Self-Resonant Dark Matter

Workshop on particle physics and cosmology
26, November, 2021, Speaker : Seongsik Kim

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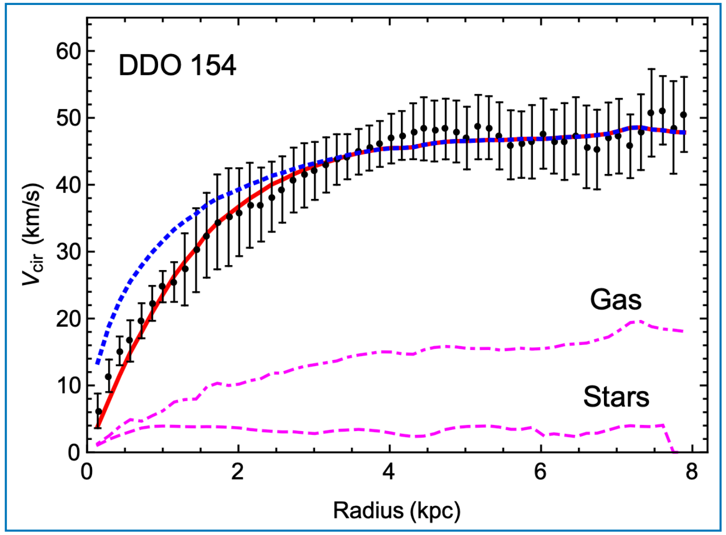
arXiv 2108.06278

Motivation - SIDM model

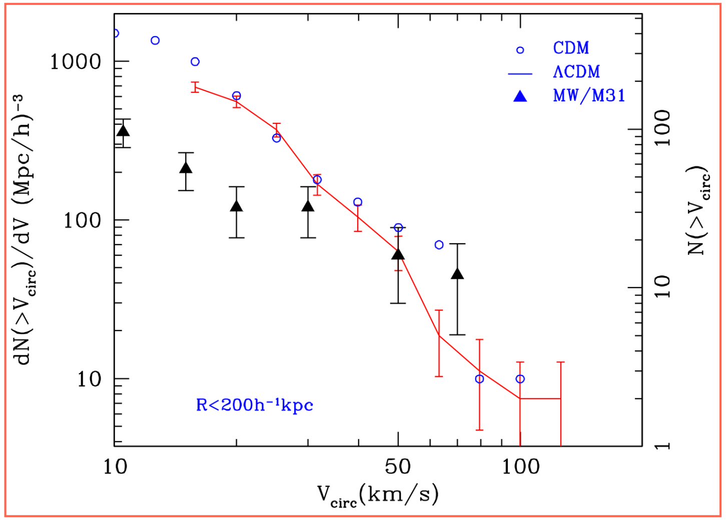
There are some mismatches between ΛCDM prediction, and observations.

Self-Interacting Dark Matter (SIDM) model could resolve these mismatches with an appropriate DM scattering cross-section.



Core Cusp Problem

Reprinted From S. Tulin, Hai-Bo Yu (2017), arXiv:1705.02358v2 [hep-ph]



Missing Satellites Problem

Reprinted From A. A. Klypin, A. V. Kravtsov, O. Valenzuela, and F. Prada, Astrophys. J. 522, 82 (1999), astro- ph/9901240.

Appropriated DM Self-Scattering Cross-section is $\sigma/m \sim \mathcal{O}(1)$cm^2/g

Sommerfeld Enhancement

Sommerfeld Enhancement : One of the cross-section enhancing effects

When?

Long-Range Attractive Interaction Between Colliding Species

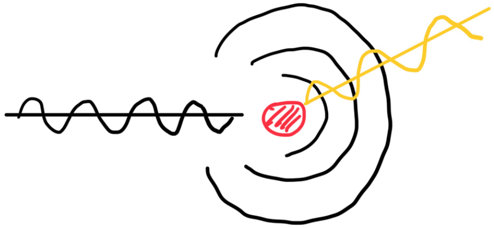
Non-relativistic collision

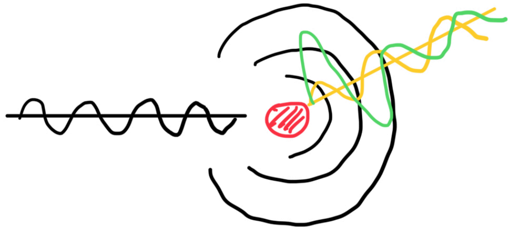
Why?

The Enhancement is so Sizable, affecting DM relic abundance significantly.

How?

By Enhancing 2 Particle-wavefunction Amplitude Near the Origin





$$\sigma = |A|^2 \sigma_0$$

σ_0 : cross-section calculated with ordinary approach

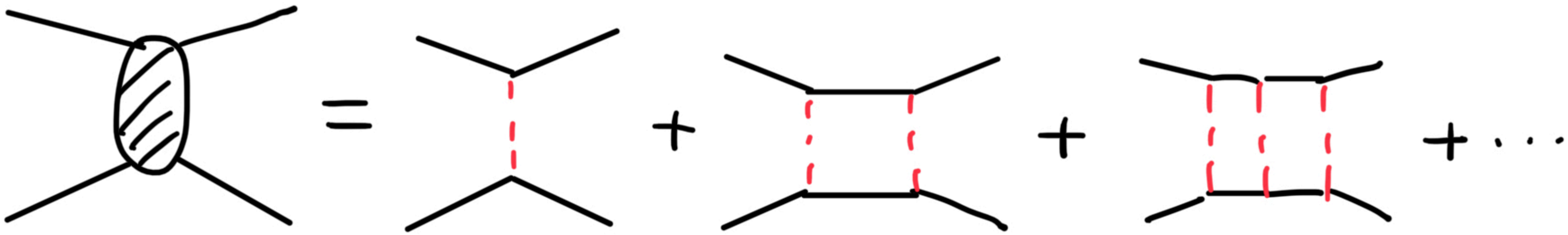
σ : Corrected Cross-section

A : Sommerfeld Factor (Enhanced Amplitude ratio)

Bethe-Salpeter Equation

Connect Enhancement Condition to QFT Condition

A Common Situation for 2→2 process



$$i\tilde{\Gamma} \qquad \mathcal{O}\left(\frac{g^2}{m_{med}^2}\right) > \mathcal{O}\left(\frac{g^2}{m_{med}^2}\right) > \mathcal{O}\left(\frac{g^2}{m_{med}^2}\right)$$

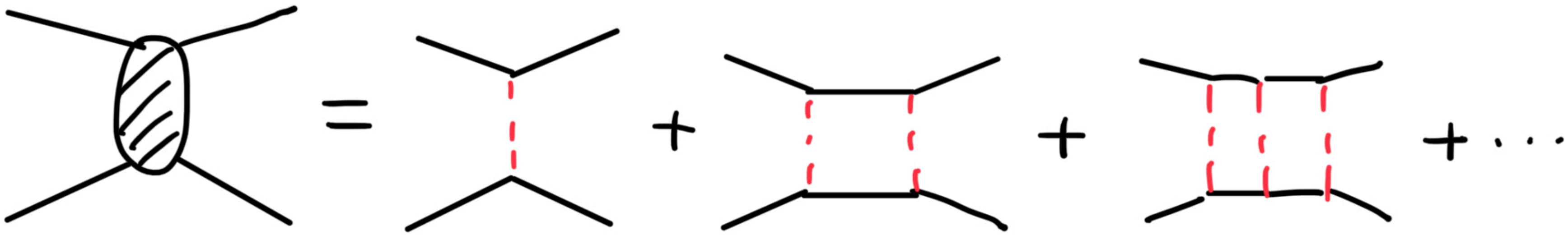
High−Order Term suppressed by Perturbative Coupling

Momentum in denominator ignored since we consider a non−relativistic limit

Bethe-Salpeter Equation

Connect Enhancement Condition to QFT Condition

Light mediator breaks perturbativity!



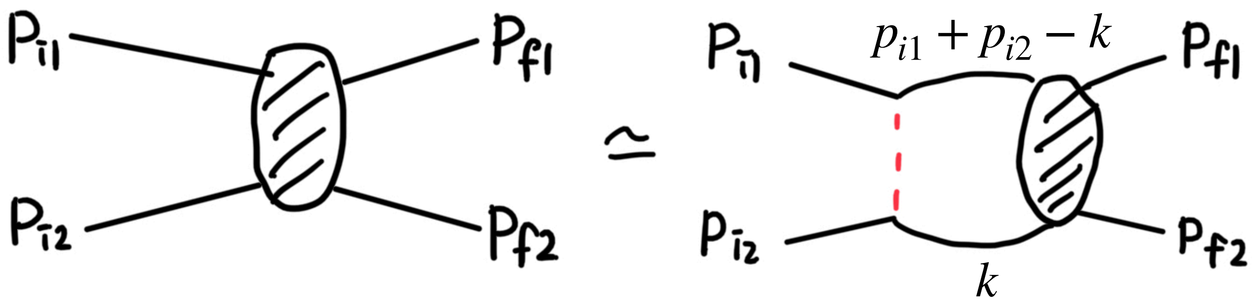
$$i\tilde{\Gamma} \qquad \mathcal{O}\left(\frac{g^2}{m_{med}^2}\right) < \mathcal{O}\left(\frac{g^2}{m_{med}^2}\right) < \mathcal{O}\left(\frac{g^2}{m_{med}^2}\right)$$

High−Order Term Enhanced!

Even momentum is included in the denominator there is still a possibility of breaking perturbativity

Bethe-Salpeter Equation

Methods for calculating Sommerfeld Factor



- $G_i(q)$ $(i = 1,2)$: particle i propagator with momentum transfer q. $\frac{i}{q^2 - m_i^2}$,
- $i\tilde{\Gamma}$: 4 point function via single mediator exchange. $q = p_{f2} - p_{i1}$
- $i\Gamma$: total 4 point function

$$i\Gamma(p_{i1}, p_{i2}; p_{f1}, p_{f2}) = -\int \frac{d^4k}{(2\pi)^4}\tilde{\Gamma}(p_{i1}, p_{i2}; k, p_{i1} + p_{i2} - k)G_2(k)G_1(p_{i1} + p_{i2} - k)\Gamma(k, p_{i1} + p_{i2} - k; p_{f1}, p_{f2})$$

Bethe−Salpeter wavefunction (in 4−momentum space)

$\chi(p_{i1}, p_{i2}; p_{f1}, p_{f2}) \equiv G_1(p_{i1})G_2(p_{i2})\Gamma(p_{i1}, p_{i2}; p_{f1}, p_{f2}) \equiv \chi(p_{i1}, p_{i2})$

BS wavefunction with CM Frame

$\chi(p_{i1}, p_{i2}) = \tilde{\chi}(P, Q)$

$$\tilde{\psi}_{BS}(\overrightarrow{Q}) \equiv \int \frac{dQ_0}{2\pi}\tilde{\chi}(Q)$$

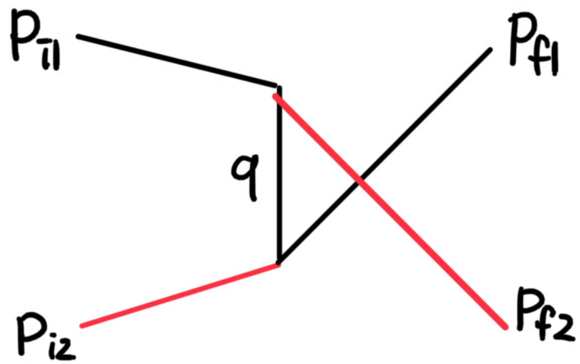
The Final Equation is the Schrodinger Equation with Yukawa potential

(scalar particle model)

$$\left(\frac{\overrightarrow{p}^2}{2\mu} - E\right)\tilde{\chi}(Q) + \int \frac{d^3q'}{(2\pi)^3}V(\overrightarrow{p} - \overrightarrow{q}')\tilde{\chi}(q') = 0$$

New Way to Induce Non-Perturbativity

Collision Mediated by MASSIVE Own



$$i\tilde{\Gamma} = -g^2 \frac{\text{Numerator}}{q^2 - m_{med}^2}$$

$$\text{denominator} = q^2 - m_{med}^2 = (E_{f2} - E_{i1})^2 - (\vec{p}_{f2} - \vec{p}_{i1})^2 - m_1^2$$

$$\simeq \left(m_2 + \frac{\vec{p}_{f2}^{\,2}}{2m_2} - m_1 - \frac{\vec{p}_{i1}}{2m_1} \right)^2 - (\vec{p}_{f2} - \vec{p}_{i1})^2 - m_1^2$$

$$\simeq m_2^2 - 2m_1 m_2 - \left(\sqrt{\frac{m_1}{m_2}} \vec{p}_{f2} - \sqrt{\frac{m_2}{m_1}} \vec{p}_{i1} \right)^2$$

$\mathcal{O}(v_{rel}^2)$

In Non−Relativistic limit,
$2m_1 \simeq m_2$ causes non−perturbativity

Simple Model for Precise Calculation

Collision Mediated by MASSIVE Own

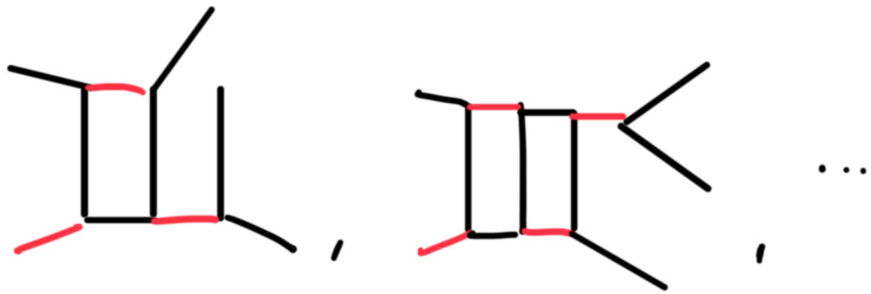
$$\mathscr{L} = |\partial_\mu \phi_1|^2 - m_1^2 \phi_1^2 + \frac{1}{2}(\partial_\mu \phi_2)^2 - \frac{1}{2} m_2^2 \phi_2^2 - 2g m_1 \phi_2 |\phi_1|^2$$

m_1 in coupling introduced to make g dimensionless.

2 DM (ϕ_1, ϕ_2) model, One Real Scalar and One Complex Scalar

We consider ($m_1 <$) $m_2 < 2m_1$ case.

Q. Why $m_2 < 2m_1$?



Significant Decay Allowed in $m_2 \geq 2m_1$ model, Leading No Particles to interact

Equation From New Mechanism

The Same Bethe–Salpeter Equation applied for this mechanism

With Different equation outcome

- $G_i(q)$ $(i = 1,2)$: particle i propagator with momentum transfer q. $\frac{i}{q^2 - m_i^2}$,
- $i\tilde{\Gamma}$: 4 point function via single mediator exchange.

  $-\frac{4im_1^2g^2}{q^2 - m_1^2} \simeq 4im_1^2g^2\left[m_2(2m_1 - m_2) + \left(\sqrt{\frac{m_1}{m_2}}\vec{p}_{f2} - \sqrt{\frac{m_2}{m_1}}\vec{p}_{i1}\right)^2\right]^{-1}$ for $q = p_{f2} - p_{i1}$

  Note that ϕ_1 always mediate u–channel interaction.
- $i\Gamma$: total 4 point function

$\tilde{\Gamma}(p_{i1}, p_{i2}; k, p_{i1} + p_{i2} - k) = \frac{4m_1^2g^2}{(p_{i2} - k)^2 - m_1^2} \equiv \tilde{U}((p_{i2} - k)^2)$

$$-\frac{1}{2\mu}\nabla^2\psi(\vec{r}) - \frac{\alpha}{r}e^{-Mr}\psi(-\frac{m_2}{m_1}\vec{r}) = E\psi(\vec{r})$$

$P = \frac{1}{2}(p_{i1} + p_{i2}) = (P_0, \vec{0})$

$, Q = \mu\left(\frac{p_{i2}}{m_2} - \frac{p_{i1}}{m_1}\right)$

$i\tilde{\psi}_{BS}(\vec{Q}) = \left[-\int\frac{dQ_0}{2\pi}G_1\left(-Q + \frac{2\mu}{m_2}P\right)G_2\left(Q + \frac{2\mu}{m_1}P\right)\right]\left[\int\frac{d^3\vec{k'}}{(2\pi)^3}\tilde{U}\left(\left|\sqrt{\frac{m_1}{m_2}}\vec{Q} + \sqrt{\frac{m_2}{m_1}}\vec{k'}\right|\right)\tilde{\psi}_{BS}(\vec{k'})\right]$

$i\tilde{\chi}(P, Q) = -G_1\left(-Q + \frac{2\mu}{m_2}P\right)G_2\left(Q + \frac{2\mu}{m_1}P\right)\int\frac{d^4k'}{(2\pi)^4}\tilde{U}((p_{i2} - k)^2)\tilde{\chi}\left(P, k'\right)$

$$E = P_0 - \frac{m_1 + m_2}{2}, M = m_2\sqrt{2 - \frac{m_2}{m_1}}, \alpha = \frac{g^2}{4\pi}$$

Schrodinger–like Equation with Long–Range potential–like term

Yukawa limit Calculation

Numerical Sommerfeld Factor for s−wave cases

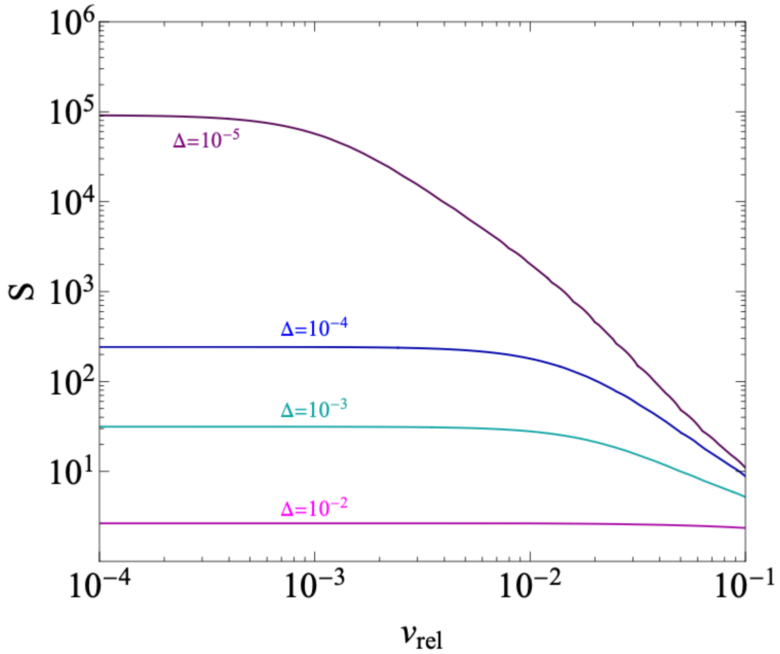


FIG. 2: Sommerfeld factor for s-wave elastic scattering, $\phi_1\phi_2 \to \phi_1\phi_2$, as a function of the relative velocity v_{rel}. We chose $\alpha = g^2/(4\pi) = 0.1$ and $\Delta = 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}$, for $m_2 = 2m_1(1-\Delta)$, in the lines from bottom to top.

$$\Delta \equiv 1 - \frac{m_2}{2m_1}$$ Mass−difference parameter

We call Yukawa limit because exponential suppression of potential remains.

- Enhancement becomes larger as $|m_2 - 2m_1|$ smaller.
- There is no absolute mass dependence in Enhancement.

Yukawa limit Result

Sommerfeld-Enhanced cross-section. s-wave cases

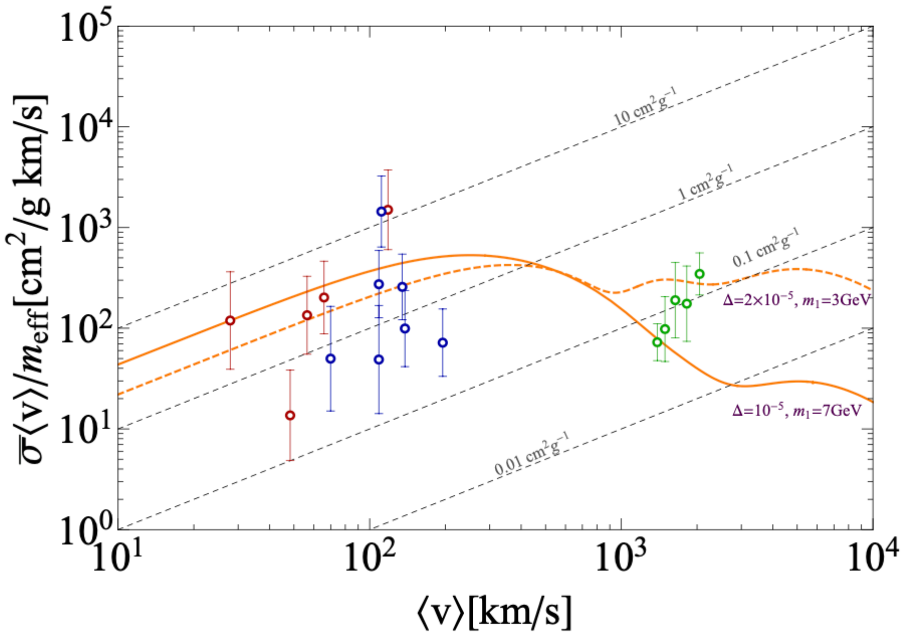


FIG. 3: Self-scattering cross section per dark matter mass for s-wave elastic scattering, $\phi_1\phi_2 \to \phi_1\phi_2$, as a function of $\langle v_{\rm rel}\rangle$. We chose $\Delta = 10^{-5}, 2\times10^{-5}$ and $m_1 = 7, 3\,{\rm GeV}$ in orange solid and dashed lines, respectively. We took $\alpha = g^2/(4\pi) = 0.1$ and $m_{\rm eff} = 2m_1(1+b)$.

M. Kaplinghat, S. Tulin and H. B. Yu, (2016), Phys. Rev. Lett. 116 no.4, 041302, arXiv:1508.03339

$$\Delta \equiv 1 - \frac{m_2}{2m_1}$$

Experimental Observations

- Red : THINGS dwarf galaxies
- Green : clusters
- Blue : LSB galaxies

Huge DM Mass requires a smaller Delta for larger non-perturbativity.

Summary

- Dark Matter Scattering Cross-sections play important role in comparing observation and theory.
- Sommerfeld Effect, which is traditionally considered due to light mediator, enhances cross-section, especially lower velocity.
- We present a new Sommerfeld enhancement mechanism without a light mediator. Instead, the particle itself became a propagator.
- We Calculated Sommerfeld Factor and Enhanced Cross-section with a simple model.