

Symplectic modular symmetry in heterotic string vacua: flavor, CP, and R-symmetries

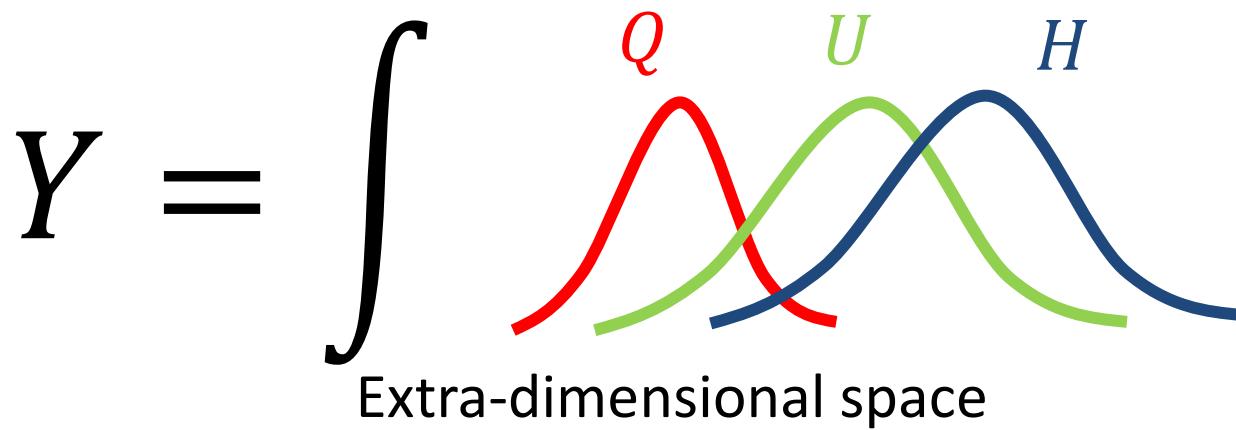
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References :

K. Ishiguro, T. Kobayashi and H.O., arXiv: 2010.10782, 2107.00487

Introduction

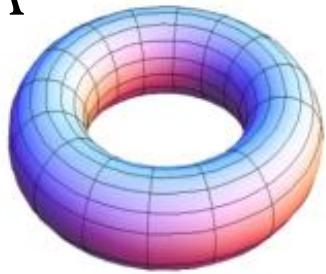
- Origin of flavor and CP symmetries : important issue in the SM
- 4D Yukawa couplings (in the higher-dimensional theory)
= Overlap integrals of zero-mode wavefunctions on the internal space



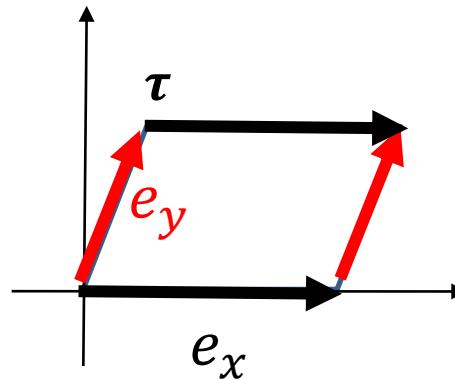
- Geometrical symmetries (and stringy selection rule)
→ Flavor structure of quarks/leptons
- Yukawa couplings $Y(\tau)$ depend on moduli τ (geometric parameters)
— Moduli VEVs determine the flavor structure and CP violation

T^2 torus : $SL(2, \mathbb{Z})$ geometrical (modular) symmetry

$$T^2 = \mathbb{C}/\Lambda$$



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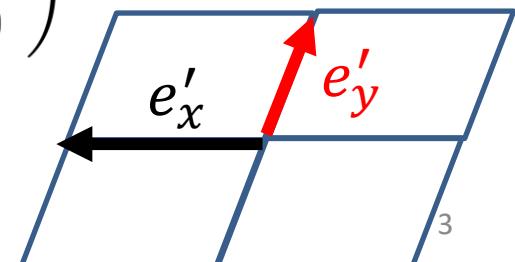
- Lattice vectors are related under $SL(2, \mathbb{Z})$ modular transformation:

$$\begin{pmatrix} e'_y \\ e'_x \end{pmatrix} = \begin{pmatrix} p & q \\ s & t \end{pmatrix} \begin{pmatrix} e_y \\ e_x \end{pmatrix}$$

$p, q, s, t \in \mathbb{Z}$ satisfying $pt - qs = 1$

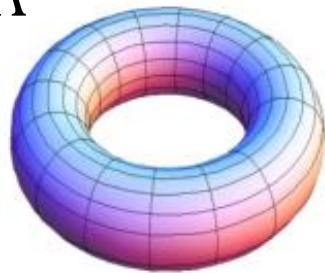
Two generators : S and T

$$S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

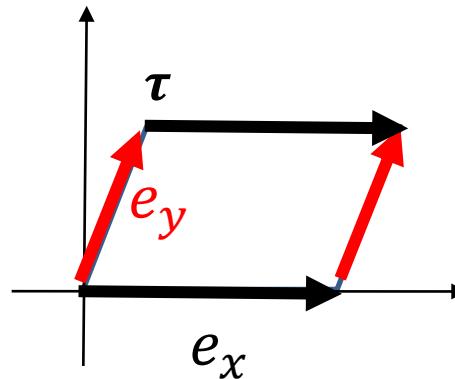


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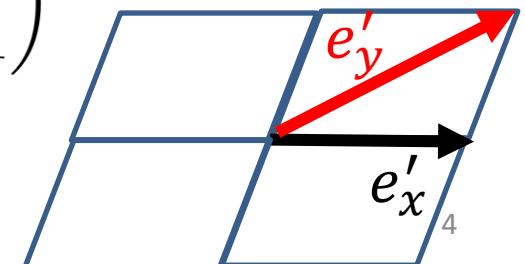
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Two generators : S and T

$$T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$



Finite subgroups of modular group

$SL(2, \mathbb{Z})$

$$\{S, T \mid S^2 = -\mathbb{I}, S^4 = (ST)^3 = \mathbb{I}\}$$

Finite subgroups :

$$\Gamma_N = \{S, T \mid S^2 = (ST)^3 = T^N = \mathbb{I}\}$$

$$\Gamma_2 \simeq S_3, \Gamma_3 \simeq A_4, \Gamma_4 \simeq S_4, \Gamma_5 \simeq A_5, \dots$$

useful to reproduce the data, in particular, the lepton sector

- Matter fields and Yukawa couplings : non-trivial representations of Γ_N

$$y_{ij}(\tau) Q_i H U_j$$

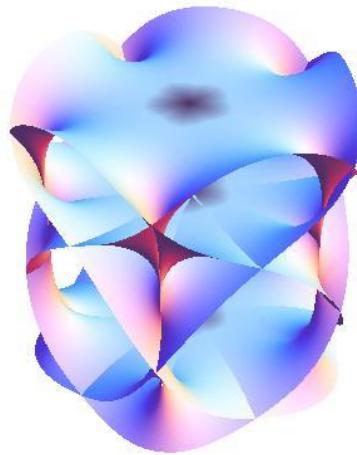
(developed in heterotic orbifold models, magnetized D-brane models)

$$Y = \int_{\text{Torus}} Q \cup U \cup H$$

Modular symmetry on 6D Calabi-Yau threefolds ?
(Vacuum solutions in string theory)

$$Y = \int_{\text{Calabi-Yau}} Q \cup U \cup H$$

6D CY : $Sp(2h + 2, \mathbb{Z})$ symplectic modular symmetry



h : # of moduli fields

$$Sp(2, \mathbb{Z}) \simeq SL(2, \mathbb{Z})$$

*A. Strominger ('90),
P. Candelas, X. de la Ossa ('91)*

- (dual) basis $\{\alpha^I, \beta_I\}$ of three cycles is related under $Sp(2h + 2, \mathbb{Z})$

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \quad I = 0, 1, \dots, h$$

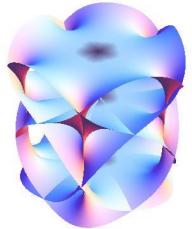
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ preserves the symplectic matrix: } \Sigma = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$E_8 \times E_8$ Heterotic string on 6D CY (standard embedding)

Candelas-Horowitz-Strominger-Witten ('85)

- 4D Gauge symmetry :

$$E_6 \times E_8$$



- Moduli \approx Matters (E_6 : 27 or $\overline{27}$)

27^i : Kaehler moduli

$\overline{27}^a$: Complex structure moduli

- Yukawa couplings (27^3)

$$W = F_{ijk} 27^i 27^j 27^k$$

$$\begin{aligned} F_{ijk} &= \partial_i \partial_j \partial_k F \\ F &: \text{prepotential} \end{aligned}$$

Symplectic modular symmetry in CY moduli space
~ Flavor symmetry

- Symplectic transf. of matters (27^i) and Yukawa couplings (F_{ijk})

$$27^i \rightarrow \widetilde{27}^i = (\tilde{X}^0)^{1/3} \frac{\partial \tilde{X}^i}{\partial X^j} 27^j$$

$$F_{ijk} \rightarrow \tilde{F}_{ijk} = \frac{\partial X^l}{\partial \tilde{X}^i} \frac{\partial X^m}{\partial \tilde{X}^j} \frac{\partial X^n}{\partial \tilde{X}^k} F_{lmn}$$

(X^0, X^i) : projective coordinates
with the gauge $X^0 = 1$
 $(i = 1, 2, \dots, h)$

- Non-trivial representations of $Sp(2h + 2, \mathbb{Z})$
- Flavor symmetry : $G_{\text{flavor}} \subset Sp(2h + 2, \mathbb{Z})$
- $U(1)_R$ symmetry : $U(1)_R \subset Sp(2h + 2, \mathbb{C})$

$\tilde{X}^0 (= e^{2i\alpha})$: R-symmetric trf. with R-charge 2/3 for matters

- S_4 flavor symmetry on 6D toroidal orbifolds with $h = 3$

$$W = 27^1 27^2 27^3$$

$$F_{123} = 1, \text{ otherwise } 0$$

- Invariant under two generators

$$P : 1 \rightarrow 3, 2 \rightarrow 1, 3 \rightarrow 2 \quad (\text{permutation of three } T^2)$$

$$Q : 1 \rightarrow -1, 2 \rightarrow -3, 3 \rightarrow 2$$

- S_4 triplet : $\{27^1, 27^2, 27^3\}$

$$S_4 \subset Sp(2 \times 3 + 2, \mathbb{Z}) = \boxed{Sp(8, \mathbb{Z})} \quad \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \rightarrow \boxed{\begin{pmatrix} a & b \\ c & d \end{pmatrix}} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

- Some classes of 6D Calabi-Yaus have S_4 flavor symmetry

Geometrical origin of 4D CP

- Assumption : 10D = 4D + 6D Calabi-Yau (CY)

Consider simultaneous transformations of

- 4D parity
- 6D orientation reversing : $z_i \rightarrow -\bar{z}_i$ ($i = 1, 2, 3$)

(z_i : local coordinates of 6D space)

(Volume form : $dV \rightarrow -dV$)

$$dV \propto dz_1 \wedge dz_2 \wedge dz_3 \wedge d\bar{z}_1 \wedge d\bar{z}_2 \wedge d\bar{z}_3$$

*Strominger-Witten ('85)
Dine-Leigh-MacIntire ('92)
Choi-Kaplan-Nelson ('92)*

10D Majorana-Weyl spinor under $SO(1,9) = SO(1,3) \times SO(6)$:

$$16 = (2, 4_+) \oplus (2', \bar{4}_-)$$

$2, 2'$: left- and right-handed spinors of $SL(2, \mathbb{C})$
 $4_+, \bar{4}_-$: + and - chirality spinors of $SU(4)$

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$$(2, 4_+) \rightarrow (2', \bar{4}_-)$$

E.g., in heterotic string, E_6 :

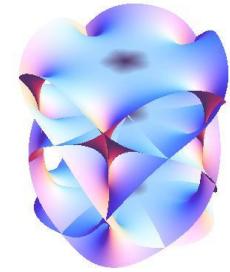
$$27 \rightarrow \bar{27}$$

- Such transformations correspond to 4D CP

- $Sp(2h + 2, \mathbb{Z})$ modular transformation:

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$z_i = x_i + iy_i$$



- CP transformation (6D orientation reversing: $x_i \rightarrow -x_i, y_i \rightarrow y_i$)

CP-even basis : $\{dx_i \wedge dx_j \wedge dy_k, dy_i \wedge dy_j \wedge dy_k\}$

CP-odd basis : $\{dx_i \wedge dx_j \wedge dx_k, dx_i \wedge dy_j \wedge dy_k\}$

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \rightarrow \mp \mathcal{CP} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$\mathcal{CP} = \pm \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- $GSp(2h + 2, \mathbb{Z}) = Sp(2h + 2, \mathbb{Z}) \rtimes \mathbb{Z}_2^{CP}$
generalized symplectic modular symmetry

- Flavor sym. is enlarged to $G_{\text{flavor}} \rtimes \mathbb{Z}_2^{CP}$ (e.g., $S_4 \rtimes \mathbb{Z}_2^{CP}$)

Conclusion

- Common origin of 4D flavor, CP, and $U(1)_R$ symmetries
 - Moduli \approx Matters (Heterotic string with standard embedding)
 - Symplectic modular flavor symmetry (e.g., S_4)
$$G_{\text{flavor}} \subset Sp(2h + 2, \mathbb{Z})$$
 - Generalized symplectic modular symmetry
$$G_{\text{flavor}} \rtimes \mathbb{Z}_2^{CP} \subset GSp(2h + 2, \mathbb{Z})$$
 - R-symmetry: $U(1)_R \subset Sp(2h + 2, \mathbb{C}) \subset GSp(2h + 2, \mathbb{C})$
- Stringy origin of Minimal Flavor Violation

T. Kobayashi and H.O., arXiv: 2108.XXXXXX

$$y_{ijkl} = \sum_m y_{ijm} y_{\bar{m}kl}$$