

Modular flavor symmetry of three-generation modes in magnetized orbifold models

Hikaru Uchida (Hokkaido Univ.)

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- arXiv:2101.00826

Collaborates: Shota Kikuchi, Tatsuo Kobayashi, et al.

Introduction : Flavor Symmetry

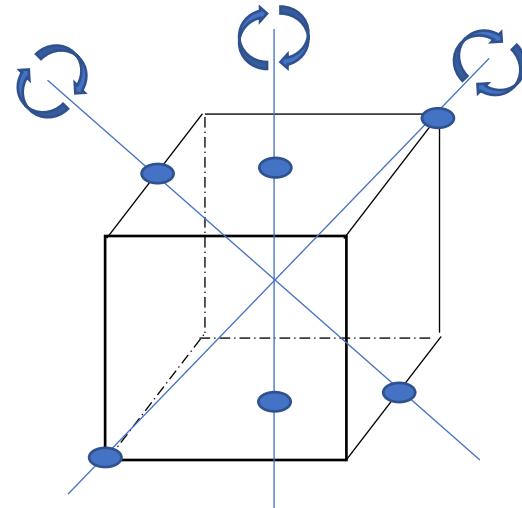
4D Standard Model

Mysteries of Flavor Structure

- Origin of 3 generation
- Origin of mass hierarchy
- Origin of flavor mixing
- . . .

Flavor Symmetry
Non-Abelian Discrete Groups
(e.g.) S_4

$$\begin{array}{c} a^4 = (bc)^4 = \mathbf{1} \\ b^2 = \mathbf{1} \qquad \qquad \qquad c^3 = \mathbf{1} \end{array}$$



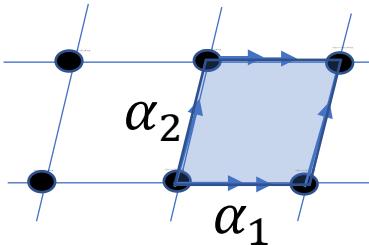
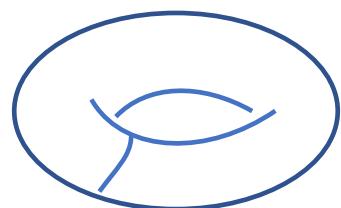
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+ Compact space (e.g.) $T^2 \simeq \mathcal{C}/\Lambda$



Complex Structure Modulus : $\tau = \frac{\alpha_2}{\alpha_1}$

Modular Flavor Symmetry
Finite Modular Subgroups
 $\Gamma_N (\subset \bar{\Gamma})$ (e.g.) $\Gamma_4 \cong S_4$

↑
Geometrical Structure
(e.g.) Modular symmetry

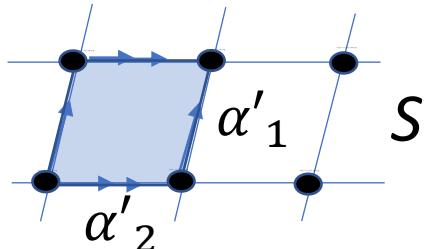
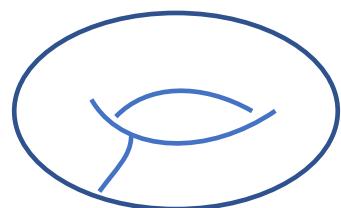
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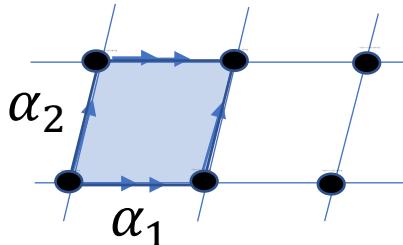
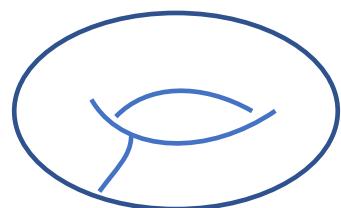
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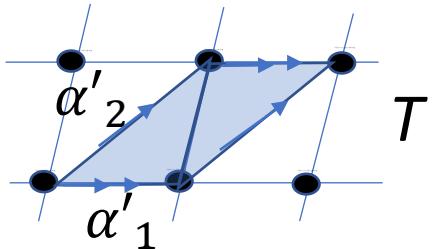
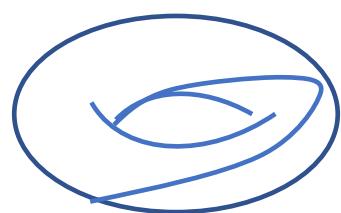
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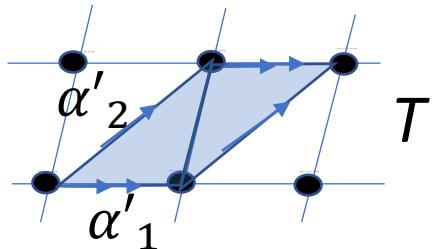
$$T: \tau \rightarrow \tau + 1$$

Introduction : Modular Flavor Symmetry

4D Standard Model Mysteries of Flavor Structure

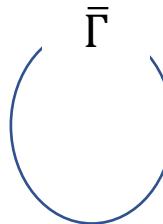
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$$\bar{\Gamma} \equiv SL(2, \mathbf{Z})/\mathbf{Z}_2$$

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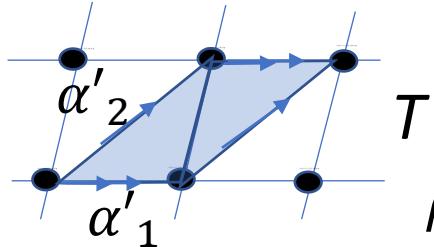
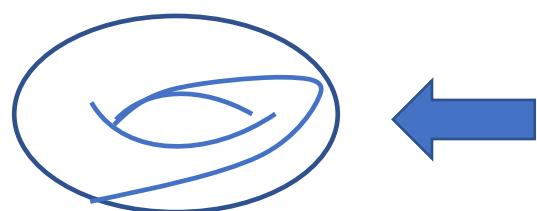
$$[S^2 = (ST)^3 = 1]$$

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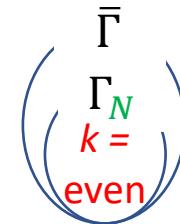
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Modular Forms of Weight k ($\in 2\mathbb{Z}$): $Y(\tau)$

Image



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$$\uparrow + [\rho(T)^N = 1]$$

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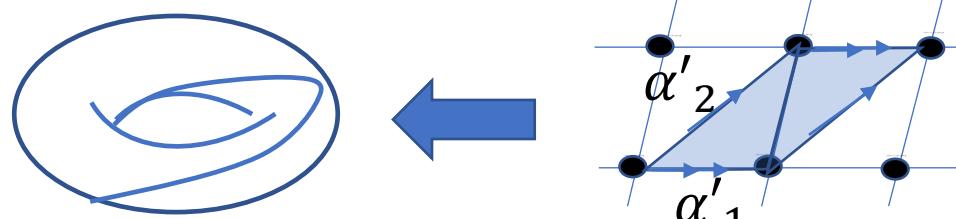
$$[\rho(S)^2 = [\rho(S)\rho(T)]^3 = 1]$$

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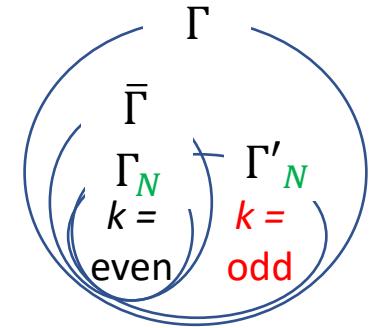
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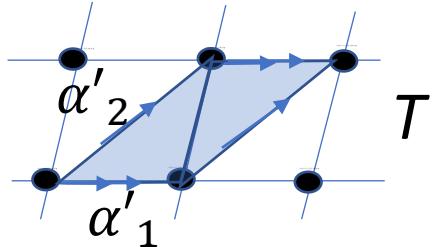
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Assumption:

Flavor group, Representation, Weight

How can they be determined?

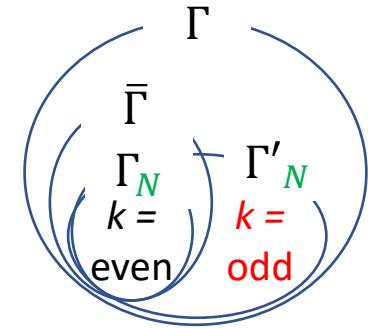
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Modular Flavor Symmetry on magnetized T^2

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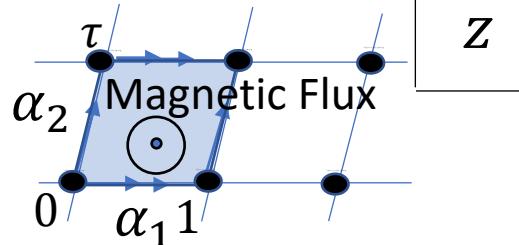
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(Coordinate, Modulus) : (z, τ)

Magnetic Flux : $(2\pi)^{-1} \int_{T^2} F = M \in \mathbb{Z}$

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(e.g.) *Modular symmetry

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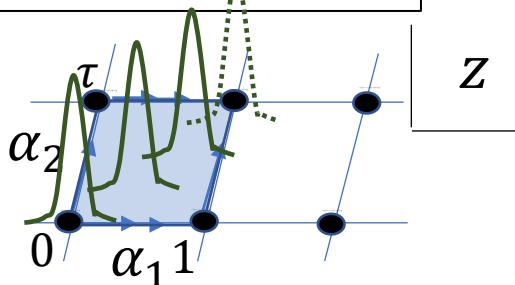
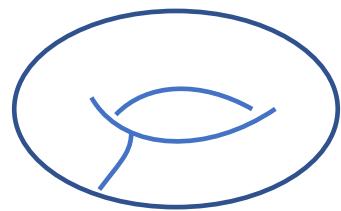
M generations on magnetized T^2

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z

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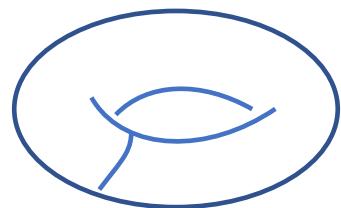
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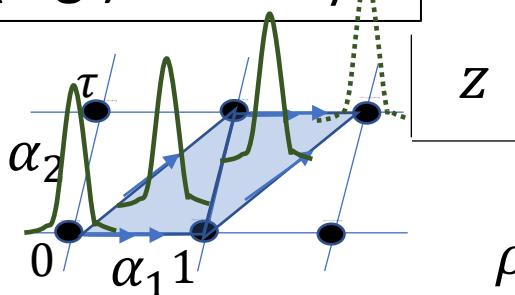
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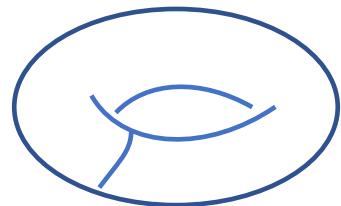
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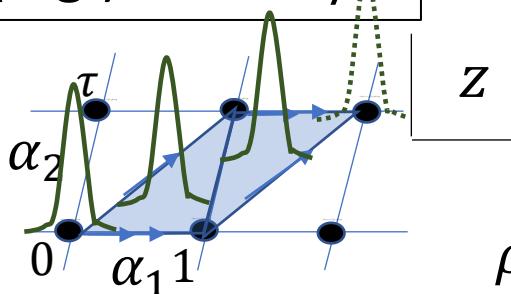
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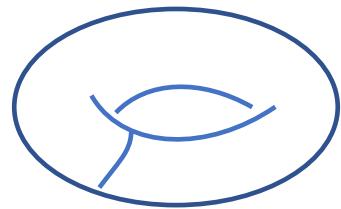
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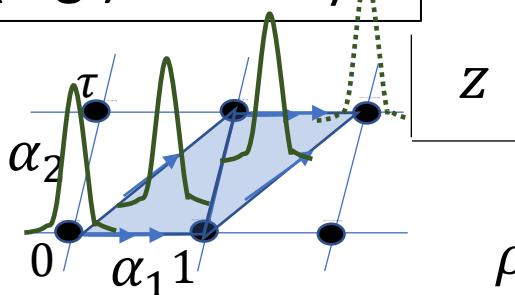
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Modular Flavor Symmetry on magnetized T^2/Z_2

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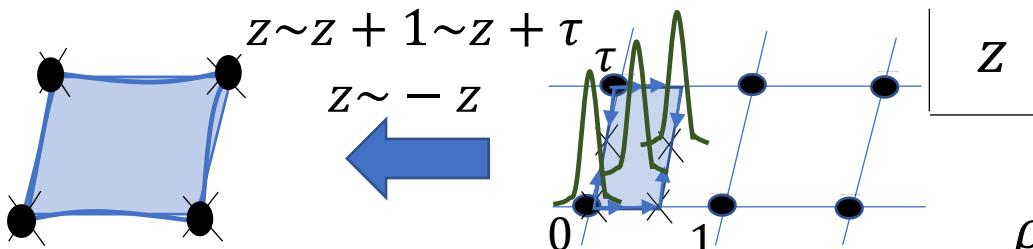
3 generations on magnetized T^2/Z_2

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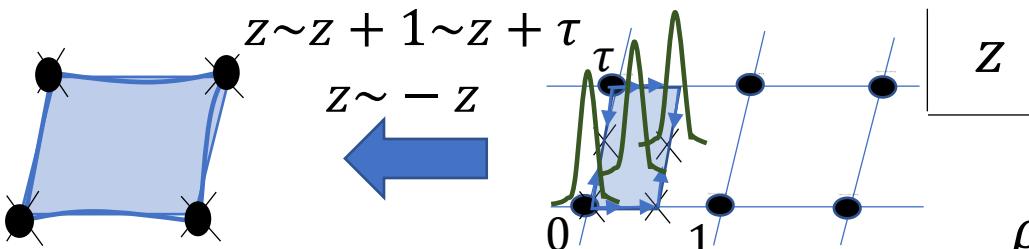
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Modular Flavor Symmetry
Finite Modular Subgroups

$$\Gamma_M \times Z_8 \quad (M = 5, 7)$$

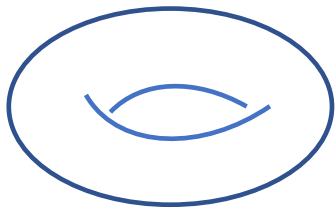
$$\uparrow + \quad [\rho(T)^M = e^{i\pi/4} \mathbf{1}]$$

Geometrical Structure
(e.g.) *Modular symmetry

$$\rho: \text{representation of } \tilde{\Gamma} \equiv \widetilde{SL}(2, \mathbb{Z})$$

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Magnetized T^2

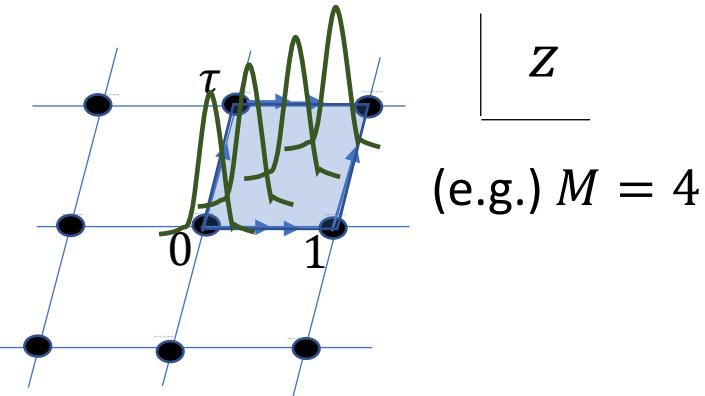


T^2

$z \sim z + 1 \sim z + \tau$



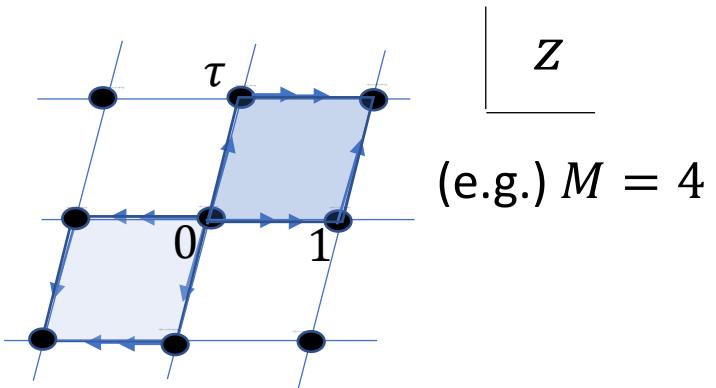
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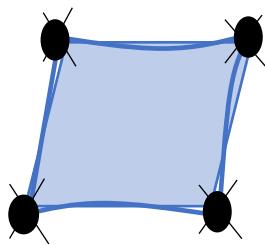
Magnetized T^2/Z_2

T^2/Z_2
 $z \sim z + 1 \sim z + \tau$
 \leftarrow
 $z \sim -z$

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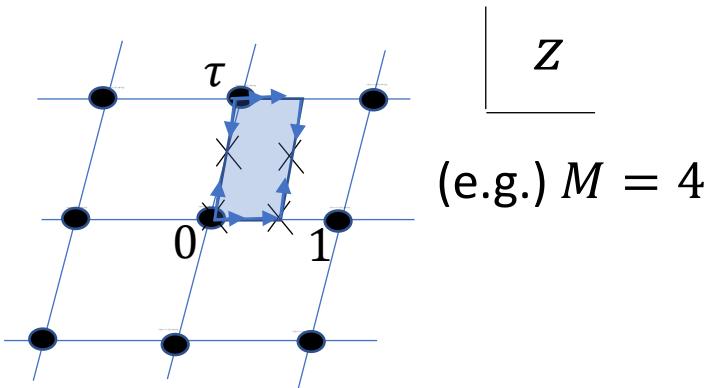


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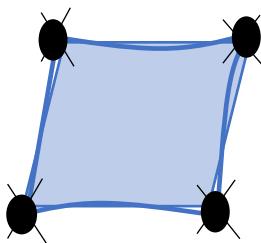


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$$T^2/Z_2$$

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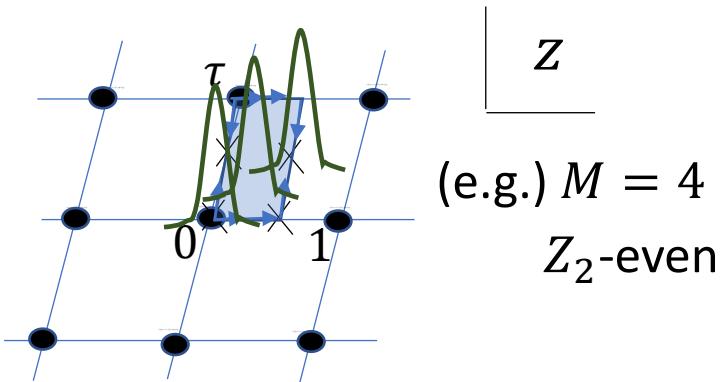


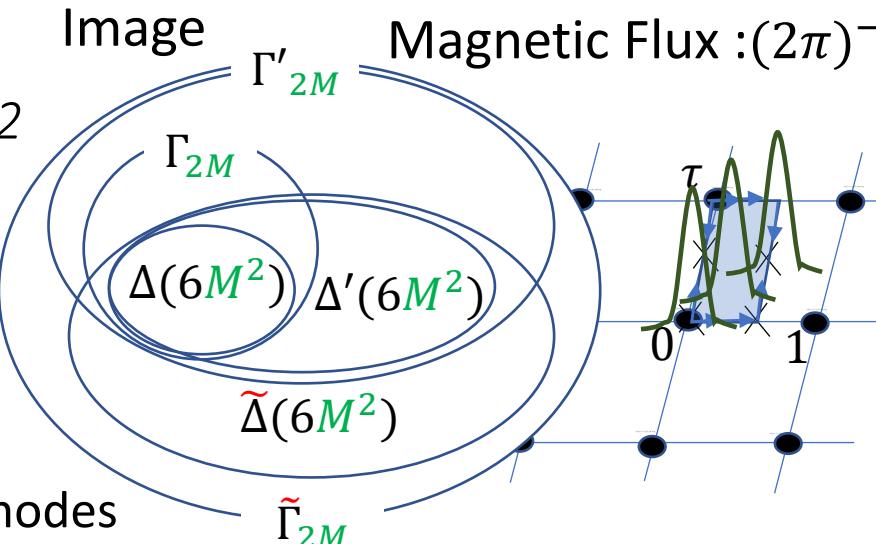
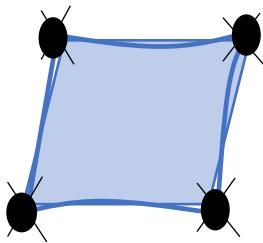
Table.1: The number of zero-modes
[with Scherk-Schwarz phase
 $(\alpha_1, \alpha_\tau) = (0,0)$]

M	2	4	6	8
T^2/Z_2 even	2	3	4	5
T^2/Z_2 odd	0	1	2	3

(e.g.)

$$M = 4 \begin{pmatrix} \psi_{T^2/Z_2^+}^{0,4} \\ \psi_{T^2/Z_2^+}^{1,4} \\ \psi_{T^2/Z_2^+}^{2,4} \end{pmatrix}$$

Magnetized T^2/Z_2



Magnetic Flux : $(2\pi)^{-1} \int_{T^2} F = M \in \mathbb{Z}$
 Z
(e.g.) $M = 4$
 Z_2 -even

Table.1: The number of zero-modes
[with Scherk-Schwarz phase
 $(\alpha_1, \alpha_\tau) = (0,0)$]

M	2	4	6	8
T^2/Z_2 even	2	3	4	5
T^2/Z_2 odd	0		2	3

$\tilde{\Delta}(6M^2)$ $\tilde{\Delta}(96)$ $\tilde{\Delta}(384)$

with
modular
weight
 $1/2$

$$\tilde{\Gamma}_{2M} \quad \rho(S)^8 = [\rho(S)\rho(T)]^6 = \rho(T)^{2M} = 1$$

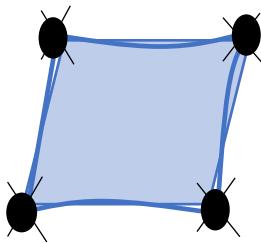


$$\tilde{\Delta}(6M^2) \quad (\rho(S)^{-1}\rho(T)^{-1}\rho(S)\rho(T))^3 = 1$$

$$\left. \begin{aligned} \Delta(6M^2) &\cong (Z_M \times Z'_M) \rtimes Z_3 \rtimes Z_2 \\ \tilde{\Delta}(6M^2) &\cong (Z_M \times Z'_M) \rtimes Z_3 \rtimes Z_8 \end{aligned} \right\}$$

$$(e.g.) \quad M = 4 \begin{pmatrix} \psi_{T^2/Z_2^+}^{0,4} \\ \psi_{T^2/Z_2^+}^{1,4} \\ \psi_{T^2/Z_2^+}^{2,4} \end{pmatrix} \quad \rho(S) = \frac{e^{i\pi/4}}{\sqrt{2}} \begin{pmatrix} 1 & \sqrt{2} & 1 \\ \sqrt{2} & 0 & -\sqrt{2} \\ 1 & -\sqrt{2} & 1 \end{pmatrix} \quad \rho(T) = \begin{pmatrix} 1 & & \\ & e^{i\pi/4} & \\ & & -1 \end{pmatrix}$$

Magnetized T^2/Z_2



$$T^2/Z_2$$

$$z \sim z + 1 \sim z + \tau$$

$$z \sim -z$$

Magnetic Flux : $(2\pi)^{-1} \int_{T^2} F = M \in \mathbb{Z}$

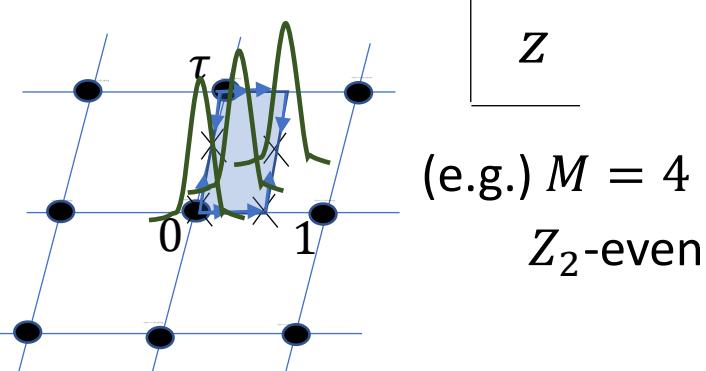


Table.1: The number of zero-modes
[with Scherk-Schwarz phase
 $(\alpha_1, \alpha_\tau) = (0,0)$]

M	2	4	6	8
T^2/Z_2 even	2	3	4	5
T^2/Z_2 odd	0	2	3	

with
modular
weight
 $1/2$

$$\tilde{\Delta}(6M^2) \quad \tilde{\Delta}(96) \quad \tilde{\Delta}(384)$$

Table.2: The number of zero-modes
[with Scherk-Schwarz phase
 $(\alpha_1, \alpha_\tau) = (1/2, 1/2)$]

M	1	3	5	7
T^2/Z_2 even	0	1	2	3
T^2/Z_2 odd	1	2	3	1

$$(\rho(\tilde{T}))^M = e^{i\pi/4} \mathbf{1}$$

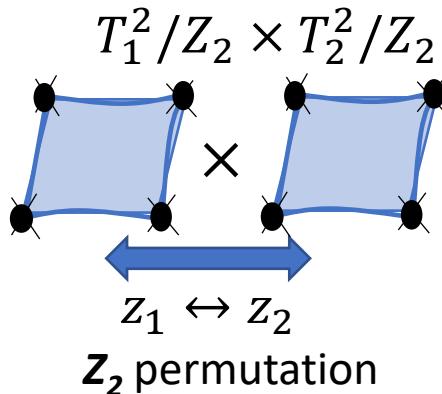
$$\Gamma_M \times Z_8 \quad A_5 \times Z_8 \quad PSL(2, \mathbb{Z}_7) \times Z_8$$

$$(e.g.) \quad M = 4 \begin{pmatrix} \psi_{T^2/Z_2^+}^{0,4} \\ \psi_{T^2/Z_2^+}^{1,4} \\ \psi_{T^2/Z_2^+}^{2,4} \end{pmatrix}$$

$$\rho(S) = \frac{e^{i\pi/4}}{\sqrt{2}} \begin{pmatrix} 1 & \sqrt{2} & 1 \\ \sqrt{2} & 0 & -\sqrt{2} \\ 1 & -\sqrt{2} & 1 \end{pmatrix}$$

$$\rho(T) = \begin{pmatrix} 1 & & \\ & e^{i\pi/4} & \\ & & -1 \end{pmatrix}$$

3-generation modes on $(T^2 \times T^2)/(\mathbb{Z}_2 \times \mathbb{Z}_2)$



$$\tau_1 = \tau_2 = \tau$$

$$M_1 = M_2 = \textcolor{red}{M}$$

3-generation modes

SS phase $(\alpha_1, \alpha_\tau) = (0,0)$

$\textcolor{red}{M} = 2 \quad \Gamma'_4 \cong S'_4 \cong \Delta'(24)$

$\textcolor{red}{M} = 4 \quad \Gamma'_8 \supset \Delta'(96)$

$\textcolor{red}{M} = 8 \quad \Gamma'_{16} \supset \Delta'(384)$

3-dimensional representations of Γ_N

$$\Gamma_4 \cong S_4 \cong \Delta(24)$$

$$\Gamma_8 \supset \Delta(96)$$

$$\Gamma_{16} \supset \Delta(384)$$

$$\Gamma_3 \cong A_4$$

$$\Gamma_5 \cong A_5$$

$$\Gamma_7 \cong PSL(2, \mathbb{Z}_7)$$

$$\Gamma'_{2\textcolor{red}{M}} \supseteq \Delta'(6\textcolor{red}{M}^2)$$

SS phase $(\alpha_1, \alpha_\tau) = (1/2, 1/2)$

$\textcolor{red}{M} = 3 \quad \Gamma_3 \times \mathbb{Z}_4 \cong A_4 \times \mathbb{Z}_4$

$\textcolor{red}{M} = 5 \quad \Gamma_5 \times \mathbb{Z}_4 \cong A_5 \times \mathbb{Z}_4$

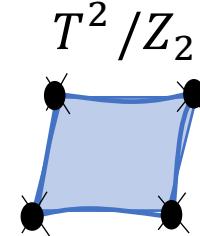
$\textcolor{red}{M} = 7 \quad \Gamma_7 \times \mathbb{Z}_4 \cong PSL(2, \mathbb{Z}_7) \times \mathbb{Z}_4$

$$\Gamma_{\textcolor{red}{M}} \times \mathbb{Z}_4$$

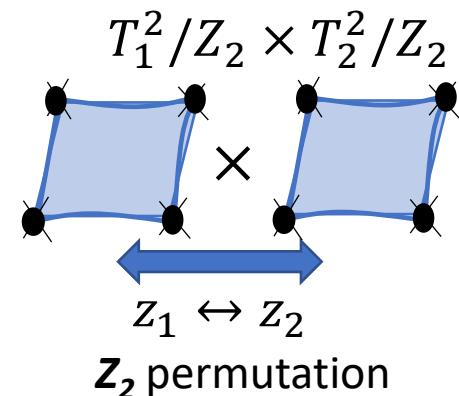
with modular weight 1

Conclusion

- 3-generation modes on magnetized T^2/Z_2 orbifold transform as modular $\tilde{\Delta}(6M^2)$ ($M = 4, 8$) triplets or $\Gamma_M \times Z_8$ ($M = 5, 7$) triplets with modular weight $1/2$.



- 3-generation modes on magnetized $(T^2 \times T^2)/(Z_2 \times Z_2)$ orbifold transform as modular $\Delta'(6M^2)$ ($M = 2, 4, 8$) triplets or $\Gamma_M \times Z_4$ ($M = 3, 5, 7$) triplets with modular weight 1 .



Flavor group, Representation, Weight can be determined in magnetized orbifold models!

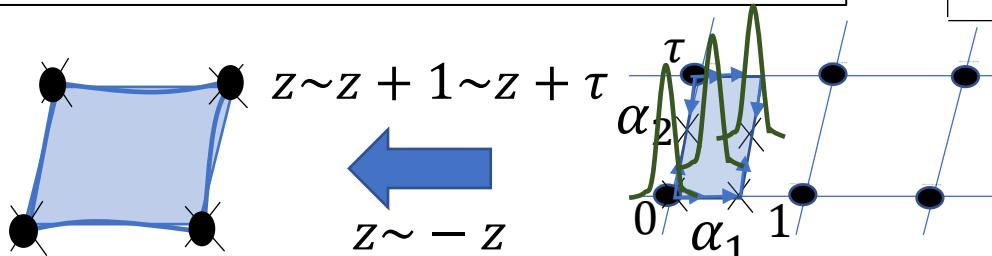
Future Work

4D Standard Model

Mysteries of Flavor Structure

- Origen of 3 generation
- Origen of mass hierarchy
- Origen of flavor mixing
- . . .

+ Compact space (e.g.) T^2/Z_2



(Coordinate, Modulus) : (z, τ)

Wavefunctions : $\psi(z, \tau)$

Magnetic Flux : $(2\pi)^{-1} \int_{T^2} F = M \in 2Z$

3 generations on magnetized T^2/Z_2

Modular Flavor Symmetry
Finite Modular Subgroups
 $(\tilde{\Gamma}_{2M} \supset) \tilde{\Delta}(6M^2)$ ($M = 4, 8$)

z

Geometrical Structure
(e.g.) *Modular symmetry

ρ : representation of $\tilde{\Gamma} \equiv \widetilde{SL}(2, \mathbf{Z})$

$T: \psi(z, \tau) \rightarrow \psi(z, \tau + 1) = \rho(T)\psi(z, \tau)$

$S: \psi(z, \tau) \rightarrow \psi(-\frac{z}{\tau}, -\frac{1}{\tau}) = (-\tau)^{1/2}\rho(S)\psi(z, \tau)$

$[\rho(S)^8 = [\rho(S)\rho(T)]^6 = 1]$