Quark-hadron duality for neutral meson mixings in the ’t Hooft model

$$D^0 - \bar{D}^0, B_d^0 - \bar{B}_d, B_s^0 - \bar{B}_s$$

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Based on arXiv:2106.06215

Introduction

- **<u>Charm quark mass is a unique scale</u>.**

$$m_c \approx 1.3 - 1.7 \text{ GeV}$$

— too heavy for ChPT

— too light for $\Lambda_{\rm QCD}/m_c$ expansion?

Theoretically challenging

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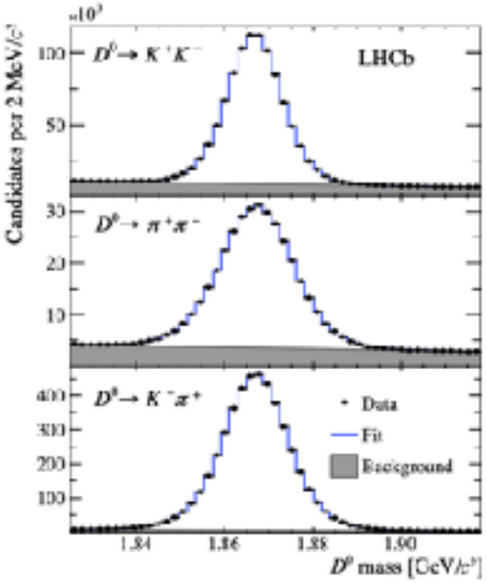
Theoretically challenging

- **<u>High statistics data are provided</u>.**

— in a good stage to test theories



LHCb [1810.06874]

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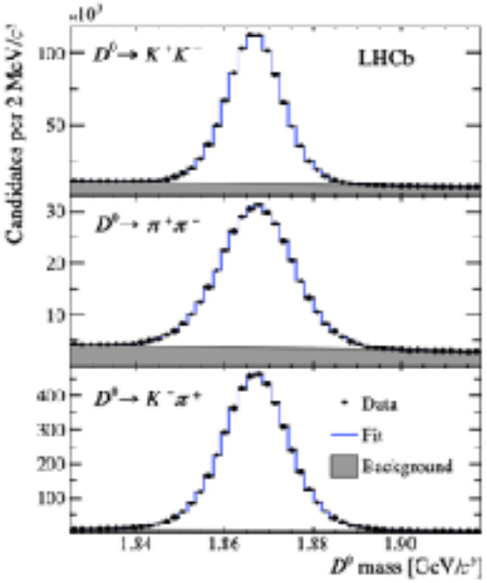


LHCb [1810.06874]

- **<u>$D^0 - \bar{D}^0$ mixing</u>.**

— $\Lambda_{\rm QCD}/m_c$ expansion is not successful

— experimental data are not quantitatively reproduced yet

perhaps, quark-hadron duality is violated?

Outline

(A) $D^0 - \bar{D}^0$ mixing

— theoretical method (OPE) and comparison with experiments

(B) Quark-hadron duality for neutral meson mixings

— quark-hadron duality in the ’t Hooft model

— numerical results for duality violation

(C) Summary

$D^0 - \bar{D}^0$ mixing

time evolution

$\bar{D}^0 \rightarrow D^0$

Time evolution Eq. $$i\frac{\partial}{\partial t}\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix} = \left(\mathbf{M} - \frac{i}{2}\mathbf{\Gamma}\right)\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix}$$

CP-conserving limit

Mass eigenstate $|D_{1,2}\rangle = |D^0\rangle \pm |\bar{D}^0\rangle$

observables

$$\begin{cases} x = (M_1 - M_2)/\Gamma = 2M_{12}/\Gamma & \text{mass difference} \\ y = (\Gamma_1 - \Gamma_2)/2\Gamma = \Gamma_{12}/\Gamma & \text{width difference} \end{cases}$$

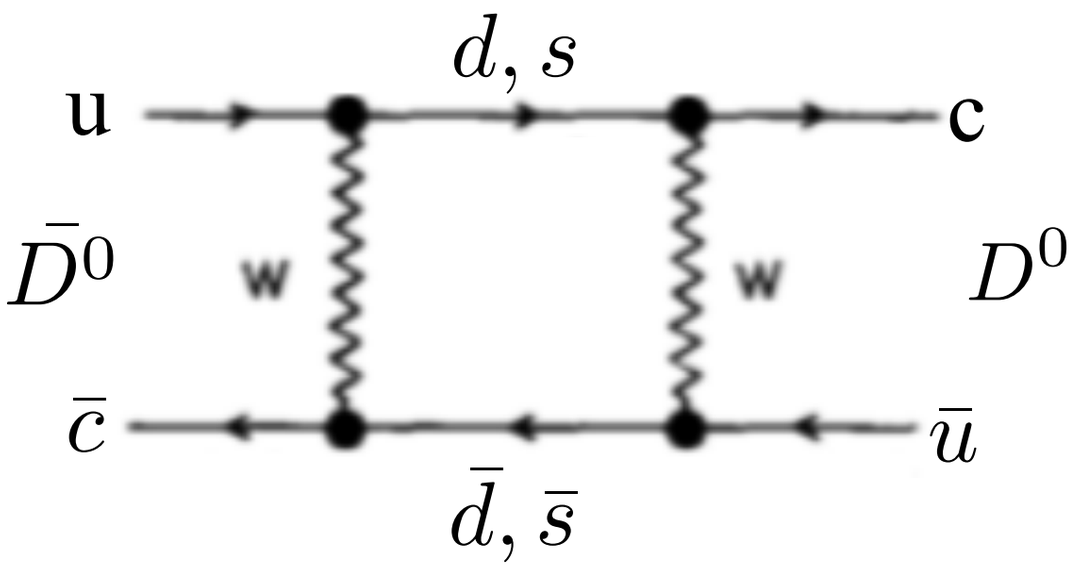
Γ : total width

$D^0 - \bar{D}^0$ mixing: theory

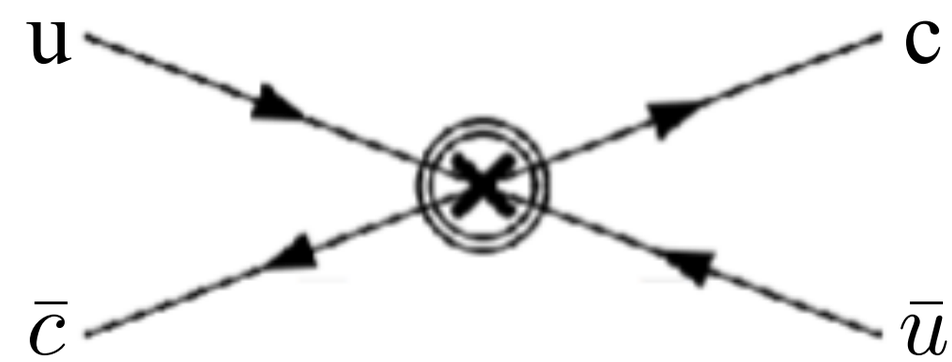
Two methods

(1) Exclusive approach (hadronic-level)

✔ (2) Inclusive approach (quark-level)



box diagrams



4-fermi interaction

$$A_{12} = M_{12} - \frac{i}{2}\Gamma_{12}$$

Heavy quark expansion (HQE)

$$\Gamma_{12} = \sum_n \frac{C_n}{m_c^n} \langle D^0 | \mathcal{O}_n^{\Delta C=2} | \bar{D}^0 \rangle$$

C_n : Wilson coefficient

n : dimension of operator

Theory / Experiment comparison (for inclusive)

	D meson	B_s meson	B_d meson
Box diagram	Hagelin 1981, Cheng 1982 Buras, Slominski and Steger 1984	Lenz and Tetlalmatzi-Xolocotzi 2019	Lenz and Tetlalmatzi-Xolocotzi 2019
NLO QCD	✔Golowich and Petrov 2005 Bobrowski *et al.* 2010		
SM	$x \simeq 6 \cdot 10^{-7}$ $y \simeq 6 \cdot 10^{-7}$ suppressed by GIM cancellation	$\Delta m_s = (18.77 \pm 0.86)\ \mathrm{ps}^{-1}$ $\Delta\Gamma_s = (0.091 \pm 0.013)\ \mathrm{ps}^{-1}$	$\Delta m_d = (0.543 \pm 0.029)\ \mathrm{ps}^{-1}$ $\Delta\Gamma_d = (2.6 \pm 0.4) \times 10^{-3}\ \mathrm{ps}^{-1}$
Exp.	HFLAV 2019 $x = (3.9^{+1.1}_{-1.2}) \times 10^{-3}$ $y = (6.51^{+0.63}_{-0.69}) \times 10^{-3}$	HFLAV 2019 $\Delta m_s = (17.757 \pm 0.021)\ \mathrm{ps}^{-1}$ $\Delta\Gamma_s = (0.090 \pm 0.005)\ \mathrm{ps}^{-1}$	HFLAV 2019 $\Delta m_d = (0.5065 \pm 0.0019)\ \mathrm{ps}^{-1}$ $\Delta\Gamma_d = (0.002 \pm 0.020)\ \mathrm{ps}^{-1}$

- For B_s meson, the experimental data are reproduced.
- For B_d meson, $\Delta\Gamma_d$ is reproduced within the experimental error.
- For D meson, the order of magnitude is not reproduced within four-quark operators.

*$\Delta m_{d,s}$ is not calculated in the HQE formalism.

Theory / Experiment comparison (for inclusive)

D meson

Box diagram — Hagelin 1981, Cheng 1982; Buras, Slominski and Steger 1984

NLO QCD — ✔Golowich and Petrov 2005; Bobrowski *et al.* 2010

SM
$$x \simeq 6 \cdot 10^{-7}$$
$$y \simeq 6 \cdot 10^{-7}$$

suppressed by GIM cancellation

Exp. (HFLAV 2019)
$$x = (3.9^{+1.1}_{-1.2}) \times 10^{-3}$$
$$y = (6.51^{+0.63}_{-0.69}) \times 10^{-3}$$

Possibilities discussed in the literature

✔ Violation of quark-hadron duality?

— 20% violation explains the data. (based on a simple model)

Jubb, Kirk, Lenz and Tetlalmatzi-Xolocotzi, 2017

Contribution of higher dim. operators?

suggested by Georgi, 1992, prior to the experimental measurement

— it gives a resource of SU(3) breaking linear in m_s, avoiding severe GIM cancellation?

$x, y \sim \mathcal{O}(10^{-3})$ Bigi and Uraltsev, 2001

— With some assumption about hadronic matrix elements,

$x \sim y \lesssim 10^{-3}$ Falk, Grossman, Ligeti and Petrov, 2001

Beyond the standard model?

e.g., Golowich, Pakvasa and Petrov, 2007; Golowich, Hewett, Pakvasa and Petrov, 2009

- For B_s meson, the experimental
- For B_d meson, $\Delta\Gamma_d$ is consistent within the experimental error.
- For D meson, the order of magnitude is not reproduced within four-quark operators.

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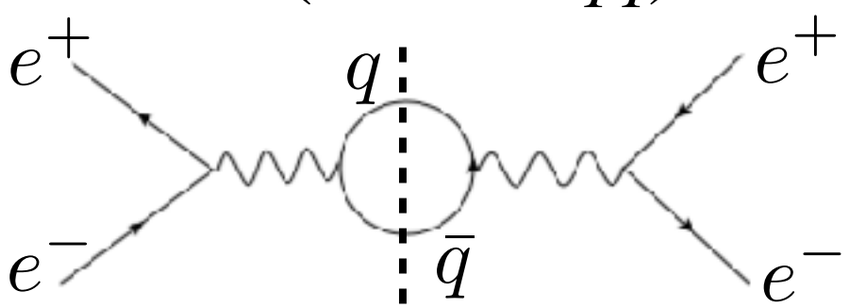
Inclusive and exclusive processes

Quark | hadron

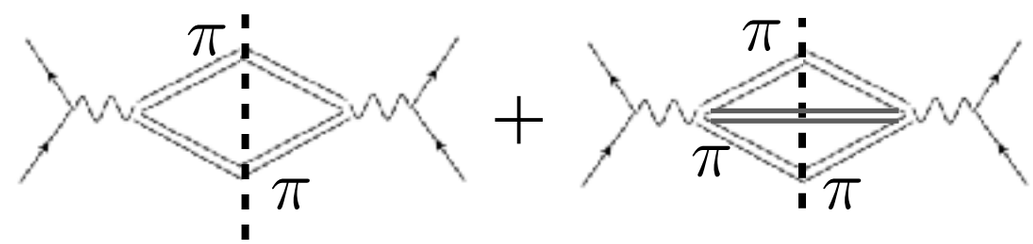
$\sigma^{(\mathrm{inc})}(e^+e^- \to q\bar{q})$



+ (condensates)

$e^+e^- \to$ hadrons

$\sigma(e^+e^- \to 2\pi) + \sigma(e^+e^- \to 3\pi) + \cdots$



Poggio, Quinn and Weinberg, 1976 for the case with smearing

Quark-hadron duality: inclusive = sum of exclusive

(one definition of)
Duality violation: inclusive $\neq$ sum of exclusive

Inclusive and exclusive processes

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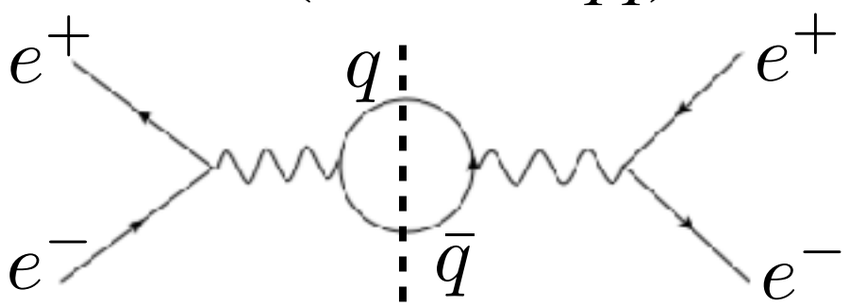

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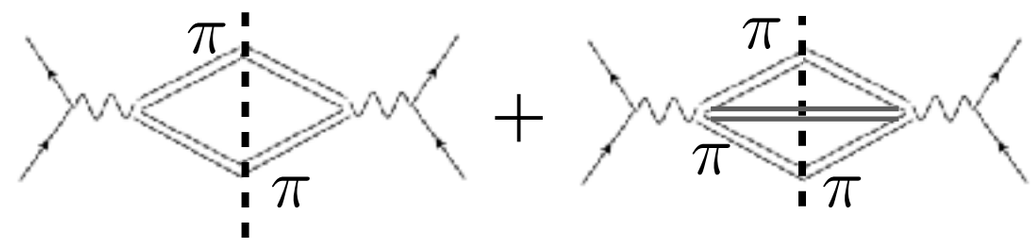
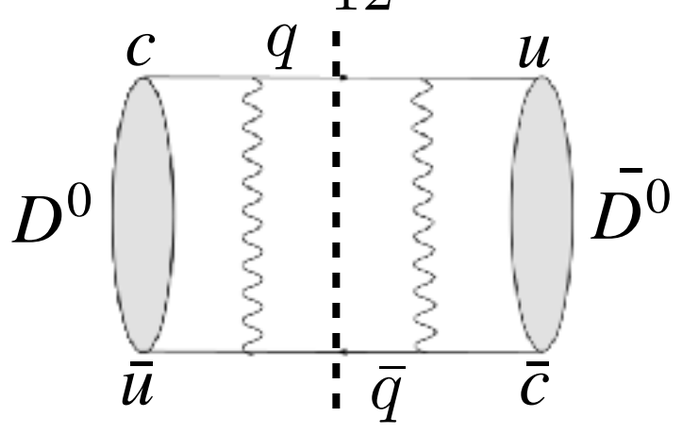


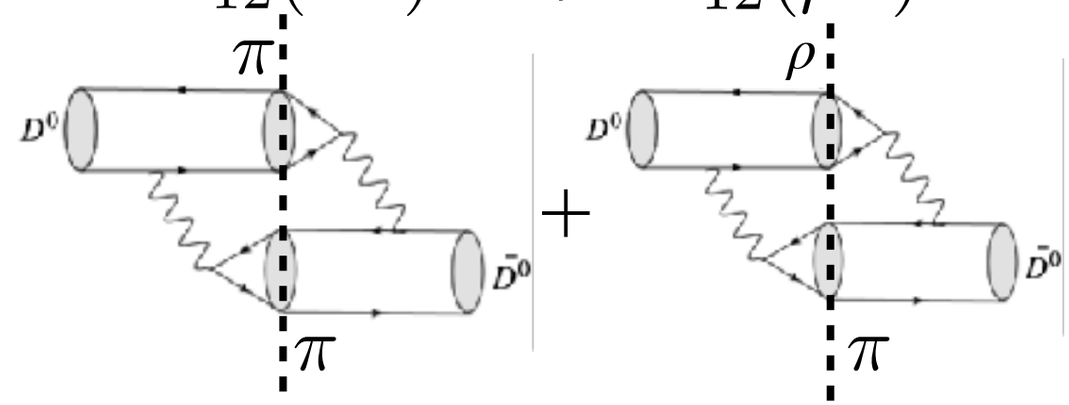
$D^0 - \bar{D}^0$

$\Gamma_{12}^{(\mathrm{inc})}$



+ (higher dimensional operators)

$\Gamma_{12}(\pi\pi) \quad + \quad \Gamma_{12}(\rho\pi) + \cdots$



+ (all the other exclusive states)

Non-trivial point

✔Does duality violation possibly give a large correction to the box diagram? This talk

Method to investigate duality violation

$$\Gamma_{12}^{(\mathrm{inc})} \stackrel{?}{=} \sum \Gamma_{12}^{(\mathrm{exc})}$$

Comparison between $d = 4$ and $d = 2$

	inclusive (HQE)	exclusive (Exp.)	exclusive (Theo.)
$d = 4$	✔	✔	✘
$d = 2$	✔	✘	✔

QCD is solvable in the large-N_c limit

’t Hooft, 1974

The ’t Hooft model (QCD$_2$ in the large-N_c limit)

◎ Asymptotic free theory. ◎ HQE is in common with d=4.

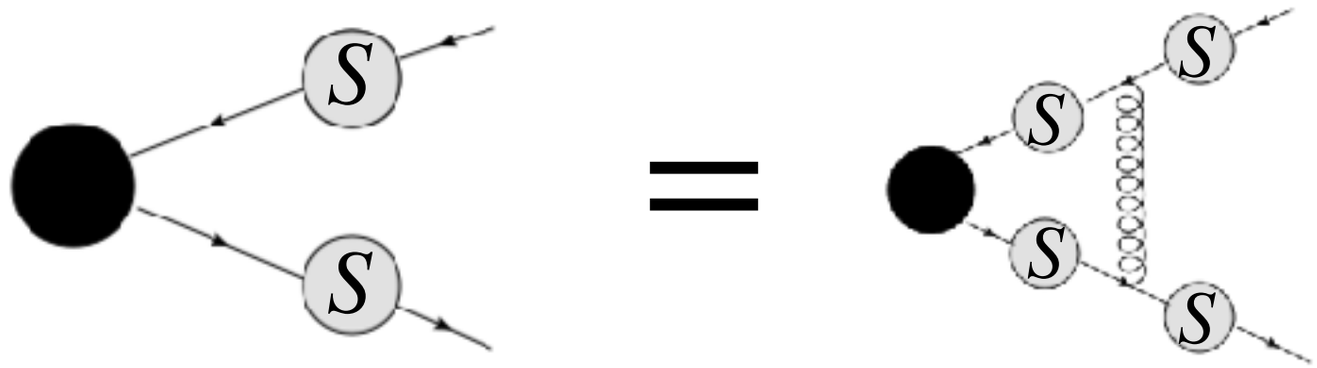
◎ Confinement is built-in. ◎ Gluon is not dynamical.

◎ Phase space offen has singularity in $d = 2$.

➡ Duality violation is qualitatively testable.

Bound state equation in the ’t Hooft model

Bethe-Salpeter equation (in the light-cone gauge):



the ’t Hooft equation: $M_k^2 \phi_k(x) = \left(\frac{m_1^2 - \beta^2}{x} + \frac{m_2^2 - \beta^2}{1-x} \right) \phi_k(x) - \beta^2 \mathrm{Pr} \int_0^1 \frac{\phi_k(y) \mathrm{d}y}{(x-y)^2}$

notation of QCD coupling: $\beta^2 = \frac{g^2}{2\pi} \left(N_c - \frac{1}{N_c} \right), \quad \lim_{N_c \to \infty} \beta = \mathrm{const.}$

Masses and wavefunctions for mesons can be determined within the formalism.

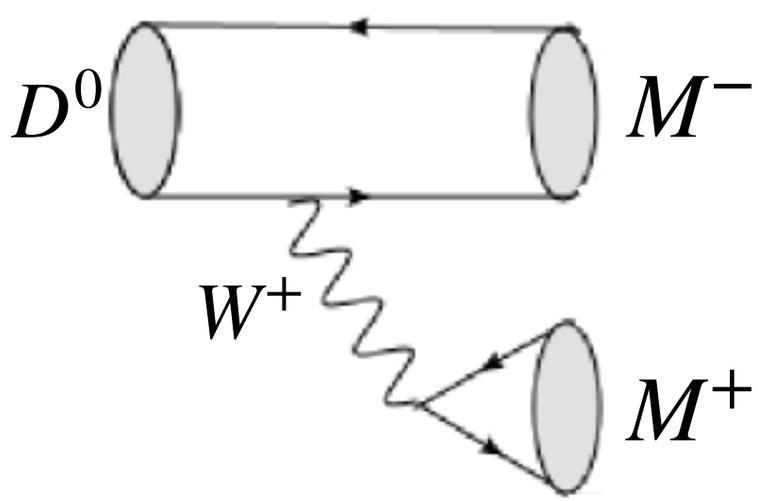
Exclusive processes for $D^0 - \bar{D}^0$ mixing

Width difference in CP conserving limit:

$$\Gamma_{12}^{(D^0)} = \sum_{k,m} (-1)^{k+m} \frac{T^{(k,m)} T^{(m,k)*}}{4M_{D^0}^2 |p_{km}|}$$

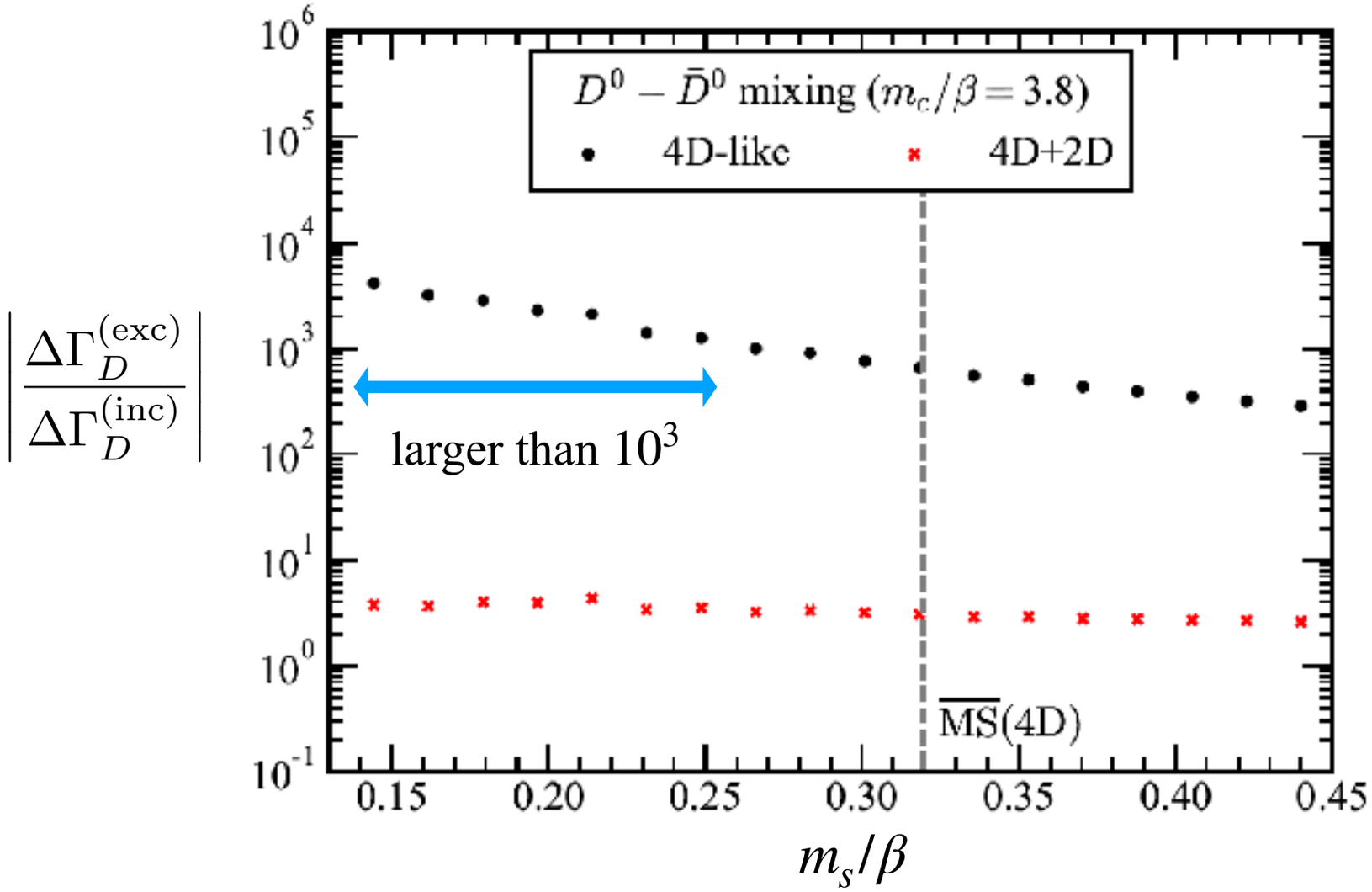
← phase space in 2D

$T^{(k,m)}$: color-allowed tree diagram



$M = \pi, K, +$ (excited states)

Numerical result: $D^0 - \bar{D}^0$ mixing



$$m_c = m_c^{\overline{\mathrm{MS}},4\mathrm{D}}$$

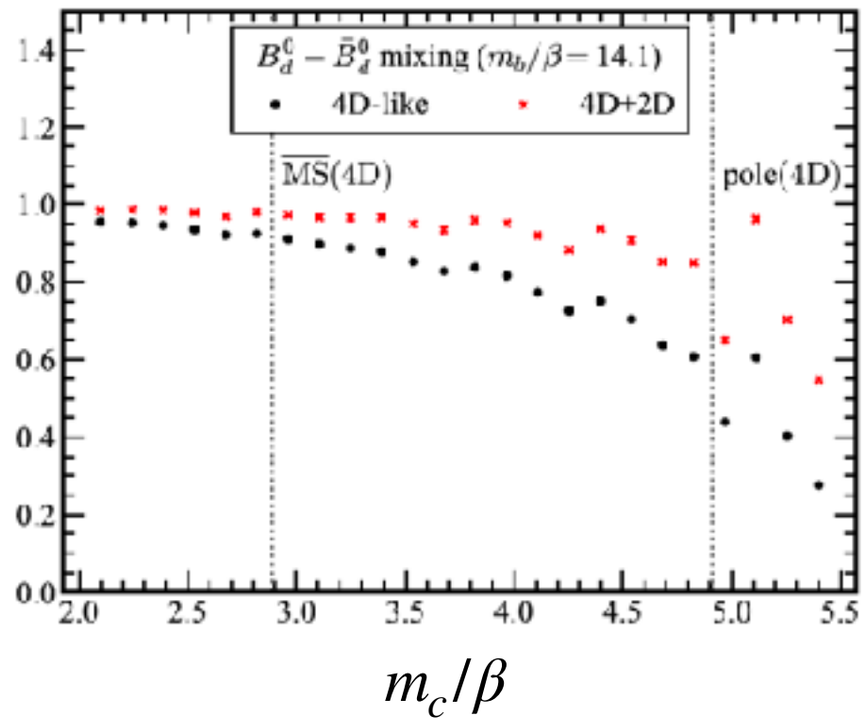
points: based only on the 4D-like phase space
crosses: based on the 4D-like phase space + 2D-specific one

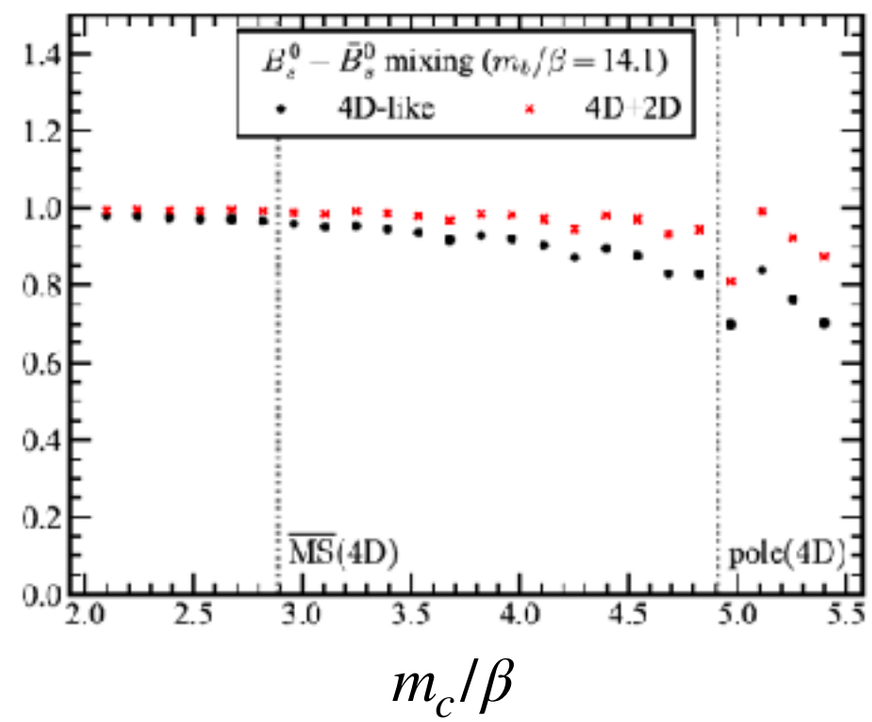
◎ The exclusive rate is enhanced by more than 10^3, confirmed for $0.14 < m_s/\beta < 0.25$, when only the 4D-like phase space function is used.

Numerical result: $B_q^0 - \bar{B}_q^0 \ (q = d, s)$

$B_d^0 - \bar{B}_d^0$ $\qquad$ $B_s^0 - \bar{B}_s^0$

$\left| \dfrac{\Delta\Gamma_q^{(\mathrm{exc})}}{\Delta\Gamma_q^{(\mathrm{inc})}} \right|$





$m_b = m_b^{\mathrm{pole,4D}}$

For $m_c < m_c^{\mathrm{pole,4D}}$

◎ For the $B_d^0 - \bar{B}_d^0$ mixing, the correction to the inclusive rate up to 40% is observed.

→ The correction of this size is not excluded yet.

(The experimental data for $\Delta\Gamma_{B_d}/\Gamma_{B_d}$ is still consistent with zero.)

◎ For the $B_s^0 - \bar{B}_s^0$ mixing, the correction to the inclusive rate up to 18% is observed.

→ The result is consistent with what is currently indicated in 4D.

(The ratio of the HFLAV data to the HQE gives $\Delta\Gamma_{B_s}^{(\mathrm{ex})}/\Delta\Gamma_{B_s}^{(\mathrm{th})} = 0.99 \pm 0.15$.)

A. Lenz and G. Tetlalmatzi-Xolocotzi, 2020

Summary

◎ We have studied local duality and its violation for heavy meson mixings on the basis of one certain dynamical mechanism.

◎ On the basis of the most color-allowed topological amplitude, we numerically evaluated duality violation.

— For the $D^0 - \bar{D}^0$ mixing, the order of magnitude for $\Delta\Gamma_D$ is enhanced by more than 10^3, confirmed for $0.14 < m_s/\beta < 0.25$, if the phase space function is given by 4D-like one.

— For the $B_s^0 - \bar{B}_s^0$ mixing, the observed difference between inclusive/exclusive is non-negligible: (20 % ,18 % ,11 % ,8%) for $m_b/\beta = (13.7, 14.1, 15.5, 17.0)$.

⟶ The results suggest that the theoretical $\Delta\Gamma_s$ should be made more precise, and motivate future measurement of $\Delta\Gamma_d$.

Backup

$D^0 - \bar{D}^0$ mixing: theory

Two methods

◉Exclusive

Hadronic-level analysis

Hard to calculate $\begin{cases} \Gamma[D \to \pi\pi] \\ \Gamma[D \to KK] \end{cases}$

↓

Data are used

✔◉Inclusive

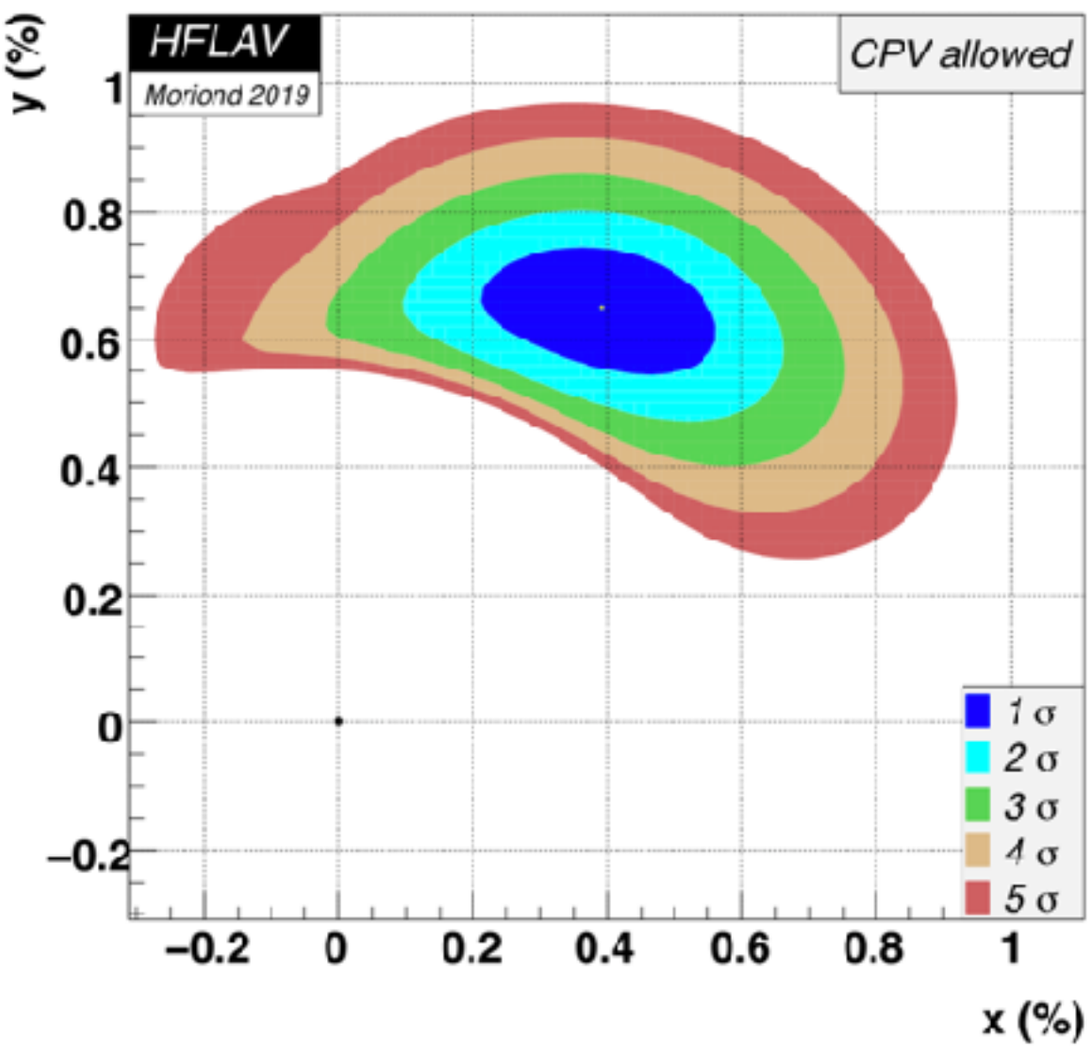
Quark-level analysis

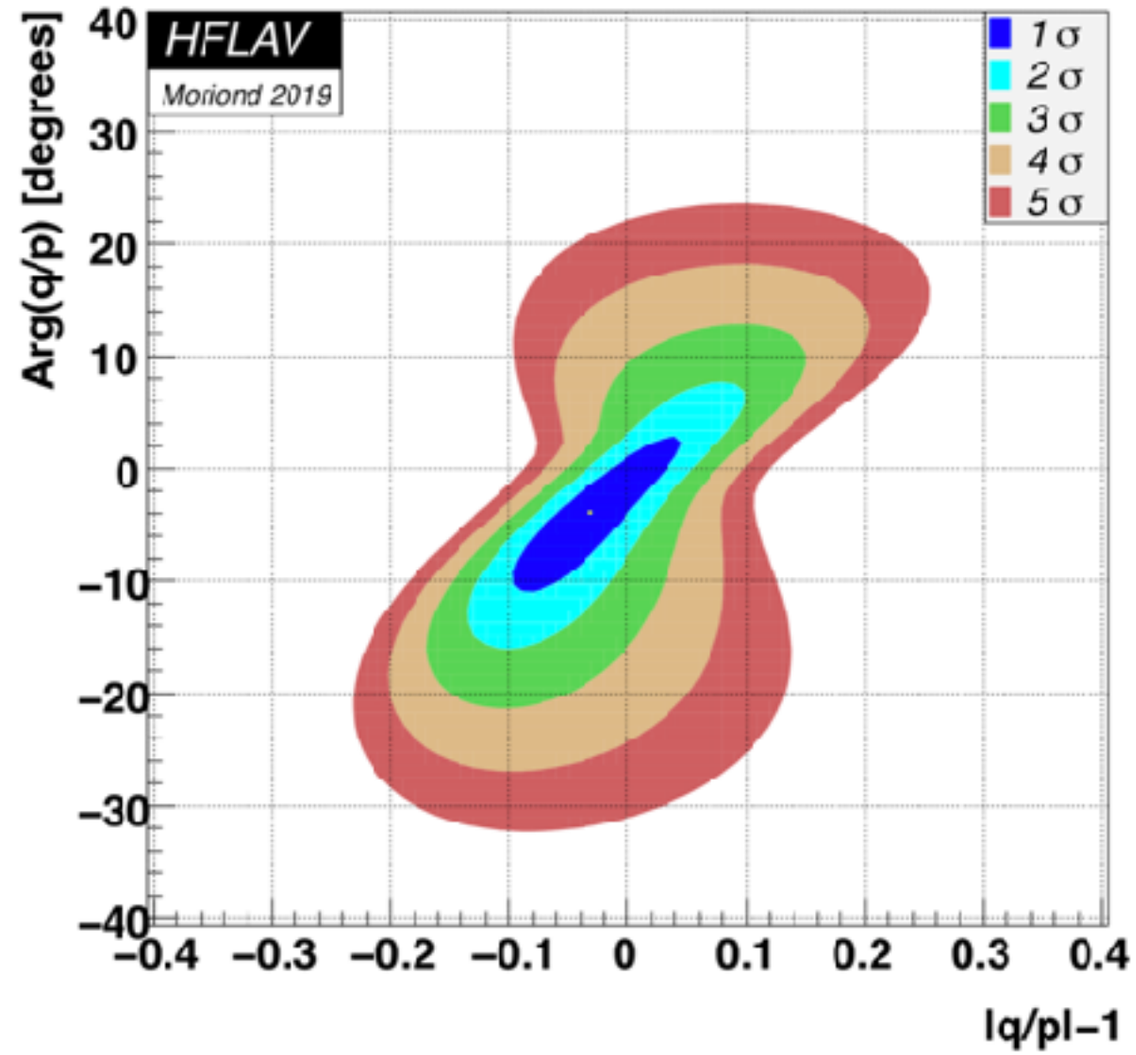
without data

purely theoretical method

(quark-hadron duality is assumed)

$D^0 - \bar{D}^0$ mixing: experiment





$(x, y) = (0, 0)$ is excluded by >>11.5σ

$(|q/p| - 1, \mathrm{Arg}(q/p)) = (0, 0)$ is allowed

→ No signal for CP violation

Previous works for heavy meson ***decays***

◉ non-leptonic decays (color-allowed tree)

[1] B. Grinstein and R. F. Lebed, Phys. Rev. D**57**, 1366-1378 (1998) [arXiv:hep-ph/9708396 [hep-ph]].

◉ semi-leptonic decay and also non-leptonic decay

[2] I. I. Y. Bigi, M. A. Shifman, N. Uraltsev and A. I. Vainshtein, Phys. Rev. D**59**, 054011 (1999) [arXiv:hep-ph/9805241 [hep-ph]].

◉ non-leptonic decay (annhilation)

[3] B. Grinstein and R. F. Lebed, Phys. Rev. D**59**, 054022 (1999) [arXiv:hep-ph/9805404 [hep-ph]].

◉ non-leptonic decays (weak annhilation, Pauli interference)

[4] I. I. Y. Bigi and N. Uraltsev, Phys. Rev. D60, 114034 (1999) [arXiv:hep-ph/9902315 [hep-ph]]; Phys. Lett. B**457**, 163-169 (1999) [arXiv:hep-ph/9903258 [hep-ph]].

◉ semi-leptonic decay

[5] R.~F. Lebed and N. G. Uraltsev, Phys. Rev. D**62**, 094011 (2000) [arXiv:hep-ph/0006346 [hep-ph]].

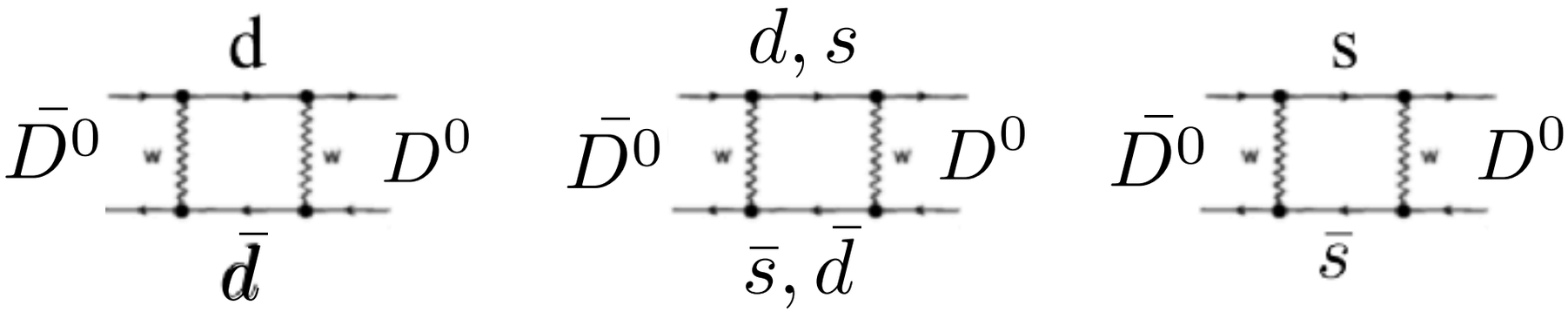
This work → heavy meson **mixings**

- Mixing is suppressed by the GIM cancellation.

➡ Tiny duality violation is possibly enlarged after cancellation.

Contributions

$$\lambda_i = V_{ci} V_{ui}^*$$



CKM suppressed
+ bottom loop
neglect

For $m_s = m_d$

CKM unitarity
$\lambda_d + \lambda_s + \lambda_b = 0$
neglect

summation $\propto \lambda_d^2 + 2\lambda_d\lambda_s + \lambda_s^2$

Contributions

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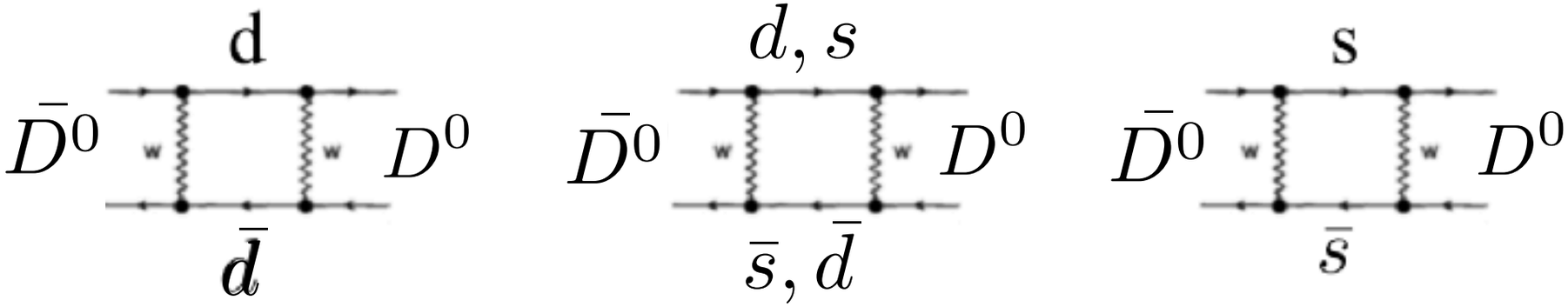

~~CKM suppressed~~
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For $m_s = m_d$

CKM unitarity
$\lambda_d + \lambda_s + \cancel{\lambda_b} = 0$
neglect

summation $\propto \lambda_d^2 + 2\lambda_d\lambda_s + \lambda_s^2 = 0$

➡ Suppressed by the GIM mechanism.

non-zero contributions to $D^0 - \bar{D}^0$ mixing $\propto$ SU(3) breaking: $\left(\frac{m_s^2 - m_d^2}{m_c^2}\right)^n$

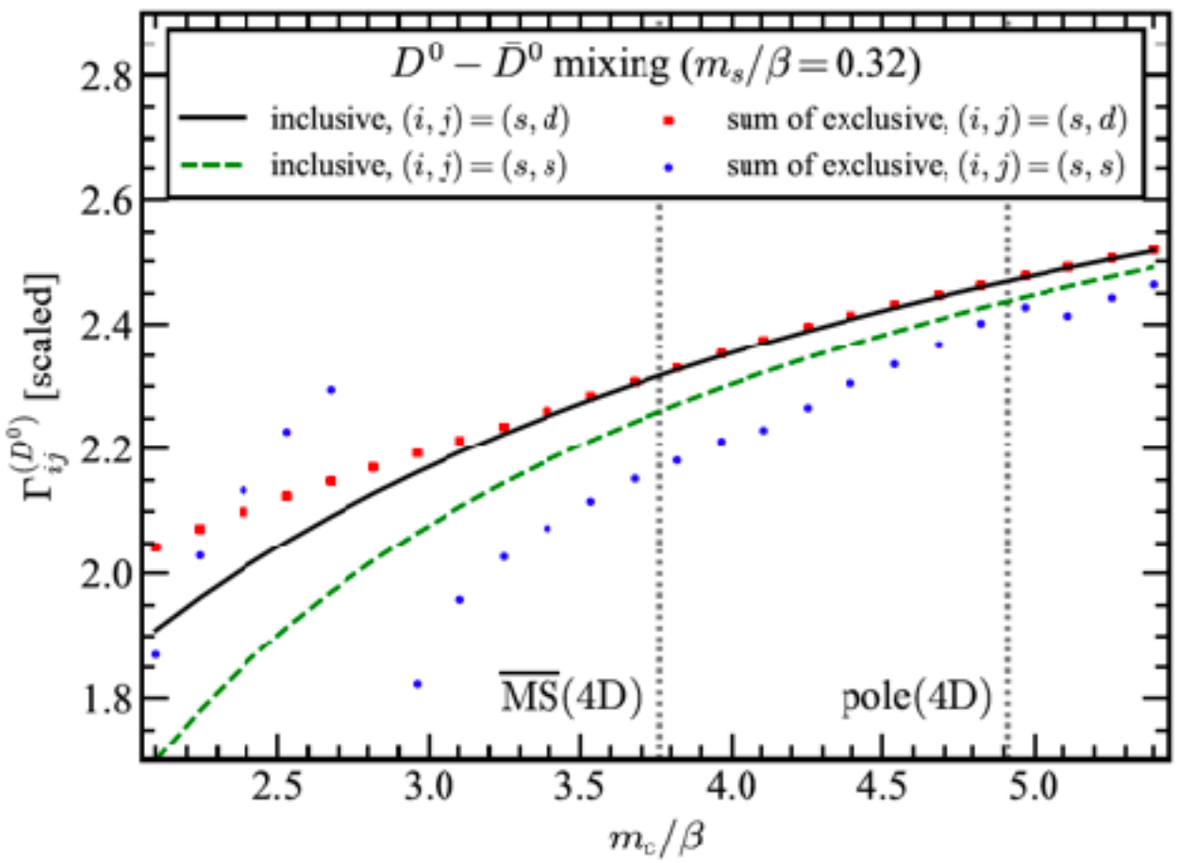
(1) $D^0 - \bar{D}^0$ mixing for individual flavors

$D^0 \to K^-\pi^+ \to \bar{D}^0$

- solid line: inclusive
- squares: sum of exclusive

$D^0 \to K^-K^+ \to \bar{D}^0$

- dashed line: inclusive
- points: sum of exclusive



Vertical dotted lines:
reference values corresponding to masses in 4D

β is fixed by 340 MeV.
(ansatz for fitting the string tension in QCD_4)

m_s/β is fixed by 0.32.
($\overline{\mathrm{MS}}$ mass at the scale of charm quark mass)

The vertical axis is normalized by $4G_F^2(c_\mathrm{V}^2 - c_\mathrm{A}^2)^2\beta N_c/\pi$.

◎ For large m_c, inclusive/exclusive agrees with each other.

◎ For small m_c, certain difference between inclusive/exclusive appears.

◎ The agreement beween inclusive/exclusive is better for $K^-\pi^+$ than K^-K^+.

◎ Obvious spikes are observed for K^-K^+ whereas it is not seen for $K^-\pi^+$.

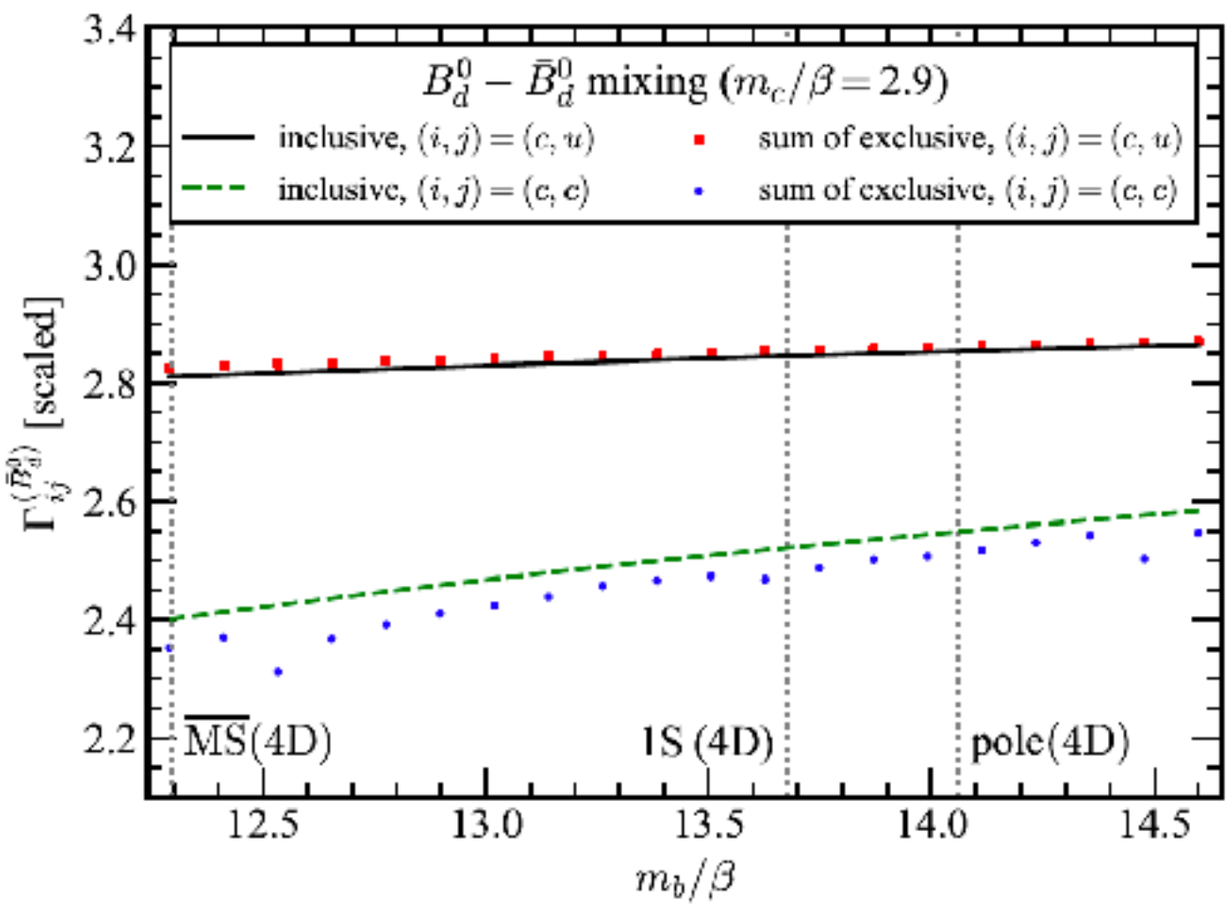
(1) $B_q^0 - \bar{B}_q^0$ $(q = d, s)$ mixing for individual flavors

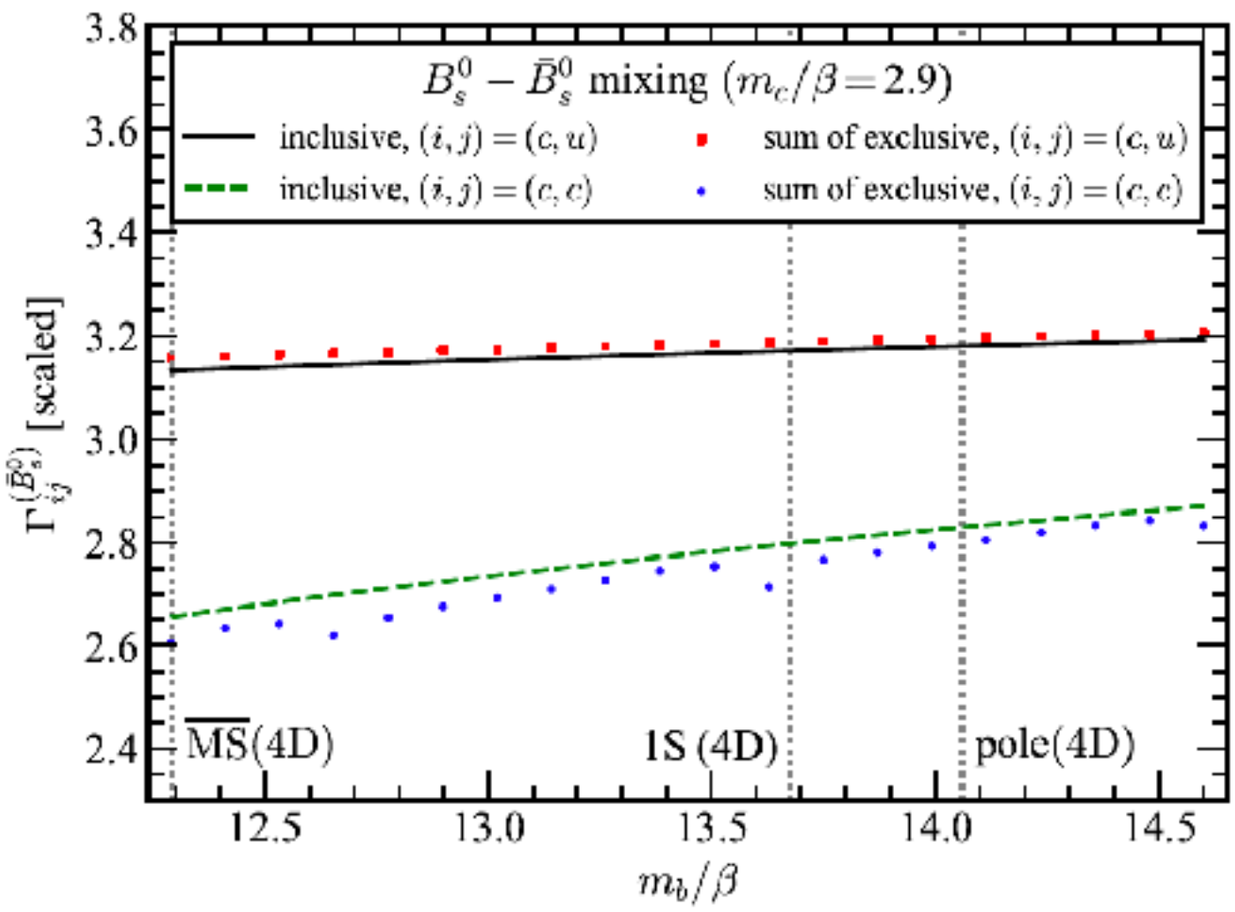
$$B_d^0 \to D^- \pi^+ \to \bar{B}_d^0$$
$$B_d^0 \to D^- D^+ \to \bar{B}_d^0$$

{ solid line: inclusive
squares: sum of exclusive

$$B_s^0 \to D_s^- K^+ \to \bar{B}_s^0$$
$$B_s^0 \to D_s^- D_s^+ \to \bar{B}_s^0$$

{ dashed line: inclusive
points: sum of exclusive





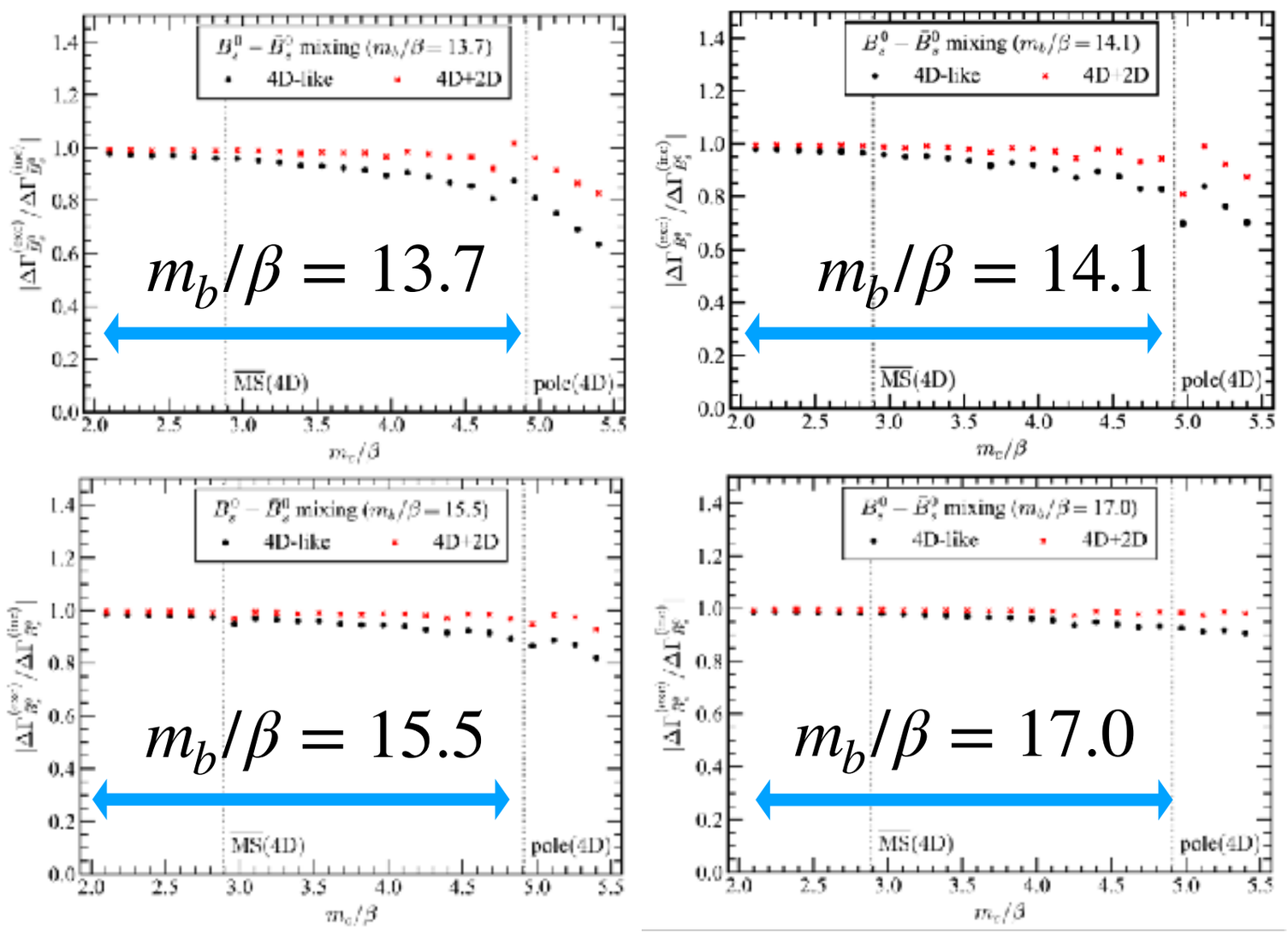
◎ The patters are similar for the $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$ mixings.

◎ The disagreement between inclusive/exclusive is larger for $B_q^0 \to DD \to \bar{B}_q^0$.

$B_s^0 - \bar{B}_s^0$ mixing

points: based only on the 4D-like phase space
crosses: based on the 4D-like phase space + 2D-specific one

If the domain of $m_c < m_c^{\text{pole,4D}}$ is considered:



◎ The corrections are up to (20 % , 18 % , 11 % , 8%) for $m_b/\beta = (13.7,\ 14.1,\ 15.5,\ 17.0)$.

→ The result is consistent with 4D within 1σ for the latter two.

(The ratio of the HFLAV data to <u>the HQE</u> gives $\Delta\Gamma_{B_s}^{(\text{ex})}/\Delta\Gamma_{B_s}^{(\text{th})} = 0.99 \pm 0.15$.)

A. Lenz and G. Tetlalmatzi-Xolocotzi, 2020

Analytical check of local duality

generalized weak vertex: $\frac{-ig_2}{\sqrt{2}} V_{\rm CKM} \gamma^\mu (c_{\rm V} + c_{\rm A}\gamma_5)$

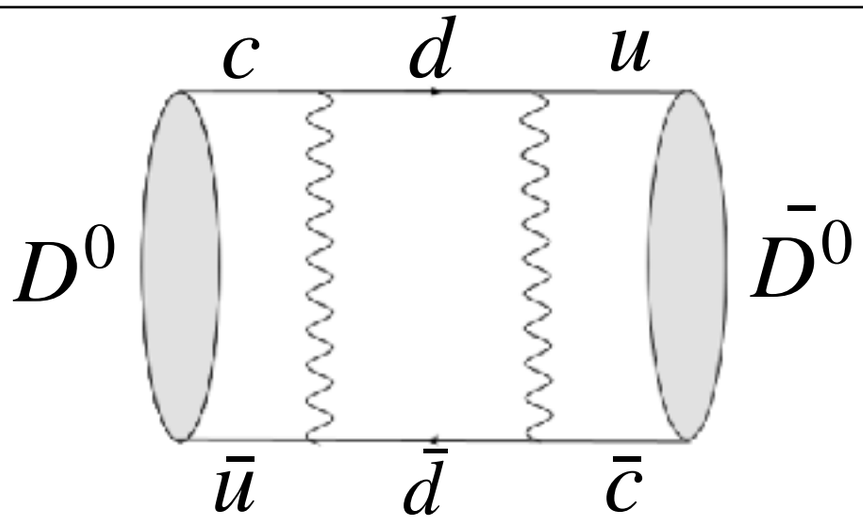
the standard model $c_{\rm V} = \frac{1}{2}, \quad c_{\rm A} = -\frac{1}{2}$

◉ Inclusive width difference

$$\Gamma_{12} = C_A < \overline{D^0} | (\bar{u}^\alpha \gamma^\mu \gamma_5 c^\alpha)(\bar{u}^\beta \gamma_\mu \gamma_5 c^\beta) | D^0 > \; + C_P < \overline{D^0} | (\bar{u}^\alpha i\gamma_5 c^\alpha)(\bar{u}^\beta i\gamma_5 c^\beta) | D^0 >$$

$$\begin{cases} C_A = + \dfrac{2G_F^2}{M_{D^0}} (c_V^2 - c_A^2) V_{cd}^* V_{ud} [(c_V^2 - c_A^2)(F_{dd}^{(\rm th)} + 2G_{dd}^{(\rm th)}) - (c_V^2 + c_A^2)(I_{dd}^{(\rm th)} + I_{dd}^{(\rm th)})] \\ C_P = - \dfrac{2G_F^2}{M_{D^0}} (c_V^2 - c_A^2) V_{cd}^* V_{ud} [(c_V^2 - c_A^2)(G_{dd}^{(\rm th)} + 2H_{dd}^{(\rm th)}) + (c_V^2 + c_A^2)(I_{dd}^{(\rm th)} + I_{dd}^{(\rm th)})] \end{cases}$$

$F_{dd}^{(\rm th)}, G_{dd}^{(\rm th)}, H_{dd}^{(\rm th)}, I_{dd}^{(\rm th)}$: phase space functions $\qquad F_{dd}^{(\rm th)} = \sqrt{1 - 4m_d^2/m_c^2}$



large-N_c factorization: $\qquad R = [M_H/(m_Q + m_q)]^2$

$$\begin{cases} \dfrac{\langle \bar{H} | (\bar{q}^\alpha \gamma^\mu \gamma_5 Q^\alpha)(\bar{q}^\beta \gamma_\mu \gamma_5 Q^\beta) | H \rangle}{2M_H} = f_H^2 M_H \\ \dfrac{\langle \bar{H} | (\bar{q}^\alpha i\gamma_5 Q^\alpha)(\bar{q}^\beta i\gamma_5 Q^\beta) | H \rangle}{2M_H} = f_H^2 M_H R \end{cases}$$

down quark massless limit: $\quad \Gamma_{12} \to 4(c_{\rm V}^2 - c_{\rm A}^2)^2 V_{cd}^* V_{ud} G_F^2 f_{D^0}^2 M_{D^0}$

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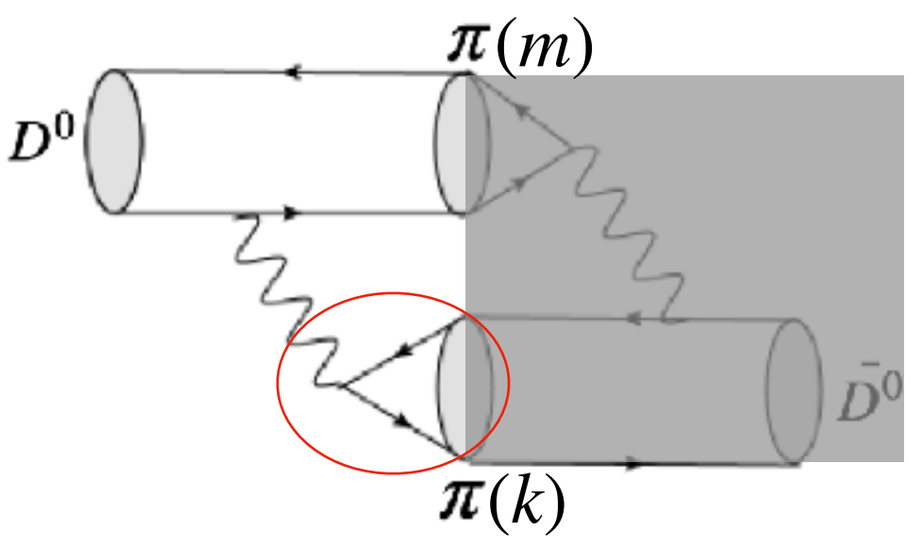
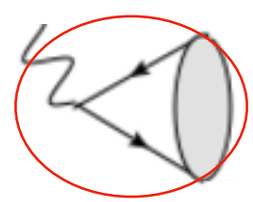
the standard model
$c_{\rm V} = \frac{1}{2}, \quad c_{\rm A} = -\frac{1}{2}$

◉ Sum of exclusive width difference

massless limits for u and d quark for $\pi(u\bar{d})$

$$\Gamma_{12}^{(D^0)} = \sum_{k,m} (-1)^{k+m} \frac{T^{(k,m)} T^{(m,k)*}}{4M_{D^0}^2 |p_{km}|}$$





$\propto f_\pi^{(k)} p_\mu \qquad f_\pi^{(k)} = \sqrt{\frac{N_c}{\pi}} \int_0^1 \phi_k(x) dx$

$$\begin{cases} \text{(a) exact solution, } \phi_0(x) = 1. \\ \text{(b) completeness: } \sum_{k=0}^{\infty} \phi_k(x)\phi_k^*(y) = \delta(x-y) \end{cases}$$

$k = 0,\ m = 0$: ground states

➡ $f_\pi^{(k)} = \begin{cases} \sqrt{N_c/\pi} & k = 0 \\ 0 & k \neq 0 \end{cases}$

Analytical check of local duality

generalized weak vertex: $\frac{-ig_2}{\sqrt{2}} V_{\rm CKM} \gamma^\mu (c_{\rm V} + c_{\rm A}\gamma_5)$

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◉ Sum of exclusive width difference

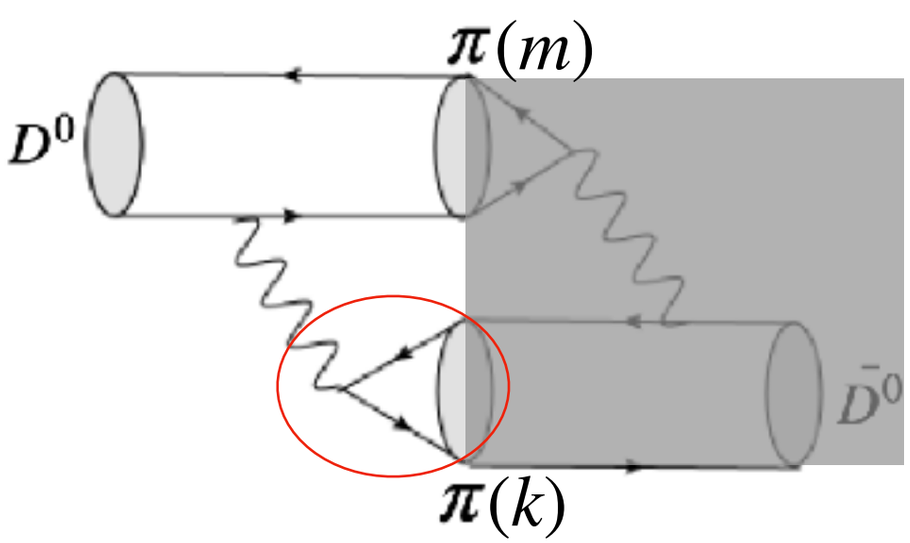
massless limits for u and d quark for $\pi(u\bar{d})$

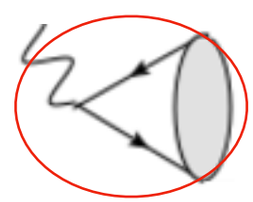
analog of the Pauli interference (Bigi and Uraltsev, 1999)

$$\Gamma_{12}^{(D^0)} = \sum_{k,m} (-1)^{k+m} \frac{T^{(k,m)} T^{(m,k)*}}{4M_{D^0}^2 |p_{km}|} \quad \rightarrow \quad \frac{T^{(0,0)} T^{(0,0)*}}{4M_{D^0}^2 |p_{00}|} = 4(c_{\rm V}^2 - c_{\rm A}^2)^2 G_F^2 V_{cd}^* V_{ud} M_{D^0} \frac{N_c}{\pi} \left(\int_0^1 \phi_{D^0}(x) \phi_\pi(x) \right)^2$$

$$= 4(c_{\rm V}^2 - c_{\rm A}^2)^2 G_F^2 V_{cd}^* V_{ud} f_{D^0}^2 M_{D^0}$$

agrees with the inclusive result





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$k = 0,\ m = 0$: ground states

$$\Rightarrow \quad f_\pi^{(k)} = \begin{cases} \sqrt{N_c/\pi} & k = 0 \\ 0 & k \neq 0 \end{cases}$$

Numerical evalutation of local duality

Motivations

(1) Check whether local duality exists for massive final states.

(2) Check the net size of observables in the presence of the GIM cancellation. (main motivation)

— We calculate $D \to \pi\pi, D \to K\pi, D \to KK$ and implement the cancellation.

Numerical method to solve the ’t Hooft equation

$$M_n^2 \phi_n^{q_1\bar{q}_2}(x) = \left(\frac{m_1^2 - \beta^2}{x} + \frac{m_2^2 - \beta^2}{1-x} \right) \phi_n^{q_1\bar{q}_2} - \beta^2 \mathrm{Pr} \int_0^1 \mathrm{d}y \frac{\phi_n^{q_1\bar{q}_2}(y)}{(x-y)^2}$$ for $q_1\overline{q_2}$ bound state

based on the BSW-improved Multhopp technique Brower, Spence and Weis, 1979

$$\phi = \sum_{k=1}^{N} a_k \sin(k\theta), \quad x = \frac{1+\cos\theta}{2}$$

eigenvalue problem: $M^2 a_i = (H_0 + V)_{ij} a_j$

M^2: eigenvalue

a_i: eigenvector

Definition of amplitude and overlap integrals

Grinstein, Lebed 1997, Bigi, Uraltsev 1999

Amplitude

$$T^{(k,m)}_{(Q\bar{q})(i,j)} = 2\sqrt{2}G_F(c_V^2 - c_A^2)\sqrt{\frac{N_c}{\pi}}\, c_k^{(q\bar{i})}\left[\sum_{n=0} \frac{[(-1)^k q^2 + (-1)^n M_n^2]c_n^{(Q\bar{j})}}{q^2 - M_n^2} F_{nm} + (-1)^{k+1} q^2 \mathcal{C}_m + m_Q m_j \mathcal{D}_m\right],$$

Overlap int.

$$\begin{cases} F_{nm} = \omega(1-\omega)\displaystyle\int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y \frac{\phi_n^{(Q\bar{j})}(x)\phi_m^{(j\bar{q})}(y)}{[\omega(1-x)+(1-\omega)y]^2} \\ \qquad \times\{\phi_0^{(Q\bar{q})}(\omega x) - \phi_0^{(Q\bar{q})}[1-(1-\omega)(1-y)]\}, \\ \mathcal{C}_m = -\dfrac{1-\omega}{\omega}\displaystyle\int_0^1 \mathrm{d}x \phi_0^{(Q\bar{q})}[1-(1-\omega)(1-x)]\phi_m^{(j\bar{q})}(x), \\ \mathcal{D}_m = -\omega\displaystyle\int_0^1 \mathrm{d}x \frac{\phi_0^{(Q\bar{q})}[1-(1-\omega)(1-x)]}{1-(1-\omega)(1-x)}\frac{\phi_m^{(j\bar{q})}(x)}{x}, \end{cases}$$

Kinematical val.

$$\omega = \frac{1}{2}\left[1 + \left(\frac{q^2 - M_m^2}{M_0^2}\right) - \sqrt{1 - 2\left(\frac{q^2 + M_m^2}{M_0^2}\right) + \left(\frac{q^2 - M_m^2}{M_0^2}\right)^2}\right]$$

$q^2 = M_k^2$ for on-shell amplitudes

Expressions in the SM

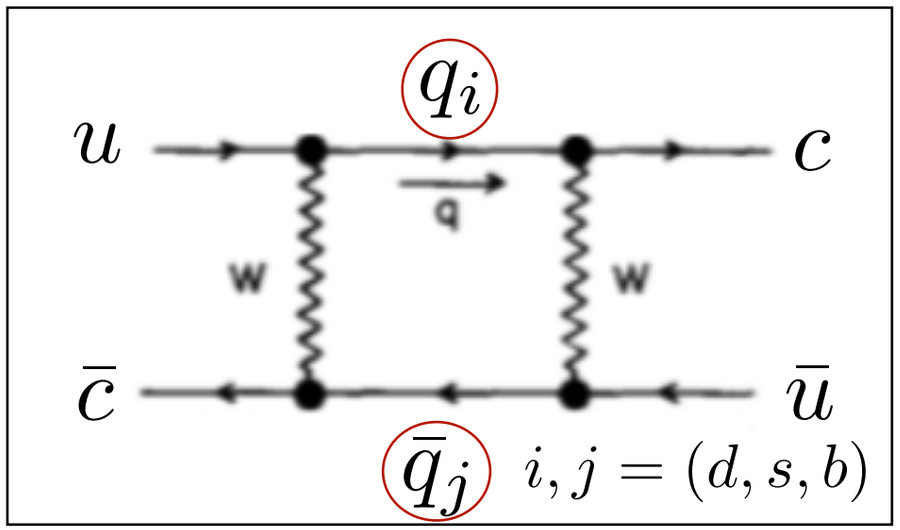
$$M_{12} - \frac{i}{2}\Gamma_{12} = B\,\langle D^0|\mathcal{O}_1|\bar{D}^0\rangle + C\,\langle D^0|\mathcal{O}_2|\bar{D}^0\rangle$$

Two contributions
$$\begin{cases} \mathcal{O}_1 = (\bar{c}u)_{\mathrm{V-A}}(\bar{c}u)_{\mathrm{V-A}} \\ \mathcal{O}_2 = (\bar{c}u)_{\mathrm{S-P}}(\bar{c}u)_{\mathrm{S-P}} \end{cases}$$

Flavor sum

$\lambda_i = V_{ci}V_{ui}^*$

Explicit formula
$$\begin{cases} M_{12}(s) = \dfrac{G_F^2 M_W^2}{32\pi^2} f_H^2 M_H \sum_{i,j}^{d,s,b} \lambda_i\lambda_j \left[\xi_1 B_1(\mu) B_{ij}^{(d)}(s) + \xi_2 R(s) B_2(\mu) C_{ij}^{(d)}(s)\right], \\ \Gamma_{12}(s) = \dfrac{G_F^2 M_W^2}{32\pi^2} f_H^2 M_H \sum_{i,j}^{d,s,b} \lambda_i\lambda_j \left[\xi_1 B_1(\mu) B_{ij}^{(a)}(s) + \xi_2 R(s) B_2(\mu) C_{ij}^{(a)}(s)\right], \end{cases}$$



CKM unitarity
$\lambda_d + \lambda_s + \lambda_b = 0 \longleftrightarrow \lambda_d = -\lambda_s - \lambda_b$

$$\begin{cases} M_{12}(s) = \dfrac{G_F^2 M_W^2}{32\pi^2} f_H^2 M_H (\lambda_s^2 U_{ss}^{(d)} + 2\lambda_s\lambda_b U_{sb}^{(d)} + \lambda_b^2 U_{bb}^{(d)}) \\ \Gamma_{12}(s) = \dfrac{G_F^2 M_W^2}{32\pi^2} f_H^2 M_H (\lambda_s^2 U_{ss}^{(a)} + 2\lambda_s\lambda_b U_{sb}^{(a)} + \lambda_b^2 U_{bb}^{(a)}) \end{cases}$$

U : loop function

Buras, Slominski and Steger, Nucl. Phys. B245, 369 (1984)

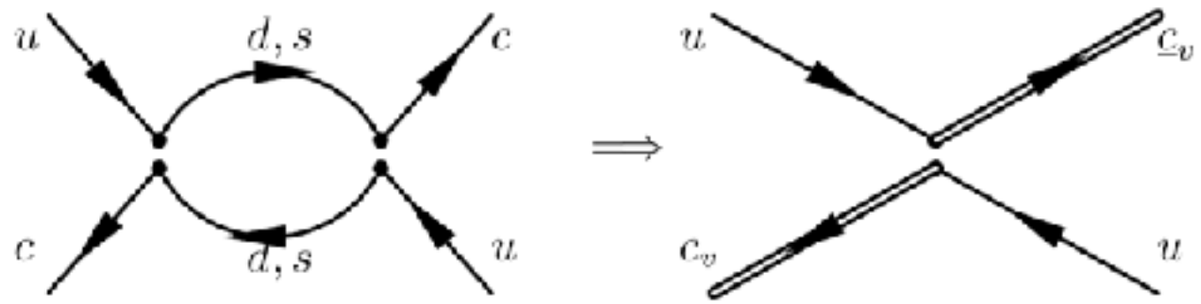
Structure of higher-dimensional operators

Ohl, Ricciardi and Simmons [9301212]

D=6

Leading in $1/m_q$

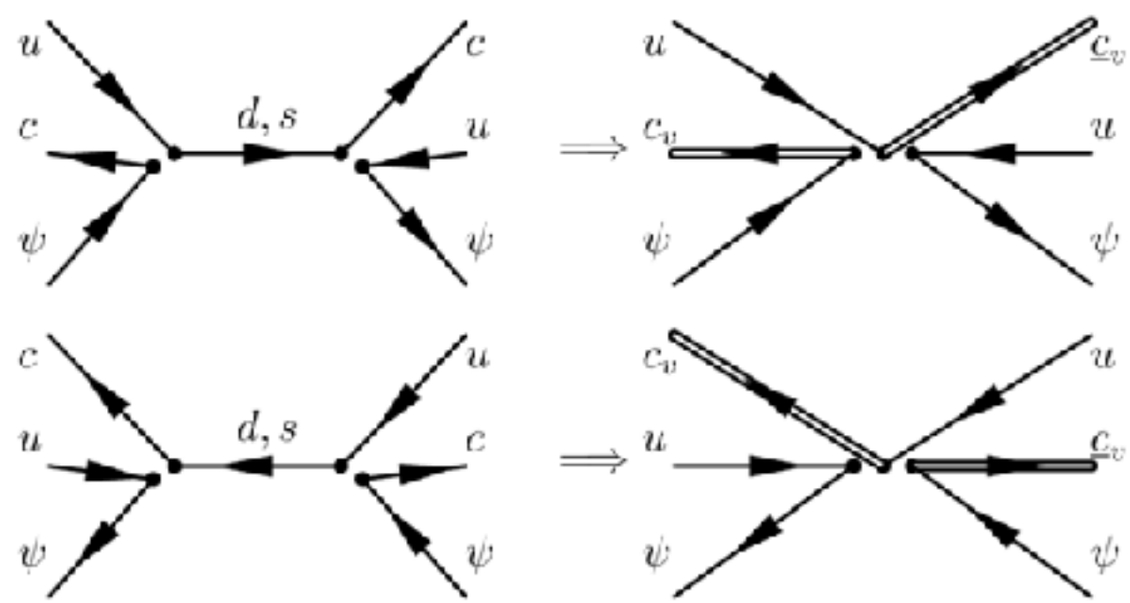
$$(\bar{c}_v \Gamma_1 u)\,(\underline{\bar{c}}_v \Gamma_2 u)$$



D=9

Subleading in $1/m_q$

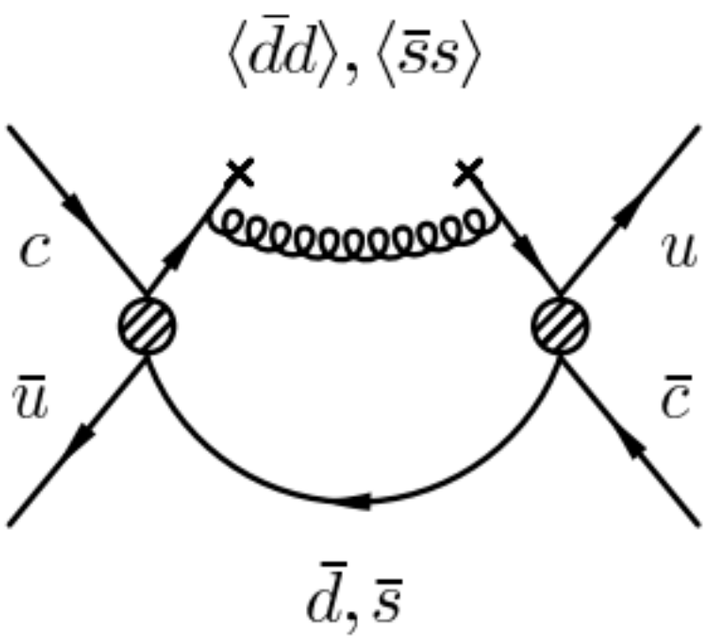
$$\left(\bar{\psi}\Gamma_1 u\right)(\bar{c}_v \Gamma_2 \psi)\,(\underline{\bar{c}}_v \Gamma_3 u)$$



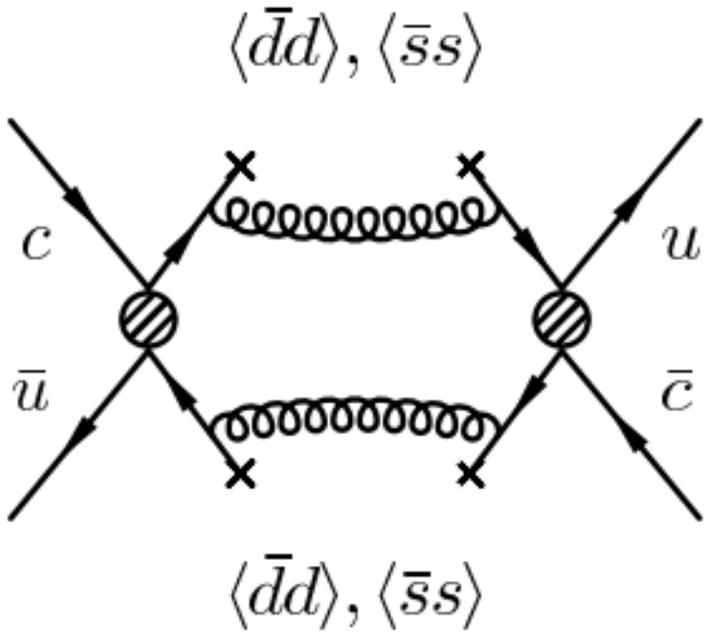
Γ_i : color/Dirac structure

Higer-dimensional operators with condensates

Bobrowski, Lenz and Riedl [1002.4794]



D=9



D=12

$\mathcal{O}(\alpha_s(4\pi)\langle\bar{q}q\rangle/m_c^3)$

$\mathcal{O}(\alpha_s^2(4\pi)^2\langle\bar{q}q\rangle^2/m_c^6)$

y	no GIM	with GIM
$D = 6, 7$	$2 \cdot 10^{-2}$	$5 \cdot 10^{-7}$
$D = 9$	$5 \cdot 10^{-4}$	?
$D = 12$	$2 \cdot 10^{-5}$	?

Motivations to study duality violation

✔

(1) For the $D^0 - \bar{D}^0$ mixing, an inclusive calculation, based on the leading operator in HQE, disagrees with data.

[0506185]

$$y \simeq 6 \cdot 10^{-7}$$

HFLAV 2019

$$y = (6.51^{+0.63}_{-0.69}) \times 10^{-3}$$

(2) For, e.g., $\bar{B} \to X_q l \bar{\nu}$ $(q = c, u)$, one can get the systematic uncertainty in the OPE.

(3) ~~Crucial to explain the short lifetime of Λ_b?~~ data updated.

before 2003

Year	Exp	Decay	$\tau(\Lambda_b)$ [ps]	$\tau(\Lambda_b)/\tau(B_d)$
1998	OPAL	$\Lambda_c l$	1.29 ± 0.25	0.85 ± 0.16
1997	ALEPH	$\Lambda_c l$	1.21 ± 0.11	0.80 ± 0.07
1995	ALEPH	$\Lambda_c l$	1.02 ± 0.24	0.67 ± 0.16
1992	ALEPH	$\Lambda_c l$	1.12 ± 0.37	0.74 ± 0.24

after 2003

Year	Exp	Decay	$\tau(\Lambda_b)$ [ps]	$\tau(\Lambda_b)/\tau(B_d)$
2010	CDF	$J/\psi\Lambda$	1.537 ± 0.047	1.020 ± 0.031
2009	CDF	$\Lambda_c + \pi^-$	1.401 ± 0.058	0.922 ± 0.038
2007	D0	$\Lambda_c \mu \nu X$	1.290 ± 0.150	0.849 ± 0.099
2007	D0	$J/\psi\Lambda$	1.218 ± 0.137	0.802 ± 0.090
2006	CDF	$J/\psi\Lambda$	1.593 ± 0.089	1.049 ± 0.059
2004	D0	$J/\psi\Lambda$	1.22 ± 0.22	0.87 ± 0.17

[1405.3601]

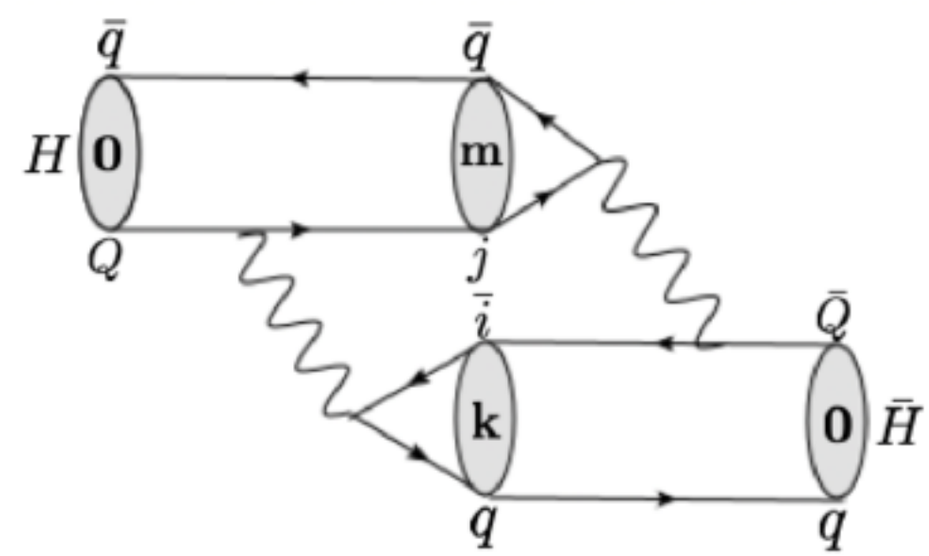
Topological amplitudes in the $1/N_c$ counting

naive countings: $$\begin{cases} T \propto N_c^{1/2} \\ C, E, P, PA \propto N_c^{-1/2} \\ PE \propto N_c^{-3/2} \end{cases}$$

(A does not contribute to the neutral meson mixings)

◉ Even in the presence of intermediate resonances,
the color-allowed tree diagram is still dominant. [9805404]

➡ dominant topology in large-N_c limit:



Inclusive analysis in the presence of the GIM mechanism

$$\Gamma_{12} = \lambda_d^2 \Gamma_{dd}^{(D,\mathrm{inc})} + 2\lambda_s\lambda_d \Gamma_{ds}^{(D,\mathrm{inc})} + \lambda_s^2 \Gamma_{ss}^{(D,\mathrm{inc})} . \qquad \lambda_k = V_{ck}V_{uk}^*$$

$$\Gamma_{ij}^{(D,\ \mathrm{inc})} \propto F_{ij}^{(\mathrm{th})}, G_{ij}^{(\mathrm{th})}, H_{ij}^{(\mathrm{th})} . \qquad i,j = d \text{ or } s$$

three types of phase space functions

$$\begin{cases} F_{ij}^{(\mathrm{th})} = \sqrt{1-2(z_i+z_j)+(z_i-z_j)^2}, & \text{: 4D-like phase space} \\ G_{ij}^{(\mathrm{th})} = \dfrac{z_i+z_j-(z_i-z_j)^2}{\sqrt{1-2(z_i+z_j)+(z_i-z_j)^2}} & \text{: 2D-specific phase space} \\ H_{ij}^{(\mathrm{th})} = \dfrac{\sqrt{z_i z_j}}{\sqrt{1-2(z_i+z_j)+(z_i-z_j)^2}} & \text{: 2D-specific phase space} \end{cases}$$

$$\begin{cases} z_d = m_d^2/m_c^2 = 0 \\ z_s = m_s^2/m_c^2 \ll 1 \end{cases}$$

expansion parameter

CKM unitarity + $\lambda_b \to 0$ limit $\Rightarrow$ $\Gamma_{12} \simeq \lambda_s^2 \Gamma_{(\mathrm{GIM},1)}^{(D,\mathrm{inc})}$, $\Gamma_{(\mathrm{GIM},1)}^{(D,\mathrm{inc})} = \Gamma_{dd}^{(D,\ \mathrm{inc})} + \Gamma_{ss}^{(D,\ \mathrm{inc})} - 2\Gamma_{sd}^{(D,\ \mathrm{inc})}$

inclusive observable:
$$\begin{cases} \Gamma_{(\mathrm{GIM},1)}^{(D,\mathrm{inc})}\big|_{\mathrm{4D-like}} \propto z_s^2 \\ \Gamma_{(\mathrm{GIM},1)}^{(D,\mathrm{inc})}\big|_{\mathrm{4D+2D}} \propto z_s \end{cases}$$

suppressed more strongly for large m_c

The order of magnitude for $\left|\Gamma^{(\mathrm{exc})}/\Gamma^{(\mathrm{inc})}\right|$ strongly depends on the two cases.

$\Rightarrow$ Both cases are presented in what follows.