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Self-interacting dark matter with u-channel resonances

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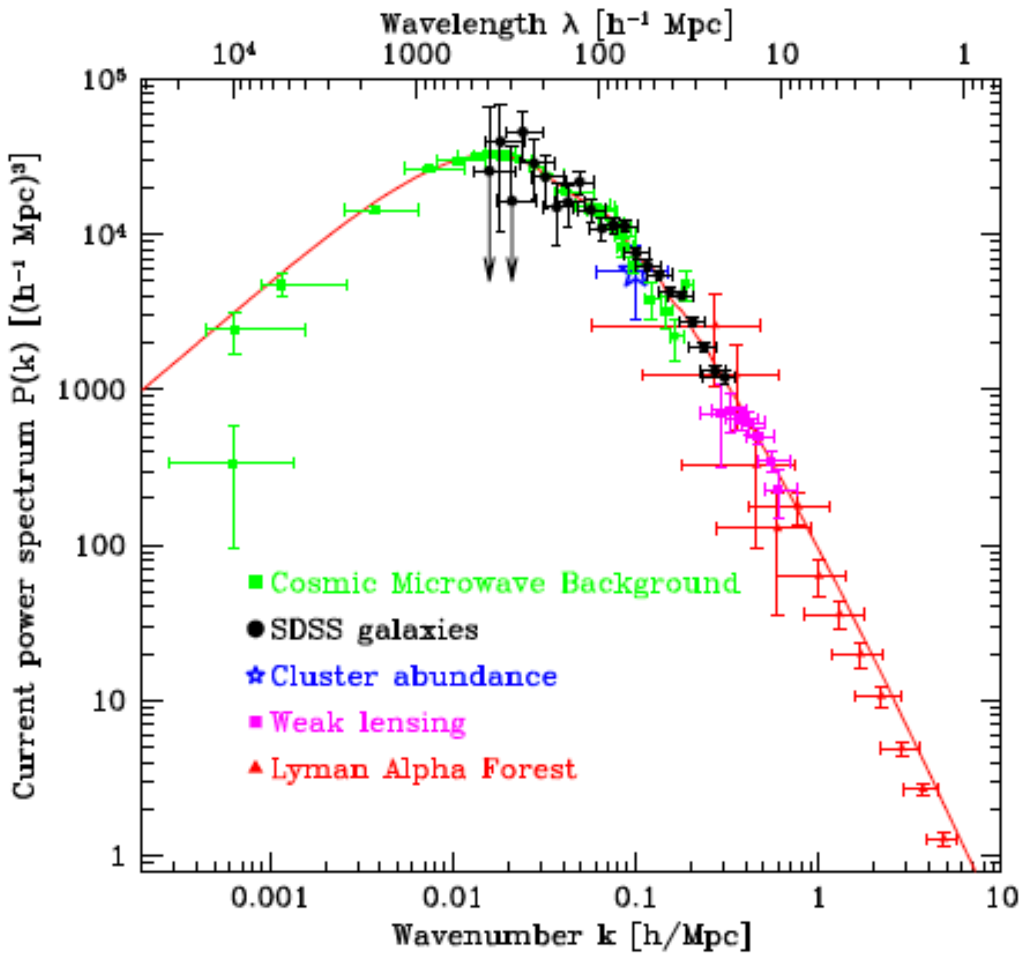
w/ Seong-Sik Kim & Bin Zhu, JHEP 10 (2021) 239 and To appear

Outline

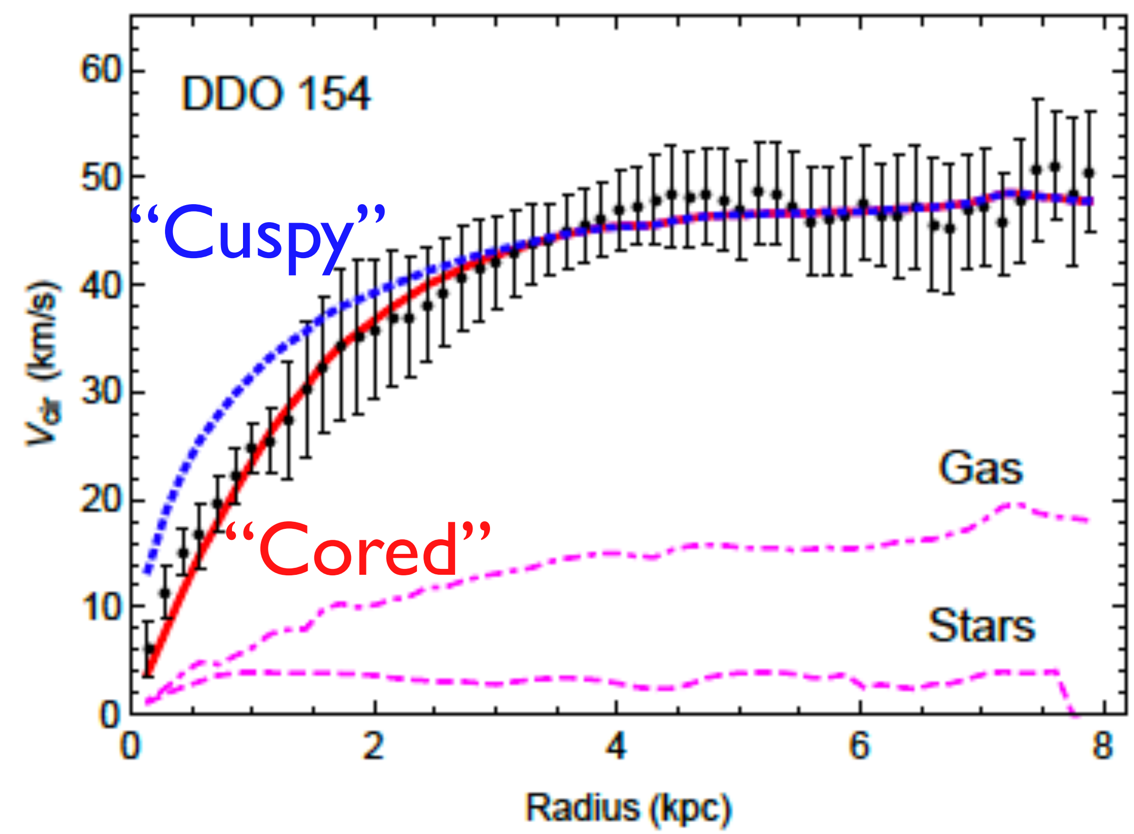
- Self-interacting dark matter
- u-channel resonances
- Models for u-channel resonances
- Conclusions

Self-interacting dark matter

Core-Cusp problem



CDM works well at >Mpc



Galaxy rotation curves at <1kpc

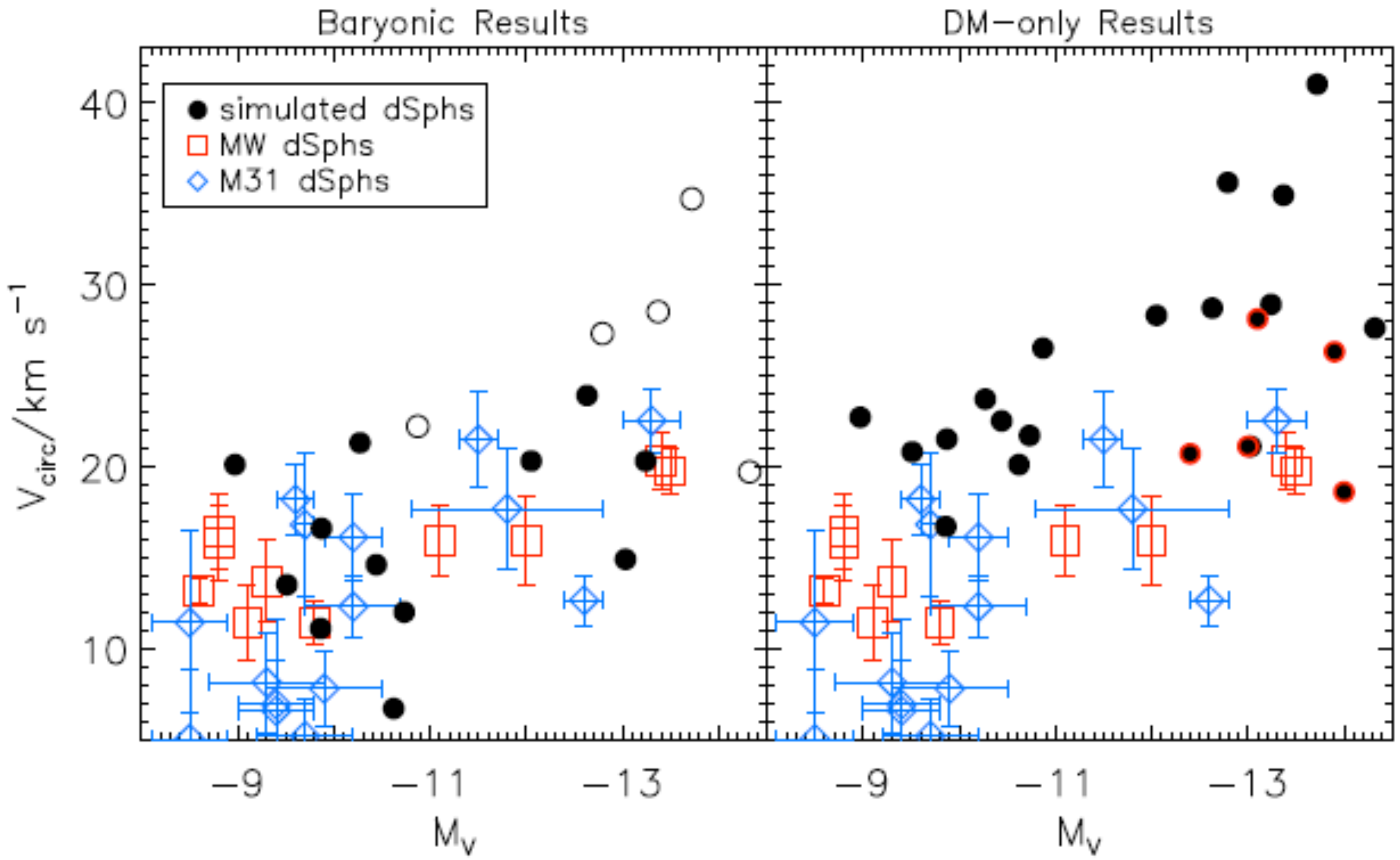
[Spergel, Steinhardt, 2000; Tulin, Yu, 2017]

- CDM N-body simulation predicts cuspy DM profile (NFW), making rotation velocities overshooting at small scales.

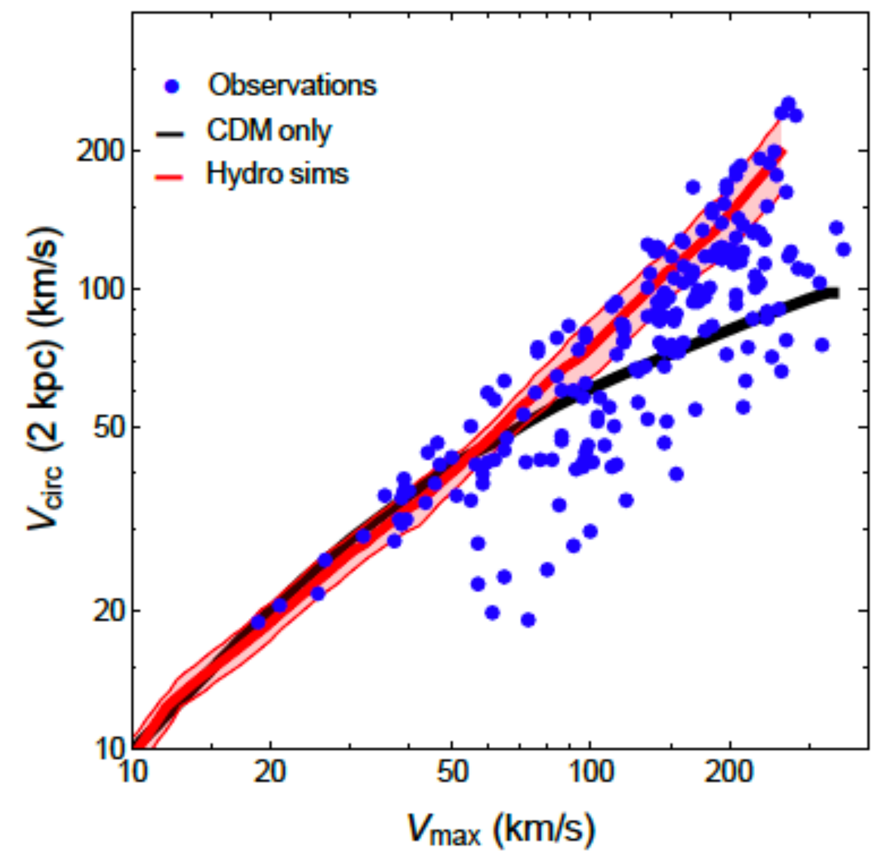
$$v_{\rm cir} \sim \sqrt{r}, \quad \rho_{\rm dm} \sim r^{-1} \quad \text{"Cuspy"}$$

$$v_{\rm cir} \sim r, \quad \rho_{\rm dm} \sim r^{0} \quad \text{"Cored"}$$

Too-big-to-fail problem



[Brooks & Zolotov, 2012]

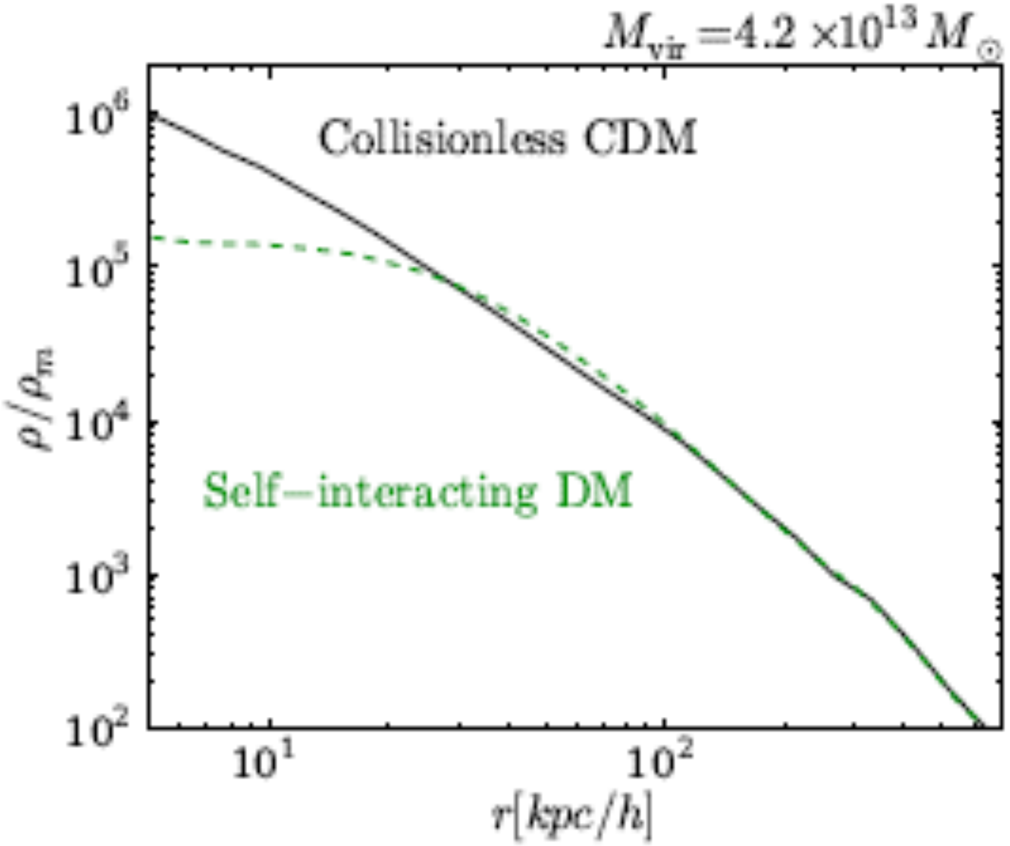


Massive subhalos are too dense to host dwarf galaxies.

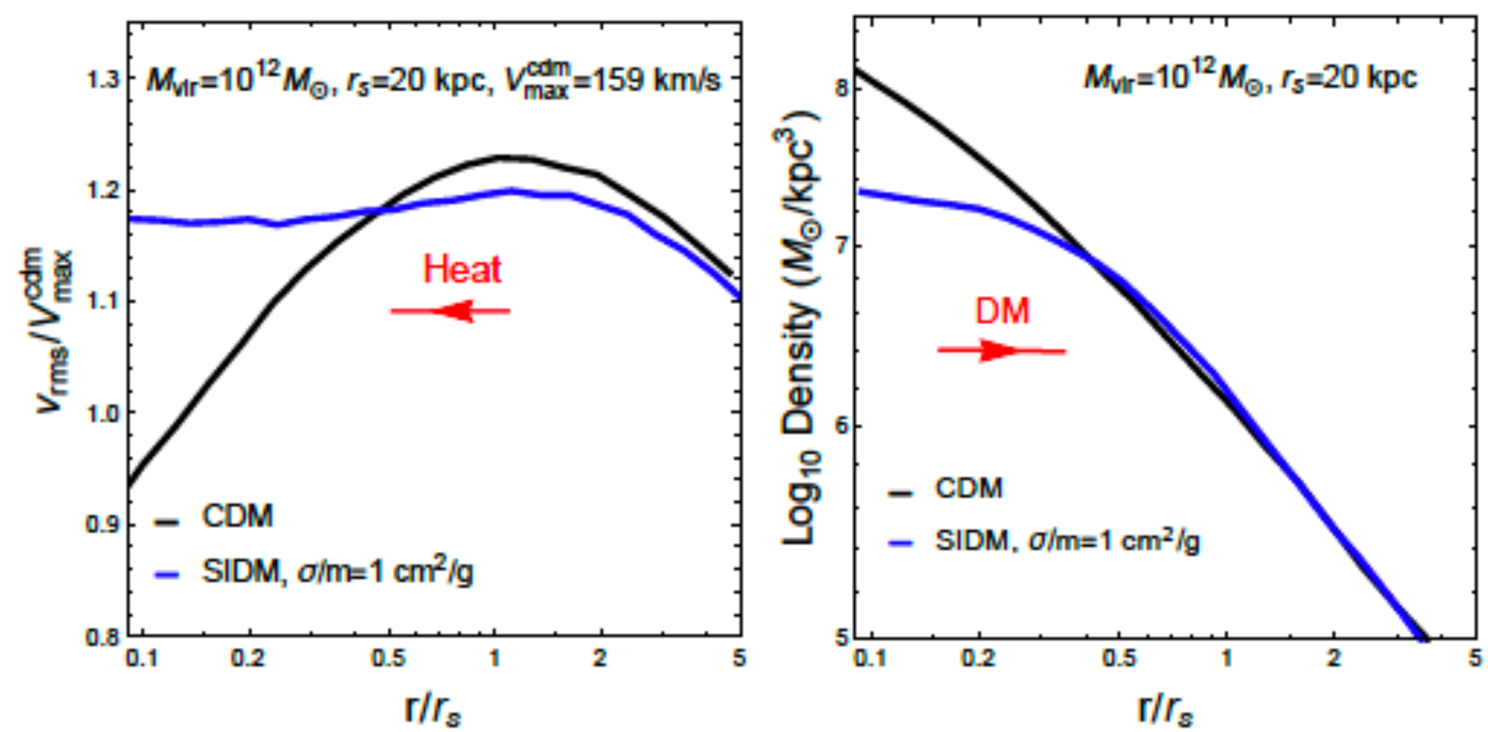
Baryonic effect (SN feedback) can make halo profile shallow.

Diversity problem: large scatter for the same maximum velocity

Self-Interacting Dark Matter -3-



[Weinberg et al, 2013]



[Tulin, Yu, 2017]

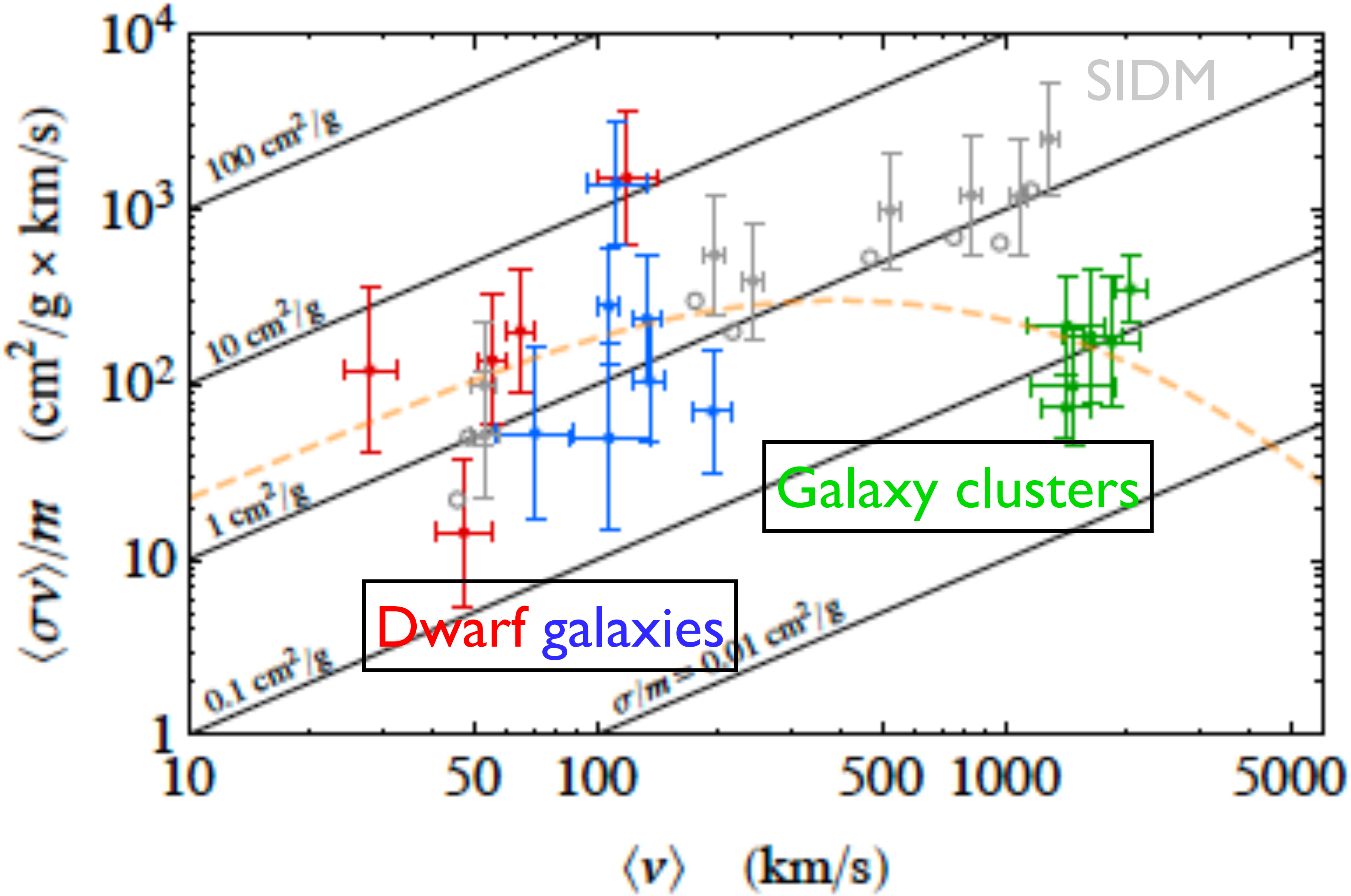
Transport heat from outside makes DM scatter and cored.

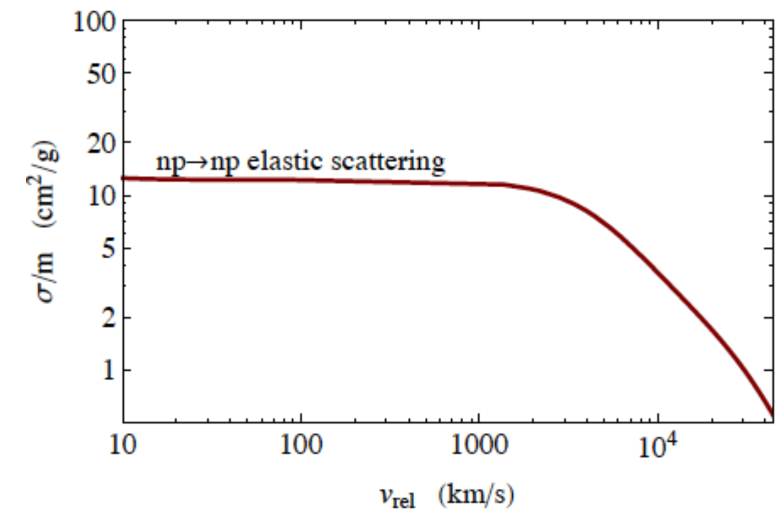
⇒ Self-Interacting cross section solves small-scale problems:

$$\frac{\sigma_{\rm self}}{m_{\rm DM}} \sim 0.1 - 10 {\rm cm}^2/{\rm g}$$

cf. Bullet cluster for DM self-scattering at clusters. $\sigma/m \lesssim 0.7\,{\rm cm}^2/{\rm g}$

Velocity-dependent SIDM





$\langle \sigma v \rangle$ =constant

$$\Rightarrow \quad \langle \sigma \rangle \propto \frac{1}{\langle v \rangle}$$

: suppressed at clusters

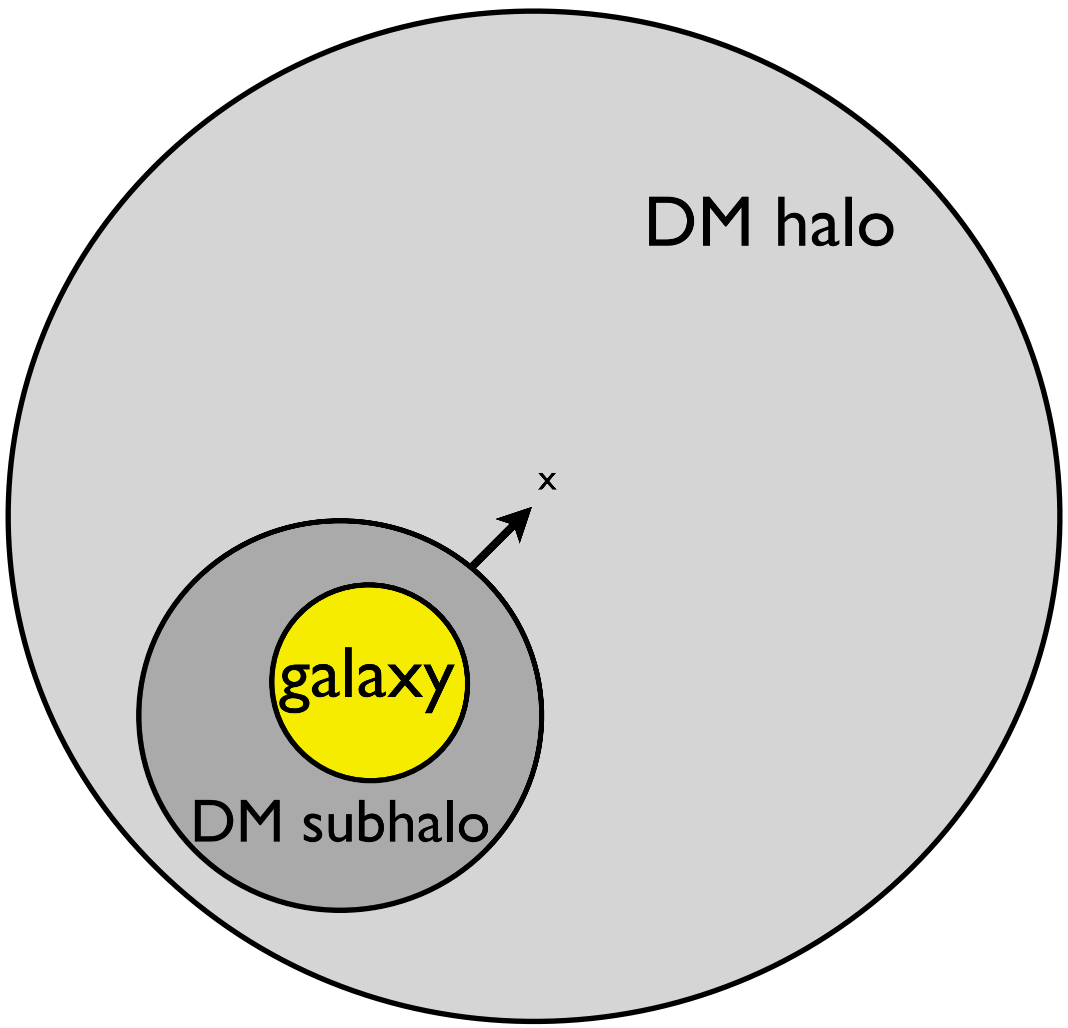
THINGS dwarf galaxies (red), LSB galaxies (blue), and clusters (green).

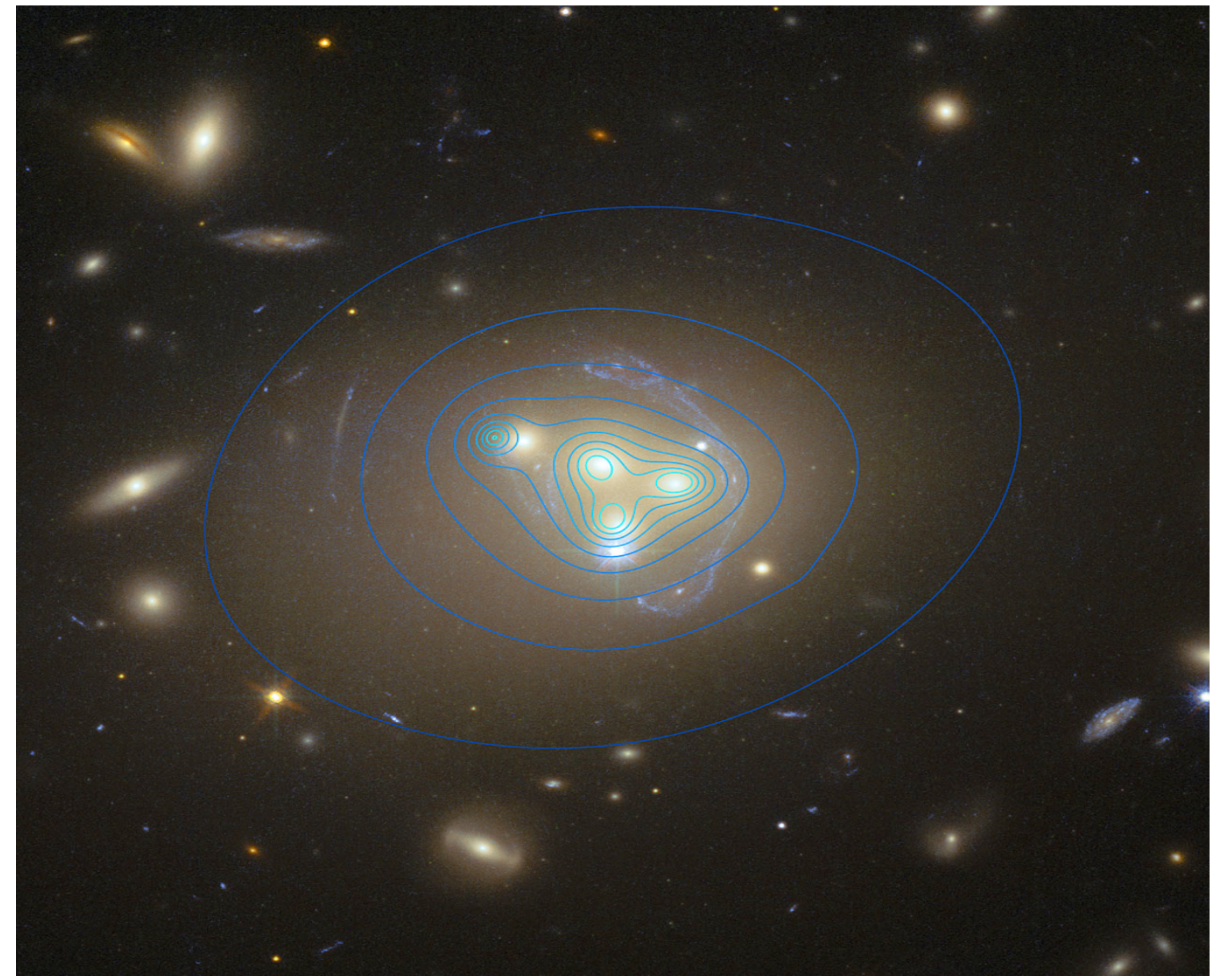
SIDM N-body simulations, with $\sigma/m = 1\ \mathrm{cm}^2/\mathrm{g}$, *(gray)*

[M. Kaplinghat et al, 2015]

- Velocity-dependence resolves tension galaxies & clusters.

SIDM: Halo separation





Friction between halos > Gravity between sub-halo and galaxy

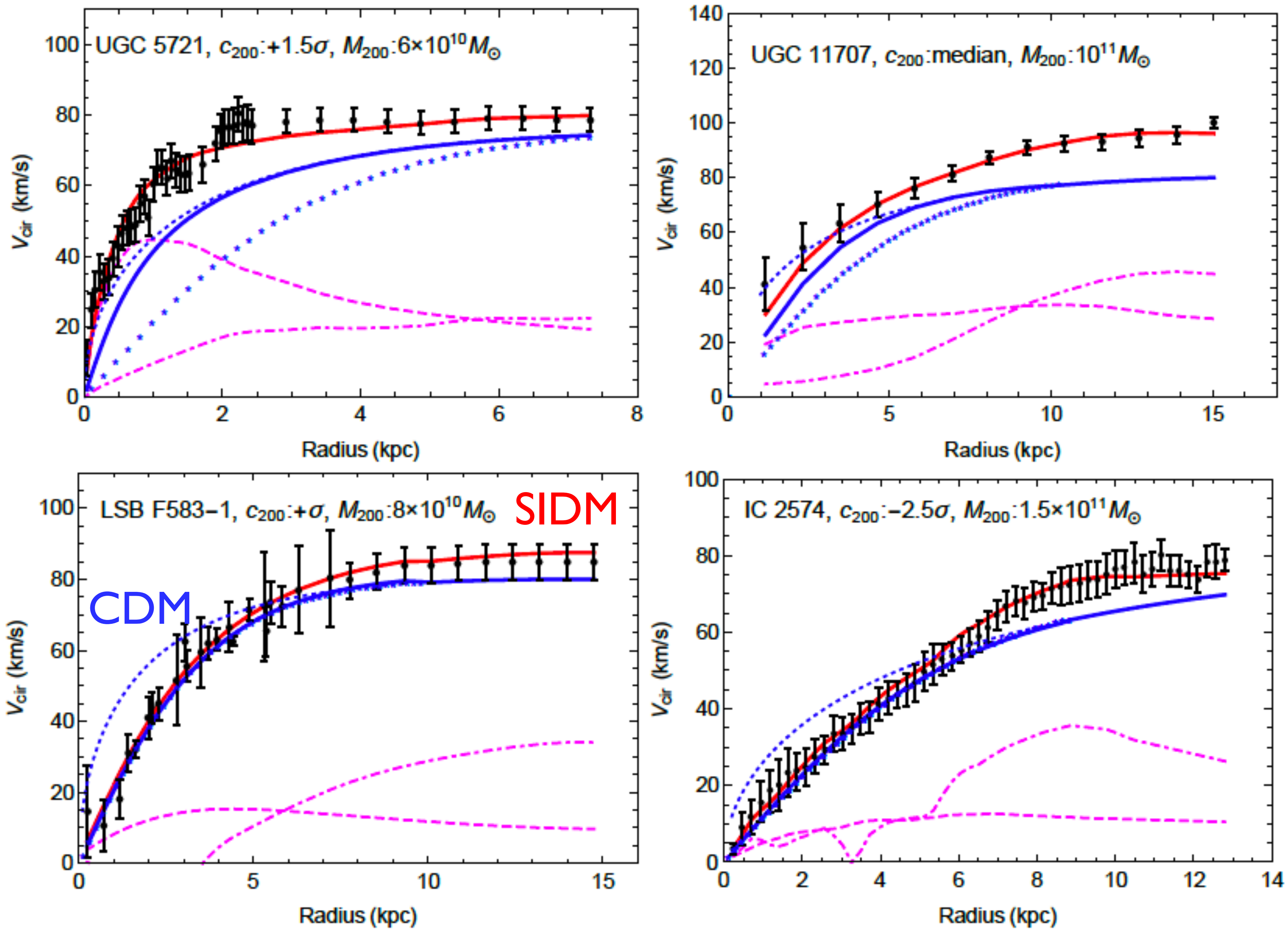
➡ splitting of sub-halo from galaxy

e.g. Abell 3827 cluster

off-set: $\Delta = 1.62^{+0.47}_{-0.49}\,\mathrm{kpc}$

[Massey et al(2015); F. Kalhoefer et al (2015)]

Diversity solved



$\sigma/m = 3\ \mathrm{cm}^2/\mathrm{g},$

[A. Kamada et al, 2016]

SIDM with baryons explains diversity of rotation curves at inner region for the same maximum velocity.

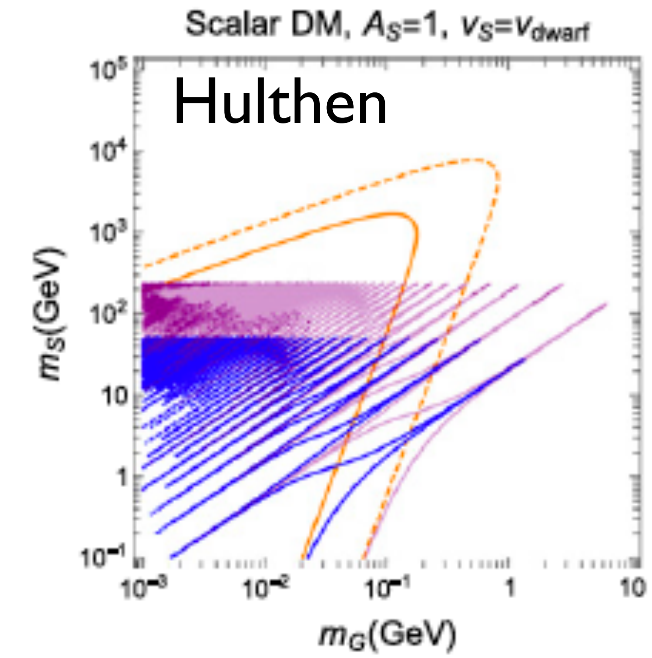
SIDM: resonances

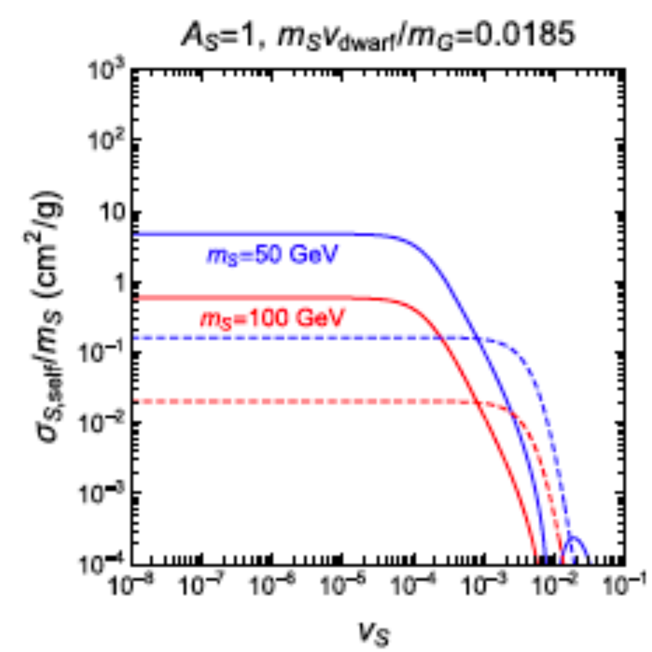
$\times e^{-m_\phi r}$

		Yukawa	spin-spin	dipole
Mediator	Interaction	$1/r$	$(\vec{s}_1 \cdot \vec{s}_2)/r$	D_{12}/r^3
Scalar	$\lambda_s \bar{\chi}\chi s$	$-\lambda_s^2$	0	0
Pseudoscalar	$i\lambda_a \bar{\chi}\gamma^5\chi a$	0	$\frac{\lambda_a^2 m_a^2}{3m_\chi^2}$	$\frac{\lambda_a^2}{m_\chi^2} h(m_a, r)$
Goldstone	$\frac{1}{f}\bar{\chi}\gamma^\mu\gamma^5\chi\partial_\mu a$	0	$\frac{4}{3}\frac{m_a^2}{f^2}$	$\frac{4}{f^2} h(m_a, r)$
Vector	$g_v \bar{\chi}\gamma^\mu\chi A_\mu$	$\pm g_v^2 \left(1 + \frac{m_A^2}{4m_\chi^2}\right)$	$\pm\frac{2g_v^2 m_A^2}{3m_\chi^2}$	$\mp\frac{g_v^2}{m_\chi^2} h(m_A, r)$
Axial vector	$g_a \bar{\chi}\gamma^\mu\gamma^5\chi A_\mu$	0	$-\frac{8g_a^2}{3}\left(1 - \frac{m_A^2}{8m_\chi^2}\right)$	$g_a^2 \left(\frac{1}{m_\chi^2} + \frac{4}{m_A^2}\right) h(m_A, r)$
Field strength	$\frac{i}{2\Lambda}\bar{\chi}\sigma^{\mu\nu}\chi F_{\mu\nu}$	0	$\mp\frac{2m_A^2}{3\Lambda^2}$	$\pm\frac{1}{\Lambda^2} h(m_A, r)$
Graviton	$-\frac{1}{\Lambda}T^{\chi,\mu\nu}G_{\mu\nu}$	$-\frac{2m_\chi^2}{3\Lambda^2}\left(1 - \frac{3}{2}\frac{m_G^2}{m_S^2}\right)$	$-\frac{m_G^2}{3\Lambda^2}$	$\frac{1}{2\Lambda^2} h(m_G, r)$
Graviton	$-\frac{1}{\Lambda}T^{S,\mu\nu}G_{\mu\nu}$	$-\frac{2}{3}\frac{m_S^2}{\Lambda^2}\left(1 - \frac{1}{2}\frac{m_G^2}{m_S^2}\right)$	0	0
Graviton	$-\frac{1}{\Lambda}T^{X,\mu\nu}G_{\mu\nu}$	$-\frac{2}{3}\frac{m_X^2}{\Lambda^2}\left(1 + \frac{1}{6}\frac{m_G^2}{m_X^2}\right)$	$-\frac{m_G^2}{3\Lambda^2}$	$-\frac{1}{2\Lambda^2} h(m_G, r)$

[Kang, HML, 2020]

$h(m, r) = 1 + mr + \frac{1}{3}(mr)^2$





Effective potential with light mediators:

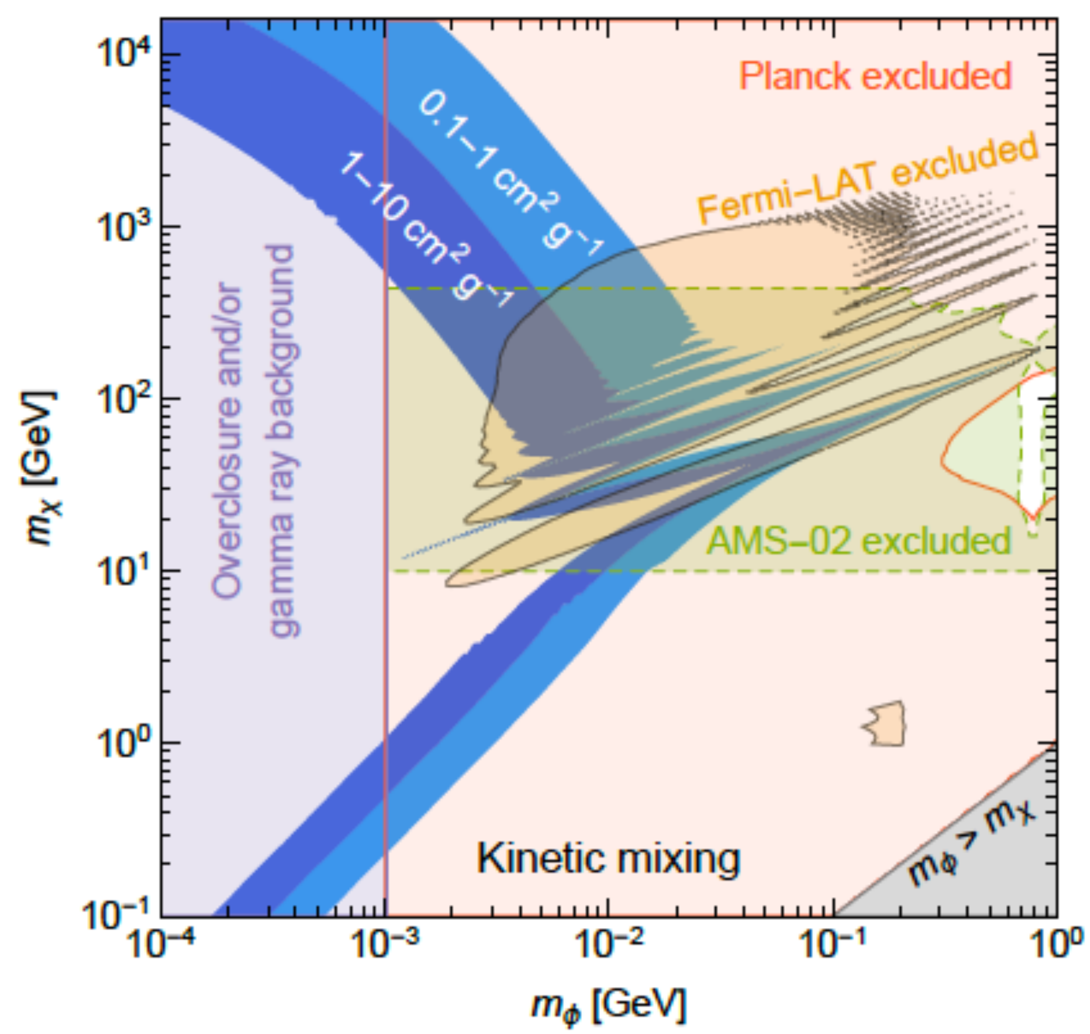
$$V_{\chi,\mathrm{eff}}(r) = -\frac{1}{4m_\chi^2}\int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} \mathcal{L}_{\chi,\mathrm{eff}}$$

$$= \frac{1}{4\pi r}\left\{c_1(r) + c_2(r)(\vec{s}_1 \cdot \vec{s}_2) + \frac{c_3(r)}{m_\chi^2 r^2}\left[3(\vec{s}_1 \cdot \hat{r})(\vec{s}_2 \cdot \hat{r}) - \vec{s}_1 \cdot \vec{s}_2\right] + \frac{c_7(r)}{m_\chi r}(\vec{s}_1 + \vec{s}_2)\cdot(\hat{r} \times \vec{v})\right\} \times e^{-m_\phi r}$$

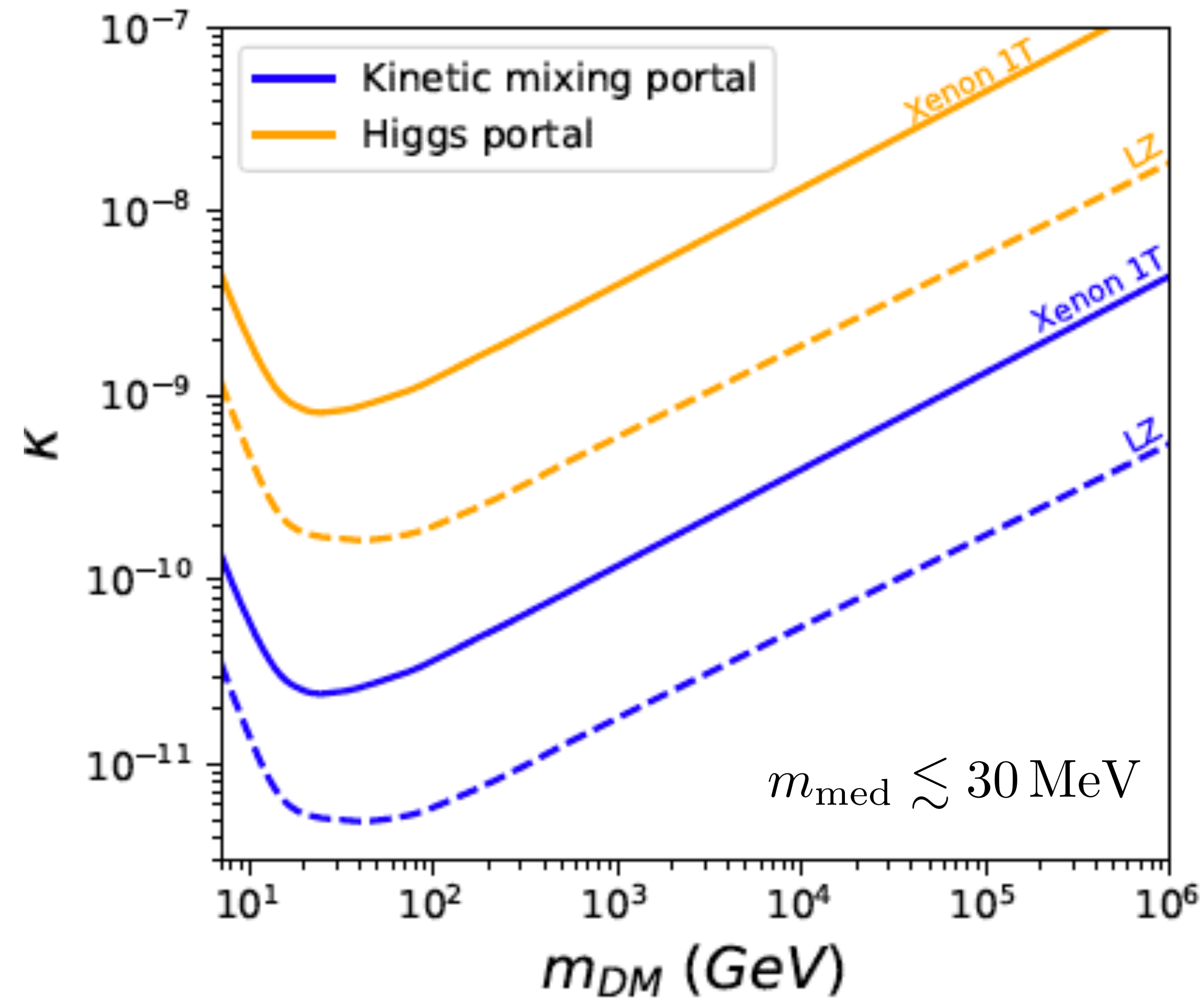
Bounds on light mediators

Light mediators enhance dark matter annihilations/detection.

→ Strong constraints from indirect and direct detections.



[Bringmann et al, 2016]



[Hambye et al, 2019]

Dark matter annihilates into neutrinos or hidden sector particles.

SIDM: sub-GeV

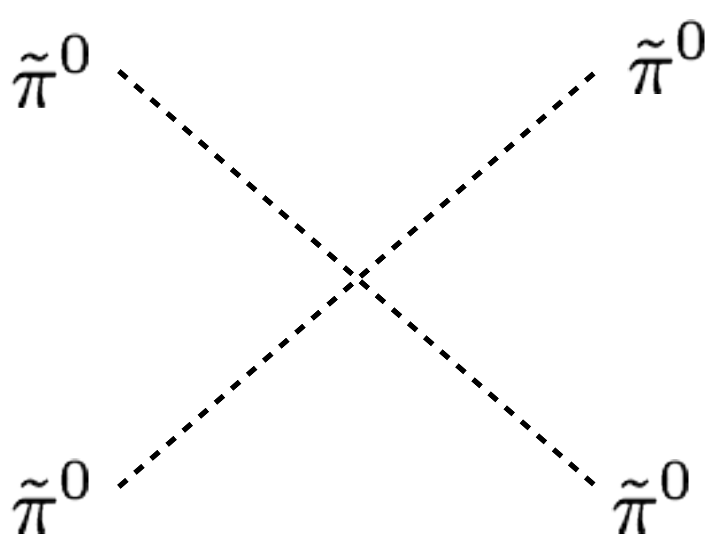
Born cross section (w/ heavy mediator):

$$\sigma_T^{\rm Born} = \frac{A_{\rm DM}^2}{2\pi m_{\rm DM}^2 v^4}\left[\ln\left(1+\frac{m_{\rm DM}^2 v^2}{m_G^2}\right) - \frac{m_{\rm DM}^2 v^2}{m_G^2 + m_{\rm DM}^2 v^2}\right] \quad , \left(V = -\frac{A_{\rm DM}}{4\pi}\frac{e^{-m_G r}}{r}\right)$$

sub-GeV DM mass => large & velocity-dependent at perturbative level.

DM relic freeze-out with 2→2 (forbidden) annihilation: CMB safe

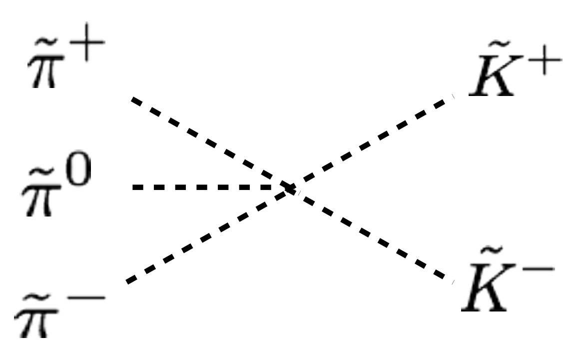
Contact interactions: e.g. Dark mesons in dark QCD



[Hochberg et al, 2014; M.-S. Seo, HML, 2015]

$$\mathcal{L}_0 = {\rm Tr}(\partial_\mu \pi \partial^\mu \pi) + \frac{1}{3f^2}{\rm Tr}[\partial_\mu \pi, \pi]^2 + \cdots$$

$$\sigma_{\rm self} = \frac{m_\pi^2}{32\pi f^4}\frac{a^2}{N_\pi^2}, \quad a^2 \sim N_\pi^4 \longrightarrow m_\pi \lesssim 1\,{\rm GeV}$$



Wess-Zumino-Witten term:

DM relic by self-interactions

$$\frac{m_\pi}{f} \sim 4, \;\; m_\pi \sim 300\,{\rm MeV}$$

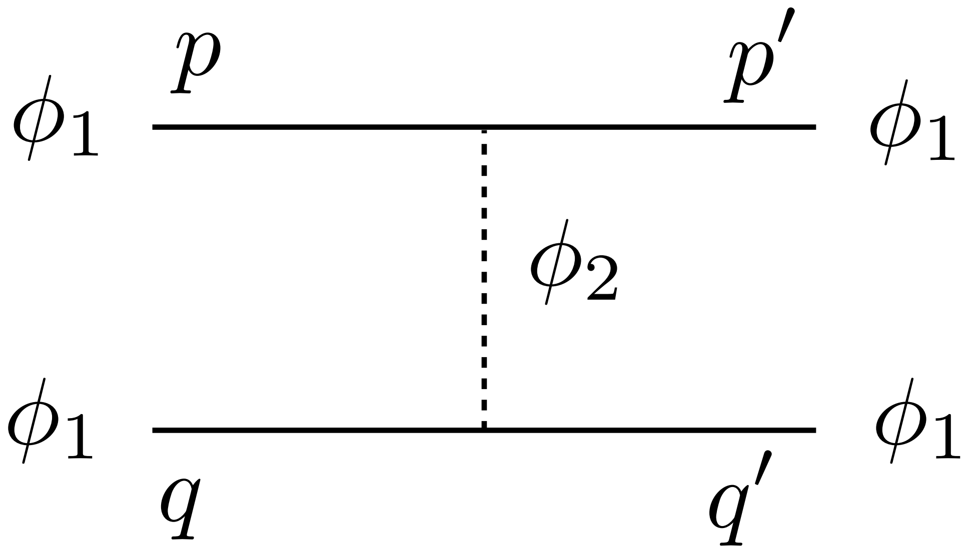
u-channel resonances

t-channel resonance

- Consider dark matter and mediator with masses, m_1 & m_2.

Triple coupling for dark matter: $\mathcal{L}_{\rm int} = -2g\, m_1 \phi_2 |\phi_1|^2$

Elastic 2→2 scattering in t-channel



$$\tilde{\Gamma}_t(p,q;p',q') = \frac{4g^2 m_1^2}{|\vec{p}-\vec{p}'|^2 + m_2^2 - \omega^2}$$

$w = p_0 - p_0' \simeq 0:$ "Instantaneous" interaction

⟶ Need of a small mass m_2 for enhanced t-channel!

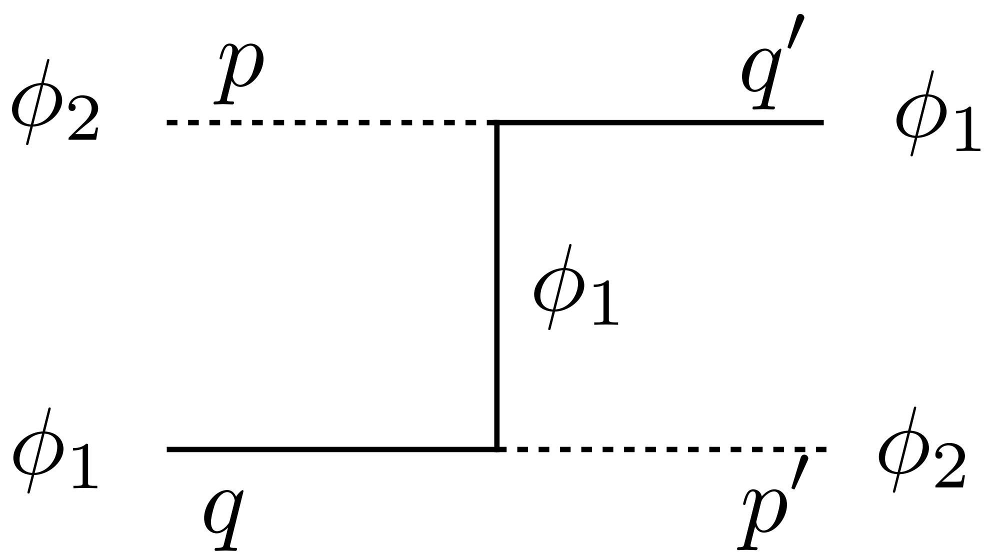
Resummation of ladder diagrams: Sommerfeld factor

u-channel resonance

- Take two component dark matter with masses, m_1 & m_2

Triple coupling between dark matter: $\mathcal{L}_{\rm int} = -2g\, m_1 \phi_2 |\phi_1|^2$

→ Elastic 2→2 scattering in u-channel



$$\tilde{\Gamma}_u(p,q;p',q') = \frac{4g^2 m_1^2}{|\vec{p}-\vec{q}'|^2 + m_1^2 - \omega^2}$$

[S. Kim, HML, B. Zhu, 2021]

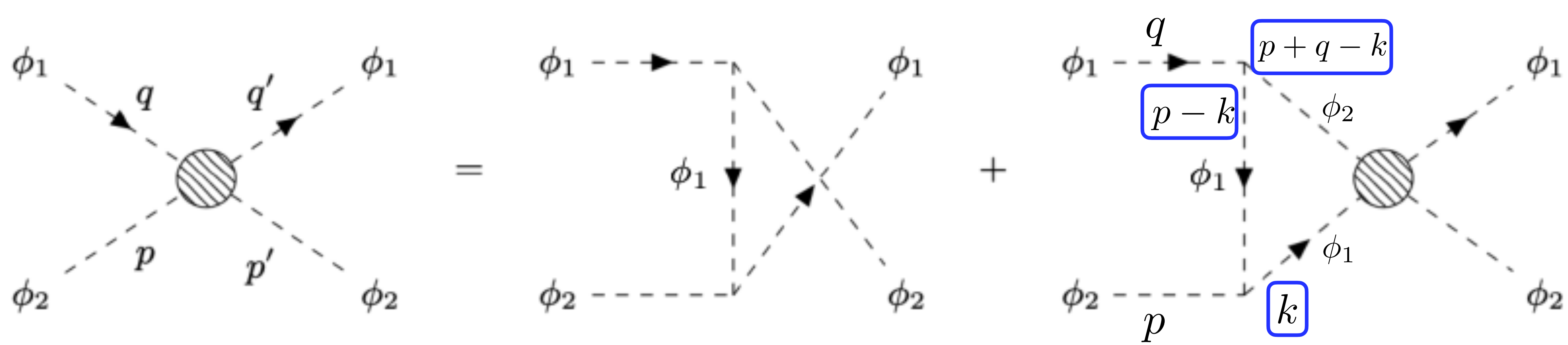
$\omega = p_0 - q_0' \approx m_2 - m_1 \neq 0$ "Delayed" interaction

$m_2 = 2m_1$ ⟶ "Effectively massless" off-shell DM

Resummation of ladder diagrams is also needed.

Non-perturbative scattering

Non-perturbative scattering in u-channel



$$i\Gamma(p,q;p',q') = i\tilde{\Gamma}(p,q;p',q') - \int \frac{d^4k}{(2\pi)^4}\tilde{\Gamma}(p,q;p+q-k,k)G_1(k)G_2(p+q-k)\Gamma(p+q-k,k;p',q')$$

Bethe-Salpeter wave function: (Tree-level contribution ignored)

$$\chi(p,q;p',q') \equiv G_2(p)G_1(q)\Gamma(p,q;p',q') \equiv \chi(p,q)$$

$\Rightarrow$ $$i\chi(p,q) = -G_2(p)G_1(q)\int \frac{d^4k}{(2\pi)^4}\tilde{\Gamma}(p,q;p+q-k,k)\,\chi(p+q-k,k)$$

Bethe-Salpeter equation

Change of variables:

$$P = \frac{1}{2}(p+q), \quad Q = \mu\Big(\frac{p}{m_2} - \frac{q}{m_1}\Big) \longrightarrow \begin{cases} \chi(p,q) = \widetilde{\chi}(P,Q) \\ \chi(p+q-k,k) = \widetilde{\chi}\Big(P, \frac{2\mu}{m_2}P - k\Big) \end{cases}$$

$$i\widetilde{\chi}(P,Q) = -G_2\Big(Q + \frac{2\mu}{m_1}P\Big)G_1\Big(-Q + \frac{2\mu}{m_2}P\Big)\int \frac{d^4k'}{(2\pi)^4}\widetilde{\Gamma}(p,q;p+q-k,k)\,\widetilde{\chi}(P,k')$$

k' : shifted loop momentum

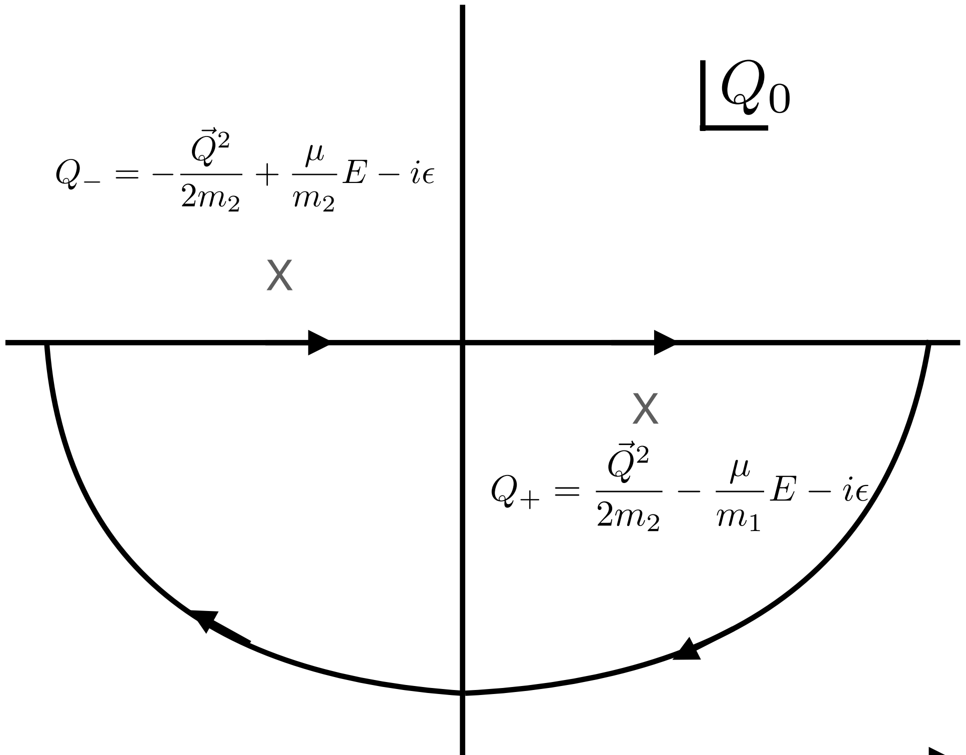
Tree-level amplitude:

$$\widetilde{\Gamma}(p,q;p+q-k,k) = \frac{4g^2m_1^2}{\Big(\sqrt{\frac{m_1}{m_2}}\vec{Q} + \sqrt{\frac{m_2}{m_1}}\vec{k}'\Big)^2 + m_2(2m_1 - m_2)} \equiv U$$

BS w.f. in momentum space: $\widetilde{\psi}_{BS}(\vec{Q}) = \int \frac{dQ_0}{2\pi}\widetilde{\chi}(P,Q)$

$$\longrightarrow \quad i\widetilde{\psi}_{BS}(\vec{Q}) = -\int \frac{dQ_0}{2\pi}G_2\Big(Q + \frac{2\mu}{m_1}P\Big)G_1\Big(-Q + \frac{2\mu}{m_2}P\Big)\int \frac{d^3k'}{(2\pi)^3}U\ \widetilde{\psi}_{BS}(\vec{k}')$$

Bethe-Salpeter equation



$$P = \tfrac{1}{2}(m_1 + m_2 + E, 0)\ ,\ Q = (Q_0, \vec{Q}):$$

$$G_2 = \frac{i}{(Q + \frac{2\mu}{m_1}P)^2 - m_2^2 + i\epsilon} \simeq \frac{i}{2m_2(Q_0 - Q_+)},$$

$$G_1 = \frac{i}{(-Q + \frac{2\mu}{m_2}P)^2 - m_1^2 + i\epsilon} \simeq \frac{i}{2m_1(Q_0 - Q_-)}$$

$$\longrightarrow \int \frac{dQ_0}{2\pi} G_2\Big(Q + \frac{2\mu}{m_1}P\Big) G_1\Big(-Q + \frac{2\mu}{m_2}P\Big) = \frac{i}{4m_1 m_2\Big(E - \frac{\vec{Q}^2}{2\mu}\Big)}$$

Bethe-Salpeter equation:

$$\left(\frac{\vec{Q}^2}{2\mu} - E\right)\widetilde{\psi}_{BS}(\vec{Q}) = -\frac{1}{4m_1 m_2}\int \frac{d^3k'}{(2\pi)^3} U\ \widetilde{\psi}_{BS}(\vec{k}')$$

[S. Kim, HML, B. Zhu, 2021]

or

$$\boxed{\left(-\frac{1}{2\mu}\nabla^2 - E\right)\psi_{BS}(\vec{x}) = -V(\vec{x})\psi_{BS}\Big(-\frac{m_2}{m_1}\vec{x}\Big)}$$

$$\begin{cases} V(\vec{x}) = -\dfrac{\alpha}{r}e^{-Mr} \\ \alpha \equiv \frac{g^2}{4\pi}, \quad M \equiv m_2\sqrt{2 - \dfrac{m_2}{m_1}} \end{cases}$$

Difference from t-channel

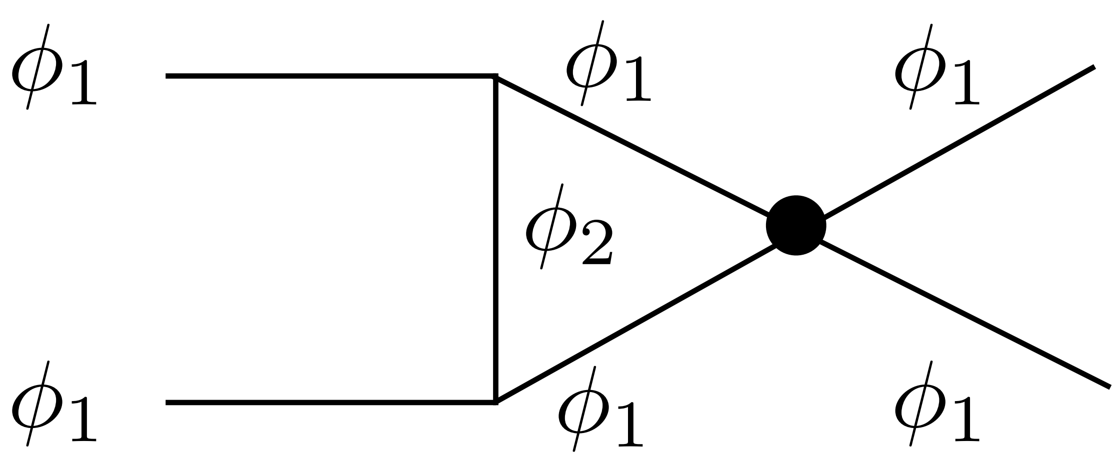
t-channel:

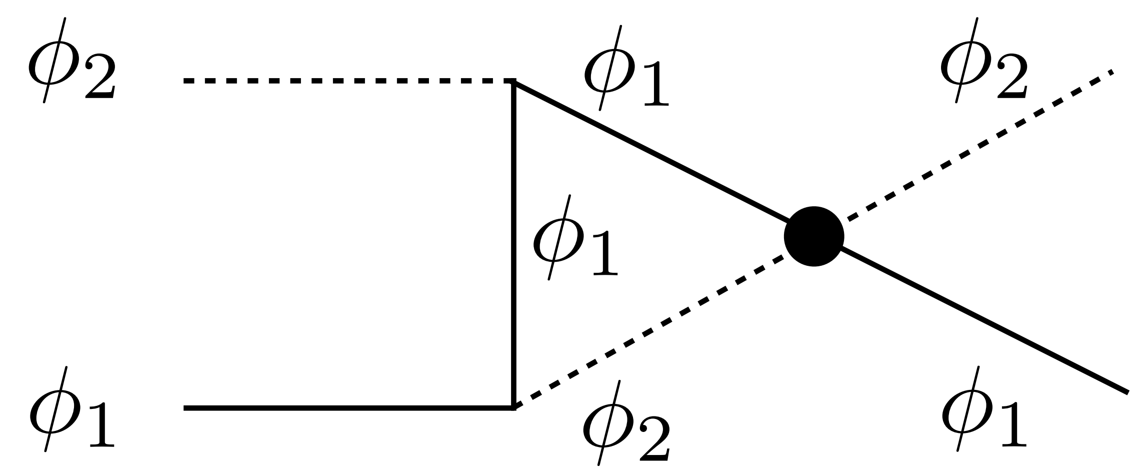
$$\tilde{\Gamma}_t = \frac{4g^2 m_1^2}{|\vec{p} - \vec{p}'|^2 + m_2^2}$$

u-channel:

$$\tilde{\Gamma}_u = \frac{4g^2 m_1^2}{\left|\sqrt{\frac{m_1}{m_2}}\vec{p} - \sqrt{\frac{m_2}{m_1}}\vec{q}'\right|^2 + m_2(2m_1 - m_2)}$$

Non-perturbative part:





$$\chi(1, 1') \sim \tilde{\Gamma}_t \chi(1, 1')$$

$$\chi(1, 2) \sim \tilde{\Gamma}_u \chi(2, 1)$$

$$\left(\frac{\nabla^2}{2\mu} + E\right)\psi(\vec{x}) = V(\vec{x})\psi(\vec{x})$$

$$\left(\frac{\nabla^2}{2\mu} + E\right)\psi(\vec{x}) = V(\vec{x})\psi\left(-\frac{m_2}{m_1}\vec{x}\right)$$

Wave-function flips sign!

Delay diff. eq.

BS w.f. in spherical coordinates:

$$\psi_{\rm BS}(\vec{x}) = R_l(r) Y_l^m(\theta,\phi) \longrightarrow \psi_{\rm BS}\Big(-\frac{m_2}{m_1}\vec{x}\Big) = \boxed{(-1)^l} R_l\Big(\frac{m_2}{m_1} r\Big) Y_l^m(\theta,\phi)$$

Radial equation : $R_l(x) = u_l(x)/x$, $\quad a = \frac{2v_{\rm rel}}{\alpha}$, $b = \frac{m_2}{m_1}$ and $c = \frac{2M}{\mu\alpha}$

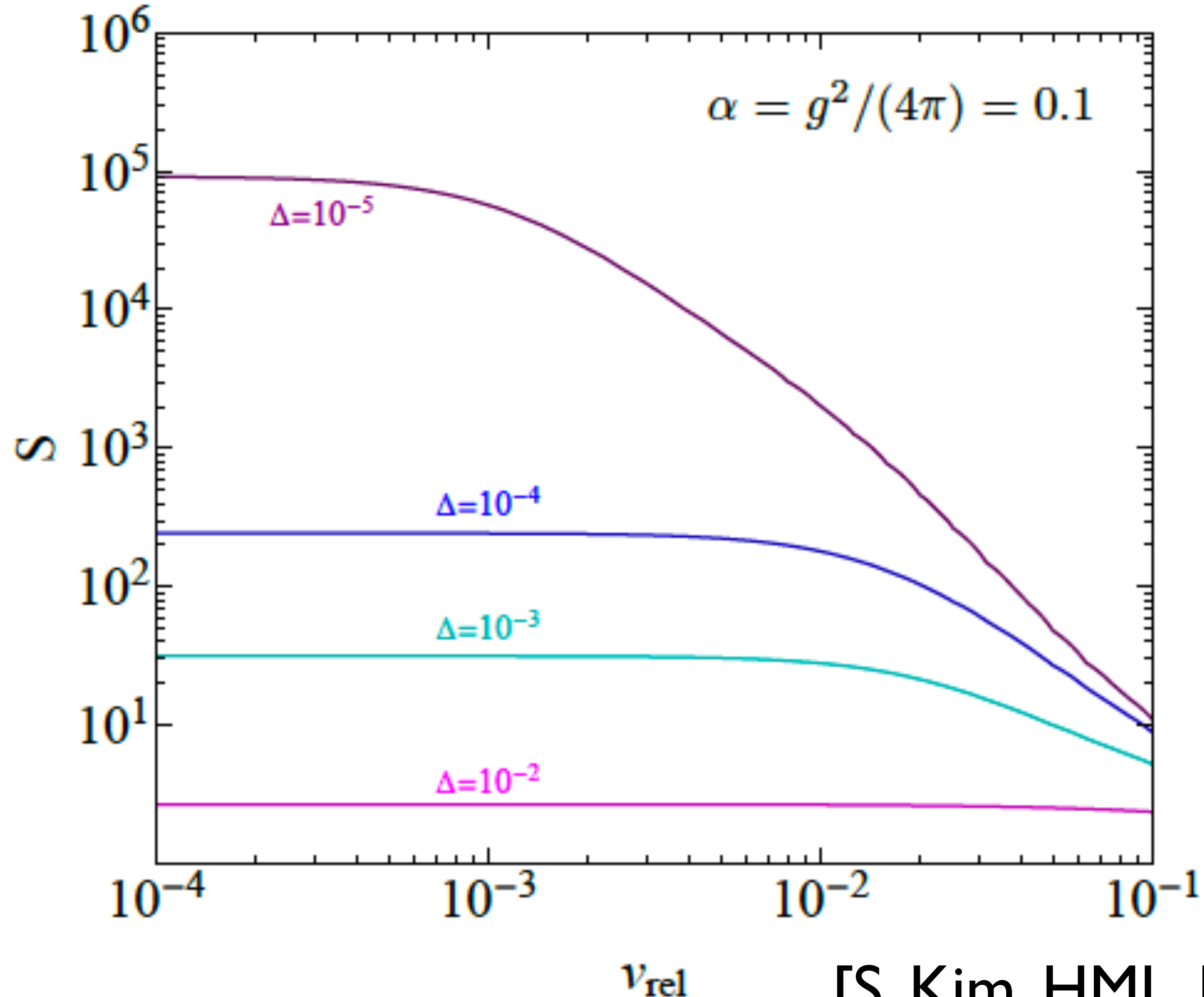
$$\Rightarrow \quad \left(\frac{d^2}{dx^2} - \frac{l(l+1)}{x^2}\right) u_l(x) + \frac{4e^{-cx}}{b\,x}(-1)^l\, u_l(bx) + a^2\, u_l(x) = 0$$

- Attractive for l=even ; repulsive for l=odd.
- Effective mediator mass: $M \equiv m_2\sqrt{2 - \frac{m_2}{m_1}} \to 0, \quad m_2 \to 2m_1.$
- Delay differential eq:

$$x = e^{-\rho} \longrightarrow \tilde{u}_0''(\rho) + \tilde{u}_0'(\rho) + 2e^{-\rho}\,\tilde{u}_0(\underline{\rho - \ln 2}) + a^2 e^{-2\rho}\,\tilde{u}_0(\rho) = 0$$

delay term

Sommerfeld factor



[S. Kim, HML, B. Zhu, 2021]

Sommerfeld factor (s-wave):

$$S = \frac{|\psi_{\rm BS}(0)|^2}{|\psi_{\rm pert}(0)|^2} = A^2$$

Effective mediator mass

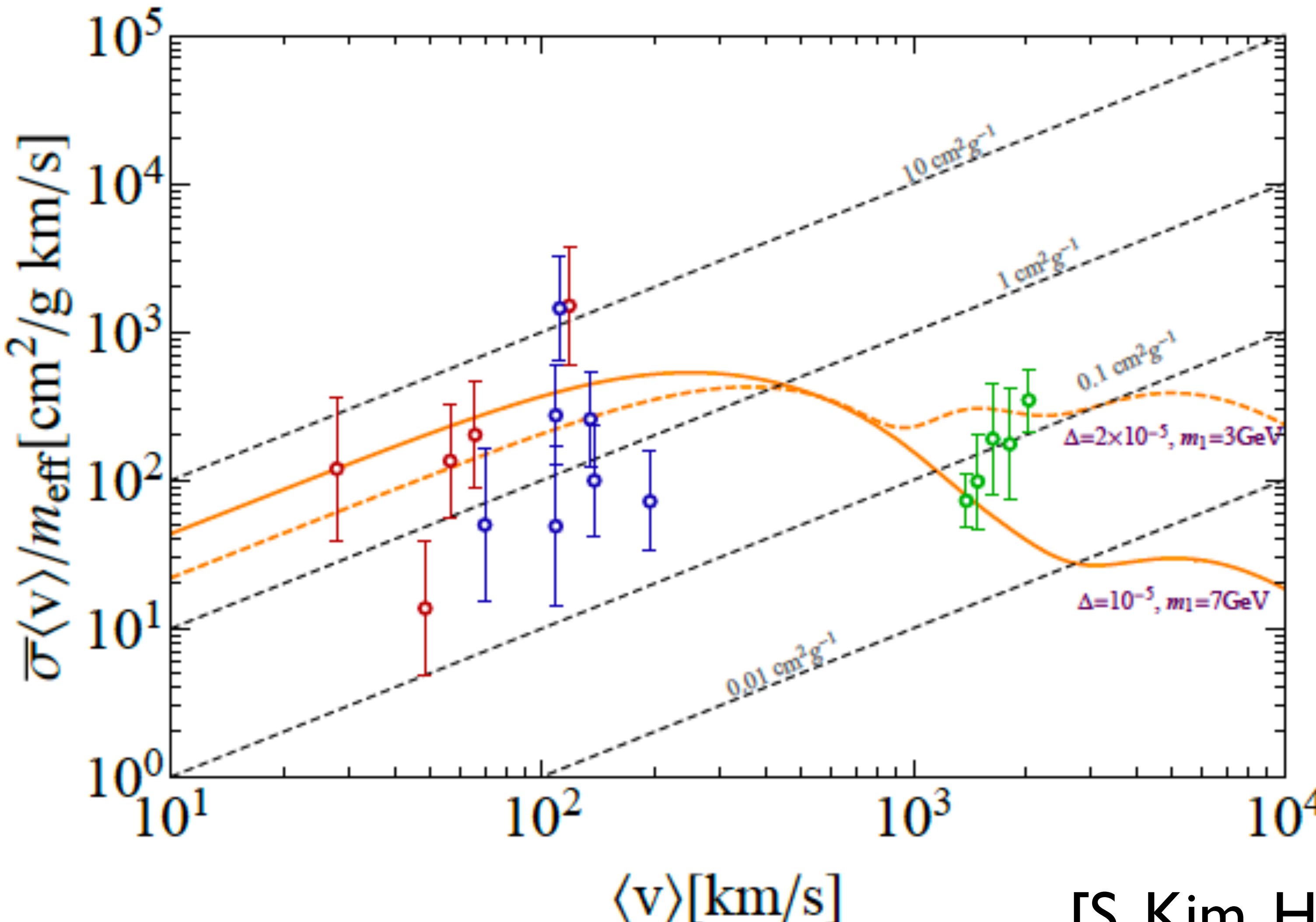
$$\longleftrightarrow \quad \Delta = 1 - \frac{m_2}{2m_1} \geq 0$$

Boundary conditions (s-wave):

$$\tilde{u}_0(\rho) \longrightarrow \frac{1}{a}\sin(a\,e^{-\rho} + \delta_0), \qquad \rho \to -\infty, \qquad \text{"plane-wave"}$$

$$\tilde{u}_0(\rho) \longrightarrow A\,e^{-\rho}, \qquad \rho \to +\infty \qquad \text{"constant R"}$$

u-channel self-scattering



$\alpha = g^2/(4\pi) = 0.1$

[S. Kim, HML, B. Zhu, 2021]

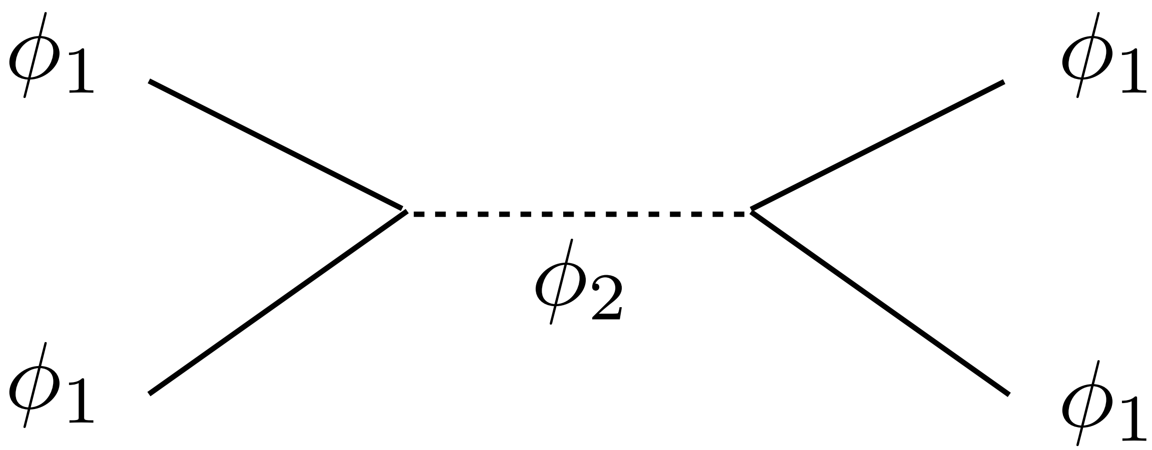
$\phi_1\phi_2 \to \phi_1\phi_2$: $\sigma = \frac{4\pi}{k^2}\sin^2\delta_0$, total cross section

$\bar{\sigma} = \langle\sigma v_{\rm rel}^3\rangle/(24/\sqrt{\pi}v_0^3)$: energy-transfer average

⇨ Self-scattering for SRDM is velocity-dependent.

Other channels for SIDM

s-channel

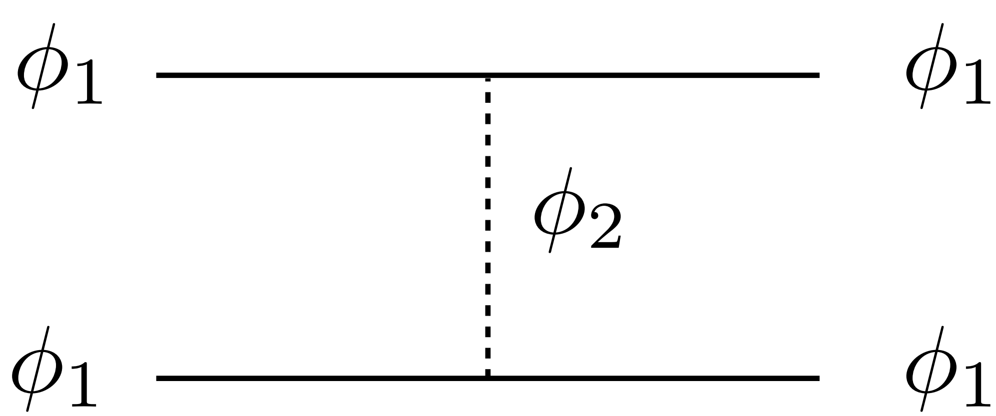


$$\sim \frac{1}{(m_1^2(4+v_{\rm rel}^2)-m_2^2)^2+\Gamma_2^2 m_2^2}$$

u-channel mass: $M \equiv m_2\sqrt{2-\frac{m_2}{m_1}}$ $\longrightarrow$ $m_2 < 2m_1$

off resonance

t-channel



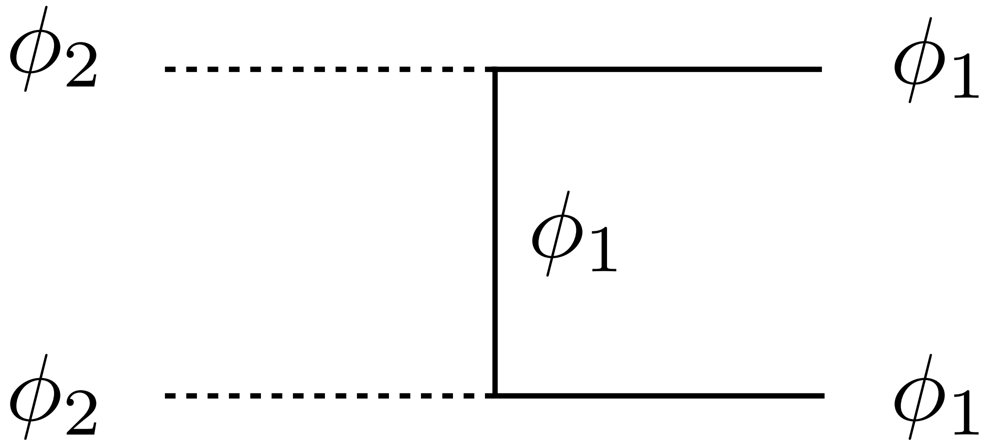
$$m_2 \approx 2m_1$$

No enhancement

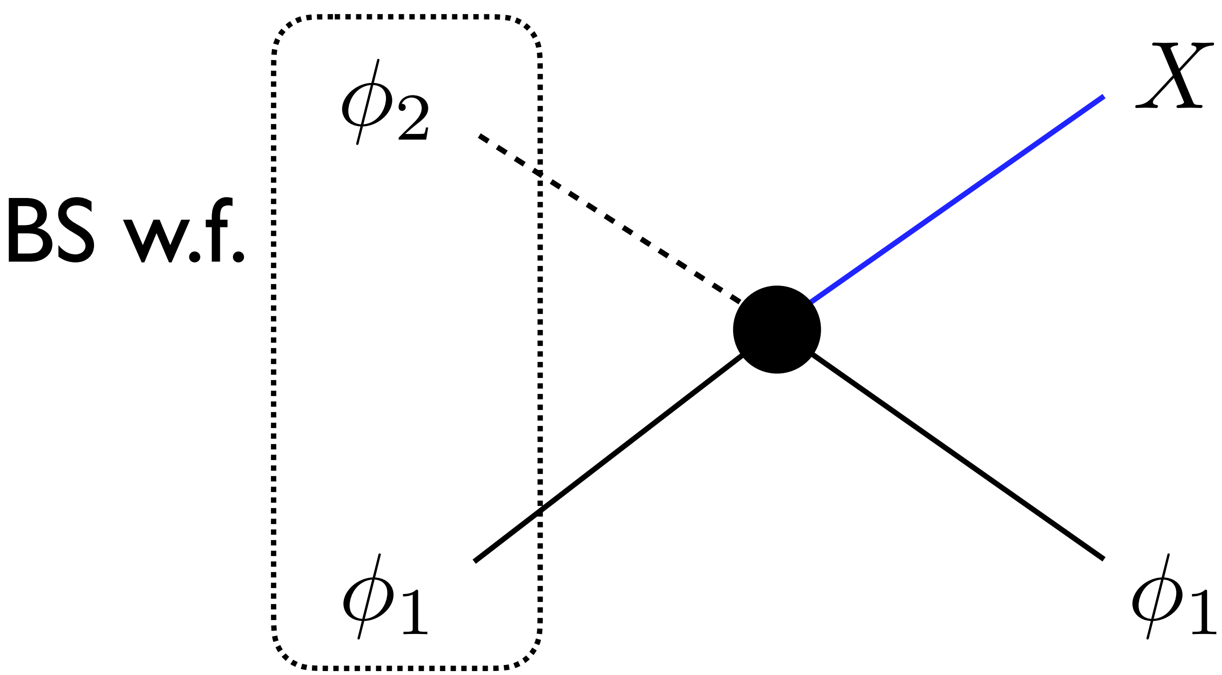
Models for u-channel resonances

DM annihilation

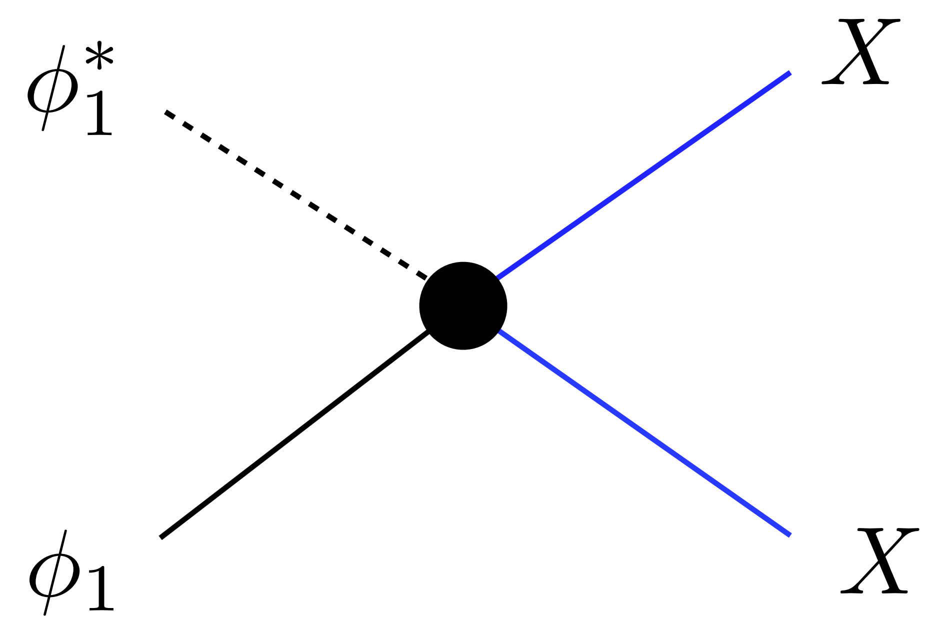
2→2 annihilation



No enhancement in the initial state



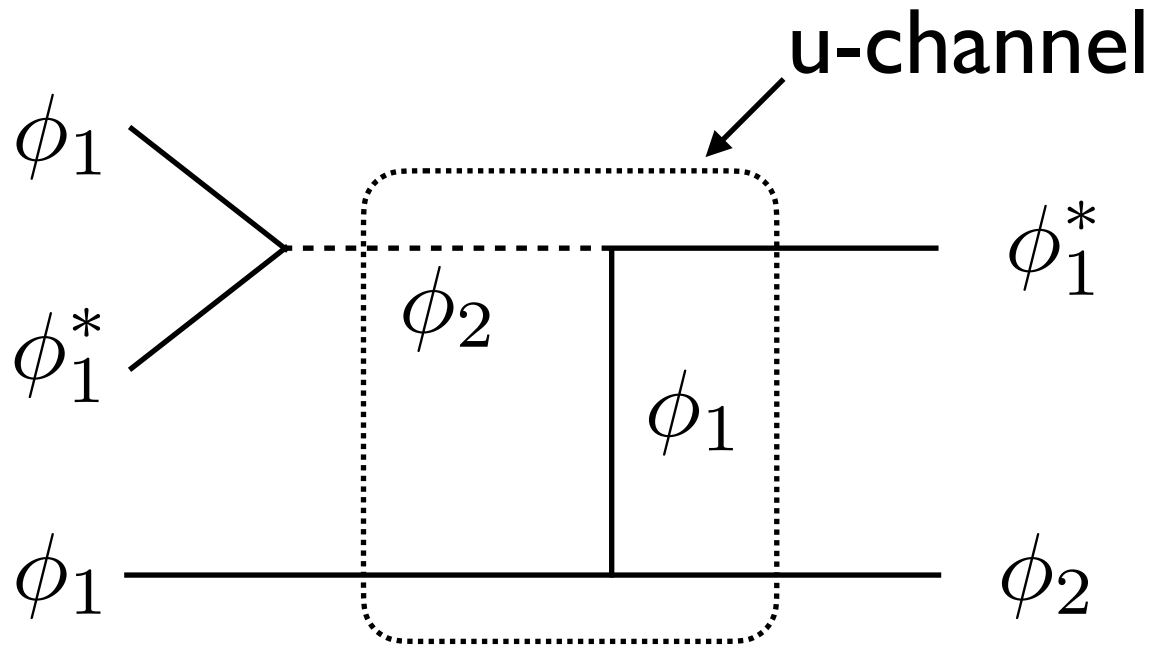
Sommerfeld enhancement



X : Extra mediator for ϕ_1

DM annihilation

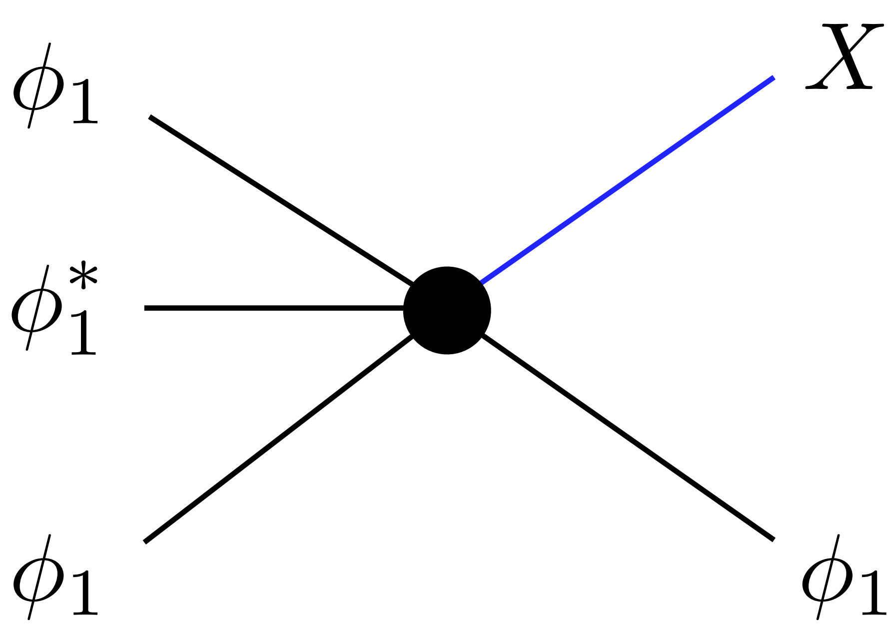
3→2 annihilation



$$\sim \frac{1}{(4m_1^2 - m_2^2)^4}$$

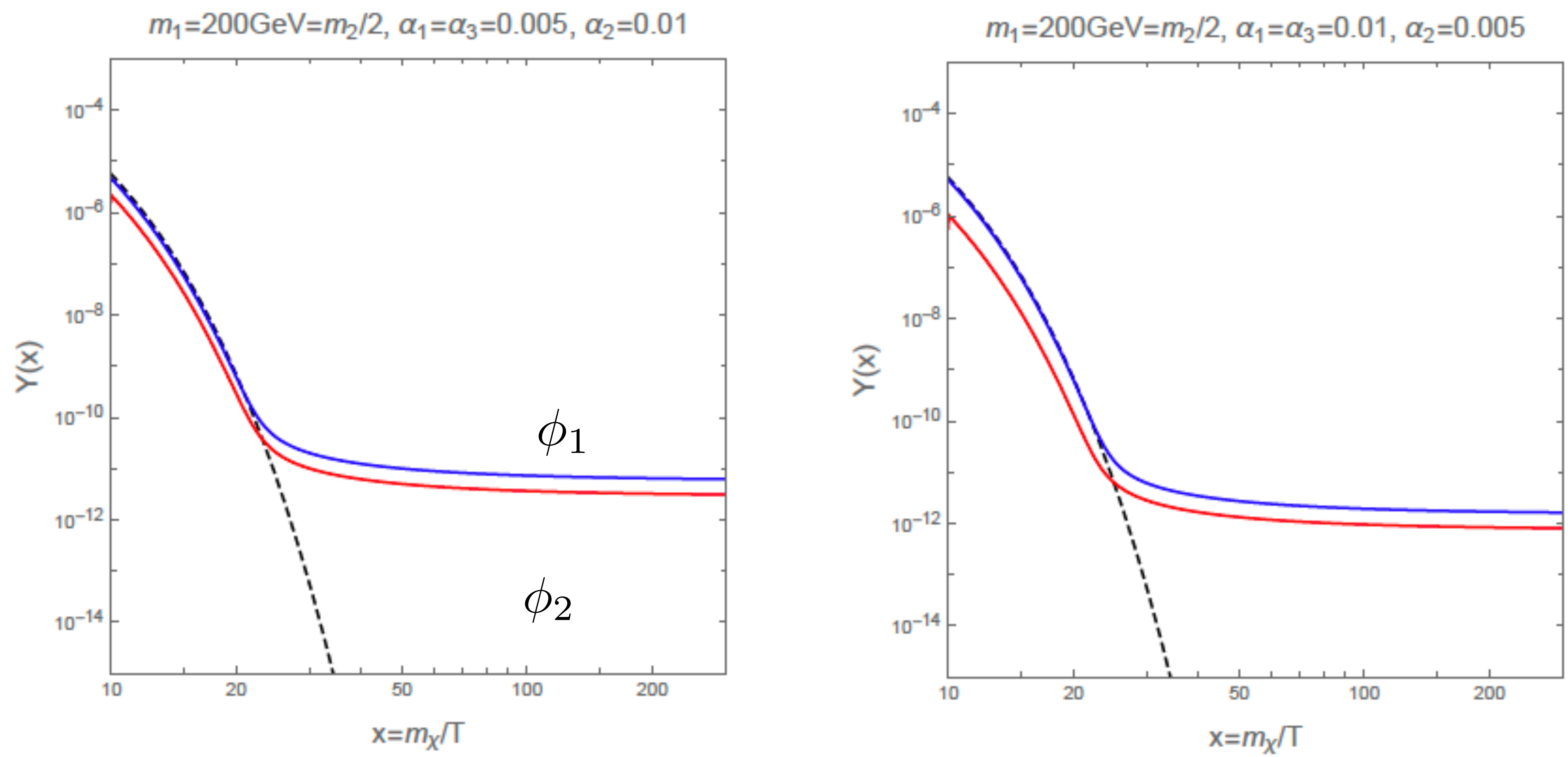
s-, u-channel enhanced.

But, no enhancement in the initial state.



No enhancement in the initial state.

DM relic density



[S. Kim, HML, B. Zhu, To appear]

$$\langle\sigma v\rangle_{\phi_1\phi_1^*\to XX} = \frac{2\alpha_1^2}{m_1^2}, \qquad \langle\sigma v\rangle_{\phi_2\phi_2\to\phi_1\phi_1^*} = \frac{2\alpha_2^2}{m_1^2}, \qquad \langle\sigma v\rangle_{\phi_1\phi_2\to\phi_1 X} = \frac{2\alpha_3^2}{m_1^2}$$

Two-component dark matter can be equally abundant.

EFT for double DM

u-channel amplitudes for general two-comp dark matter:

N

dark matter	scalar	pseudo-scalar	fermion	vector	axial-vector
scalar(ϕ)	$+4g^2m_\phi^2$	$+4g^2m_\phi^2$	$\pm 2y_\chi^2 m_\chi(2m_\chi - m_\phi)$	NA	NA
pseudo-scalar(a)	–	–	$\mp 2\lambda_\chi^2 m_\chi m_a$	NA	NA
fermion(χ)	–	–	NA	$\mp 2g_{Z'}^2 m_\chi m_{Z'}$	$\pm 2g_{A'}^2 m_\chi(2m_\chi - m_{A'})$
vector(Z')	NA	NA	–	$-6g_X^2 m_X(2m_X - m_{X_3})$	NA
axial-vector(A')	NA	NA	–	–	NA

$\pm$: fermion or anti-fermion

$$\tilde{\Gamma}_u(p,q;p',q') = \frac{N}{\left(\sqrt{\frac{m_1}{m_2}}\vec{p} - \sqrt{\frac{m_2}{m_1}}\vec{q}'\right)^2 + m_2(2m_1 - m_2)}$$

[S. Kim, HML, B. Zhu, 2021]

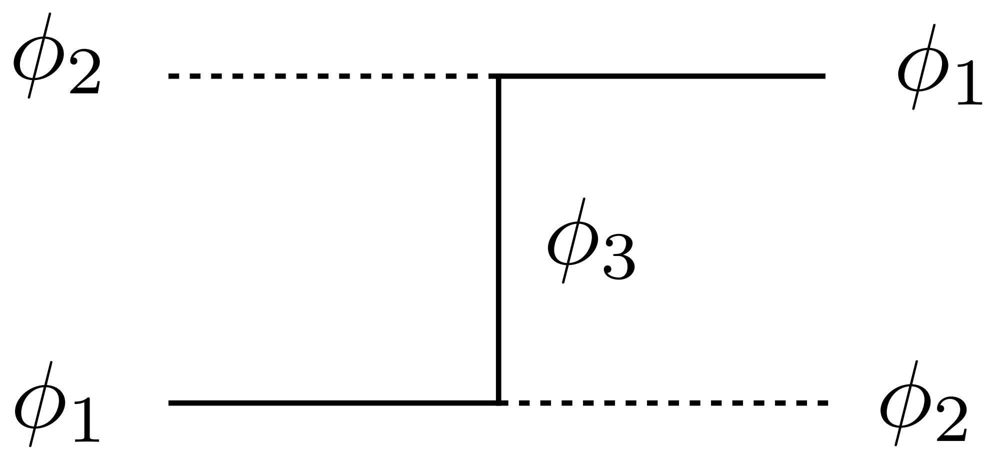
Scalar-(pseudo)scalar, fermion-pseudoscalar(vector)

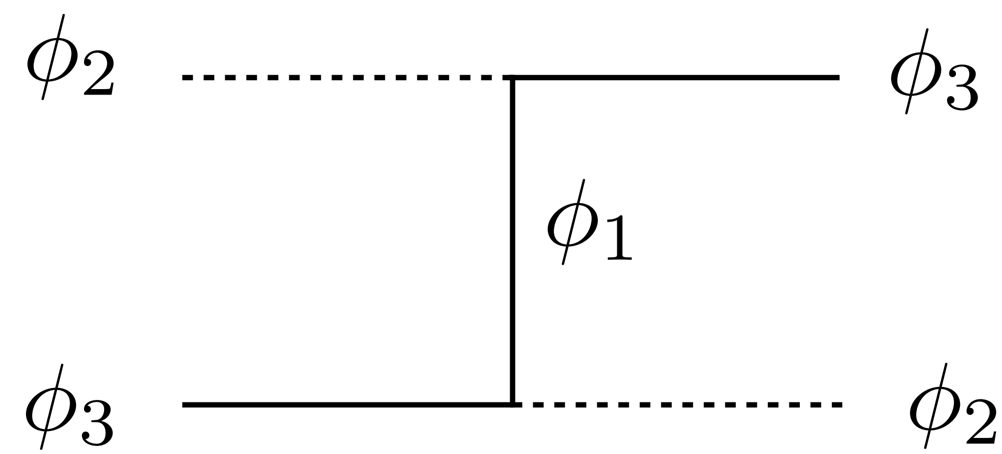
u-channel enhanced self-scattering

Triple DM

- Take three component dark matter with masses, m_i (I=1,2,3).

$$\mathcal{L}_{\rm int} = -2g\, m_1 \phi_1 \phi_2 \phi_3^* + {\rm h.c.}$$





u-channel resonance masses:

[S. Kim, HML, B. Zhu, 2021]

$$M = \sqrt{\frac{m_2}{m_1}}\sqrt{m_3^2-(m_1-m_2)^2}$$

$$M = \sqrt{\frac{m_2}{m_3}}\sqrt{m_1^2-(m_3-m_2)^2}$$

$$M=0: \quad m_3 = |m_1 - m_2|$$

$$M=0: \quad m_1 = |m_3 - m_2|$$

➡ u-channel resonance conditions are generalized.

Conclusions

- Non-perturbative effects are important for understanding dark matter self-scattering and annihilation.
- Dark matter self-scattering can be delayed due to an u-channel exchange of dark matter, mimicking a long-range interaction without a light mediator.
- Non-perturbative scattering amplitude for two-component dark matter is obtained a la Bethe-Salpeter.
- Multi-component dark matter with u-channel resonances identified; concrete model building anticipated.