Cosmological PBHs as Dark Matter

Zachary S.C. Picker

Asia-Pacific workshop on particle physics and cosmology

THE UNIVERSITY OF SYDNEY

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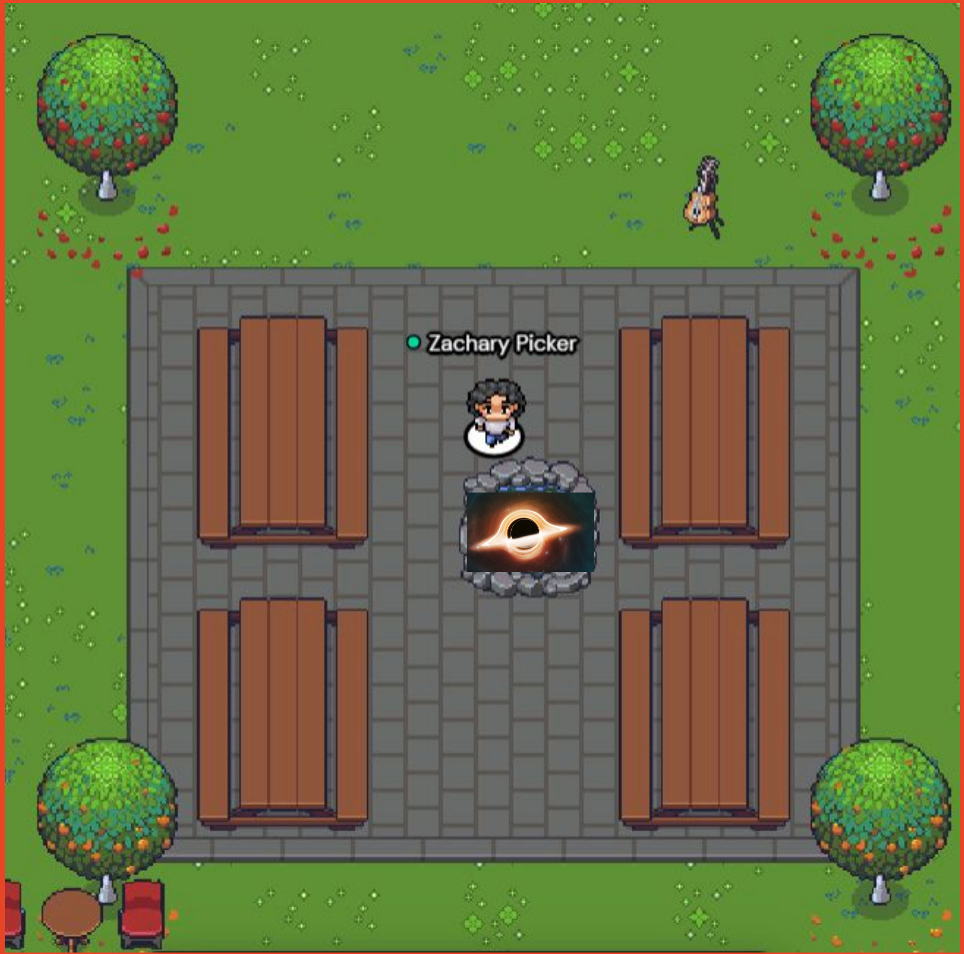

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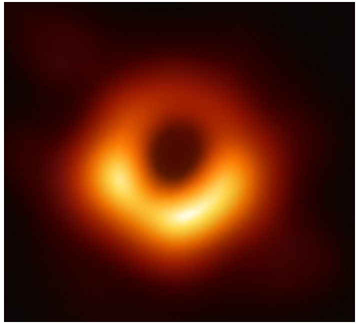
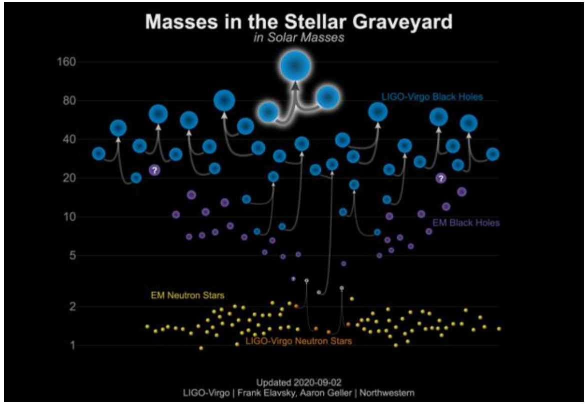
Primordial Black Holes

- Primordial black holes form from horizon-size overdensities:

$$m = \gamma m_H \qquad \gamma \approx 0.2$$



- Inhomogeneities in early universe is common mechanism
    - Also: cosmic loop collapse, domain walls, other exotic things
- Motivation?
    - Pair-instability mass gap: ~50-150 M_{sun}
    - Seeding supermassive BHs
- Dark matter?
    - Acts like cold dark matter
    - Only observed candidate...



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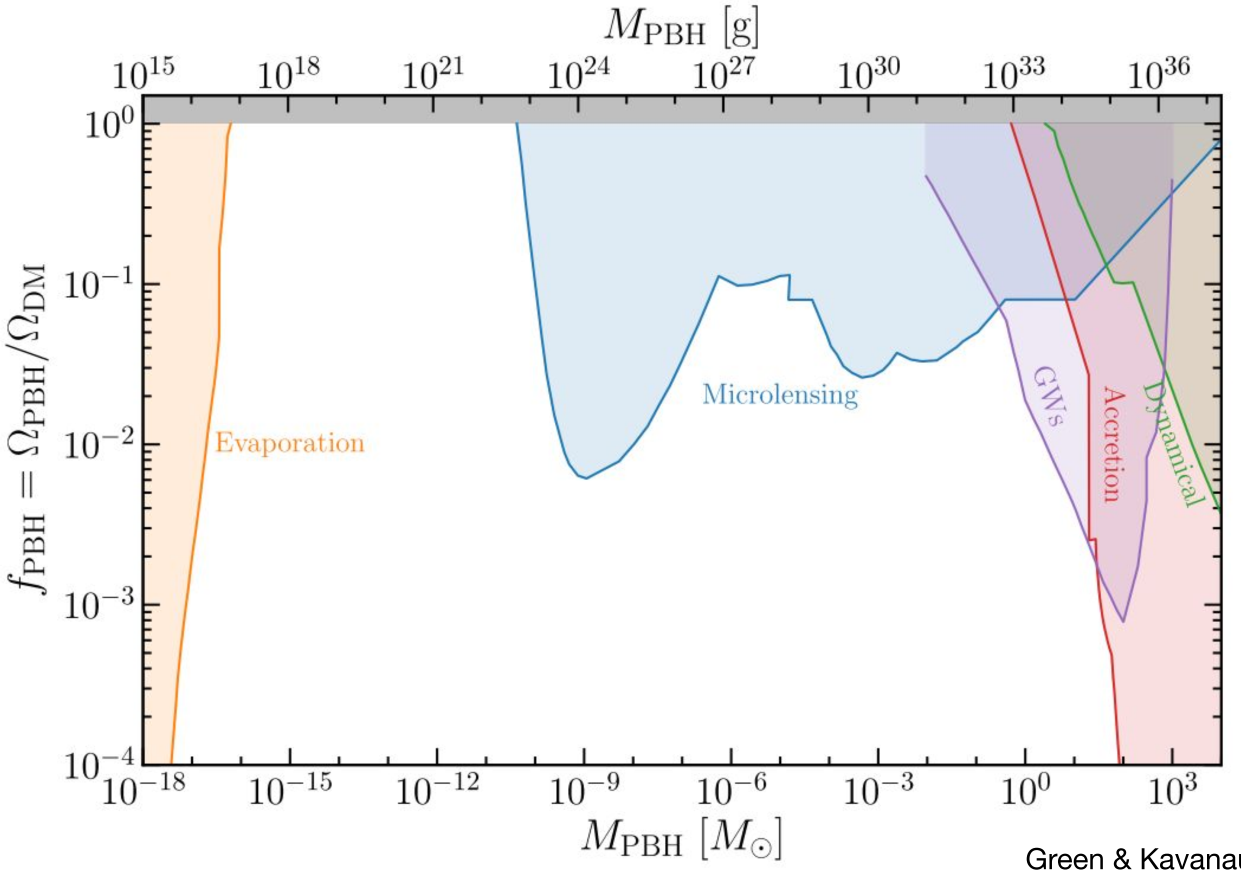
Dark Matter Constraints



Green & Kavanaugh, 2020

Which constraints rely on early-universe calculations?

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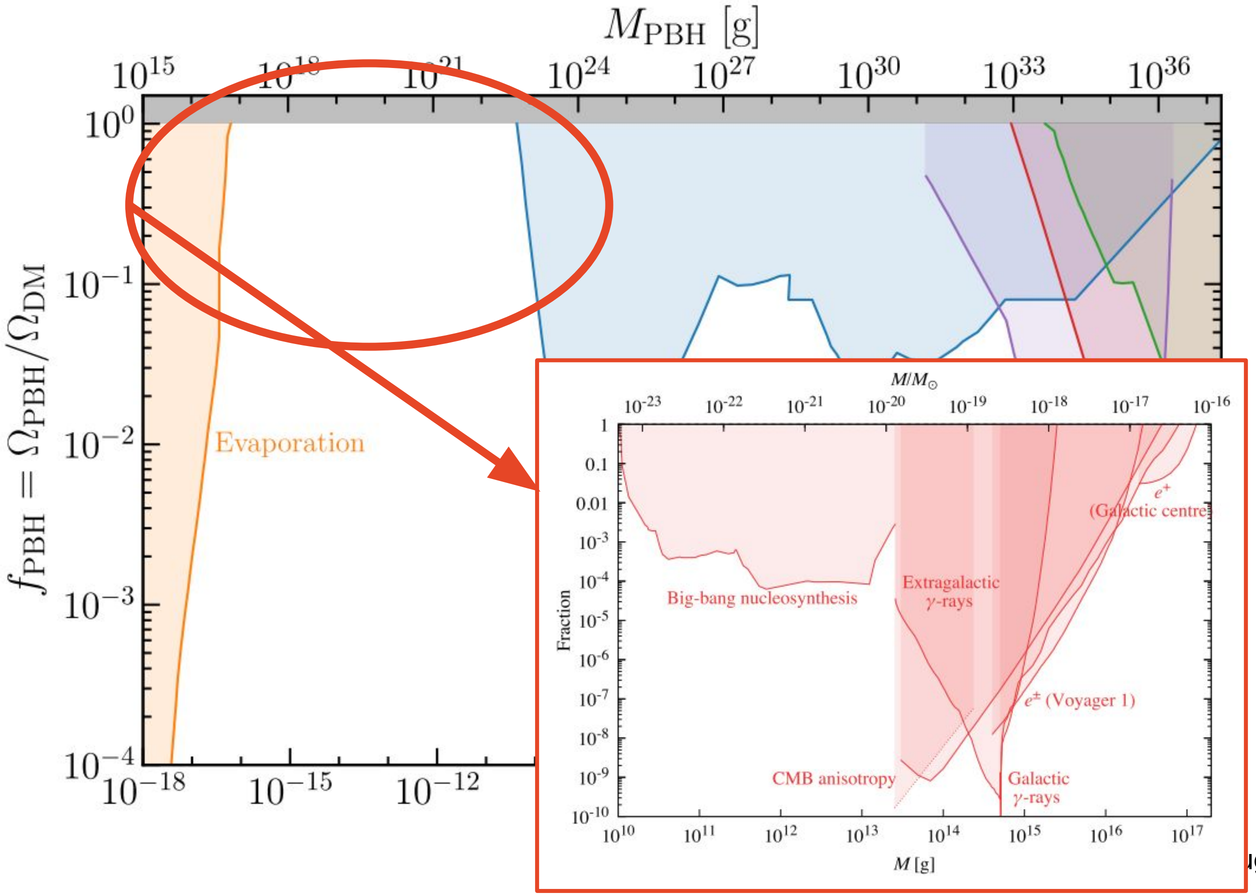
Dark Matter Constraints



...ugh, 2020

'Asteroid-mass' range:

- *No* constraints in asteroid-mass range
- Lower boundary: evaporation in early universe

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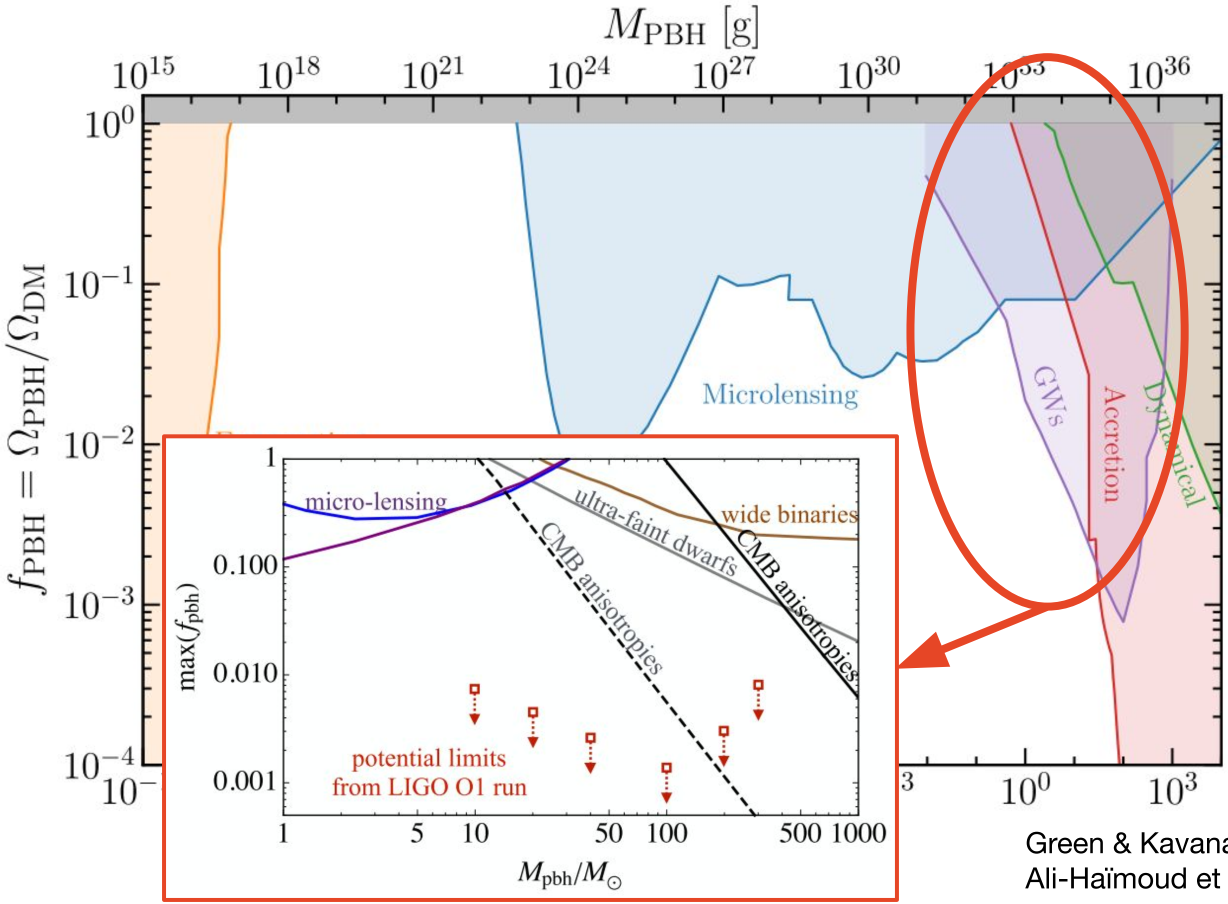
Dark Matter Constraints



'LIGO-mass' range:

- (maybe) tiny gap between microlensing and accretion
- Strongest constraint: binary formation in early universe

Green & Kavanaugh, 2020
Ali-Haïmoud et al, 2017

A Brief History...

- Birth of PBH field is intimately connected to PBHs in cosmological backgrounds

- Carr & Hawking 1974: must use a BH solution which is asymptotically FLRW

THE HYPOTHESIS OF CORES RETARDED DURING EXPANSION AND THE HOT COSMOLOGICAL MODEL

Ya. B. Zel'dovich and I. D. Novikov

Translated from Astronomicheskii Zhurnal, Vol. 43, No. 4, pp. 758-760, July-August, 1966
Original article submitted March 14, 1966

The existence of bodies with dimensions less than $R_g = 2GM/c^2$ at the early stages of expansion of the cosmological model leads to a strong accretion of radiation by these bodies. If further calculations confirm that accretion is catastrophically high, the hypothesis on cores retarded during expansion [3, 4] will conflict with observational data.

BLACK HOLES IN THE EARLY UNIVERSE

B. J. Carr and S. W. Hawking

(Received 1974 February 25)

SUMMARY

The existence of galaxies today implies that the early Universe must have been inhomogeneous. Some regions might have got so compressed that they underwent gravitational collapse to produce black holes. Once formed, black holes in the early Universe would grow by accreting nearby matter. A first estimate suggests that they might grow at the same rate as the Universe during the radiation era and be of the order of 10^{15} to 10^{17} solar masses now. The observational evidence however is against the existence of such giant black holes. This motivates a more detailed study of the rate of accretion which shows that black holes will not in fact substantially increase their original mass by accretion. There could thus be primordial black holes around now with masses from 10^{-5} g upwards.

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Cosmological PBH metrics

- Schwarzschild metric:
    - Embedded in flat, empty space
    - Mass is defined at infinity
- Cosmological solution:
    - Should be embedded in cosmological fluid
    - Asymptotically FLRW
    - *Local* mass definition
    - Still valid when cosmological horizon is close to BH horizon

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Brief survey of metrics

Require dust backgrounds:

- Sultana-Deyer
- Einstein-Strauss (swiss-cheese vacuole)

Both (overlap):

- Lemaître- Tolman-Bondi metrics
  - Shell-crossing singularities

Pesky singularities:

- McVittie
  - Spacelike singularity at horizon

Generalized McVittie metrics ⇒ Thakurta metric

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Thakurta black holes

$$\boxed{\mathrm{d}s^2 = \left(1 - \frac{2Gm_{\mathrm{MS}}}{R}\right)\mathrm{d}\tau^2 - \left(1 - \frac{2Gm_{\mathrm{MS}}}{R}\right)^{-1}\mathrm{d}R^2 - R^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right)}$$

- Simple + elegant metric: $\mathrm{d}s^2 = a^2\mathrm{d}s^2_{schw.}$
- Misner-sharp mass: $m_{\mathrm{MS}} = ma(t) + \dfrac{H^2R^3}{2Gf(R)}$ $\quad f(R) = 1{-}2Gma(t)/R$
- Apparent horizon: $R_{\mathrm{b}} = \dfrac{1}{2H}\left(1 - \sqrt{1 - 8HGma(t)}\right) \approx 2ma$
- Only valid until black hole decouples from Hubble flow
  - Eg binary formation, galaxy formation,...

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Thakurta metric: theoretical uncertainties

- Sourced by radial heat flow (imperfect fluid)
  - Can presumably imagine as temperature gradient towards BH
  - Is this physical for the cosmological plasma?
- How do we computer global energy density?
  - If we use the ‘comoving’ mass, it doesn’t work as dark matter
  - MS mass is local, must use the physical mass m
- What is the horizon structure?
  - Thakurta metric is *not* eternal, it interpolates to ~Schwarzschild metric at decoupling
  - Some find white hole horizon (arXiv:2106.06651), some find black hole (arXiv:1103.0750)...

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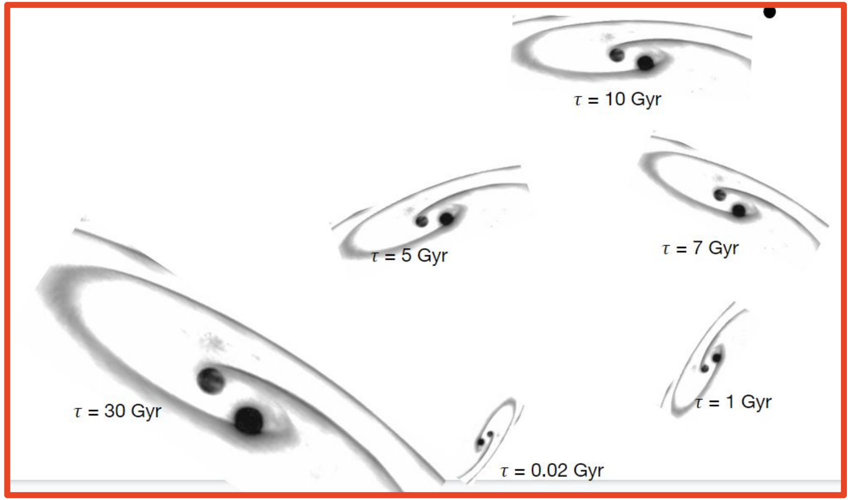
Binary abundances

Schwarzschild PBHs:

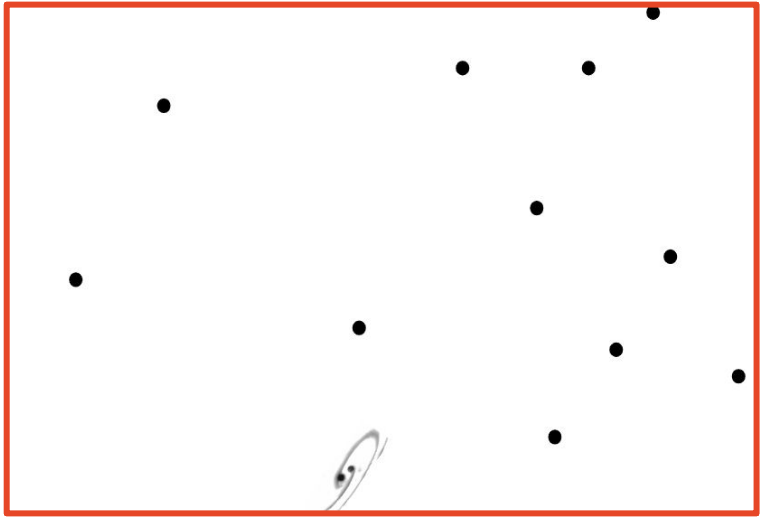
At matter-radiation equality:



Many of these coalesce ~today

Thakurta PBHs:

At matter-radiation equality:



τ_{max} ~ 100 sec (!)

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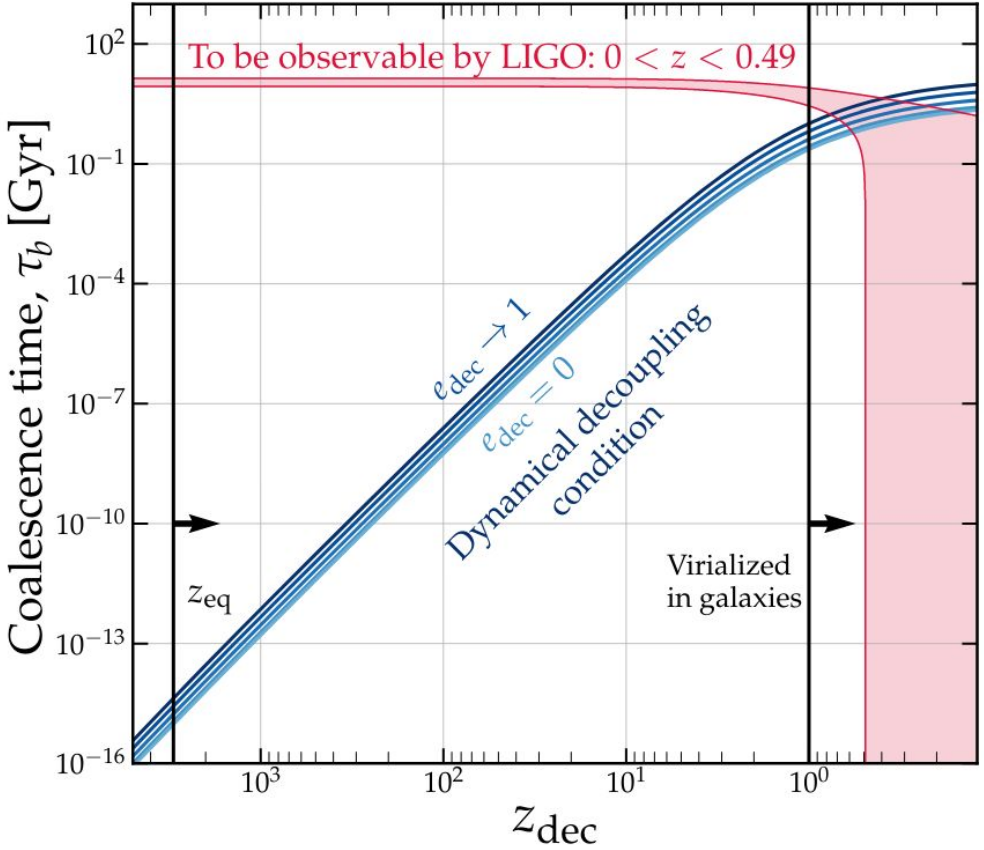
Binary abundances



arxiv:2008.10743:

Eliminating the LIGO bounds on primordial black hole dark matter

Céline Bœhm,[a] Archil Kobakhidze,[a] Ciaran A. J. O'Hare,[a] Zachary S. C. Picker,[a] Mairi Sakellariadou[b]

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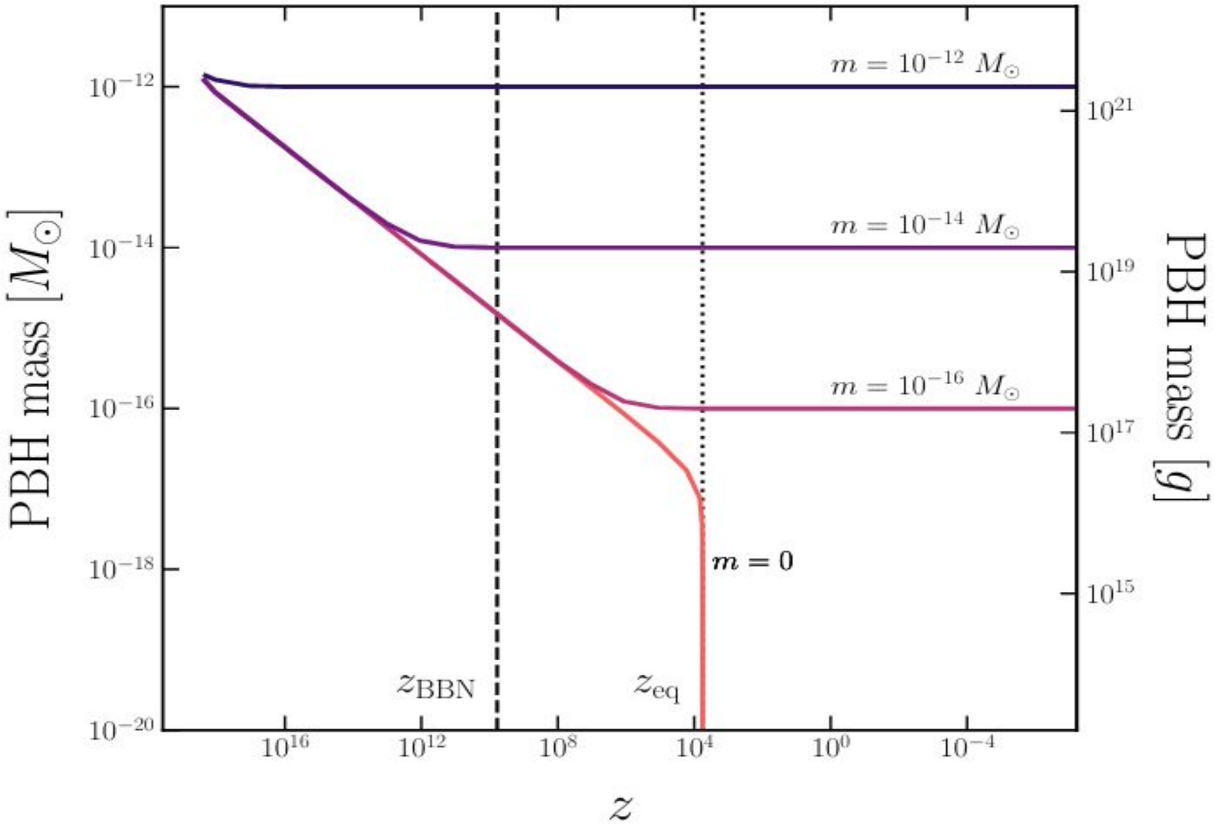
Hawking evaporation

- Much smaller horizon
  - Mass loss $\propto$ 1/S.Area
- 'Critical mass' BH which evaporates at m-r equality:

$$m_* \sim 9.6 \times 10^{-13}\ M_\odot$$

- 'Stability constraint' on dark matter today with masses smaller than critical mass
  - Too sensitive to initial conditions to populate masses in between



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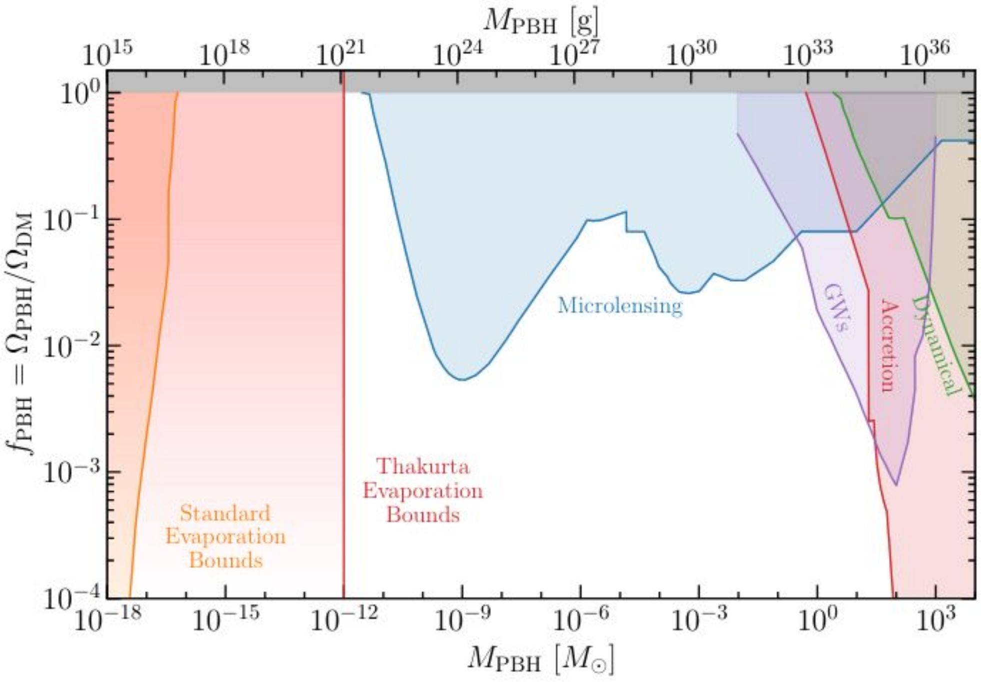
Hawking evaporation - constraints



arxiv:2103.02815:

Navigating the asteroid field: New evaporation constraints for primordial black holes as dark matter

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Conclusions

- PBHs are (still) exciting dark matter candidates
- We *need* a proper cosmological embedding of black holes
  - Schwarzschild will not do!
- Such metrics may have large phenomenological consequences
  - For example, the Thakurta metric
  - LIGO bounds evaded, asteroid-mass range excluded...
- Lots of work left to do!
  - Better/different metrics
  - Accretion, exotic processes, etc...

Thanks for listening!

(Many) Extra slides

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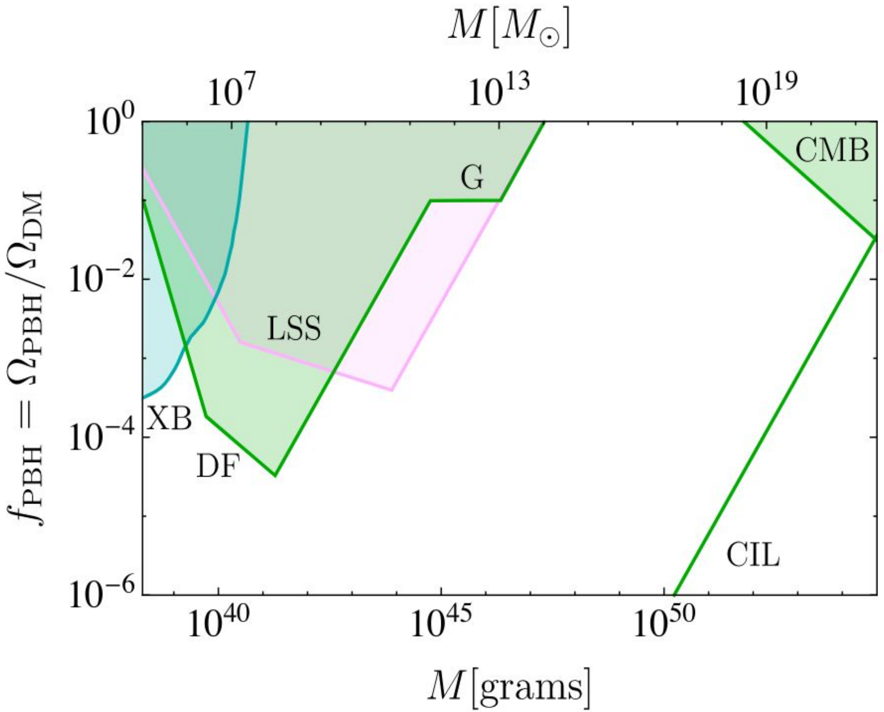
“Stupendously Large” PBH Constraints

$M > 10^5 M_{sun}$

- DF: halo dynamical friction
- G: galaxy tidal distortions
- CMB: dipole
- LSS: large scale structure formation
- XB: X-ray binary accretion
- CIL: cosmological incredulity



Carr et al, 2020

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"LIGO Range" bounds

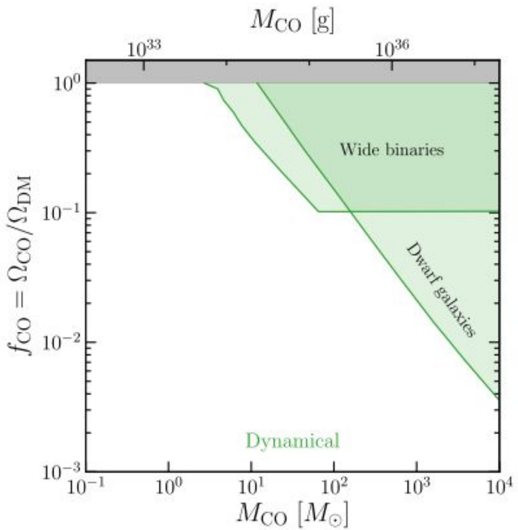
$\sim 1 M_{sun} < M < \sim 1000 M_{sun}$

Dynamical:

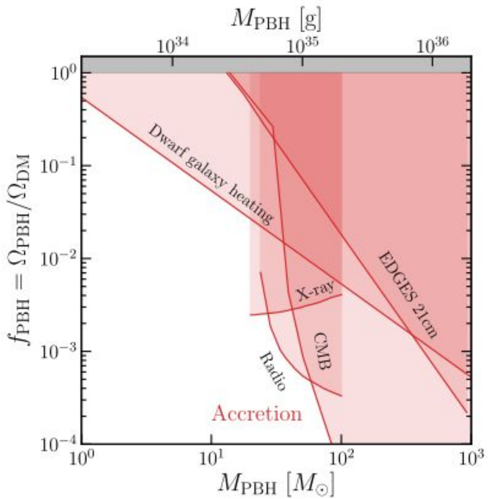
- Wide binaries
- Dwarf galaxies/star clusters (Eridanus II)

Accretion:

- Recombination
- CMB
- X-ray/radio



Green & Kavanaugh, 2020



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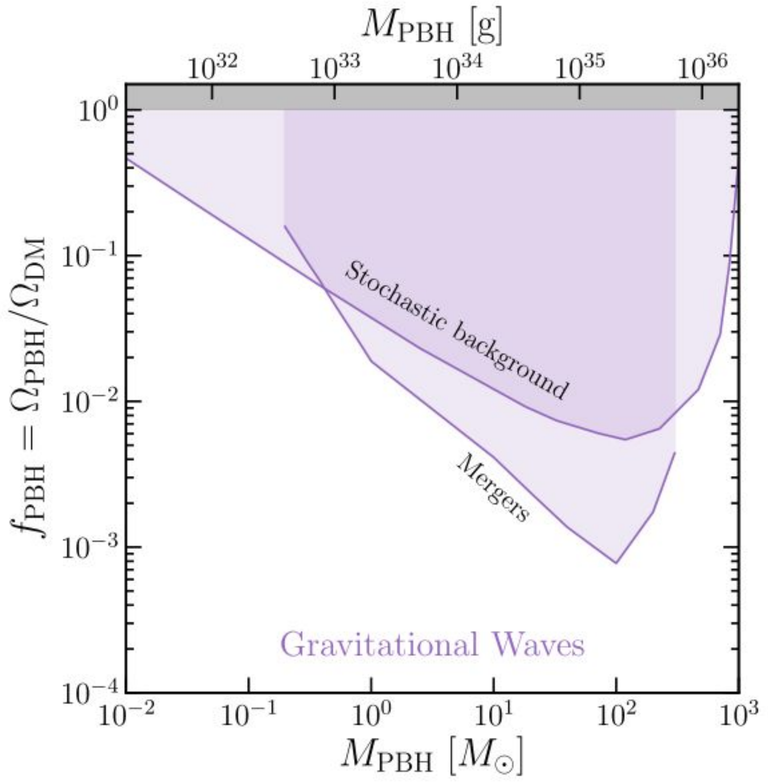
"LIGO Range" bounds

$\sim 1 M_{sun} < M < \sim 1000 M_{sun}$

Gravitational waves

- Binary abundance
    - Mergers today
    - Stochastic background

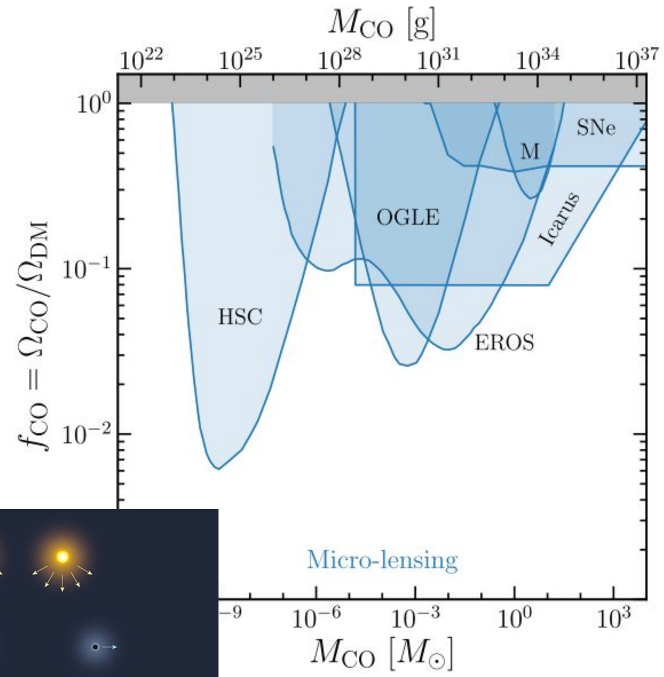


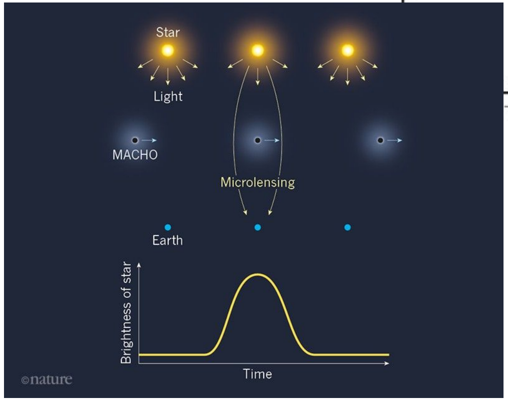
Green & Kavanaugh, 2020

Microlensing- range constraints

$\sim 10^{-12}\,M_{sun} < M < \sim 10\,M_{sun}$

- MACHO,EROS,OGLE,HSC:
  - stellar microlensing
  - LMC, galactic bulge, M31
- Icarus: lensing event
  - Would be dimmer with compact object pop.
- SNe: supernova magnification distribution
  - Depends on DM smoothness





Green & Kavanaugh, 2020

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Asteroid-mass range

$10^{-17}\, M_{sun} < M < 10^{-12}\, M_{sun}$

- Too large for usual Hawking radiation constraints
- Too small for microlensing
- Unconstrained?
  - Femtolensing: finite size, diffraction effects
  - Capture in stars
  - Ignition of white dwarfs

Monetero-Camacho et al, 2019

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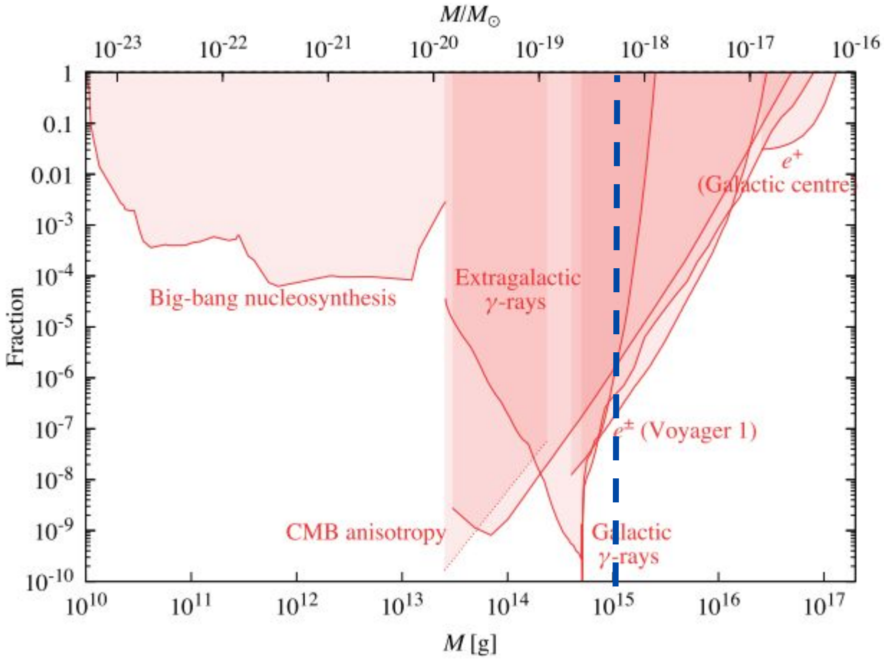
Evaporation-range constraints

$M < 10^{-16}\, M_{sun}$

- PBH Lifetime
  - Stability constraint
- BBN
- CMB
- Gamma rays
- Galactic centre



Carr et al 2020

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Spherically symmetric metrics

$$\mathrm{d}s^2 = \left(1 - \frac{2Gm_{\mathrm{MS}}}{R}\right)\mathrm{d}\tau^2 - \left(1 - \frac{2Gm_{\mathrm{MS}}}{R}\right)^{-1}\mathrm{d}R^2 - R^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right)$$

- Can put any spherically symmetric metric in this form
  - (after suitable coordinate transformation)
- m_{MS} is known as the "Misner-Sharp" mass
  - Invariant measure of internal energy
  - Easy to see in these coordinates:

$$m_{\mathrm{MS}} = \int_V \mathrm{d}^3x\sqrt{-g}\,T^0_0$$

- (quasi-) local and effective description of mass

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Spherically symmetric metrics - kodama time

$$\mathrm{d}s^2 = \left(1 - \frac{2Gm_{\mathrm{MS}}}{R}\right)\mathrm{d}\tau^2 - \left(1 - \frac{2Gm_{\mathrm{MS}}}{R}\right)^{-1}\mathrm{d}R^2 - R^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right)$$

- Dynamical metrics don't have a killing field
    - Instead we use the 'Kodama vector' to define timelike direction
    - Parallel to Killing field in the static limit
- Can then define a *geometrically preferred* time coordinate τ
    - In the above, the norm of the Kodama time translation vector is taken to coincide with the norm of the Kodama vector
    - Allows us to calculate surface gravity, four-acceleration with less ambiguity

Abreu & Visser, 2010

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Thakurta metric - metric details

- Attractor solution of generalized McVittie metrics
- Simple & Elegant: $\mathrm{d}s^2 = a^2 \mathrm{d}s^2_{schw.}$
- $$\mathrm{d}s^2 = f(R)\left(1 - \frac{H^2R^2}{f^2(R)}\right)\mathrm{d}t^2 + \frac{2HR}{f(R)}\mathrm{d}t\,\mathrm{d}R - \frac{\mathrm{d}R^2}{f(R)} - R^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right)$$
- $f(R) = 1 - 2Gma(t)/R$
- m is the ‘physical’ mass of the overdensity (or in static limit, today)
- Sourced by imperfect fluid with radial heat flow:

$$\begin{aligned} T_{\mu\nu} &= (\rho + P)\,u_\mu u_\nu + g_{\mu\nu}P + q_{(\mu}u_{\nu)} \\ q_\mu &= (0, q, 0, 0)\ , \\ u_\mu &= (u, 0, 0, 0)\ . \end{aligned}$$

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Thakurta metric - metric details

- Would like the Thakurta in the general form,

$$\mathrm{d}s^2 = \left(1 - \frac{2Gm_{\rm MS}}{R}\right)\mathrm{d}\tau^2 - \left(1 - \frac{2Gm_{\rm MS}}{R}\right)^{-1}\mathrm{d}R^2 - R^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right)$$

- Use the transformation:

$$\mathrm{d}\tau = \mathrm{d}t + \frac{HR}{f(R)}\frac{\mathrm{d}R}{1 - \frac{2Gm_{\rm MS}}{R}}$$

  - n.b. away from the horizon, Kodama time and cosmic time ~coincide

- Quasi-local Misner-Sharp mass: $\boxed{m_{\rm MS} = ma(t) + \dfrac{H^2R^3}{2Gf(R)}}$

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Thakurta metric - metric details

- Quasi-local Misner-Sharp mass: $$m_{\rm MS} = ma(t) + \frac{H^2 R^3}{2Gf(R)}$$
- Apparent horizon: $$R_{\rm b} = \frac{1}{2H}\left(1 - \sqrt{1 - 8HGma(t)}\right) \approx 2ma(1+2\delta)$$
    - Gives us a small parameter: $1/8 > \delta := HGma$
- Look at the two extremes
    - Near the horizon: $m_{MS} \approx ma$
    - Far from the horizon: $m_{\rm MS} \approx \frac{H^2 R^3}{2G} = \frac{4\pi}{3} R^3 \rho$

      $R \gg \left(2Gma/H^2\right)^{1/3}$

Total energy density

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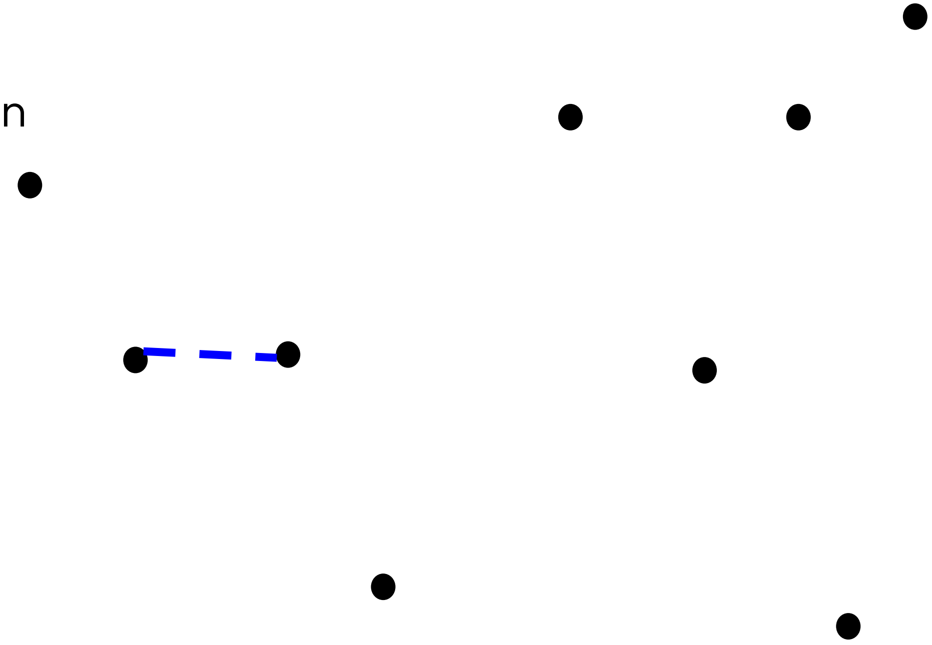
Binary formation

- Average BH separation today

$$\bar{x}_0 = \left(\frac{m}{f_{\rm PBH}\rho_{\rm cr}\Omega_{\rm DM}}\right)^{1/3}$$

$$\approx \frac{1.2\ \rm kpc}{f_{\rm PBH}^{1/3}}\left(\frac{m}{30M_\odot}\right)^{1/3}$$



- At earlier times,

$$\bar{x} = a(t)\ \bar{x}_0$$

Gives the physical average distance

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Binary formation

- Semimajor axis:

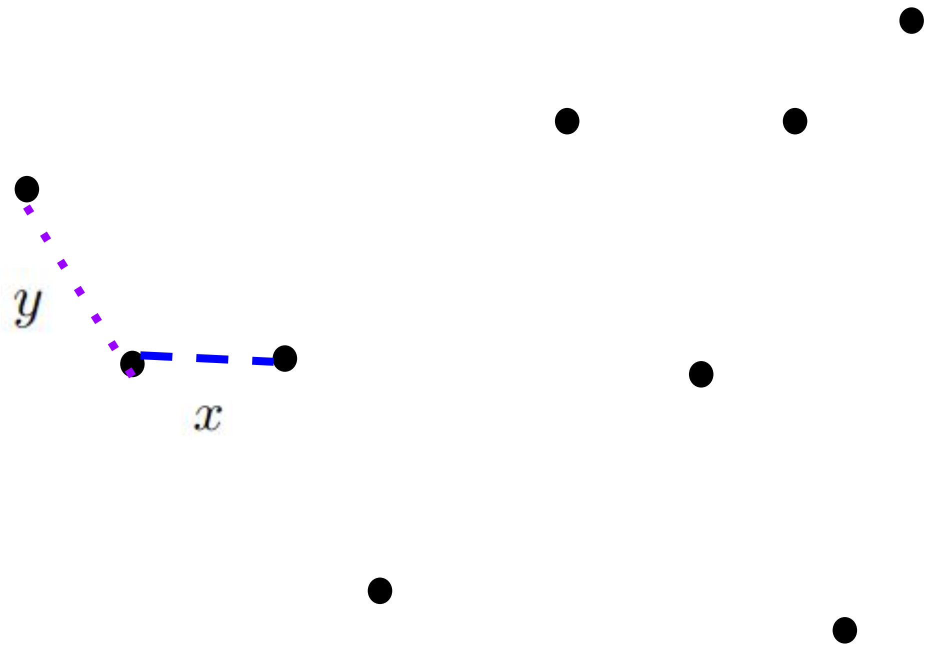
$$\mathfrak{a} = \alpha x$$

- Semiminor axis:

$$\mathfrak{b} = \beta \left(\frac{x}{y}\right)^3 \mathfrak{a}$$

- Eccentricity:

$$e = \sqrt{1 - \left(\frac{\mathfrak{b}}{\mathfrak{a}}\right)^2}$$



- Distribution function for BH separations in $(x, x + dx)$, $(y, y + dy)$:

$$dP = \frac{9}{\bar{x}^6} x^2 y^2 dx dy$$

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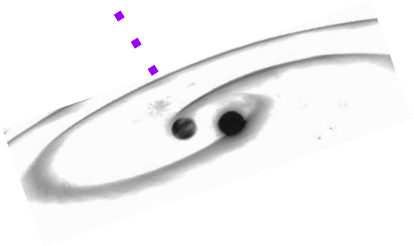
Binary formation

- If decoupling condition is satisfied at

$$z = z_{\rm dec}$$

- It coalescences with time

$$\tau_{\rm b} = \frac{3}{85}\frac{\mathfrak{a}_{\rm dec}^4(1-e_{\rm dec}^2)^{7/2}}{r_{\rm s}^3}$$



- Can rewrite the distribution function in terms of coalescence times and integrate over eccentricities

  - Event rate = $n_{\rm BH}\, dP\left(\tau_b \approx t_0\right)$

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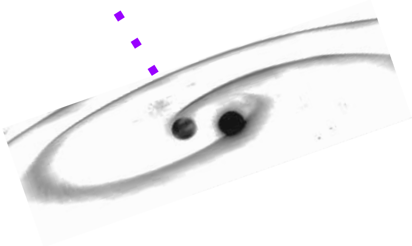
Decoupling condition in detail



Schwarzschild decoupling:

$$\ddot{R} = \frac{\ddot{a}}{a}R - \frac{Gm}{R^2}$$

$$\frac{m}{V} \gtrsim \rho$$

Thakurta Decoupling:

$$\ddot{R} = -\frac{Gma}{R^2} + \frac{\ddot{a}}{a}R$$

$$\frac{ma(t)}{V} \gtrsim \rho$$

$$E = -GM\mu a^2/(2R)$$

$$\dot{R}/R = -\dot{E}/E + 2H$$

$$\boxed{\dot{E}/E > 2H}$$

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Thakurta BH quadrupole power

- For Keplerian motion: $$d = \frac{\mathfrak{a}\left(1-e^2\right)}{1+e\cos\psi} \qquad \dot{\psi} = \frac{\sqrt{2Gm\mathfrak{a}\mathfrak{a}\left(1-e^2\right)}}{d^2}$$
- The nonzero elements of the quadrupole tensor are:

$$Q_{xx} = \frac{1}{2}ma\ d^2\cos^2\psi \qquad Q_{yy} = \frac{1}{2}ma\ d^2\sin^2\psi \qquad Q_{xy} = Q_{yx} = \frac{1}{2}ma\ d^2\cos\psi\sin\psi$$

- The gravitational wave power is given by:

$$P = \frac{G}{5}\left(\frac{\mathrm{d}^3Q_{ij}}{\mathrm{d}t^3}\frac{\mathrm{d}^3Q_{ij}}{\mathrm{d}t^3} - \frac{1}{3}\frac{\mathrm{d}^3Q_{ii}}{\mathrm{d}t^3}\frac{\mathrm{d}^3Q_{ij}}{\mathrm{d}t^3}\right)$$

- Power for Thakurta black holes:

$$P = -\frac{64}{5}\frac{G^4m^5a^5}{\mathfrak{a}^5\left(1-e^2\right)^{7/2}}\left[\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)+\ldots\right] \approx P_{\mathrm{Schw.}}a^5$$

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Thakurta decoupling

- After decoupling, however, the PBHs follow the *usual* Schwarzschild coalescence time
- So we can combine the decoupling condition $\dot{E}/E > 2H$ with:

$$P = -\frac{64}{5}\frac{G^4 m^5 a^5}{\mathfrak{a}^5\left(1-e^2\right)^{7/2}}\left[\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)+\ldots\right] \approx P_{\mathrm{Schw.}}a^5$$

$$\tau_{\mathrm{b}} = \frac{3}{85}\frac{\mathfrak{a}_{\mathrm{dec}}^4\left(1-e_{\mathrm{dec}}^2\right)^{7/2}}{r_{\mathrm{s}}^3}$$

$$\boxed{\left(1+z_{\mathrm{dec}}\right)^3 H\left(z_{\mathrm{dec}}\right) < \frac{1}{\tau_b}\frac{96}{425}\left(1+\frac{73}{24}e_{\mathrm{dec}}^2+\frac{37}{96}e_{\mathrm{dec}}^4\right)}$$

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Thakurta surface gravity

- Black hole horizon: $$R_{\rm b} = \frac{1}{2H}\left(1 - \sqrt{1 - 8HGma(t)}\right) \quad \approx 2ma(1+2\delta)$$

$$1/8 > \delta := HGma$$

- Surface gravity is relatively well-defined in our coordinate system:

$$\kappa = \frac{1 - 2\left(\frac{\partial}{\partial R} M_{\rm MS}(R_{\rm b})\right)}{2R_{\rm b}}$$

- For the Thakurta metric:

$$\kappa \approx \frac{1}{2Gma}(1-6\delta) \approx \frac{2\kappa_{\rm Schw.}}{a}$$

Abreu and Visser 2010

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Thakurta Temperature

- We define temperature in the standard way: $T = \kappa/2\pi$
- Assume radiation follows the Stefan-Boltzman law: $\dot{U}_H = -\sigma T^4 A$
  - Measured by an observer far from BH and cosmological horizon
- T is explicitly time-dependent, so need to check thermality:

$$\left|\dot{T}\right| < \left|\dot{U}_H\right|$$

$$\downarrow$$

$$240\ Gma \lesssim 1/H$$

- We assume black holes form when $2Gm \sim 1/H$, so this is easily satisfied

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Thermodynamic identity

- 'Easy' derivation of Hawking radiation using

$$\frac{\mathrm{d}U}{\mathrm{d}\tau} = T\frac{\mathrm{d}S}{\mathrm{d}\tau} - P\frac{\mathrm{d}V}{\mathrm{d}\tau} \qquad S = A/4,$$

- Internal energy U changes as a result of radiation *and* from growing horizon:

$$\frac{\mathrm{d}U}{\mathrm{d}\tau} = -\sigma T^4 A + 2\delta$$

- Can solve the thermodynamic identity for the physical mass loss:

$$\frac{\mathrm{d}m}{\mathrm{d}t} = -\frac{1}{1920\pi G^2 m^2 a^2} = \frac{8}{a^2}\left(\frac{\mathrm{d}m}{\mathrm{d}t}\right)_{\mathrm{Schw}}$$

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New critical mass

- Want to find critical mass m_* which evaporates by matter-radiation equality:

$$\int_0^{m_*} \mathrm{d}m \; m^2 \propto \int_{t_{eq}}^{t_f} \frac{\mathrm{d}t}{a(t)^2}$$

- The formation time (or redshift) is also a function of mass:

$$z_\mathrm{f}(m) = \left(\frac{2GmH_0\sqrt{\Omega_r}}{\gamma}\right)^{-1/2}$$

- Approximate analytic solution for critical mass:

$$m_* \sim 9.6 \times 10^{-13} \; M_\odot \left(\frac{\gamma}{0.2}\right)^{\frac{1}{7}} \left(\frac{h}{0.67}\right)^{-\frac{3}{7}} \left(\frac{\Omega_\mathrm{r}}{5.4 \times 10^{-5}}\right)^{-\frac{3}{14}}$$