chen (2019)

Eikonal QNMs and BH shadows

- A simple relation formula (non-rotating BH)

$$\omega_R = \lim_{l \gg 1} \frac{1}{R_S}$$

Jusufi (2020), Cuadros-Melgar *et al.* (2020)

- A general mapping exists for Kerr BHs

Jusufi (2020), Yang (2021)

■ Beyond GR?

- QNM spectrum
- Eikonal QNM and photon orbits
- Isospectrality

Regge-Wheeler & Zerilli

$$\left(\frac{d^2}{dr_*^2} + \omega^2 \right) \Psi = V_{a/p} \Psi$$

■ Schwarzschild black hole:

- Odd parity (axial): **Regge-Wheeler equation**

$$V_a = \left(1 - \frac{2M}{r}\right) \left[\frac{l(l+1)}{r^2} - \frac{6M}{r^3} \right]$$

- Even parity (polar): **Zerilli equation**

$$V_p = \frac{2\left(1-\frac{2M}{r}\right)[\lambda^2(\lambda+1)r^3 + 3M\lambda^2r^2 + 9M^2\lambda r + 9M^3]}{r^3(\lambda r + 3M)^2}$$
$$\lambda = (l+2)(l-1)/2$$

Isospectrality: They share the same spectrum!

Isospectrality

- Chandrasekhar transformation

$$V_{a/p} = W^2 \mp \frac{dW}{dr_*} - \frac{\lambda^2(\lambda + 1)^2}{9M^2}$$
$$W = \frac{6M(2M - r)}{r^2(6M + 2\lambda r)} - \frac{\lambda(\lambda + 1)}{3M}$$

W: superpotential

- Wave functions:

$$\Psi_{a/p} = \frac{1}{\beta - \omega^2} (\mp W \Psi_{p/a} + \frac{d\Psi_{p/a}}{dr_*})$$

Chandrasekhar (1983)

- Isospectrality is very fragile

$$V_{a/p} \rightarrow V_{a/p} + \delta V_{a/p}$$

Cardoso *et al.* (2019)

Coupled to additional d.o.f

- In many modified gravity, Schwarzschild and Kerr are still solutions
- There may be additional physical d.o.f in the theory:

$$\left(\frac{d^2}{dr_*^2} + \omega^2 \right) \Psi - V_{a/p} \Psi = S_{a/p} \quad S_{a/p}: \text{source term from extra d.o.f}$$

- In $f(R)$: $S_s = 0$, $S_p \neq 0$ Bhattacharyya, Shankaranarayanan (2017) (2018)

- In dCS gravity: $S_s \neq 0$, $S_p = 0$ Cardoso, Gualtieri (2009)

- EdGB gravity Blázquez-Salcedo *et al.* (2016)

- Scalar-tensor theory

Kobayashi, Motohashi, Suyama (2012), Tattersall, Ferreira (2018)

- Others: Nonlocal gravity, Hybrid-metric-Palatini gravity

Chen, Park (2021), Chen, Kung, Chen (2020)

Conclusions

- Black hole QNMs are important tool to probe strong gravity
 - QNM spectra are directly altered when additional parameters enter
 - Eikonal QNMs and spherical photon orbits
 - Non-minimal matter-curvature couplings
 - Isospectrality
 - Very fragile
 - $f(R)$ gravity, dCS gravity, nonlocal gravity, HMPG