Dynamically emergent gravity from hidden local Lorentz symmetry

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and in progress
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Quantum gravity

- One says
  - Quantum gravity based on the Einstein-Hilbert action is not renormalizable.
  - Because the Newton constant is mass-dimension -2.

Quantum gravity

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    perturbatively

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Key points in this talk

- Quantum gravity could be formulated ***non*-perturbatively** within QFT!
- Strong-Weak correspondence!?
- Emergence of spacetime by spontaneous symmetry breaking.

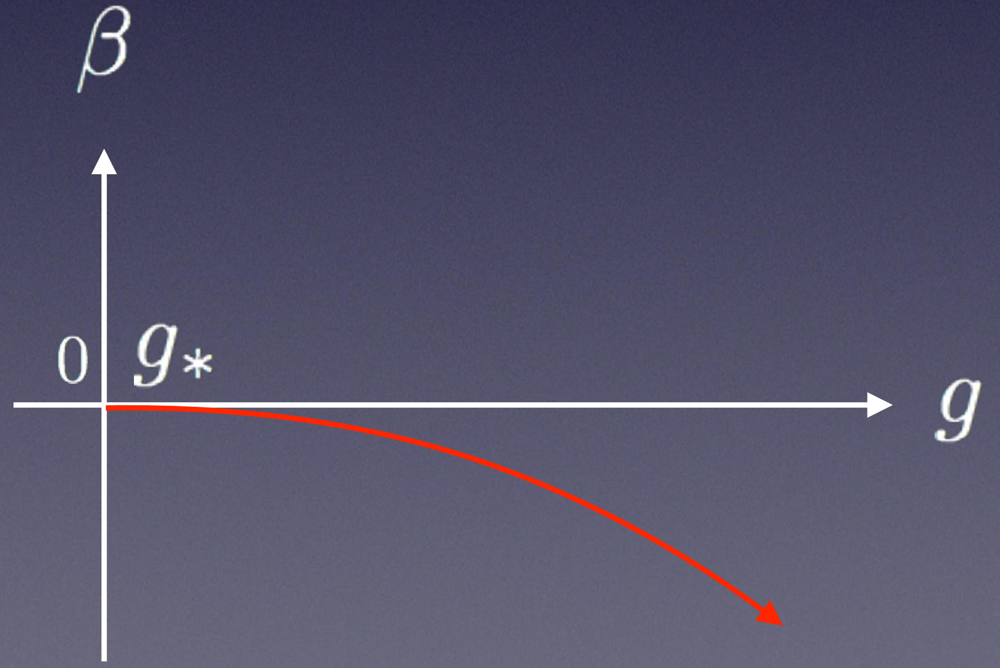
Contents

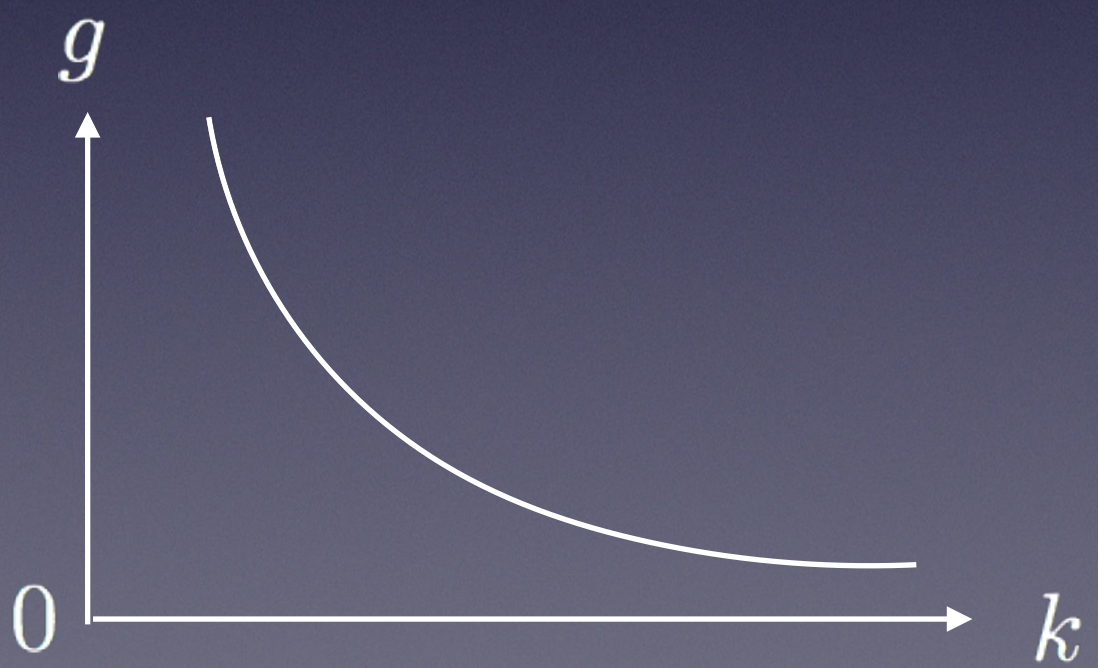
- Asymptotically safe gravity:
  - non-perturbatively renormalizable quantum gravity
- Universality class at fixed point
- First-order formalism with degenerate limit

Asymptotic freedom

- Asymptotic freedom

$$\partial_t g = -\beta_0 g^3, \quad \beta_0 > 0$$

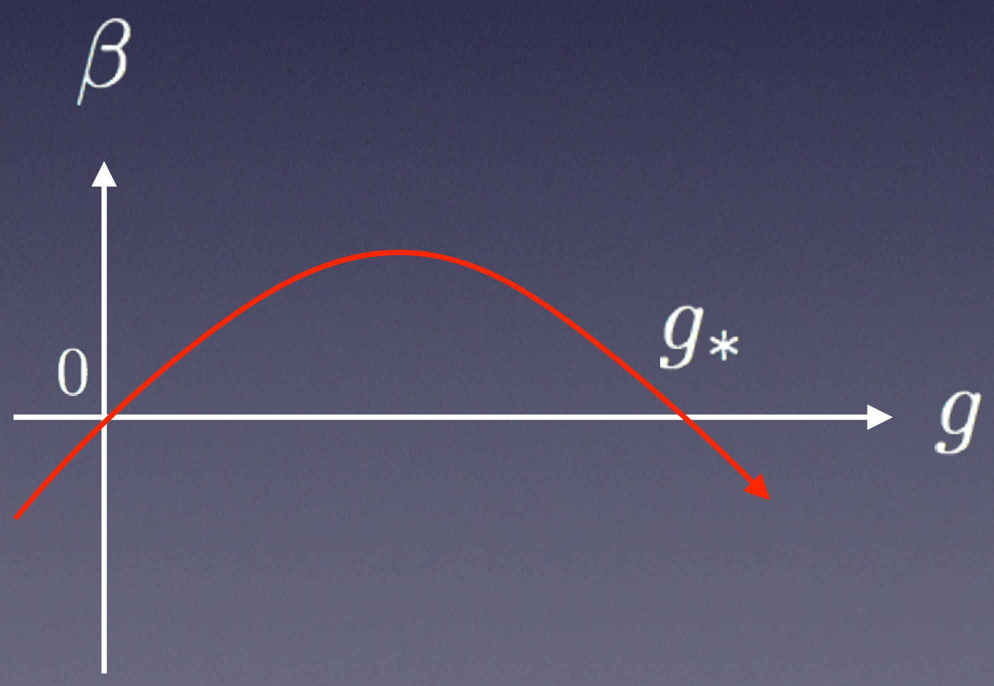


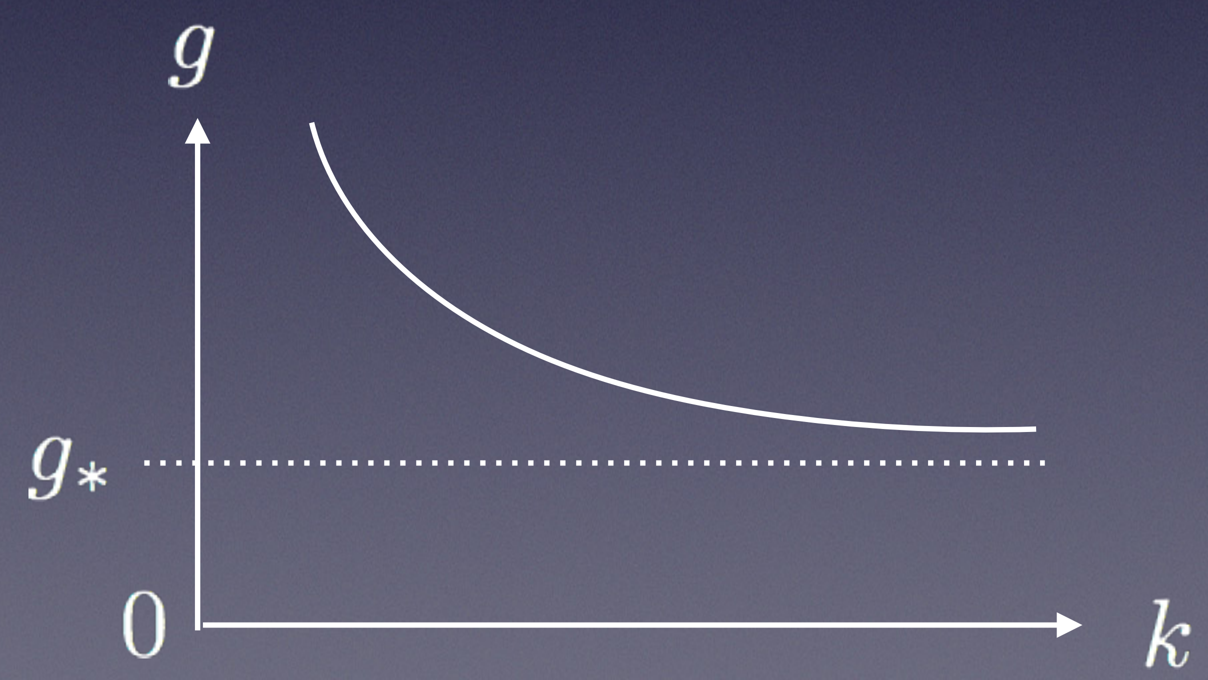


Asymptotic safety

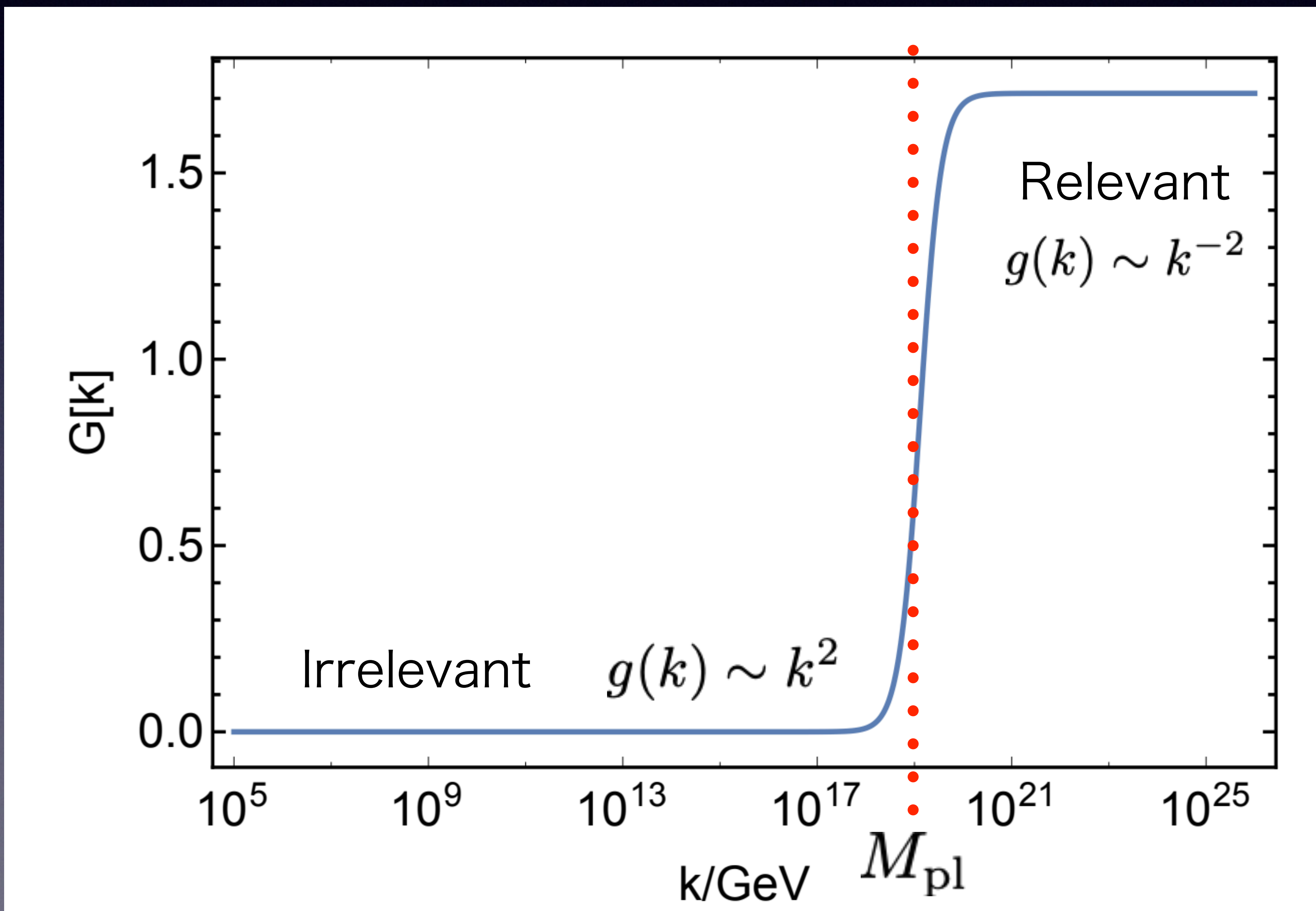
- Asymptotic safety $\quad g = G_N k^2$

$$\partial_t g = 2g - \beta_0 g^2, \quad \beta_0 > 0$$





RG flow of Newton constant



Non-trivial FP

$[G_N]=+2$

large anomalous dimension induced

$[G_N]=-2$

Gaussian FP

Asymptotically safe theories

- D=3 non-linear σ model
- D=3 Gross-Neveu model
- D=4 gravity???
- D=5 Yang-Mills theory???

These theories are perturbatively NON-renormalizable but (could be) NON-perturbatively renormalizable.

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An Example of asymptotically safe theories

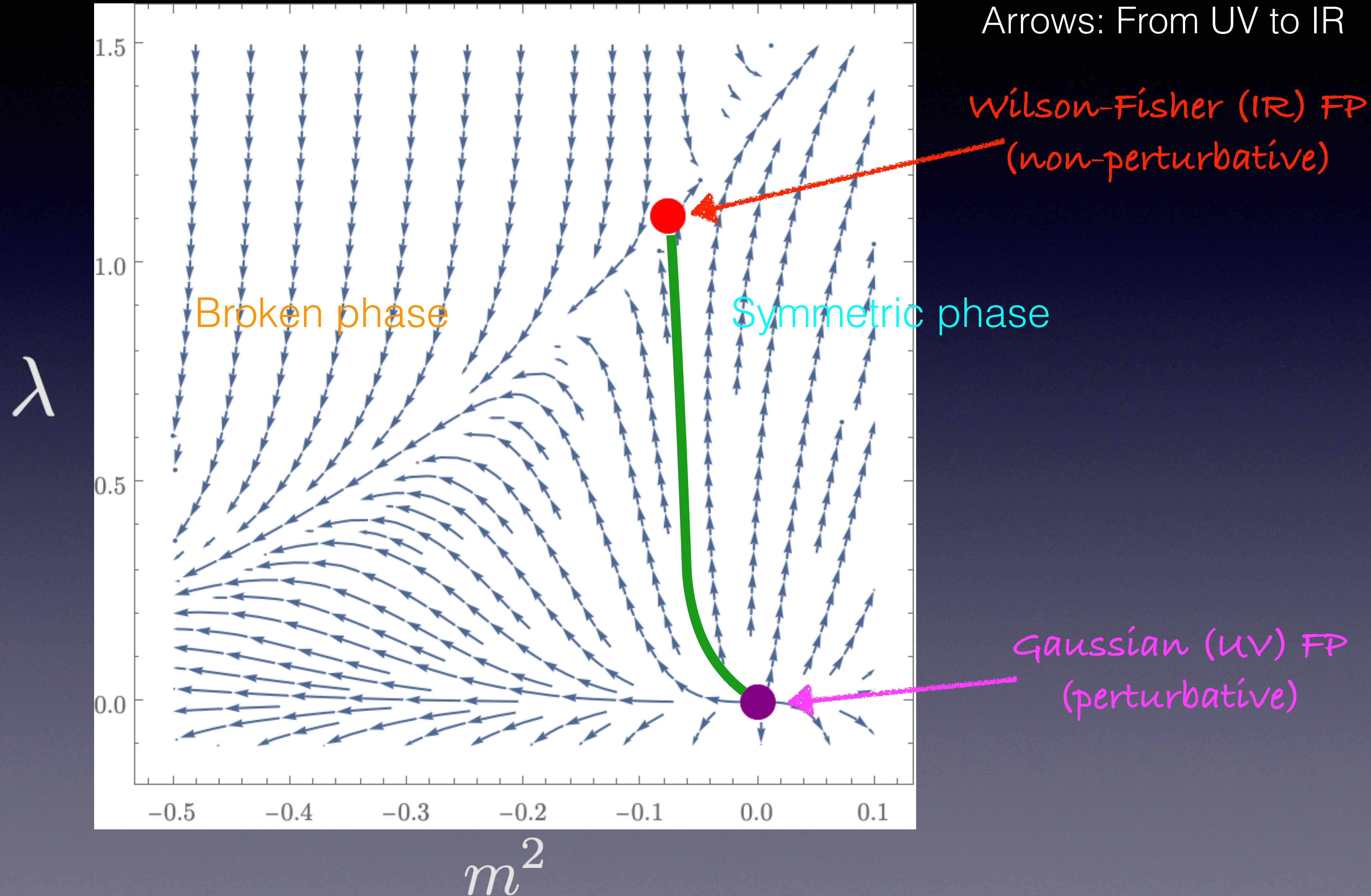
- Non-linear σ model in 3 dim.
  - Scalar theory with a field constraint $\langle \phi^i \phi^j \rangle = f_\pi^2 \delta^{ij}$
  - Symmetry breaking *O*(*N*) →*O*(*N*-1) in the linear σ model.

$$\phi^i = (\sigma, \pi^1, \ldots, \pi^{N-1}) \qquad \langle \sigma \rangle = f_\pi$$

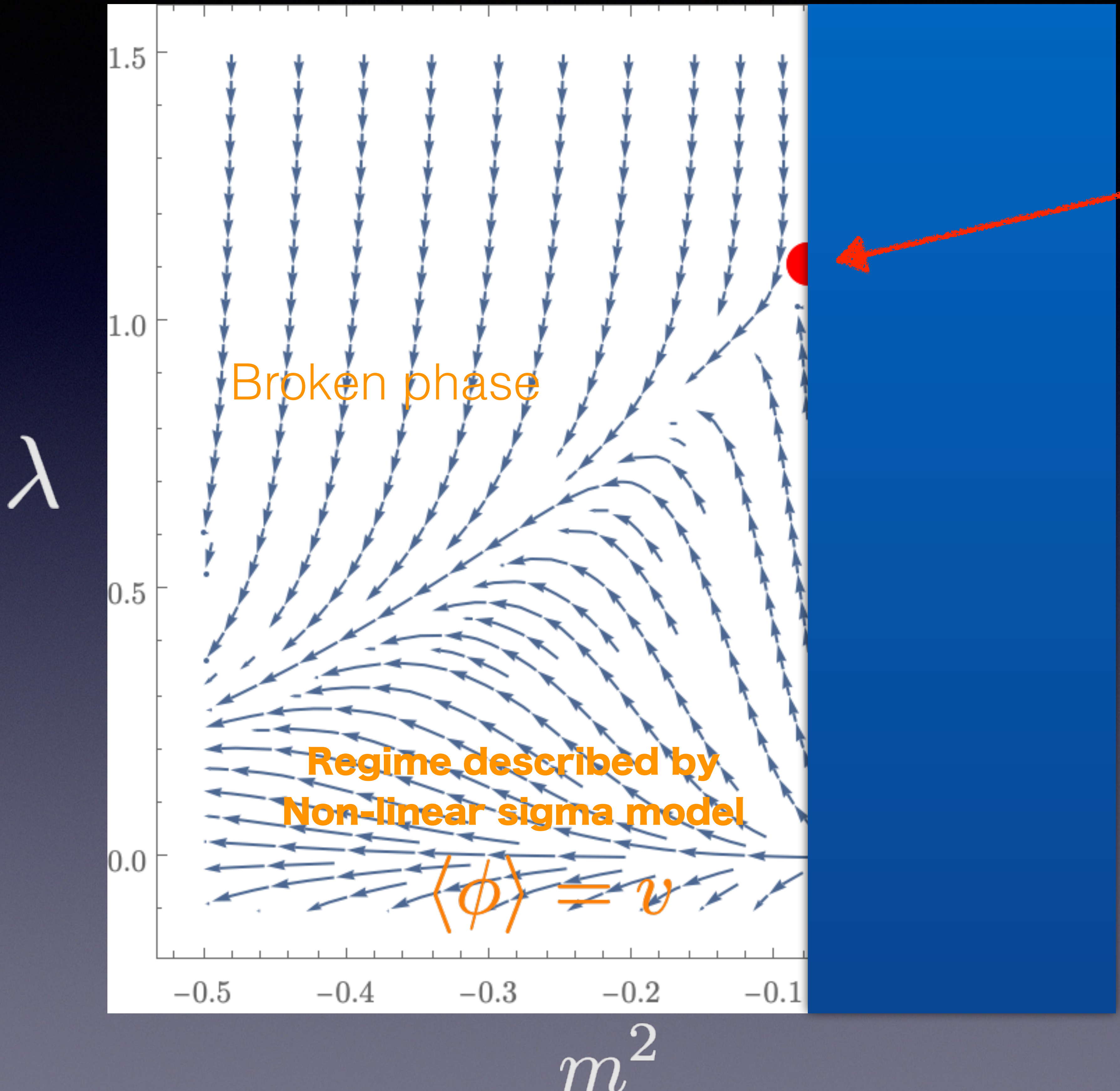
  - Describes dynamics of NG bosons (pions). $S[\pi^i]$
  - Perturbatively ***non***-renormalizable

Phase diagram of 3d linear σ model

Arrows: From UV to IR



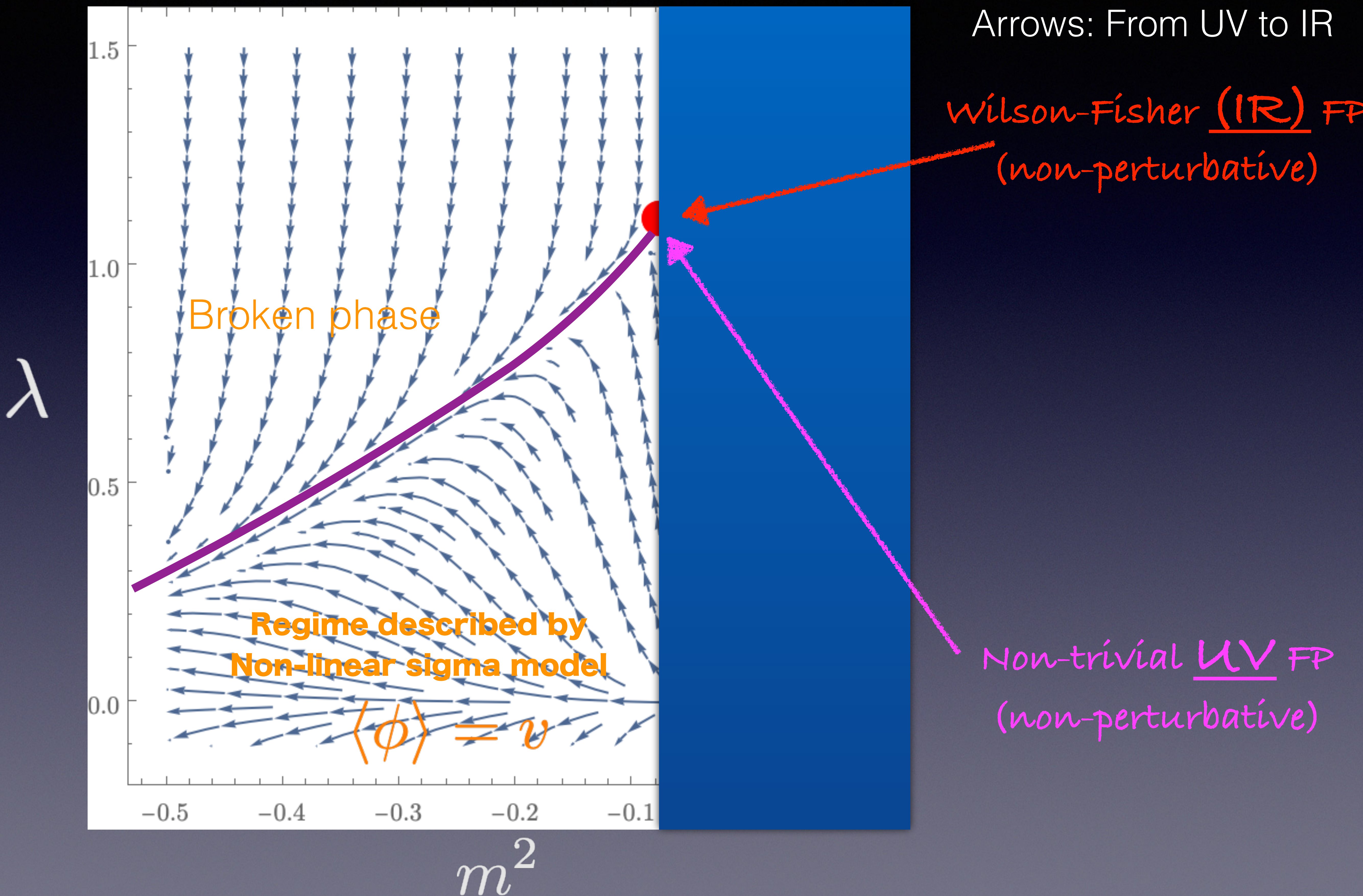
Phase diagram of 3d linear σ model



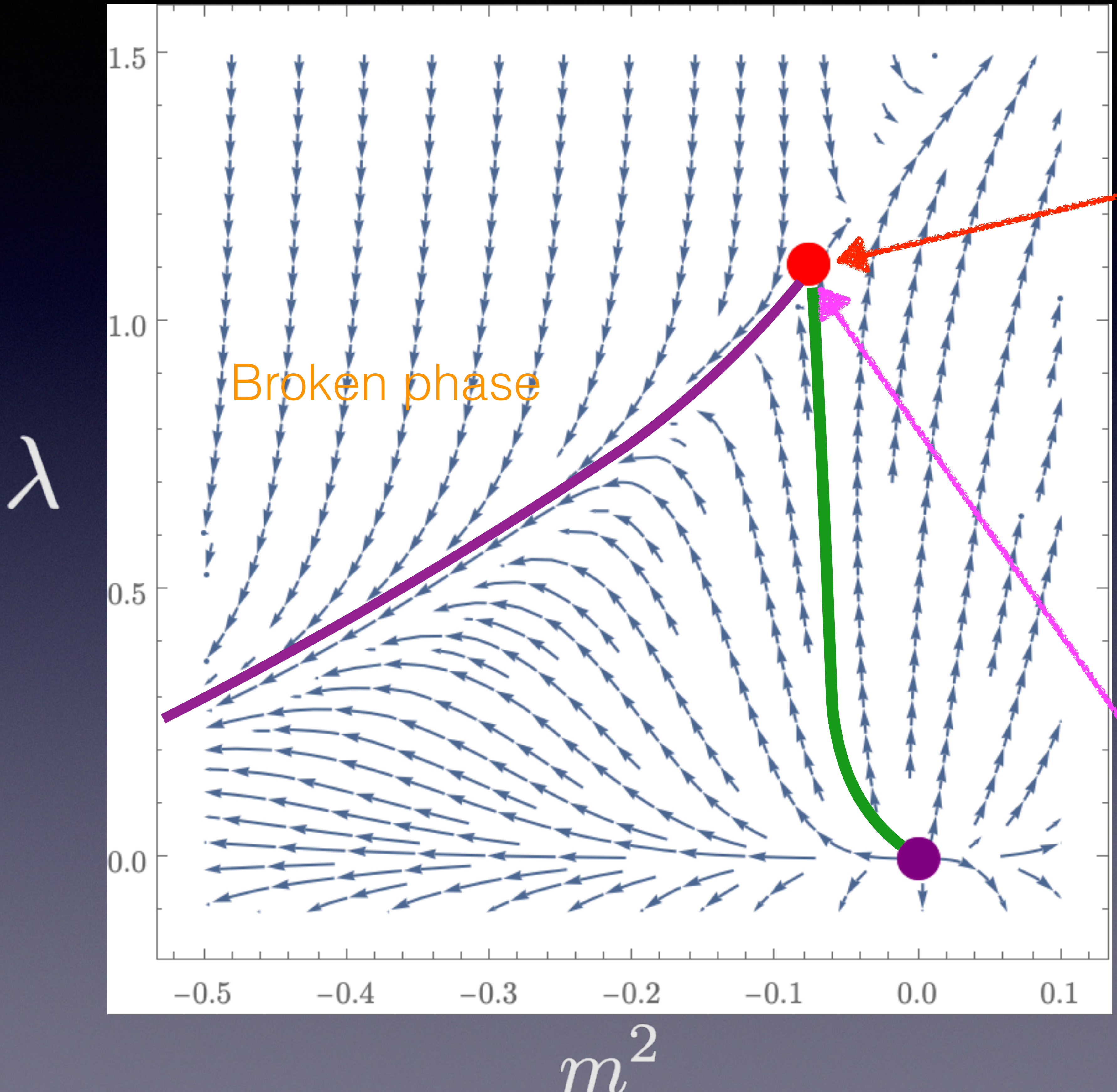
Arrows: From UV to IR

Wilson-Fisher (IR) FP
(non-perturbative)

Phase diagram of 3d linear σ model



Phase diagram of 3d linear σ model



Arrows: From UV to IR

Wilson-Fisher (IR) FP
(non-perturbative)

linear σ model

Same universality class

non-linear σ model

Non-trivial UV FP
(non-perturbative)

To summarize

Non-linear σ model in 3 dim
O(N-1)

- Perturbatively non-renormalizable
- Asymptotically safe (UV FP)
- Constraint on fields

$$\langle \phi^i \phi^j \rangle = f_\pi^2 \delta^{ij}$$

Asymptotically safe gravity

- Perturbatively non-renormalizable
- Asymptotically safe (UV FP)
- Constraint on fields

$$g_{\mu\alpha} g^{\alpha\nu} = \delta^\nu_\mu$$

Same universality class

O(N) linear σ model in 3 dim

- Perturbatively renormalizable
- Unitary
- Asymptotically free (Gaussian FP)
- IR fixed point (Wilson-Fisher FP)

?

Contents

- Asymptotically safe gravity:
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How to formulate?

- Metric theories are diffeomorphism invariant.

$$? \xrightarrow{\text{SSB}} \text{Diffeomorphism}$$

$$g_{\mu\alpha}g^{\alpha\nu} = \delta^{\nu}_{\mu}$$

- In this work, we consider local Lorentz SO(1,3):

$$SO(1,3)_{\text{local}} \times \text{Diff.} \xrightarrow[\cancel{SO(1,3)}_{\text{local}}]{\text{SSB}} \text{Diffeomorphism}$$

$$g_{\mu\alpha}g^{\alpha\nu} = \delta^{\nu}_{\mu}$$

First-order formalism

- Based on SO(1,3) local Lorentz symmetry (and diff.)
  - Vierbein $e_\mu{}^a$
  - Local-Lorentz (LL) gauge field $(A_\mu)^a{}_b$
- Action (Einstein-Hilbert)

$$S = \int \mathrm{d}^4 x \, e \left[-\Lambda + \frac{M^2}{2} e_a{}^\mu e_b{}^\nu F^{ab}{}_{\mu\nu} \right]$$

$$F^a{}_{b\mu\nu} = (\partial A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu])^a{}_b$$

First-order formalism

$$S = \int \mathrm{d}^4x \, e \left[-\Lambda + \frac{M^2}{2} e_a{}^\mu e_b{}^\nu F^{ab}{}_{\mu\nu} \right]$$

- Equation of motion $\quad (A_\mu)^a{}_b = e_\nu{}^a D_\mu e^\nu{}_b$
  - Obtain the EH action in the vierbein formalism
  - Introducing inverse vierbein breaks $SO(1,3)_{\mathrm{local}}$ symmetry.
- Kinetic term of LL gauge field

$$\frac{1}{4} F^{ab}{}_{\mu\nu} F_{ab}{}^{\mu\nu} + \cdots \quad \to R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \cdots$$

Degenerate limit

- Non-linear σ model: O*(N-1)* invariant
  - Constraint on fields $\langle \phi^i \phi^j \rangle = f_\pi^2 \delta^{ij}$
  - $f_\pi^2 \to 0$: symmetric phase (O*(N)* invariant)
- Gravity in first-order formalism

$$\langle e^a{}_\mu \rangle = C\delta^a_\mu$$
$$\bar{g}_{\mu\nu} \propto C^2$$
$$\bar{g}^{\mu\nu} \propto C^{-2}$$

  - Constrain on metric $g_{\mu\alpha} g^{\alpha\nu} = \delta^\nu_\mu$
  - $C \to 0$: symmetric phase (SO(1,3) invariant).

Model with degenerate limit

- Including matters, at a certain scale,

$$S = \int \mathrm{d}^4 x\, e \left[-V + \frac{M^2}{2} e_a{}^\mu e_b{}^\nu F^{ab}{}_{\mu\nu} - \frac{Z_\psi}{2} \left(\bar{\psi} e_a{}^\mu \gamma^a D_\mu \psi + \mathrm{h.c.} \right) \right]$$

$$D_\mu = \partial_\mu - i g_L (A_\mu)^{ab} \Sigma_{ab} + \cdots$$

  - Invariant under $SO(1,3)_{\mathrm{local}} \times$ diff.
  - Only fermions are dynamical!
  - No kinetic terms of vierbein, gauge fields, scalar fields.
  - These fields would be dynamical via fermion quantum corrections.

Spontaneous local Lorentz symmetry breaking

- $SO(1,3)_{\rm local} \times$ diff.
- Generation of expectation value of vierbein
- A possible solution would be a flat spacetime.

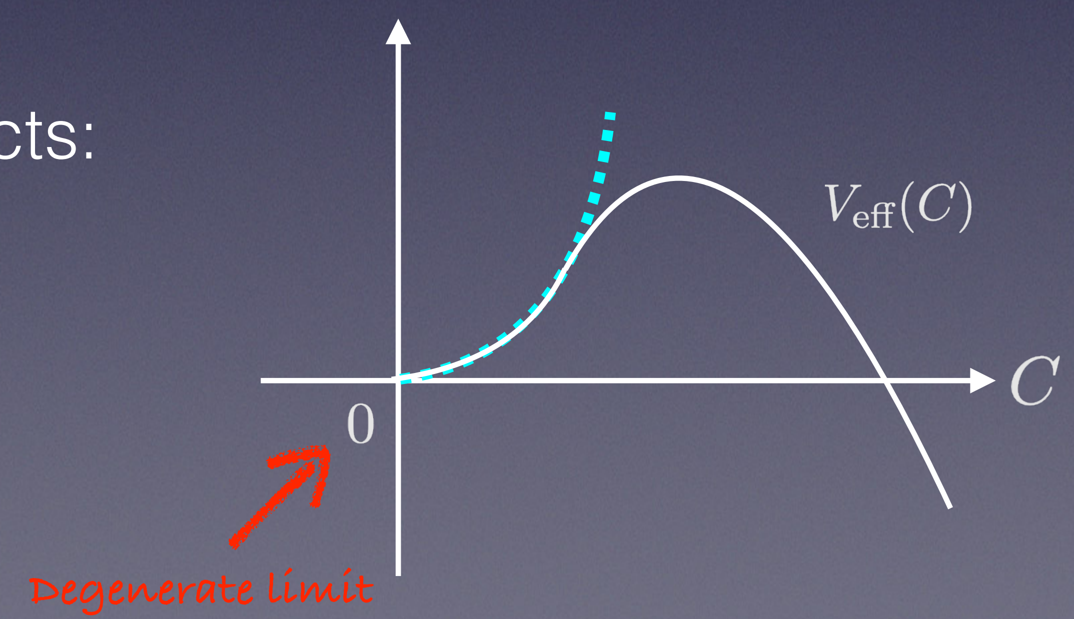
$$\langle e^a{}_\mu \rangle = C\delta^a_\mu$$

$SO(1,3)_{\rm local} \to SO(1,3)_{\rm global}$

- Effective potential from spinor loop effects:

$$V_{\rm eff}(C) = -VC^4 - \frac{(CM)^4}{2(4\pi)^2}\log\left(C^2M^2/\mu^2\right)$$

- Precise analysis is in progress.



Spontaneous local Lorentz symmetry breaking

- Local Lorentz gauge symmetry is broken.
  - Degrees of freedom (d.o.f.):

Symmetric part (metric)
(radial modes)

Anti-symmetric part (torsion)
(NG modes)

    - Vierbein $e_\mu{}^a$: 16 d.o.f. = 10 + 6 d.o.f.

eaten

    - LL gauge field $(A_\mu)^a{}_b$: 6 d.o.f.
  - LL gauge bosons become massive and decouple.
  - The symmetry parts (radial modes) are still massless thanks to diif..

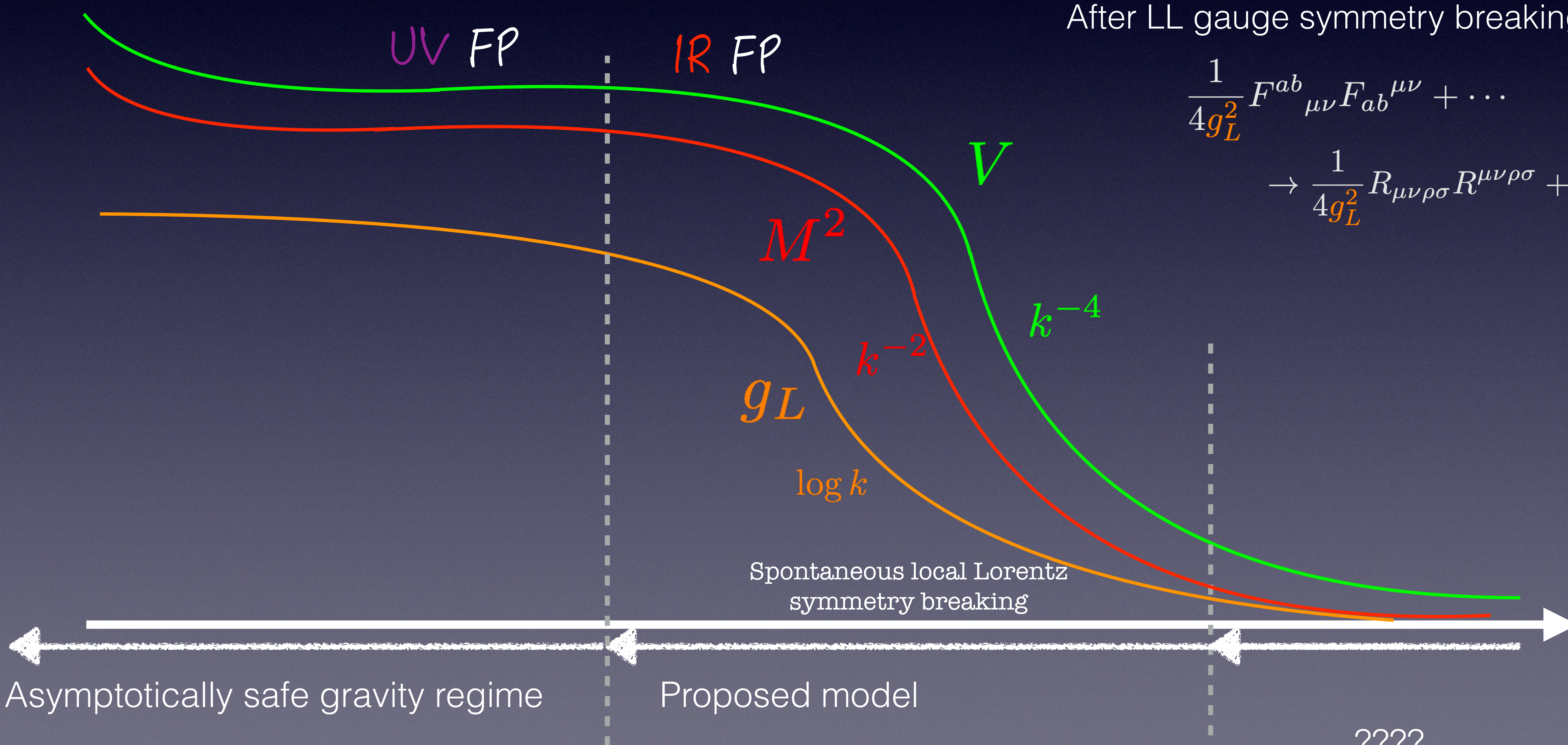
Very ideal scenario!

$$S = \int \mathrm{d}^4x\, e\Big[-V + \frac{M^2}{2} e_a{}^\mu e_b{}^\nu F^{ab}{}_{\mu\nu} - \frac{Z_\psi}{2}\left(\bar\psi e_a{}^\mu \gamma^a(\partial_\mu - i g_L A_\mu)\psi + \mathrm{h.c.}\right)\Big]$$

After LL gauge symmetry breaking

$$\frac{1}{4g_L^2} F^{ab}{}_{\mu\nu} F_{ab}{}^{\mu\nu} + \cdots$$

$$\to \frac{1}{4g_L^2} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \cdots$$



$SO(1,3)_{\text{global}} \times$ diff.

$SO(1,3)_{\text{local}} \times$ diff.

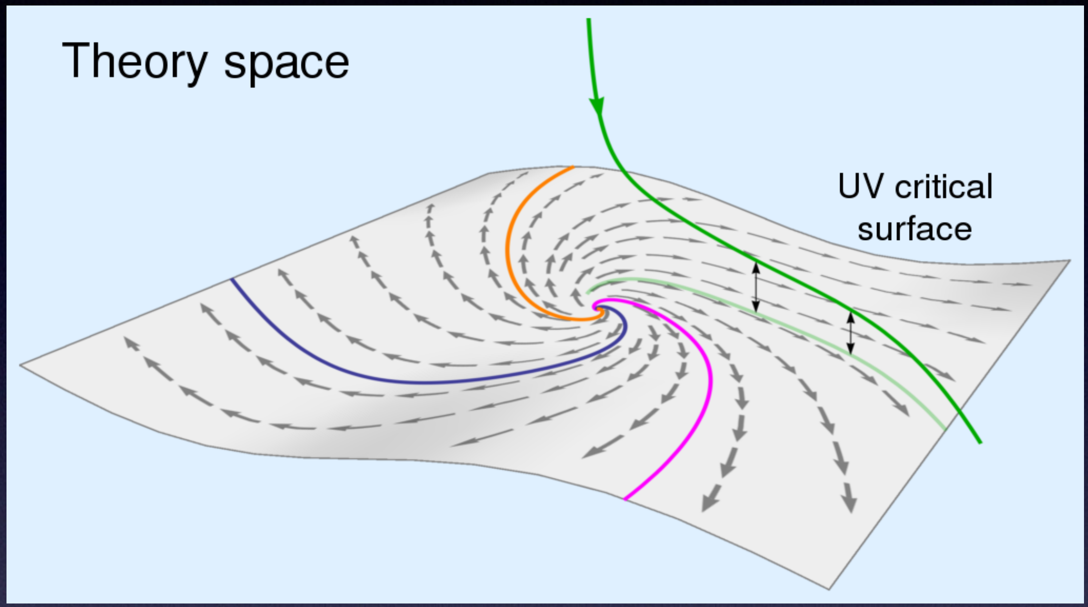
Summary

- First-order formalism with degenerate limit
  - SO(1,3) local Lorentz symmetry
- Fermionic fluctuations make other fields dynamical.
- Spontaneous LL gauge symmetry breaking: generation of spacetime

- Precise analysis on spontaneous symmetry breaking
- Perturbatively renormalizable??
- Connection to low energy quantum gravity

Appendix

Asymptotic safety



- Suggested by S. Weinberg

  S. Weinberg, Chap 16 in General Relativity

- Existence of UV fixed point
  - Continuum limit $k \to \infty$.
  - UV critical surface (UV complete theory) is defined by relevant operators.
  - Dimension of UV critical surface = number of free parameters.
- Generalization of asymptotic free

Asymptotic freedom vs safety

Asymptotic freedom

$$\beta(g) = -\left(0 + \beta_0 g^2\right) g$$

canonical scaling　　anomalous scaling

Asymptotic safety

$$\beta(g) = -\left(-2 + \beta_0 g^2\right) g$$

canonical scaling　　anomalous scaling

$$\beta(g) = 0$$

$$g_* = 0 \qquad g^2 \sim \beta_0 g_*^2 \log k$$

$$g_* = 0 \qquad g^2 \sim \beta_0 g_*^2 \log k$$

$$g_* = \sqrt{2/\beta_0} \qquad g^2 \sim k^{-2}$$

Degenerate limit

- First-order formalism
  - $e = \det e_\mu{}^a \sim C^4 \to 0$
  - Some components of vierbein degenerate.
  - Admit **topology-change processes** in path integral.

A. Tseytlin, J. Phys. **A15** (1982) L105.
G. T. Horowitz, Class. Quant. Grav. **8** (1991) 587.

- Forbids inverse vierbein
  - Remove terms divergent in the limit $e^\mu{}_a \sim C^{-1} \to \infty$.
  - Do not use inverse metric. $\bar{g}_{\mu\nu} \propto C^2, \ \bar{g}^{\mu\nu} \propto C^{-2}$

Inverse vierbein

- Metric $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$
  - Canonical normalization $g_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{1}{M_P} h_{\mu\nu}$
- Inverse metric $g_{\mu\alpha} g^{\alpha\nu} = \delta^\nu_\mu$

$$g^{\mu\nu} = \bar{g}^{\mu\nu} - h^{\mu\nu} + h^\mu{}_\alpha h^{\alpha\nu} + \cdots$$

$$g^{\mu\nu} = \bar{g}^{\mu\nu} - M_P h^{\mu\nu} + M_P^2 h^\mu{}_\alpha h^{\alpha\nu} + \cdots$$

Model with degenerate limit

- Including matters, at a certain scale,

$$S = \int \mathrm{d}^4 x\, e \Big[-V + \frac{M^2}{2} e_a{}^\mu e_b{}^\nu F^{ab}{}_{\mu\nu} - \frac{Z_\psi}{2} \left(\bar{\psi} e_a{}^\mu \gamma^a D_\mu \psi + \text{h.c.} \right) \Big]$$

$$D_\mu = \partial_\mu - i g_L (A_\mu)^{ab} \Sigma_{ab} + \cdots$$

- $$e = \frac{1}{4!} \epsilon_{abcd} \epsilon^{\mu\nu\rho\sigma} e^a{}_\mu e^b{}_\nu e^c{}_\rho e^d{}_\sigma \sim C^4 \to 0$$
- $$e e_a{}^\mu = \frac{1}{3!} \epsilon_{abcd} \epsilon^{\mu\nu\rho\sigma} e^b{}_\nu e^c{}_\rho e^d{}_\sigma \sim C^3 \to 0$$
- $$e e_{[a}{}^\mu e_{b]}{}^\nu = \frac{1}{2!2!} \epsilon_{abcd} \epsilon^{\mu\nu\rho\sigma} e^c{}_\rho e^d{}_\sigma \sim C^2 \to 0$$
- $$\mathrm{AntiSym}[e e_a{}^\mu e_b{}^\nu e_c{}^\rho] = \epsilon_{abcd} \epsilon^{\mu\nu\rho\sigma} e^d{}_\sigma \propto C \to 0$$ No invariant term
- $$\mathrm{AntiSym}[e e_a{}^\mu e_b{}^\nu e_c{}^\rho e_d{}^\sigma] = \epsilon_{abcd} \epsilon^{\mu\nu\rho\sigma}$$ Topological $\epsilon_{abcd} \epsilon^{\mu\nu\rho\sigma} F^{ab}{}_{\mu\nu} F^{cd}{}_{\rho\sigma}$

Degenrate limit

- Higgs potential

$$V(H^\dagger H) = m^2(H^\dagger H) + \lambda(H^\dagger H)^2$$

- In terms of invariance and renormalizability

$$V(H^\dagger H) = \frac{c_1}{H^\dagger H} + \frac{c_2}{(H^\dagger H)^2} + \cdots + m^2(H^\dagger H) + \lambda(H^\dagger H)^2$$

- Inverse terms diverge for H→0 (symmetric phase)

LL gauge theory

- Ordinary YM theory

$$\mathcal{L} = -\frac{1}{4}\mathrm{tr}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(\partial_\mu - igA_\mu^a T^a)\psi$$

T^a commutes with γ^a.

- LL gauge theory

$$\mathcal{L} = -\frac{1}{4}\mathrm{tr}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(\partial_\mu - ig_L A_\mu^{ab}\Sigma_{ab})\psi$$

$$\Sigma_{ab} = \frac{1}{4}[\gamma^a, \gamma^b]$$

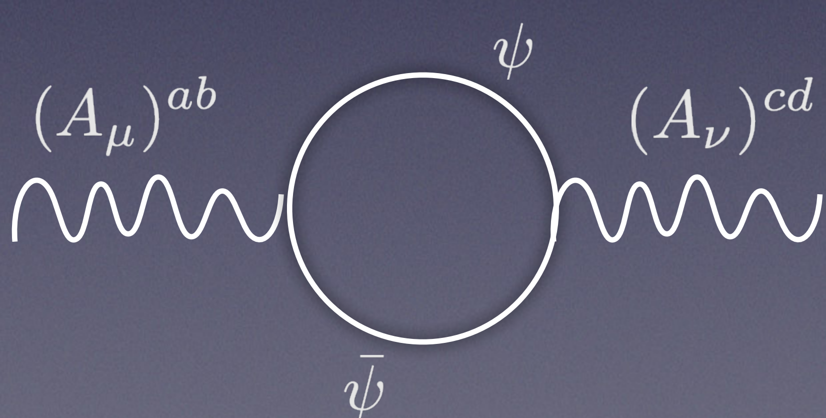
Σ^{ab} do not commute with γ^a.

Make LL gauge field Dynamical

- The staring action has no kinetic terms except for spinor fields.

- Fermionic fluctuations make other fields dynamical.

  - e.g. LL gauge field in a flat spacetime background



$$= -i\eta^{c[a}\eta^{b]d}\left[f(p^2)\left(\eta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) + g(p^2)\frac{p^\mu p^\nu}{p^2}\right]$$

Analogy to QCD

- QCD
  - Nambu-Jona-Lasinio model $\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \psi - \frac{G}{2}(\bar{\psi}\psi)^2$
  - Quark-meson model $\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \psi - \frac{m^2}{2}\phi^2 + y\phi\bar{\psi}\psi$

Bosonisation

$\phi \sim \bar{\psi}\psi$

No kinetic term of boson

- Gravity
  - Spinor gravity: Vierbein and LL gauge field are composites of spinors

    A. Hebecker, C. Wetterich, Phys.Lett. **B574** (2003) 269-275

  - Our model $$S = \int \mathrm{d}^4x\, e \left[-V + \frac{M^2}{2} e_a{}^\mu e_b{}^\nu F^{ab}{}_{\mu\nu} - \frac{Z_\psi}{2}\left(\bar{\psi} e_a{}^\mu \gamma^a D_\mu \psi + \mathrm{h.c.} \right) \right]$$