Missing final state puzzle in monopole-fermion scattering

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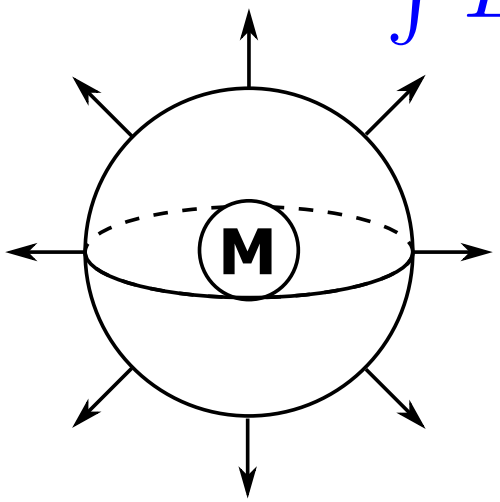
Magnetic monopoles

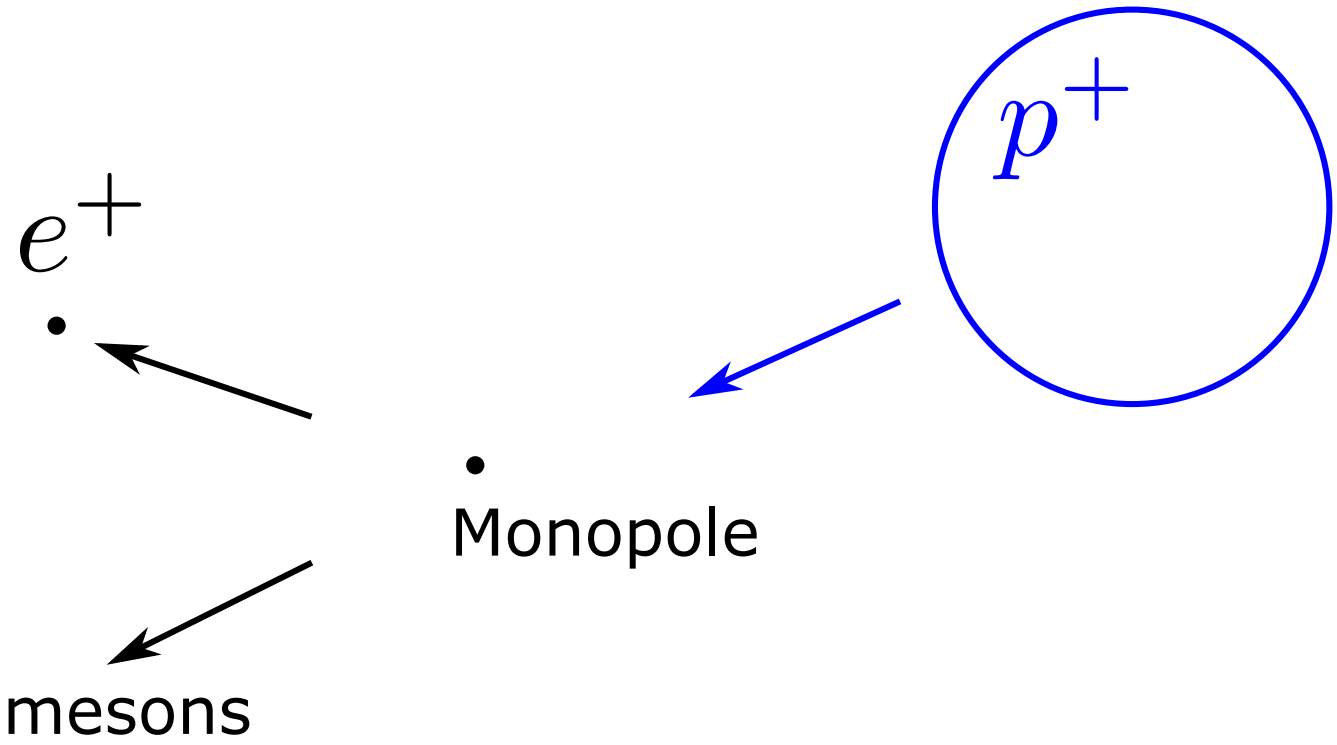
- **A Dirac monopole** has **a singularity** at the core.
- **Dirac quantization**: If there is a particle with charge e, a magnetic charge has to be $\mathbf{2\pi e^{-1} n}$.

$$\int \vec{B} \cdot d\vec{S} = 2\pi e^{-1} n$$



- When a gauge group G is spontaneously broken down to $G_{\rm sub} \times U(1)_{\rm sub}$, there are **regular magnetic monopoles corresponding to $U(1)_{\rm sub}$ as topological solitons** (**'t Hooft-Polyakov monopoles**).
- eg.) An $SU(2)$ gauge theory with an adjoint Higgs
- In **GUT theories**, there are such monopoles typically.

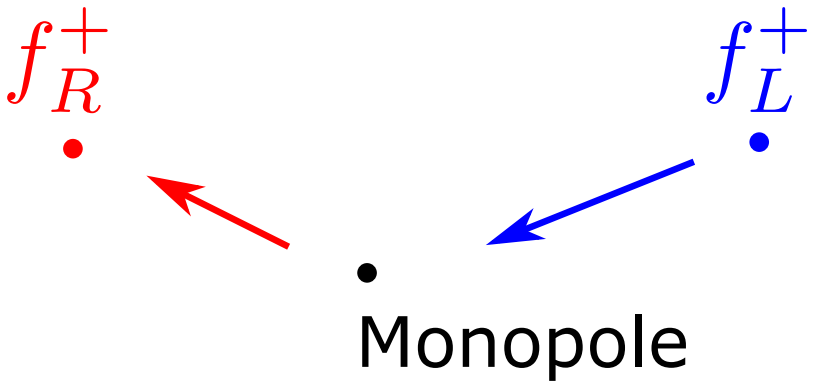
Rubakov-Callan effect



[V. A. Rubakov 1982, C. G. Callan 1982]

- When a proton collides with a GUT monopole, it decays into a positron and mesons.
- The effect has been used to set limits on the monopole flux in the Universe.

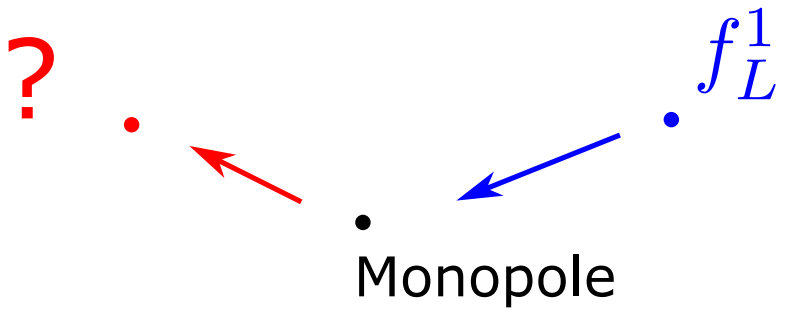
Helicity flip: One-flavor case



$$\langle \bar{f} f \rangle \propto \frac{1}{r^3}$$

- When a charged fermion collides with a monopole, **its helicity has to flip**.
- The important point is that it occurs even in **the massless theory**, where there are no mixing terms of left-handed and right-handed fermions.
- Thus an operator that mixes them has to have a non-zero expectation value. $\Leftrightarrow$ **The fermion condensation**

Puzzle: Two-flavor massless case

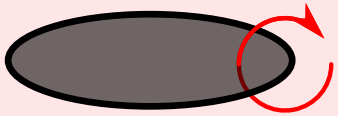


$$\langle \bar{f}^1 f^1 \rangle = 0$$

$$\langle (\bar{f}^1 f^1)(\bar{f}^2 f^2) \rangle \propto \frac{1}{r^6}$$

- The final state has to have the opposite helicity to the initial state and the same flavor charge.
- **There are no such fermions in the action.**

? = A wall bounded by a string



of the phase of the fermion condensate

Outline

1. Missing final state puzzle
2. Soliton of the phase of the fermion condensation
3. Soliton picture of the scattering

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Set up

- We consider an $SU(2)$ gauge theory with an adjoint Higgs and **4 flavors of Weyl fermions**, where $SU(2)$ is spontaneously broken down to $U(1)$.
- The global symmetry is $SU(4)$.

$$SU(2)\text{ doublets}\left\{\overbrace{\left(\begin{pmatrix}a_1^+\\ b_1^-\end{pmatrix},\begin{pmatrix}a_2^+\\ b_2^-\end{pmatrix},\begin{pmatrix}a_3^+\\ b_3^-\end{pmatrix},\begin{pmatrix}a_4^+\\ b_4^-\end{pmatrix}\right)}^{SU(4)\text{ quadruplets (fund. rep.)}}\right.$$

- The theory can be regarded as an approximation of $SU(5)$ GUT:

$$\begin{pmatrix}a_1\\ b_1\end{pmatrix}=\begin{pmatrix}e_L^+\\ d_L^3\end{pmatrix},\quad \begin{pmatrix}a_2\\ b_2\end{pmatrix}=\begin{pmatrix}\bar{d}_L^3\\ e_L^-\end{pmatrix},\quad \begin{pmatrix}a_3\\ b_3\end{pmatrix}=\begin{pmatrix}u_L^1\\ \bar{u}_L^2\end{pmatrix},\quad \begin{pmatrix}a_4\\ b_4\end{pmatrix}=\begin{pmatrix}\bar{u}_L^2\\ \bar{u}_L^1\end{pmatrix}.$$

The low energy effective theory

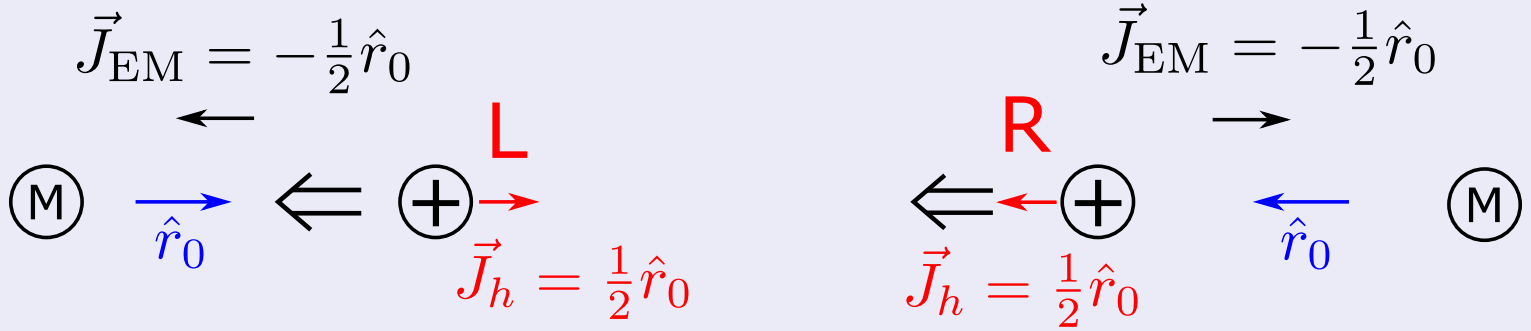
- We approximate the theory as the gauge theory of the unbroken $U(1)$.

$$
\begin{aligned}
&a : \text{The } U(1) \text{ gauge field,}\\
&f = da,\\
&a_j : \text{The Weyl fermions with charge } +1,\\
&b_j : \text{The Weyl fermions with charge } -1.
\end{aligned}
$$

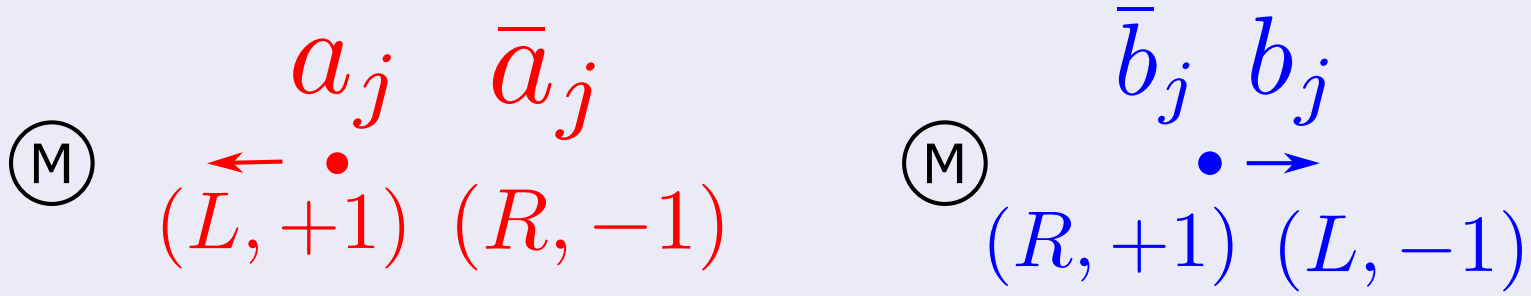
- The 't Hooft-Polyakov monopole is approximated by **the (background) Dirac monopole**.
- X, Y bosons, GUT Higgs bosons are considered to be infinitely heavy.

Helicity flip

- In a monopole background, there are **s-wave states** of fermions (the total angular momentum is zero).



- The higher partial waves cannot reach the monopole core.
  [R. Jackiw & C. Rebbi, 1976]
- In our setup, for s-wave fermions, only a_j and $\bar{a}_j$ can be incoming particles, and only b_j and $\bar{b}_j$ can be outgoing particles.

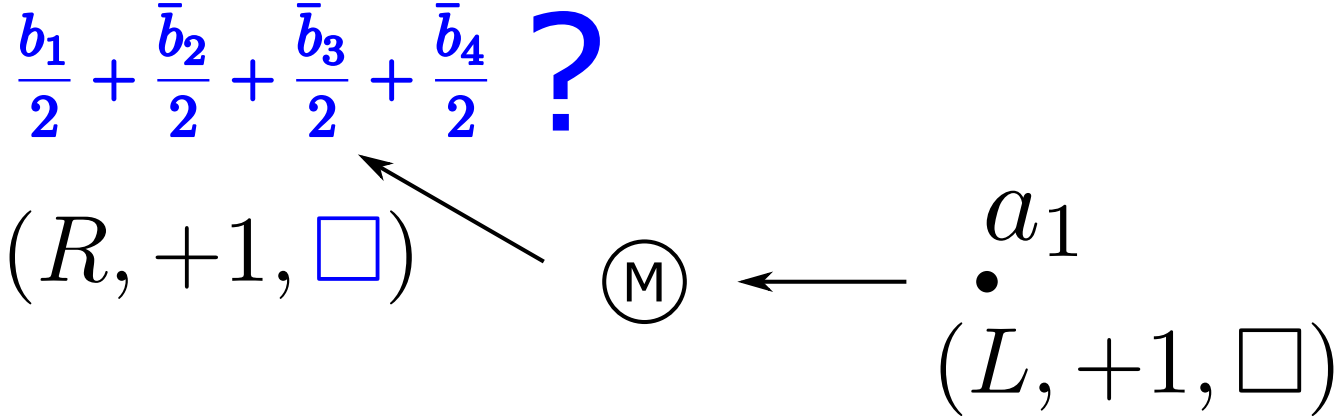


The missing final state puzzle

- The helicity, the $U(1)$ charge and the representation of $SU(4)$ are

$$a_j : (L, +1, \square), \quad b_j : (L, -1, \square), \quad \bar{a}_j : (R, -1, \overline{\square}), \quad \bar{b}_j : (R, +1, \overline{\square})$$

- If the initial state is a_1, the quantum number of the final state has to be $(R, +1, \square)$. However, there are no particles with this quantum number.
- "semitons"?



$b_j/2$: "Semiton", the state with **halves of the quantum numbers of b_j**.

What is the "semiton state"

$$|\frac{b_1}{2}, \frac{\bar{b}_2}{2}, \frac{\bar{b}_3}{2}, \frac{\bar{b}_4}{2}, M\rangle \ ?$$

- The state has **half fermion numbers**. (The state is the eigenstate of $\int d^3x \bar{b}_1 \sigma^0 b_1$ with the eigenvalue $1/2$.)
- The state is **orthogonal to any multi-particle state of $a_j, b_j, \bar{a}_j, \bar{b}_j$.**
- By projecting the fields to the s-wave component (**the s-wave approximation**), it was found that the state is actually the final state.
  [C. G. Callan 1984, J. Polchinski 1984, J. M. Maldacena & A. W. W. Ludwig 1997]

The problem is what an appropriate **interpretation** of the state is. A non-particle state? A multi-particle state?
Our claim is that it is **a new single-particle state that can exist only in the monopole background and only when the fermions are massless**.

Outline

1. Missing final state puzzle
2. Soliton of the phase of the fermion condensation
3. Soliton picture of the scattering

The fermion condensate

- The fermion condensation occurs due to the helicity flip.
  Fact: The following fermionic operators have **nonzero expectation values**:

$$\left\langle (\bar{b}^i \bar{\sigma}_\mu a_j)(\bar{a}^k \bar{\sigma}^\mu b_l) \right\rangle = \frac{1}{r^6}(c_1 \delta^i_j \delta^k_l + c_2 \delta^i_l \delta^k_j),$$

$$\langle (a_{i_1} b_{i_2})(a_{i_3} b_{i_4}) \rangle = \frac{1}{r^6} c_3 \varepsilon_{i_1 i_2 i_3 i_4}.$$

  Let us consider the effective theory of the operators' phases.

- The variables of the effective theory is the following four:

$$\theta_A : \text{the sum of the phases of } (a_1 b_2)(a_3 b_4) \text{ and } (a_2 b_1)(a_4 b_3),$$

$$\theta_{1j} : \text{the phase of } (\bar{b}^1 \bar{\sigma}_\mu a_1)(\bar{a}^j \bar{\sigma}^\mu b_j) \text{ for } j = 2, 3, 4$$

- It is convenient to express them using **the phases of the fermion fields.**
  Let $\boldsymbol{\alpha_j}$ be the phase of $\boldsymbol{a_j}$ and $\boldsymbol{\beta_j}$ be that of $\boldsymbol{b_j}$.
- The genuine variables are expressed as

$$\theta_A = \sum_j (\alpha_j + \beta_j), \quad \theta_{1j} = \alpha_1 - \beta_1 - \alpha_j + \beta_j \quad \text{for } j = 2, 3, 4.$$

The effective theory and Monopole bags

- To reproduce the chiral anomaly, the effective Lagrangian contains

$$\boldsymbol{\theta_A f \wedge f/(8\pi^2) + \theta_{12} f \wedge F_{12}/(4\pi^2)} + \cdots ,$$

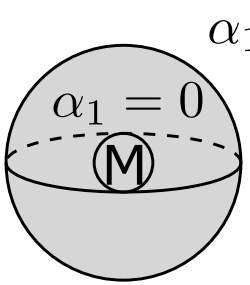
F_{12} : $U(1) \in SU(4)$ field strength corresponding to $\mathrm{diag}(1,-1,0,0)$

$$\Rightarrow \quad j^\mu = \varepsilon^{\mu\nu\rho\sigma} \frac{1}{4\pi^2} \partial_\nu \theta_A f_{\rho\sigma}, \quad j^\mu_{12} = \varepsilon^{\mu\nu\rho\sigma} \frac{1}{4\pi^2} \partial_\nu \theta_{12} f_{\rho\sigma} \cdots$$

(cf. **Axion**)

- **A monopole bag** has the electric and flavor charges:

$$Q = \int d^3x \; j^0 = \frac{1}{4\pi^2} \int_0^\infty dr \partial_r \theta_A \int_{S^2_r} \vec{B} \cdot d\vec{S} = 1.$$

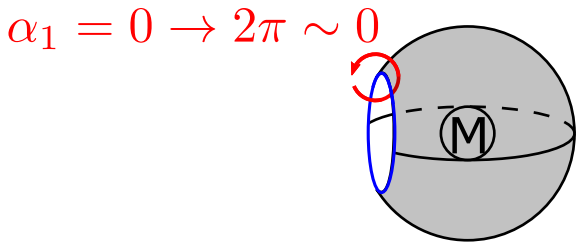


Witten effect:
$Q = +1$
$Q_{1j} = +1$

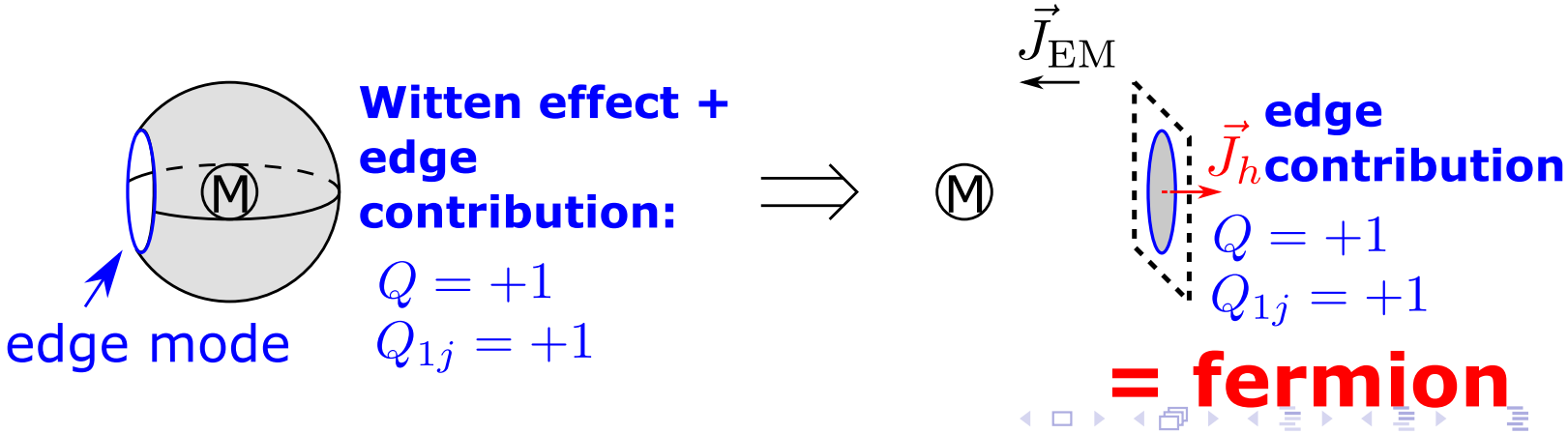
$\Longleftrightarrow$ $a_1 : Q = +1,$
$Q_{1j} = +1$

Walls bounded by strings

- Because $\boldsymbol{\alpha_1 \sim \alpha_1 + 2\pi}$, the wall of α_1 can have **a boundary**, which is **a string of** α_1.



- To regularize the singularity of α_1 at the string, the fermion condensates are zero at the string.
- There are **edge modes** to maintain the gauge invariance.
  (c.f. **axion strings [C. G. Callan & J. A. Harvey 1985]**)
- The edge modes contribute to the charge so that the total charge is an integer.

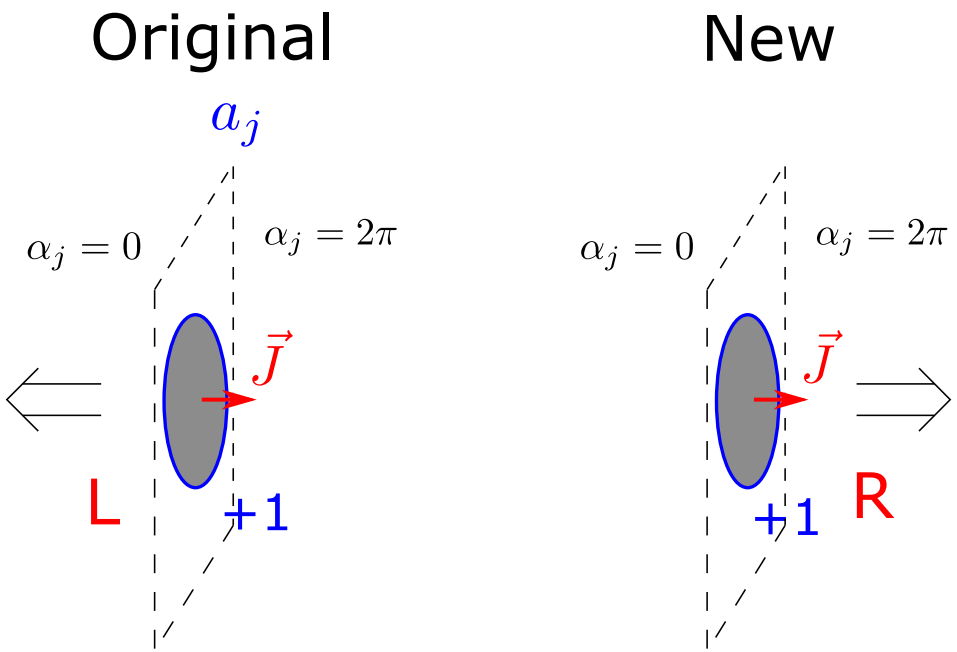


The final state of the scattering

- The objects can be regarded as **fermions**.
- Some of them have **opposite helicity** to the original fermions.

  $\Rightarrow$ **New fermions!**

- **The final state of the monopole-fermion scattering is identified with the new fermions.**



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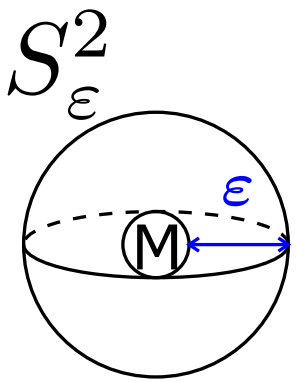
The boundary condition at the core of the monopole

- In the process of the scattering, **the boundary condition** at the core of the monopole plays an important role.
- The boundary condition is determined so that **the monopole does not have the electric and flavor charges**:

$$\int_{S^2_\varepsilon} \vec{j} \cdot d\vec{S} = 0, \quad \int_{S^2_\varepsilon} \vec{j}_{1k} \cdot d\vec{S} = 0,$$



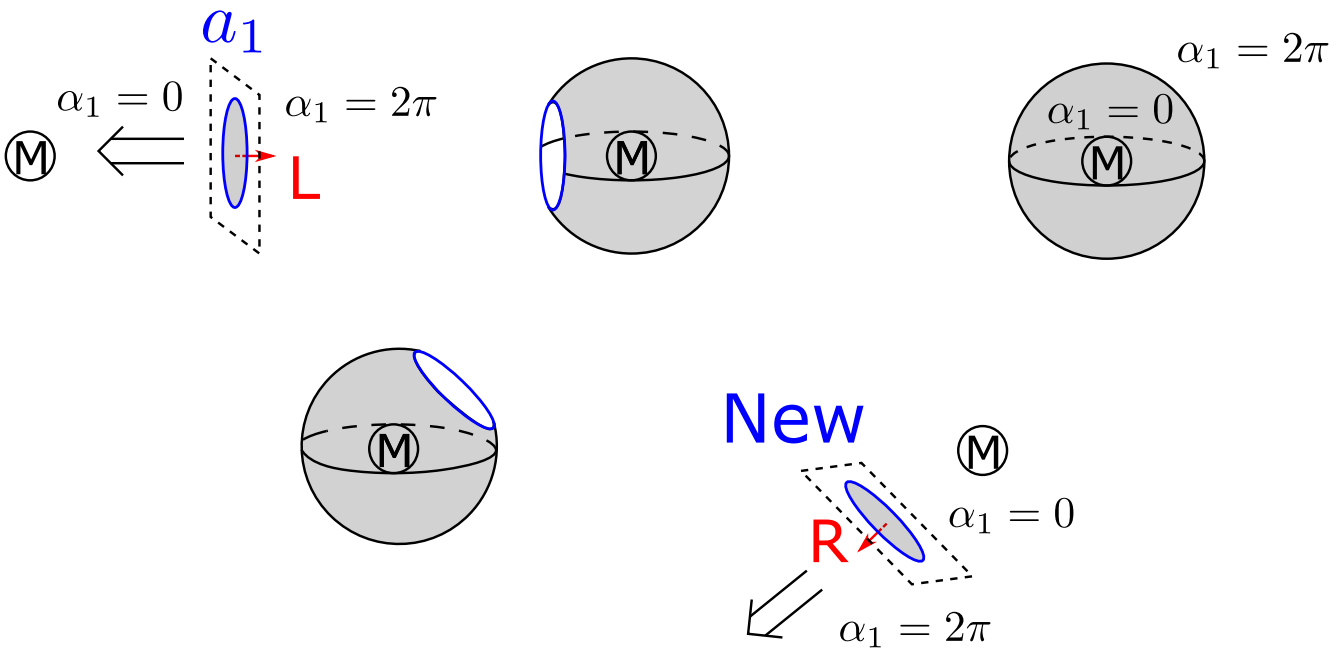
which implies that

$$\partial_t \theta_A|_{r=0} = 0, \quad \partial_t \theta_{1k}|_{r=0} = 0, \quad \forall k,$$

i.e., **the phases of the condensations can not change at the core of the monopole**.

A soliton picture of the scattering

- Due to the boundary condition, the monopole cannot penetrate the wall.
- When the wall reaches the monopole, the wall wraps it.
- The final state corresponds to the new particle with $(R, +1, \square)$.



(cf. Scattering of an axion string and a monopole **[I. Kogan 1993]**)

Summary

- When a charged fermion collides with a monopole, the helicity of the s-wave component of the fermion has to flip.
- As a consequence, there is a fermion condensate violating the fermion number conservation.
- Puzzle: If there are two flavors of massless Dirac fermions, **any fermions in the action cannot be the final state of the monopole-fermion scattering due to the flavor charge conservation and the helicity flip**.
- We solve this puzzle by identifying the final state as **a new fermion, which is a soliton of the fermion condensates**.