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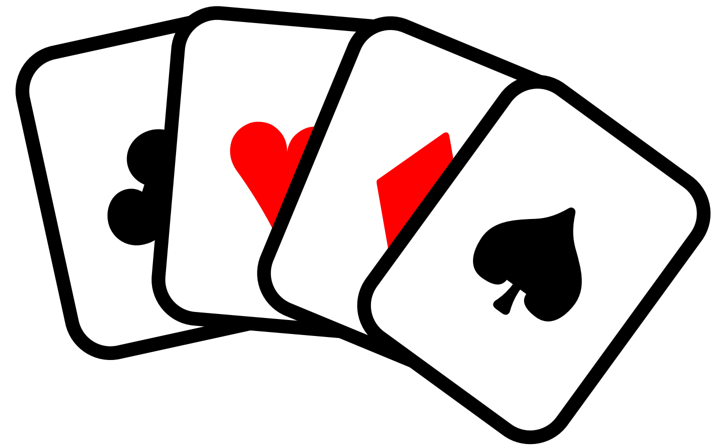

A Reshuffled SIMP Dark Matter Model with $U(1)_D$ to Z_4 symmetry

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In collaboration with Pyungwon Ko & Chih-Ting Lu (KIAS)

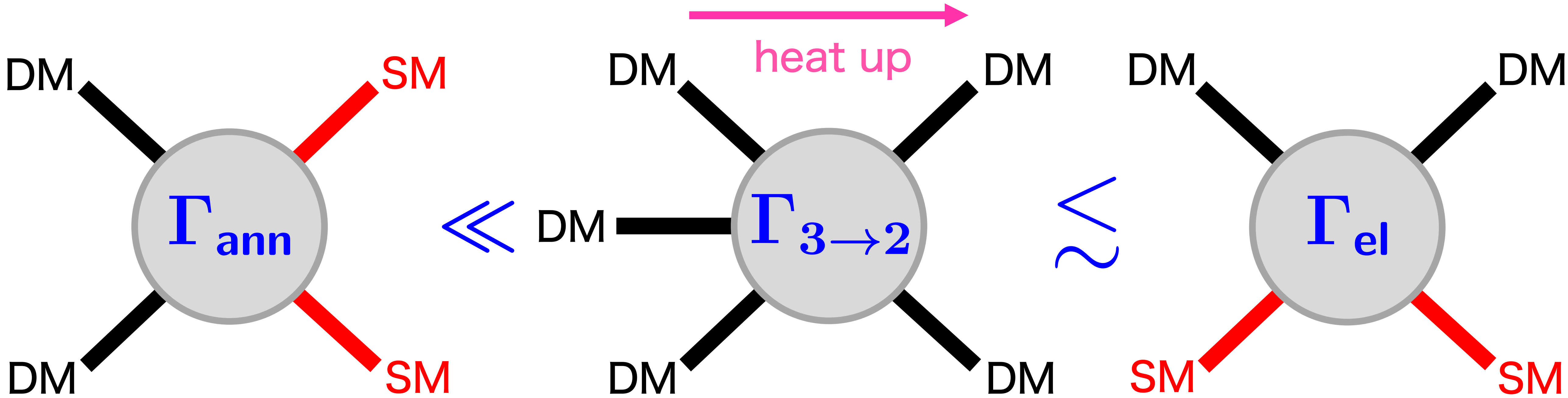
In progress (based on 2107.04375)

11/26/2021

Workshop on particle physics and cosmology 2021

I. Introduction

Strongly interacting massive particles (SIMP)



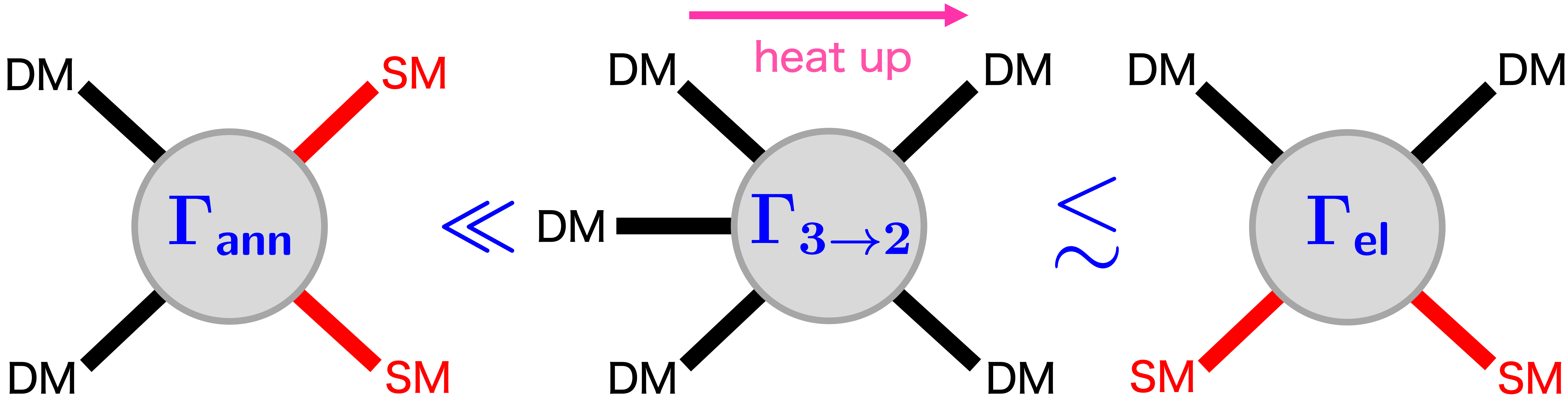
$$\Gamma_{\text{ann}} = n_{\text{DM}} \langle \sigma_{\text{ann}} v \rangle \qquad \Gamma_{3\to 2} = n_{\text{DM}}^2 \langle \sigma_{3\to 2} v^2 \rangle \qquad \Gamma_{\text{el}} = n_{\text{SM}} \langle \sigma_{\text{el}} v \rangle$$

SIMP scenario :

$$\Gamma_{\text{el}} \gtrsim \Gamma_{3\to 2} \gg \Gamma_{\text{ann}} > H_{\text{DM}}$$

Y. Hochberg, E. Kuflik, T. Volansky & J.G. Wacker (2014)

Strongly interacting massive particles (SIMP)



$$\Gamma_{\text{ann}} = n_{\text{DM}}\langle \sigma_{\text{ann}} v\rangle \qquad \Gamma_{3\to 2} = n_{\text{DM}}^2\langle \sigma_{3\to 2} v^2\rangle \qquad \Gamma_{\text{el}} = n_{\text{SM}}\langle \sigma_{\text{el}} v\rangle$$

SIMP scenario :

$$\Gamma_{\text{el}} \gtrsim \Gamma_{3\to 2} \gg \Gamma_{\text{ann}} > H_{\text{DM}}$$

$$\langle \sigma_{3\to 2} v^2\rangle \equiv \frac{\alpha_{\text{eff}}^3}{m_{\text{DM}}^5}$$

$$\alpha_{\text{ann}} \simeq 0$$

➡

$$\alpha_{\text{eff}} \simeq 1 - 10$$

(strong scale)

$$m_{\text{DM}} \sim 100\ \text{MeV}$$

Y. Hochberg, E. Kuflik, T. Volansky & J.G. Wacker (2014)

Multi-component DM

- Majority of DM models suggest that DM particle is WIMP-type and of only one kind.

- Null results of direct search detections have cornered WIMP.

- Dark sector may be plentiful as same as the visible sector.

- It is reasonable to consider a scenario containing more than one species of DM beyond the WIMP paradigm.

  e.g. hidden QCD, multi-component FIMP, multi-component SIMP,……

  A. Katz, *et al.* (2020) J Herms & A. Ibarra (2020) S. M. Choi, *et al.* (2021)

  S.P. Zakeri, *et al.* (2018) S.P. Zakeri, *et al.* (2018)

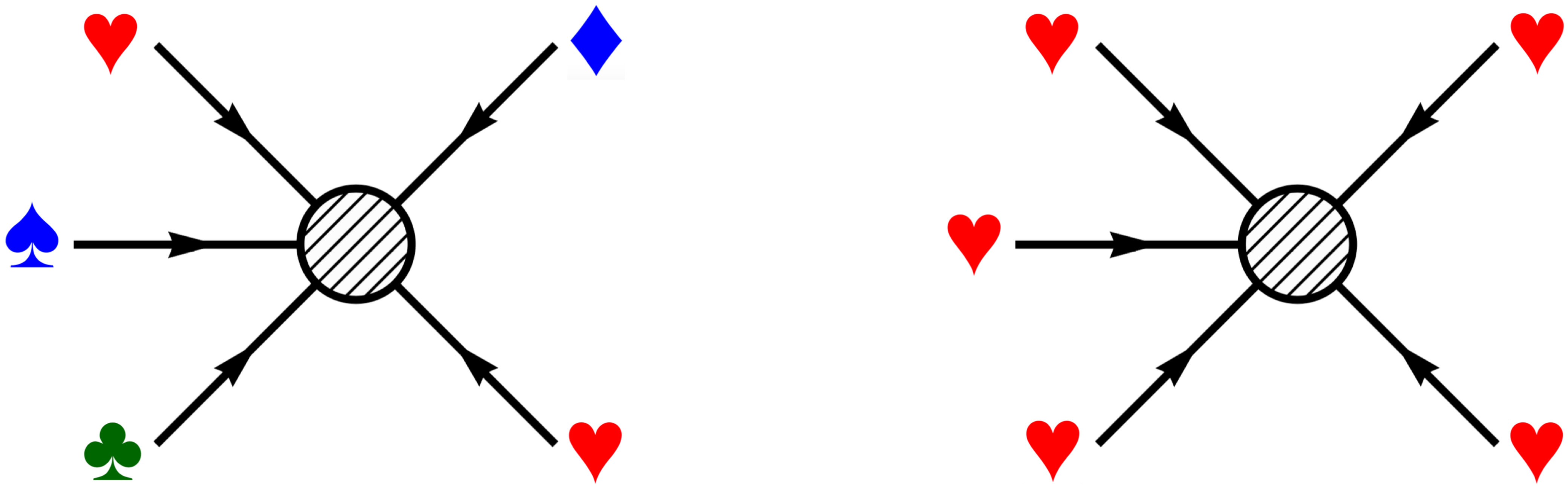
Multi-component DM

- In genuine multi-component DM models, it should be possible for different DM species to have distinctive properties such as mass, (dark) charge, and spin, etc.

- Each DM particle should contribute to a sizable abundance to the observed DM relic density.

Carl Sagan's dragon in garage

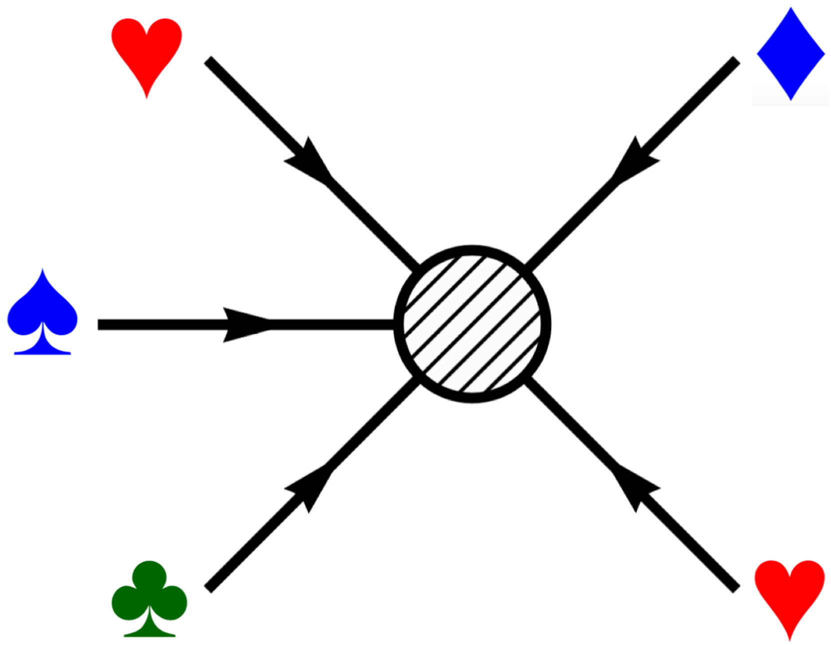
Multi-component SIMP scenario



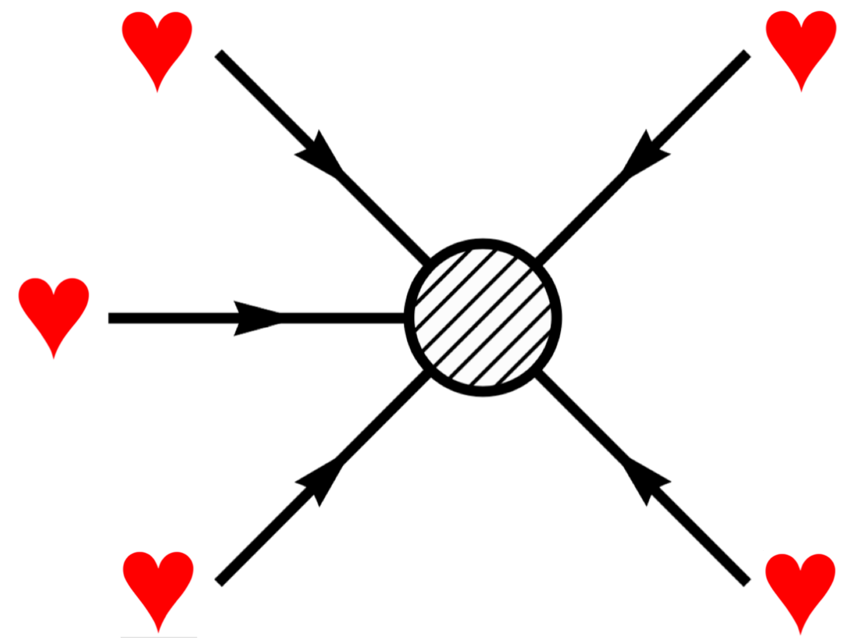
multi-component SIMP　　　　single-component SIMP

Poker notation : ♠ (spade) ♥ (heart) ♦ (diamond) ♣ (club)

Multi-component SIMP scenario



multi-component SIMP

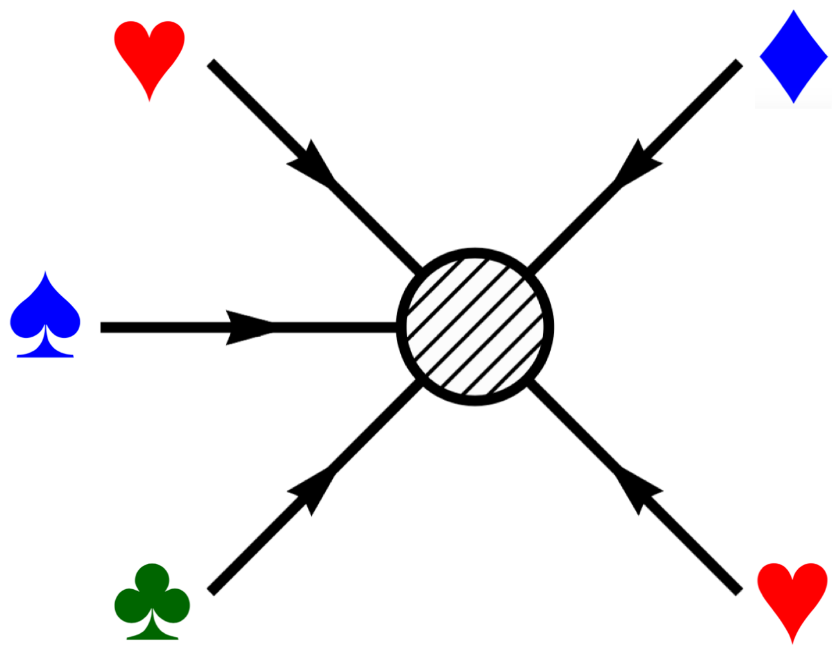


single-component SIMP

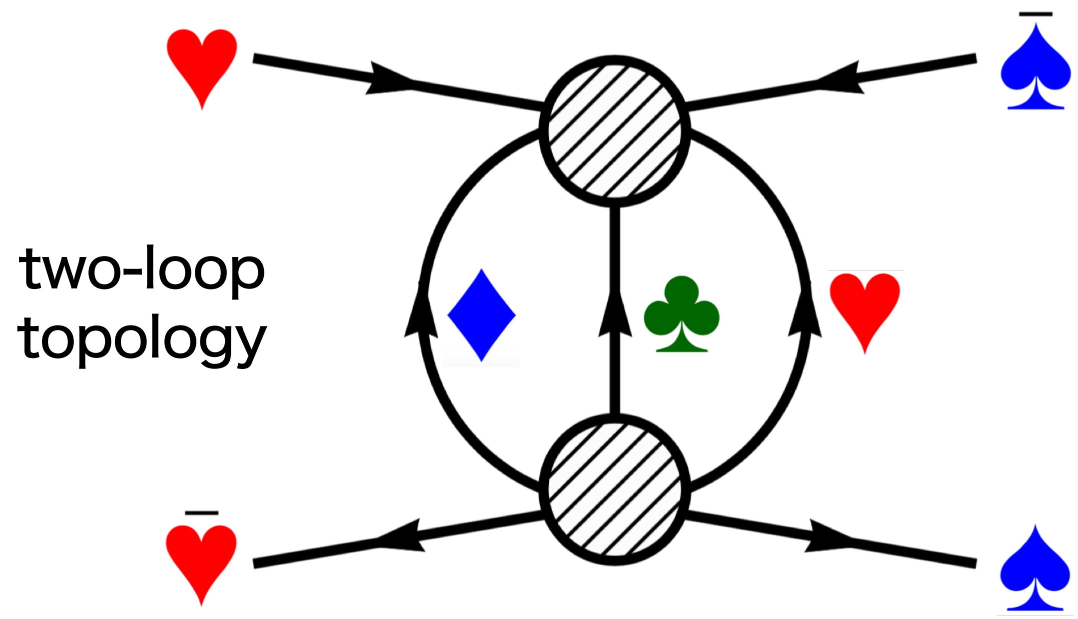
What is the difference between single-component SIMP scenario & multi-component SIMP scenario?

Poker notation : ♠(spade) ♥(heart) ♦(diamond) ♣(club)

Multi-component SIMP scenario

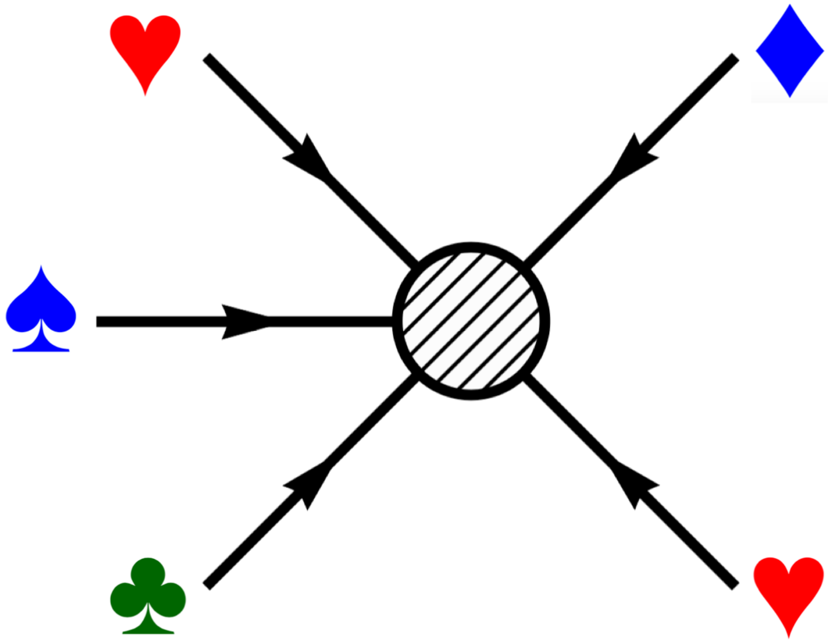


multi-component SIMP

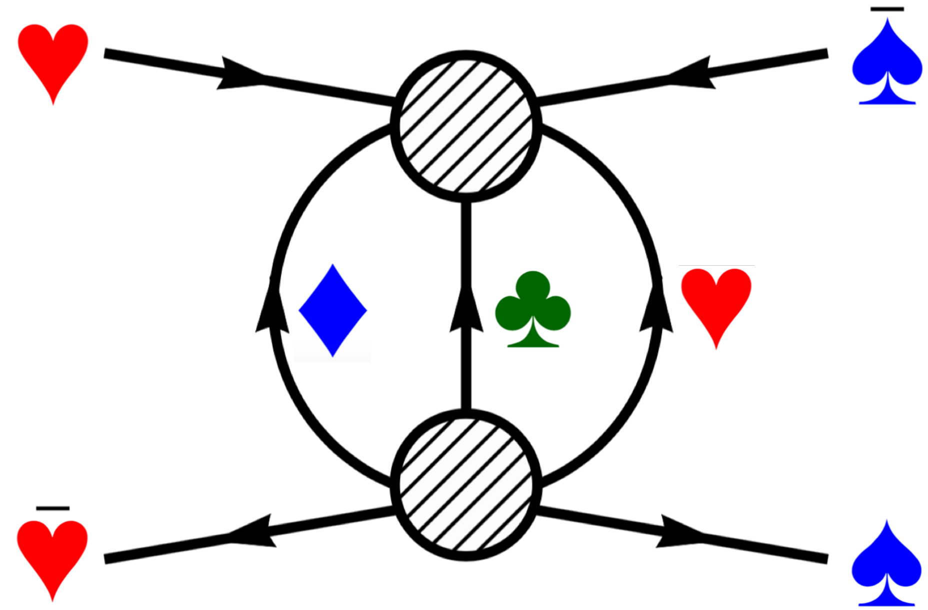


Poker notation : ♠ (spade) ♥ (heart) ♦ (diamond) ♣ (club)

Multi-component SIMP scenario



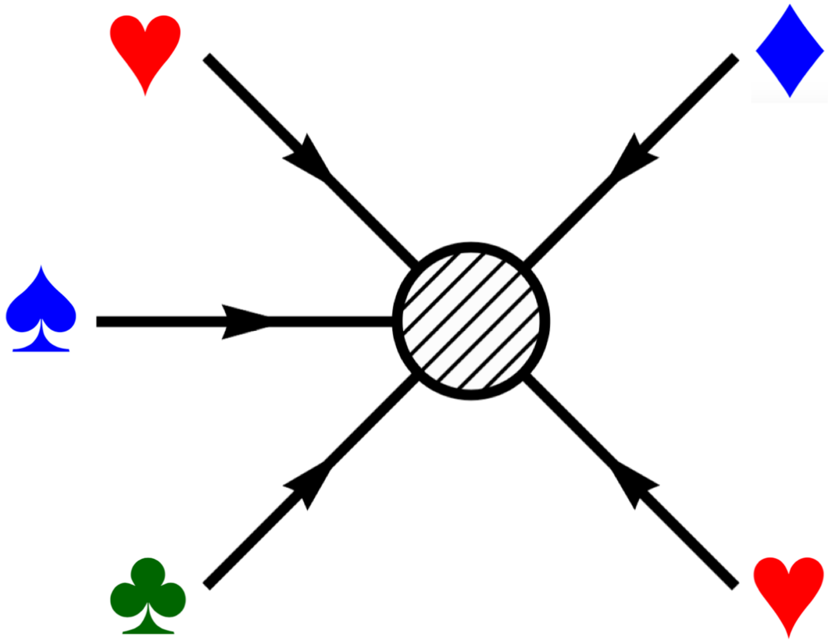
multi-component SIMP



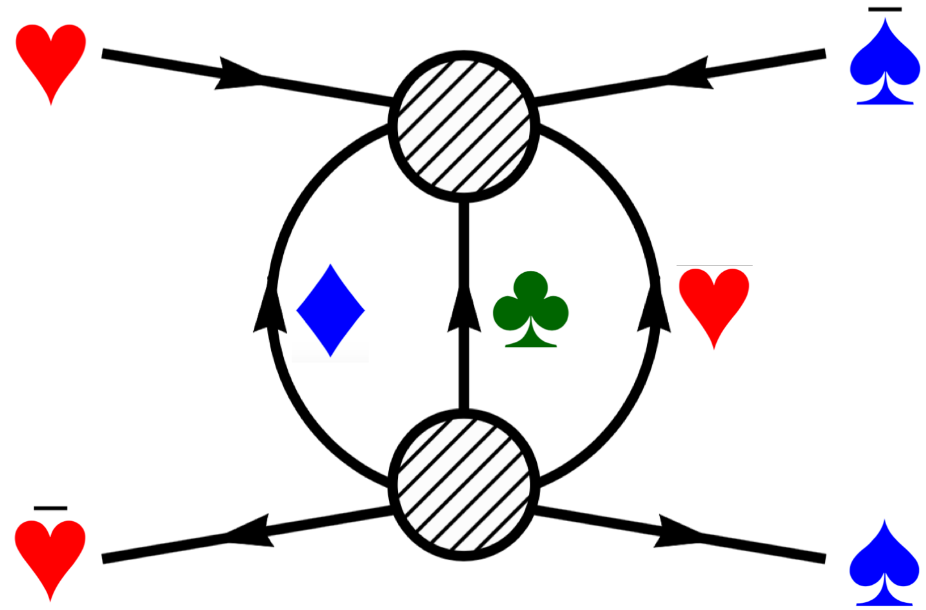
$$\Gamma_{3\to 2} \quad \gtrless ? \quad \Gamma^{2\text{-loop}}_{2\to 2}$$

Poker notation : ♠(spade) ♥(heart) ♦(diamond) ♣(club)

Multi-component SIMP scenario



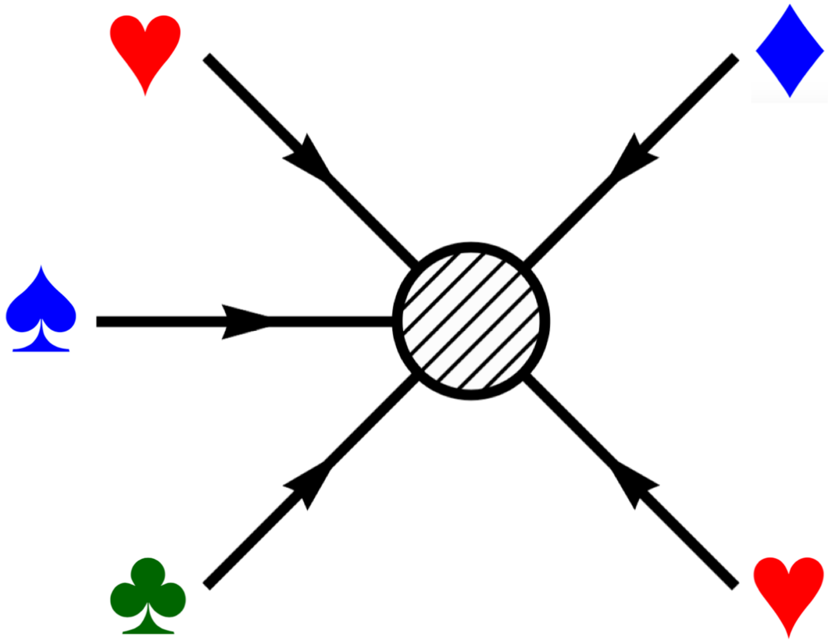
multi-component SIMP



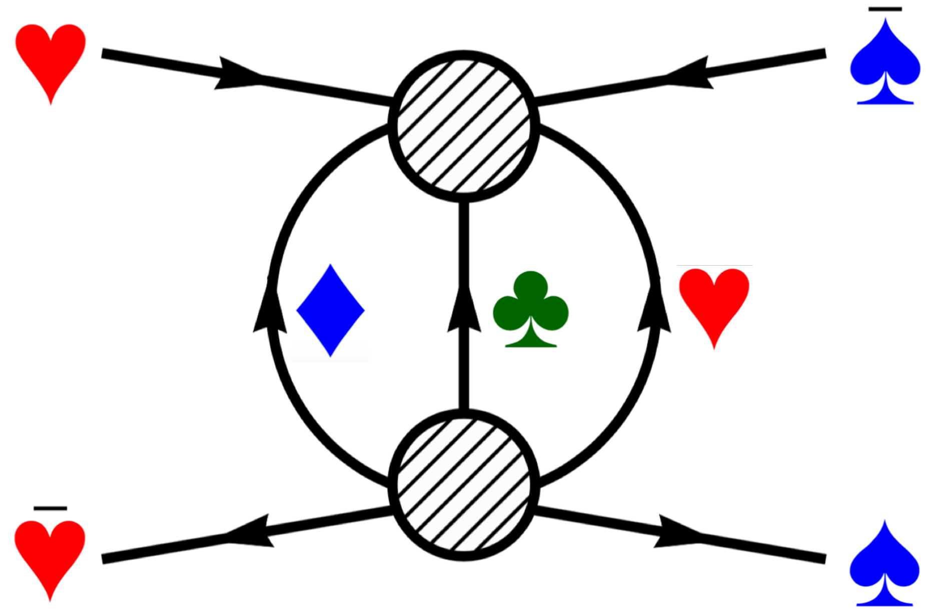
$$\left.\frac{\Gamma_{3\to2}}{\Gamma^{\text{2-loop}}_{2\to2}}\right|_{T=T_f} = \left.\frac{\qquad\qquad}{}\right|_{T=T_f}$$

Poker notation : ♠ (spade) ♥ (heart) ♦ (diamond) ♣ (club)

Multi-component SIMP scenario



multi-component SIMP

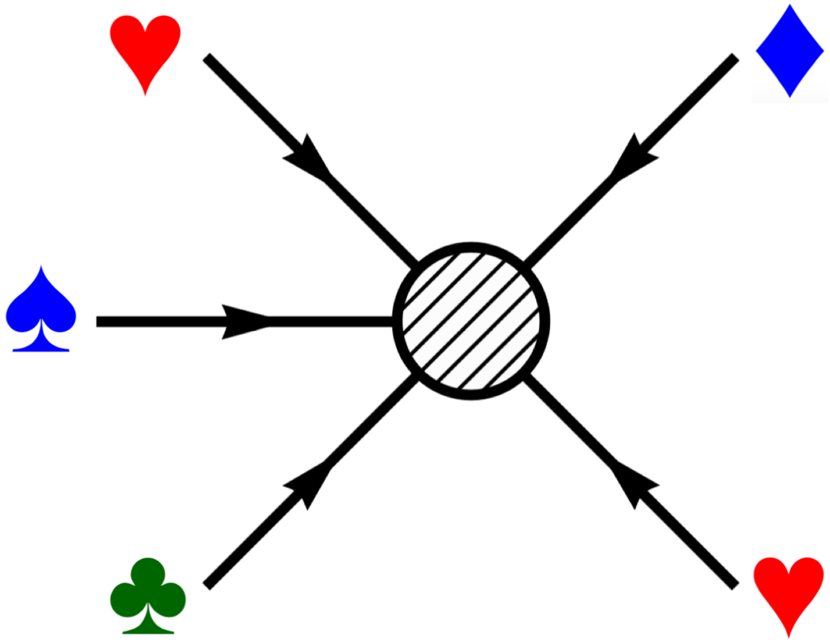


$$\left.\frac{\Gamma_{3\to2}}{\Gamma^{\text{2-loop}}_{2\to2}}\right|_{T=T_f} = \left.\frac{n_{\rm DM}^2\langle\sigma_{3\to2}v^2\rangle}{n_{\rm DM}\langle\sigma^{\text{2-loop}}_{2\to2}v\rangle}\right|_{T=T_f}$$

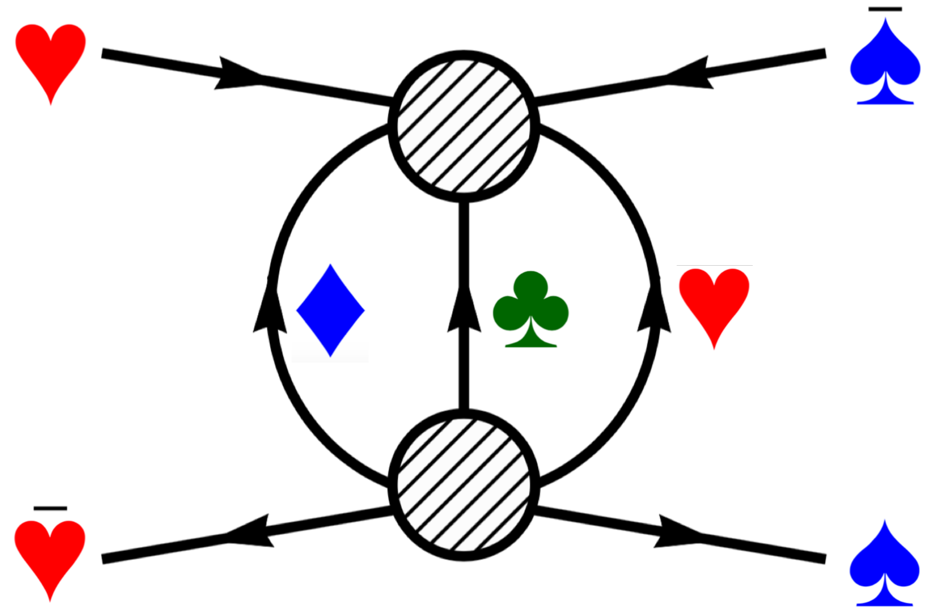
$$\langle\sigma_{3\to2}v^2\rangle \equiv \frac{\alpha_{\rm eff}^3}{m_{\rm SIMP}^5}$$

Poker notation : ♠ (spade) ♥ (heart) ♦ (diamond) ♣ (club)

Multi-component SIMP scenario



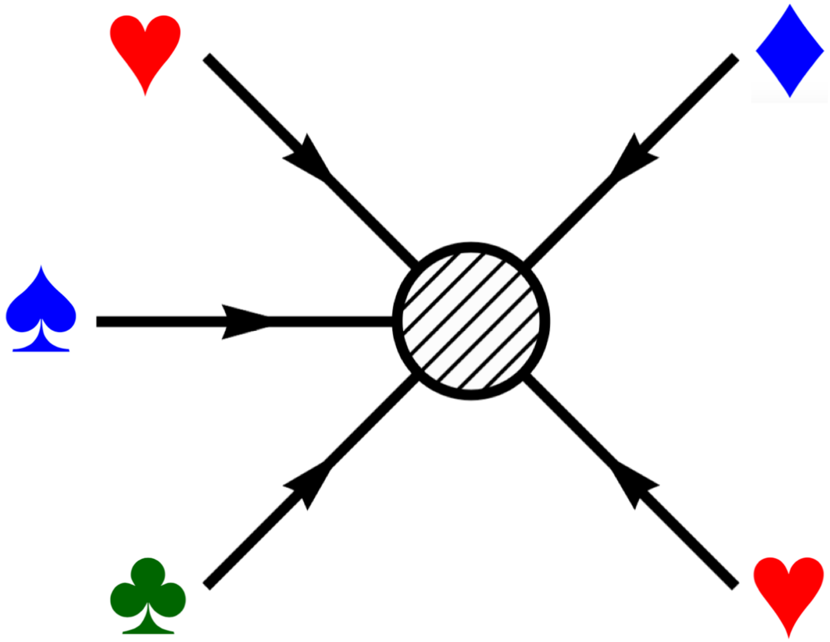
multi-component SIMP



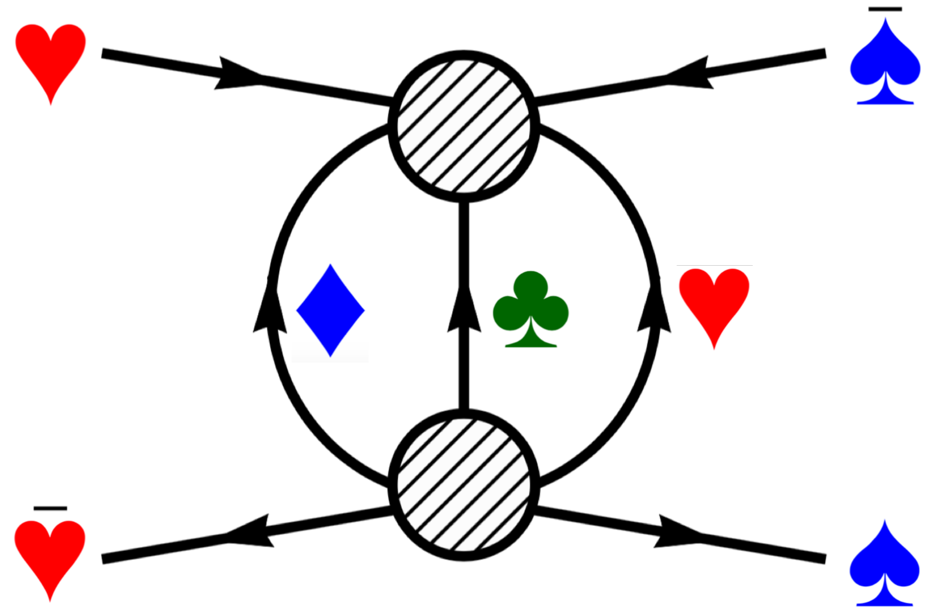
$$\left.\frac{\Gamma_{3\to2}}{\Gamma^{\text{2-loop}}_{2\to2}}\right|_{T=T_f} = \left.\frac{n_{\rm DM}^2\,\alpha_{\rm eff}^3/m_{\rm DM}^5}{n_{\rm DM}\,\alpha_{\rm eff}^6/\left[(4\pi)^8 m_{\rm DM}^2\right]}\right|_{T=T_f}$$

Poker notation : ♠(spade) ♥(heart) ♦(diamond) ♣(club)

Multi-component SIMP scenario



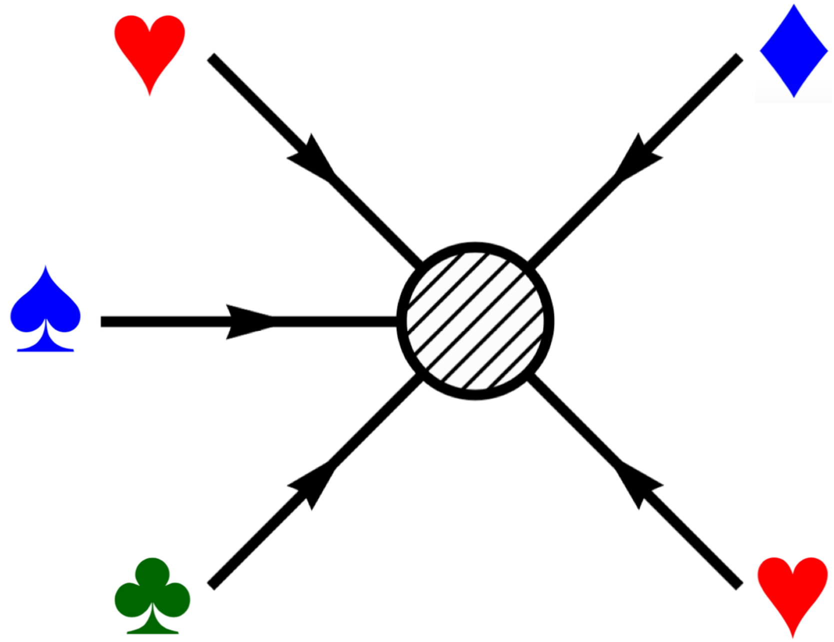
multi-component SIMP



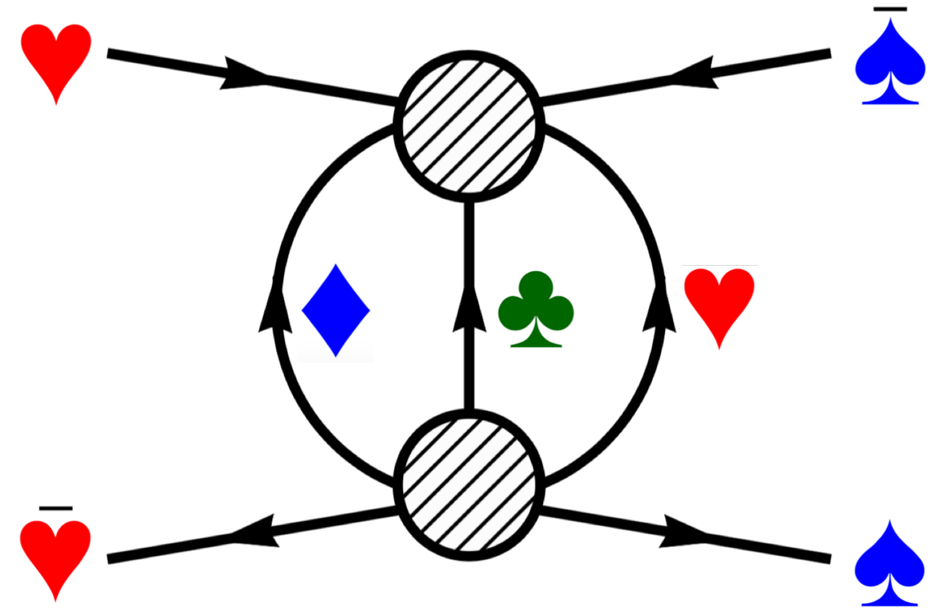
$$\left.\frac{\Gamma_{3\to2}}{\Gamma^{\text{2-loop}}_{2\to2}}\right|_{T=T_f} = \frac{(4\pi)^8}{\alpha_{\text{eff}}^3}\frac{n_{\text{DM}}(T_f)}{m_{\text{DM}}^3}$$

Poker notation : ♠ (spade) ♥ (heart) ♦ (diamond) ♣ (club)

Multi-component SIMP scenario



multi-component SIMP



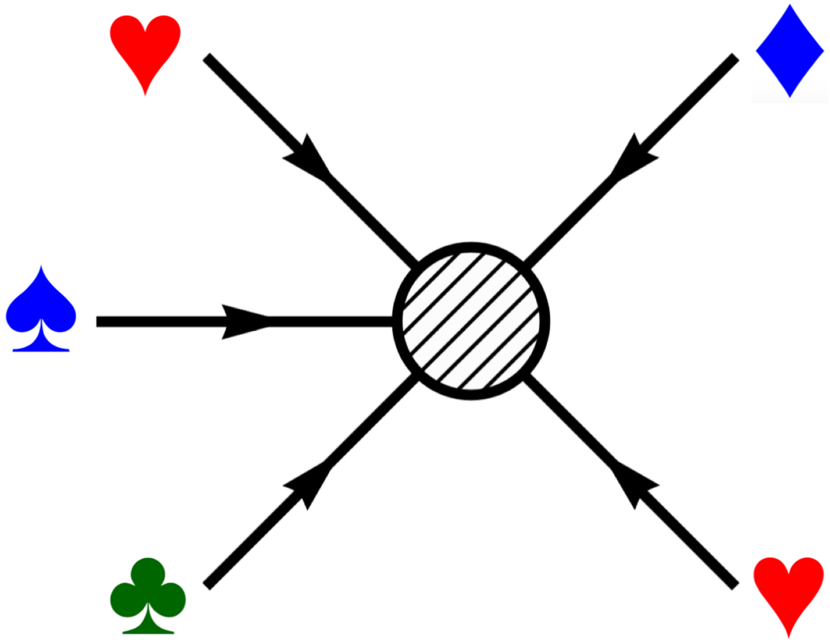
$$\left.\frac{\Gamma_{3\to2}}{\Gamma^{\text{2-loop}}_{2\to2}}\right|_{T=T_f} = \frac{(4\pi)^8}{\alpha_{\text{eff}}^3}\,\frac{n_{\text{DM}}(T_f)}{m_{\text{DM}}^3}$$

$$n_{\text{DM}}(T_f) \simeq \left(\frac{z_f}{2\pi}\right)^{3/2} e^{-z_f}\,T_f^3 \sim 10^{-8}T_f^3$$

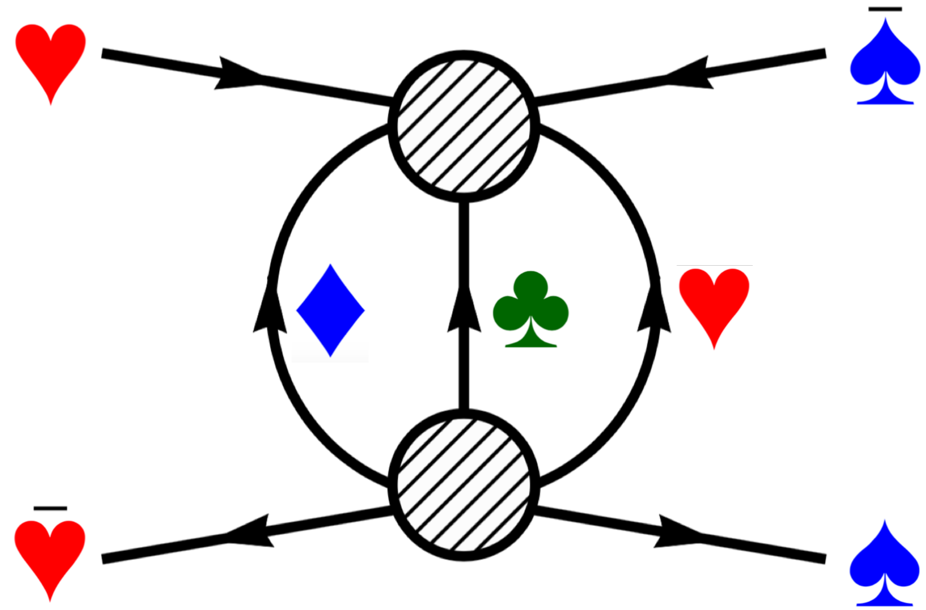
$$z_f \equiv \frac{m_{\text{DM}}}{T_f} \simeq 20$$

Poker notation : ♠ (spade) ♥ (heart) ♦ (diamond) ♣ (club)

Multi-component SIMP scenario



multi-component SIMP

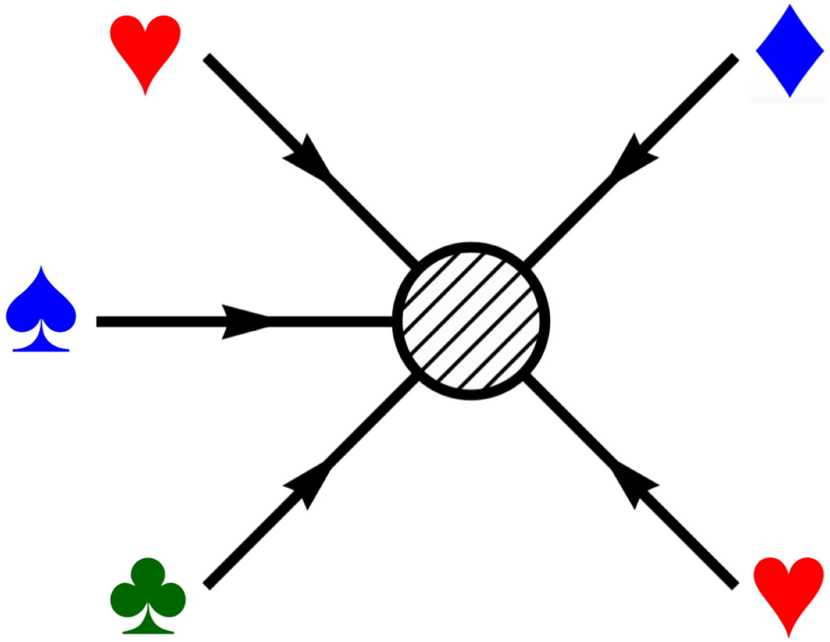


$$\left.\frac{\Gamma_{3\to2}}{\Gamma_{2\to2}^{\text{2-loop}}}\right|_{T=T_f} = \frac{(4\pi)^8}{\alpha_{\text{eff}}^3}\frac{n_{\text{DM}}(T_f)}{m_{\text{DM}}^3} \simeq \frac{1}{\alpha_{\text{eff}}^3 z_f^3} \ll 1$$

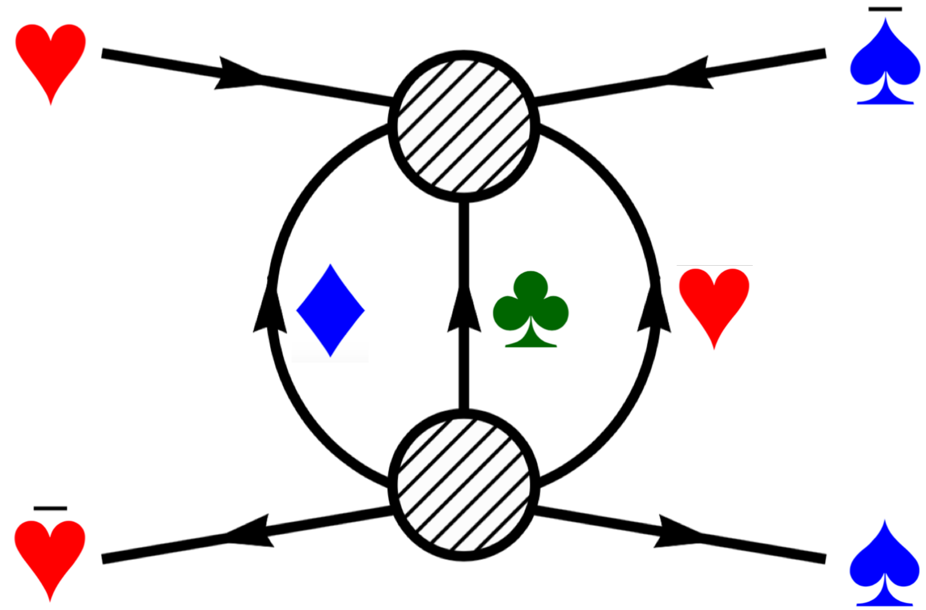
$$z_f \equiv \frac{m_{\text{DM}}}{T_f} \simeq 20$$

Poker notation : ♠ (spade) ♥ (heart) ♦ (diamond) ♣ (club)

Reshuffled SIMP (rSIMP)



number-changing process
: deplete the DM yields



number-conserving process
: redistribute/**reshuffle** the DM yields

rSIMP scenario : $\Gamma^{\text{2-loop}}_{2\to 2} > \Gamma_{\text{el}} \gtrsim \Gamma_{3\to 2} \gg \Gamma_{\text{ann}} > H_{\text{DM}}$

will be explained later

Poker notation : ♠ (spade) ♥ (heart) ♦ (diamond) ♣ (club)

II. Effective theory

EFT Model

- Three representative EFT models for the rSIMP scenario.

Model	Fields	$U(1)_D$	Interaction
A	$(\heartsuit, \spadesuit)$	$(2, -3)$	$\mathcal{O}^{(5)}_{\heartsuit\spadesuit} = \frac{c}{3!2!\Lambda} \heartsuit^3 \spadesuit^2$
B	$(\heartsuit, \diamondsuit)$	$(2, -3)$	$\mathcal{O}^{(6)}_{\heartsuit\diamondsuit} = \frac{c}{3!2!\Lambda^2} \heartsuit^3 \overline{\diamondsuit^c} \diamondsuit$
C	$(\heartsuit, \clubsuit, \diamondsuit)$	$(1, -2, -5)$	$\mathcal{O}^{(6)}_{\heartsuit\clubsuit\diamondsuit} = \frac{c}{3!\Lambda^2} \heartsuit^3 \overline{\clubsuit} \diamondsuit$

$\heartsuit \spadesuit$: complex scalar $\quad$ $\diamondsuit \clubsuit$: Dirac fermion

Annihilation process

$\heartsuit\spadesuit$: S $\clubsuit\diamondsuit$: F

- Model A & B : $3m_{\heartsuit} > 2m_{\spadesuit,\diamondsuit} > m_{\heartsuit}$
- Model C : $3m_{\heartsuit} > m_{\clubsuit} + m_{\diamondsuit} > m_{\heartsuit} > |m_{\clubsuit} - m_{\diamondsuit}|$

Model	$n \to 2$	Number-conserving/changing process
A $\heartsuit^3\spadesuit^2$	$2 \to 2$	$\heartsuit\bar{\heartsuit} \to \spadesuit\bar{\spadesuit}$ $\spadesuit\bar{\spadesuit} \to \heartsuit\bar{\heartsuit}$
	$3 \to 2$	$\heartsuit\heartsuit\heartsuit \to \bar{\spadesuit}\bar{\spadesuit}$ $\heartsuit\heartsuit\spadesuit \to \bar{\heartsuit}\bar{\spadesuit}$ $\heartsuit\spadesuit\spadesuit \to \bar{\heartsuit}\bar{\heartsuit}$
B $\heartsuit^3\overline{\diamondsuit^c}\diamondsuit$	$2 \to 2$	$\heartsuit\bar{\heartsuit} \to \diamondsuit\bar{\diamondsuit}$ $\diamondsuit\bar{\diamondsuit} \to \heartsuit\bar{\heartsuit}$
	$3 \to 2$	$\heartsuit\heartsuit\heartsuit \to \bar{\diamondsuit}\bar{\diamondsuit}$ $\heartsuit\heartsuit\diamondsuit \to \bar{\heartsuit}\bar{\diamondsuit}$ $\heartsuit\diamondsuit\diamondsuit \to \bar{\heartsuit}\bar{\heartsuit}$
C $\heartsuit^3\bar{\clubsuit}\diamondsuit$	$2 \to 2$	$\heartsuit\bar{\heartsuit} \to \diamondsuit\bar{\diamondsuit}, \clubsuit\bar{\clubsuit}$ $\diamondsuit\bar{\diamondsuit}, \clubsuit\bar{\clubsuit} \to \heartsuit\bar{\heartsuit}$ $\diamondsuit\bar{\diamondsuit} \to \clubsuit\bar{\clubsuit}$ $\clubsuit\bar{\clubsuit} \to \diamondsuit\bar{\diamondsuit}$
	$3 \to 2$	$\heartsuit\heartsuit\heartsuit \to \bar{\diamondsuit}\clubsuit$ $\heartsuit\heartsuit\diamondsuit \to \bar{\heartsuit}\clubsuit$ $\heartsuit\heartsuit\bar{\clubsuit} \to \bar{\heartsuit}\bar{\diamondsuit}$ $\heartsuit\diamondsuit\bar{\clubsuit} \to \bar{\heartsuit}\bar{\heartsuit}$

DM-SM interaction

- If the $\mathrm{U}(1)_{\mathrm{D}}$ symmetry is gauged, it is natural to introduce vector-portal interactions between the dark sector and the SM sector after SSB as

$$\overleftrightarrow{\partial_\mu} \equiv \overrightarrow{\partial_\mu} - \overleftarrow{\partial_\mu}$$

$$\mathcal{O}^{(6)}_{\phi e} = \frac{c_{\phi e}}{\Lambda^2_{Z'}} \left(i \phi^\dagger \overleftrightarrow{\partial_\mu} \phi \right) \left(\bar{e} \gamma^\mu e \right) \ , \quad \phi = \heartsuit, \spadesuit$$

B. V. Lehmann & S. Profumo (2020)

$$\mathcal{O}^{(6)}_{\psi e} = \frac{c_{\psi e}}{\Lambda^2_{Z'}} \left(\overline{\psi} \gamma_\mu \psi \right) \left(\bar{e} \gamma^\mu e \right) \ , \quad \psi = \clubsuit, \diamondsuit$$

- As we shall see soon, the rSIMP masses are tens of MeV scale, we then focus on the DM and $e^\pm$ interactions.

III. Cosmological evolution

Model A : $\heartsuit^3 \spadesuit^2$

$\heartsuit \spadesuit$: S

The Boltzmann equations of comoving number yields $Y_\heartsuit$, $Y_\spadesuit$

$$\frac{dY_\heartsuit}{dx} = -\frac{s^2}{Hx}\Bigg\{ 3\langle \sigma v^2\rangle_{\heartsuit\heartsuit\heartsuit\to\bar{\spadesuit}\bar{\spadesuit}}\left[Y_\heartsuit^3 - Y_\spadesuit^2\frac{(Y_\heartsuit^{\rm eq})^3}{(Y_\spadesuit^{\rm eq})^2}\right] + \langle \sigma v^2\rangle_{\heartsuit\heartsuit\spadesuit\to\bar{\heartsuit}\bar{\spadesuit}}\left(Y_\heartsuit^2 Y_\spadesuit - Y_\heartsuit Y_\spadesuit Y_\heartsuit^{\rm eq}\right) - \langle \sigma v^2\rangle_{\heartsuit\spadesuit\spadesuit\to\bar{\heartsuit}\bar{\heartsuit}}\left[Y_\heartsuit Y_\spadesuit^2 - Y_\heartsuit^2\frac{(Y_\spadesuit^{\rm eq})^2}{Y_\heartsuit^{\rm eq}}\right]\Bigg\} - \frac{s}{Hx}\Bigg\{\langle \sigma v\rangle_{\heartsuit\bar{\heartsuit}\to\spadesuit\bar{\spadesuit}}\left[Y_\heartsuit^2 - Y_\spadesuit^2\frac{(Y_\heartsuit^{\rm eq})^2}{(Y_\spadesuit^{\rm eq})^2}\right] - \langle \sigma v\rangle_{\spadesuit\bar{\spadesuit}\to\heartsuit\bar{\heartsuit}}\left[Y_\spadesuit^2 - Y_\heartsuit^2\frac{(Y_\spadesuit^{\rm eq})^2}{(Y_\heartsuit^{\rm eq})^2}\right]\Bigg\}$$

$$\frac{dY_\spadesuit}{dx} = -\frac{s^2}{Hx}\Bigg\{ 2\langle \sigma v^2\rangle_{\heartsuit\spadesuit\spadesuit\to\bar{\heartsuit}\bar{\heartsuit}}\left[Y_\heartsuit Y_\spadesuit^2 - Y_\heartsuit^2\frac{(Y_\spadesuit^{\rm eq})^2}{Y_\heartsuit^{\rm eq}}\right] - 2\langle \sigma v^2\rangle_{\heartsuit\heartsuit\heartsuit\to\bar{\spadesuit}\bar{\spadesuit}}\left[Y_\heartsuit^3 - Y_\spadesuit^2\frac{(Y_\heartsuit^{\rm eq})^3}{(Y_\spadesuit^{\rm eq})^2}\right]\Bigg\} - \frac{s}{Hx}\Bigg\{\langle \sigma v\rangle_{\spadesuit\bar{\spadesuit}\to\heartsuit\bar{\heartsuit}}\left[Y_\spadesuit^2 - Y_\heartsuit^2\frac{(Y_\spadesuit^{\rm eq})^2}{(Y_\heartsuit^{\rm eq})^2}\right] - \langle \sigma v\rangle_{\heartsuit\bar{\heartsuit}\to\spadesuit\bar{\spadesuit}}\left[Y_\heartsuit^2 - Y_\spadesuit^2\frac{(Y_\heartsuit^{\rm eq})^2}{(Y_\spadesuit^{\rm eq})^2}\right]\Bigg\}$$

s : entropy density $\quad$ H : Hubble parameter

$$x \equiv \frac{m_\heartsuit}{T} \qquad Y_j^{\rm eq} = \frac{45\sqrt{2}}{8\pi^{7/2}}\frac{g_j}{g_{\star s}(x)}(r_j x)^{3/2} e^{-r_j x} \qquad r_j \equiv \frac{m_j}{m_\heartsuit}$$

Model A : $\heartsuit^3 \spadesuit^2$

$\heartsuit\,\spadesuit$: S

- The Boltzmann equations of comoving number yields $Y_\heartsuit$, $Y_\spadesuit$

$$\frac{dY_\heartsuit}{dx} = -\frac{s^2}{Hx}\Bigg\{3\,\langle\sigma v^2\rangle_{\heartsuit\heartsuit\heartsuit\to\bar{\spadesuit}\bar{\spadesuit}}\left[Y_\heartsuit^3 - Y_\spadesuit^2\frac{(Y_\heartsuit^{\mathrm{eq}})^3}{(Y_\spadesuit^{\mathrm{eq}})^2}\right] + \langle\sigma v^2\rangle_{\heartsuit\heartsuit\spadesuit\to\bar{\heartsuit}\bar{\spadesuit}}\left(Y_\heartsuit^2 Y_\spadesuit - Y_\heartsuit Y_\spadesuit Y_\heartsuit^{\mathrm{eq}}\right) - \langle\sigma v^2\rangle_{\heartsuit\spadesuit\spadesuit\to\bar{\heartsuit}\bar{\heartsuit}}\left[Y_\heartsuit Y_\spadesuit^2 - Y_\heartsuit^2\frac{(Y_\spadesuit^{\mathrm{eq}})^2}{Y_\heartsuit^{\mathrm{eq}}}\right]\Bigg\}$$

$$-\frac{s}{Hx}\Bigg\{\langle\sigma v\rangle_{\heartsuit\bar{\heartsuit}\to\spadesuit\bar{\spadesuit}}\left[Y_\heartsuit^2 - Y_\spadesuit^2\frac{(Y_\heartsuit^{\mathrm{eq}})^2}{(Y_\spadesuit^{\mathrm{eq}})^2}\right] - \langle\sigma v\rangle_{\spadesuit\bar{\spadesuit}\to\heartsuit\bar{\heartsuit}}\left[Y_\spadesuit^2 - Y_\heartsuit^2\frac{(Y_\spadesuit^{\mathrm{eq}})^2}{(Y_\heartsuit^{\mathrm{eq}})^2}\right]\Bigg\}$$

$$\langle\sigma v^2\rangle_{\heartsuit\heartsuit\heartsuit\to\bar{\spadesuit}\bar{\spadesuit}} = \frac{c^2(m_\heartsuit/\Lambda)^2}{2304\pi m_\heartsuit^5}\sqrt{9-4r_\spadesuit^2}$$

$$\langle\sigma v^2\rangle_{\heartsuit\heartsuit\spadesuit\to\bar{\heartsuit}\bar{\spadesuit}} = \frac{\sqrt{3}\,c^2(m_\heartsuit/\Lambda)^2}{128\pi m_\heartsuit^5}\,\frac{\sqrt{4r_\spadesuit^2+8r_\spadesuit+3}}{r_\spadesuit(r_\spadesuit+2)^2}$$

$$\langle\sigma v^2\rangle_{\heartsuit\spadesuit\spadesuit\to\bar{\heartsuit}\bar{\heartsuit}} = \frac{c^2(m_\heartsuit/\Lambda)^2}{256\pi m_\heartsuit^5}\,\frac{\sqrt{4r_\spadesuit^2+4r_\spadesuit-3}}{r_\spadesuit^2(2r_\spadesuit+1)}$$

$$\langle\sigma v\rangle_{\heartsuit\bar{\heartsuit}\to\spadesuit\bar{\spadesuit}} = \frac{c^4}{32\pi(4\pi)^8 m_\heartsuit^2}\,\mathcal{I}^2\!\left(\frac{\Lambda}{m_\heartsuit}\right)\sqrt{1-r_\spadesuit^2}$$

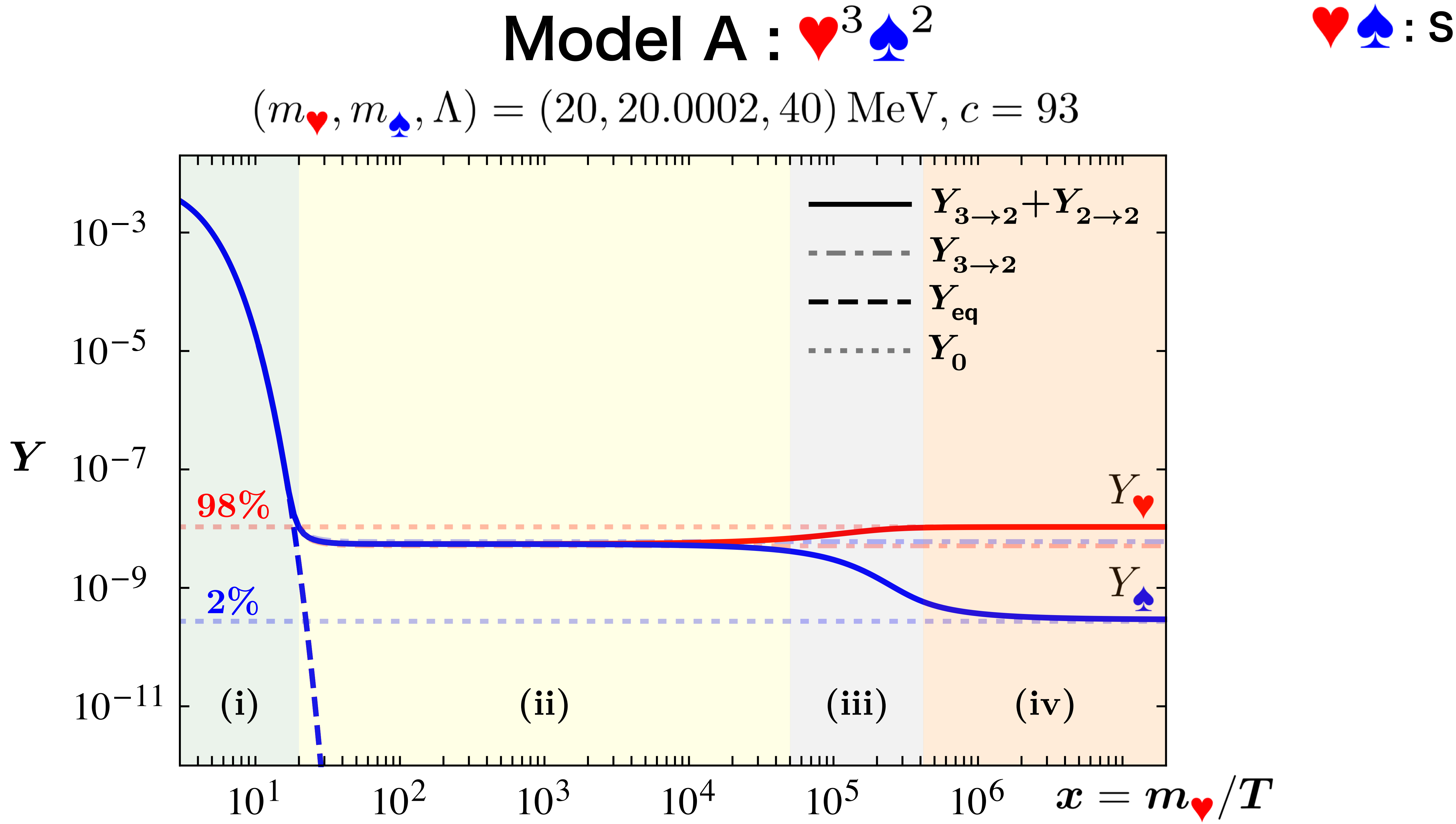
$$\langle\sigma v\rangle_{\spadesuit\bar{\spadesuit}\to\heartsuit\bar{\heartsuit}} = \frac{c^4}{32\pi(4\pi)^8 m_\heartsuit^2}\,\mathcal{I}^2\!\left(\frac{\Lambda}{m_\heartsuit}\right)\frac{\sqrt{r_\spadesuit^2-1}}{r_\spadesuit^3}$$

$$\Lambda \gtrsim m_{\heartsuit,\spadesuit}$$

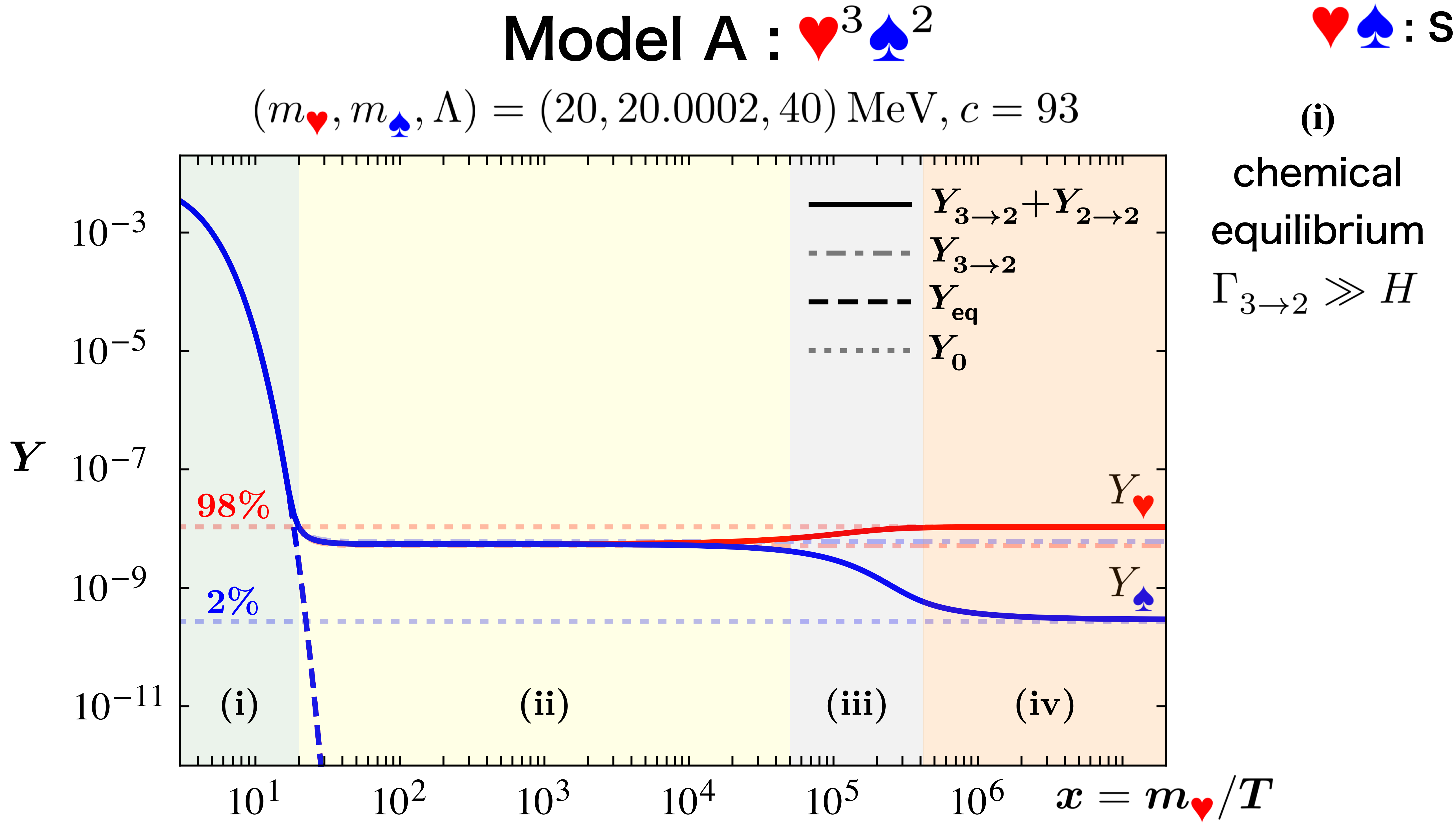
two-loop function (sunset diagram)

$$\mathcal{I}(t) \approx 1 + \frac{1}{2t^2}\left[4 - 3\ln t^2 - \frac{3}{2}\left(\ln t^2\right)^2\right]$$

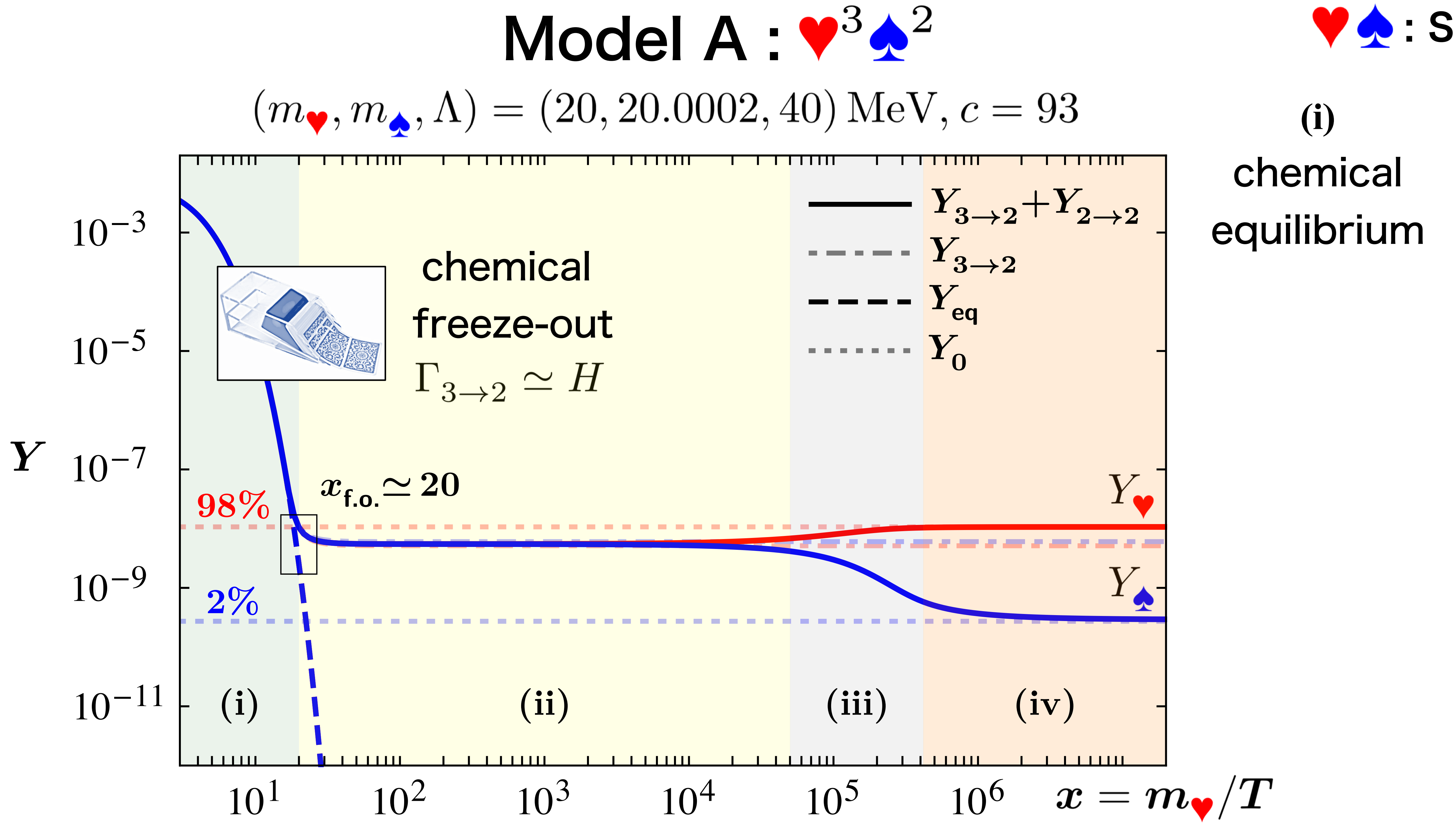
J. F. Yang, J. Zhou & C. Wu (2003)



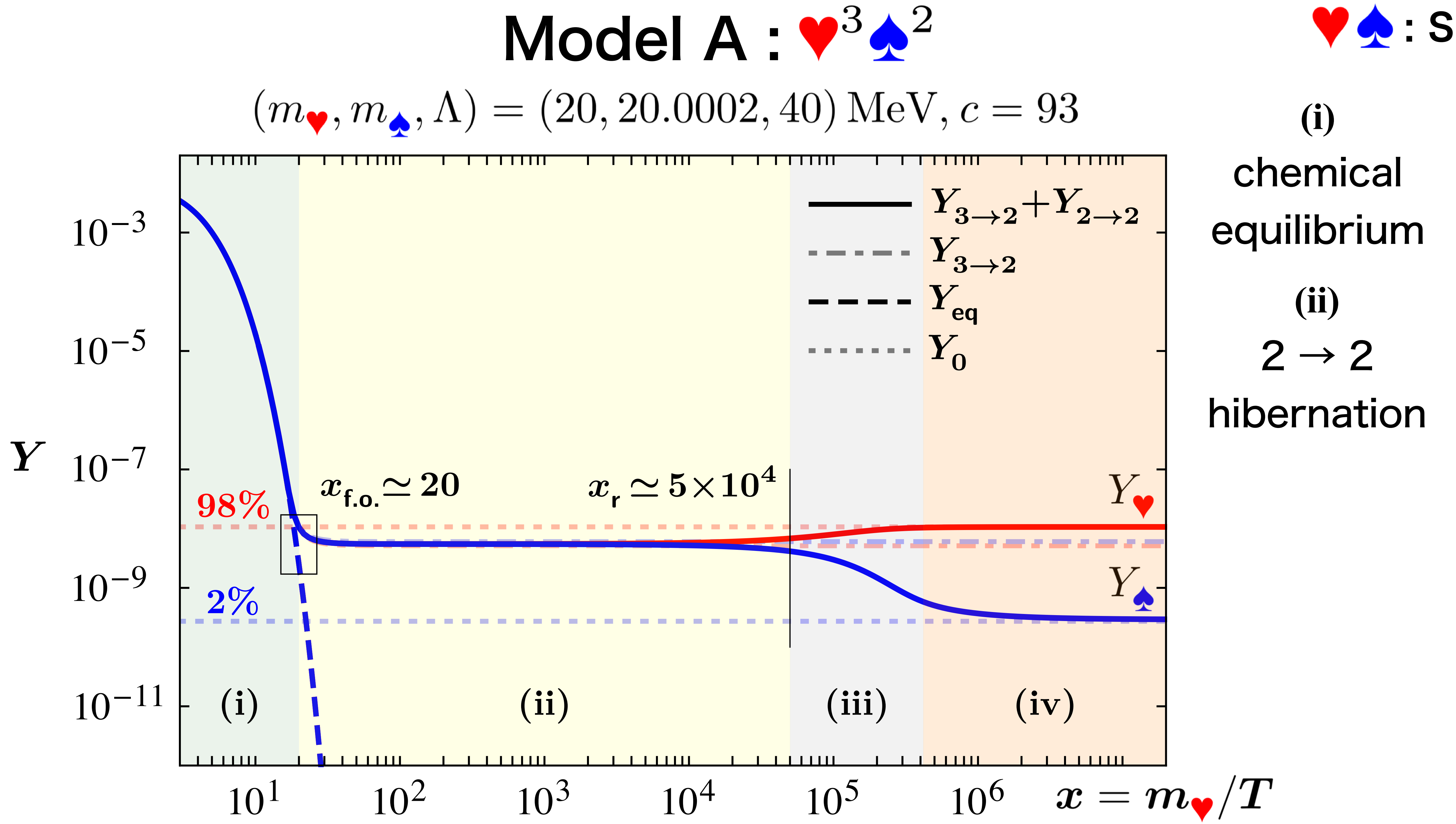
Model A : ♥³♠²
♥♠ : S
$(m_{♥}, m_{♠}, \Lambda) = (20, 20.0002, 40)$ MeV, $c = 93$
$Y_{3\to2}+Y_{2\to2}$
$Y_{3\to2}$
Y_{eq}
Y_0
Y
$Y_{♥}$
$Y_{♠}$
98%
2%
(i)
(ii)
(iii)
(iv)
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^{1}
10^{2}
10^{3}
10^{4}
10^{5}
10^{6}
$x = m_{♥}/T$



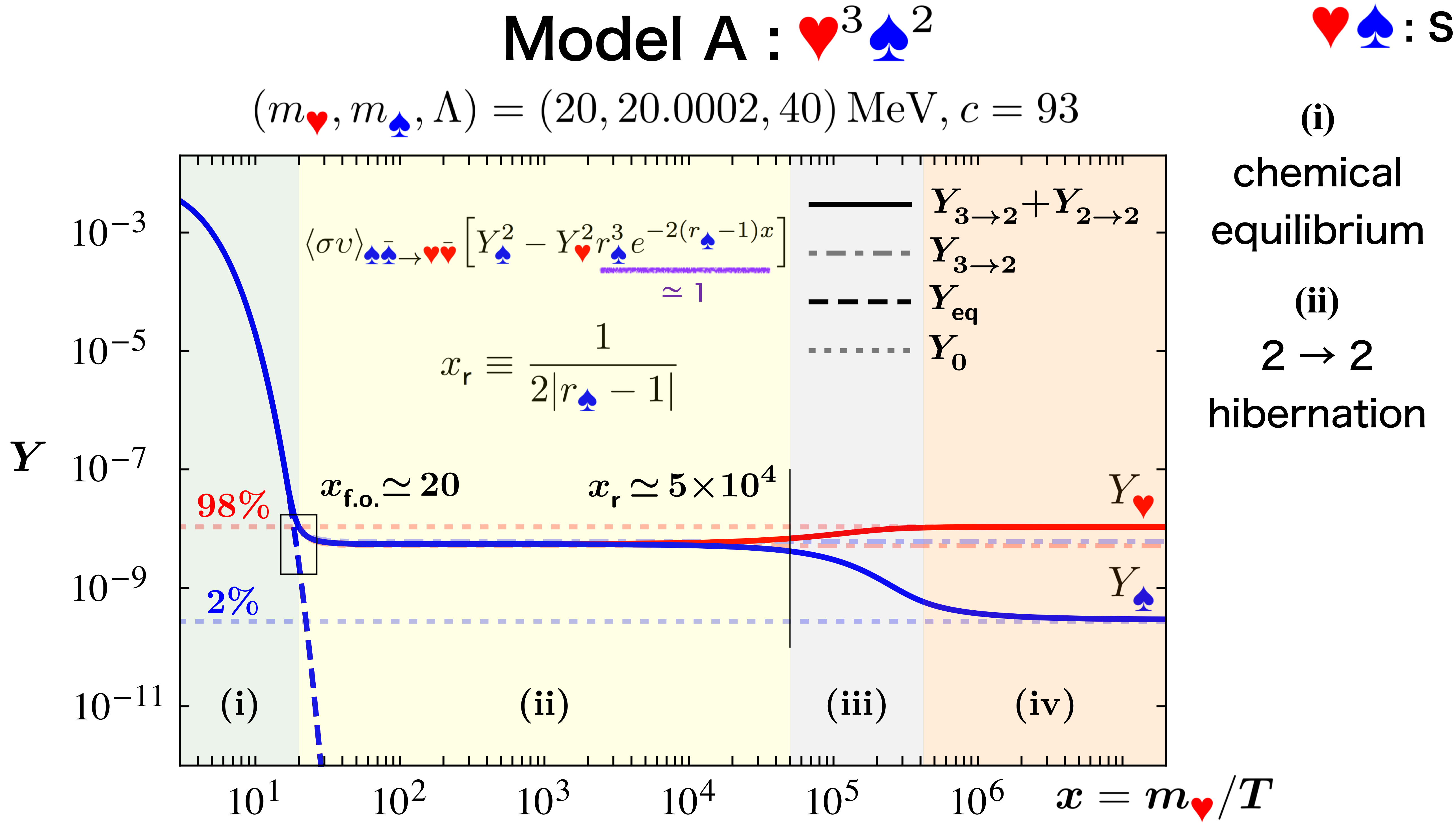
Model A : ♥³♠²
(m♥, m♠, Λ) = (20, 20.0002, 40) MeV, c = 93
♥♠ : S
(i)
chemical
equilibrium
Γ3→2 ≫ H
Y3→2+Y2→2
Y3→2
Yeq
Y0
98%
2%
Y♥
Y♠
(i)
(ii)
(iii)
(iv)
Y
10−3
10−5
10−7
10−9
10−11
10¹
10²
10³
10⁴
10⁵
10⁶
x = m♥/T



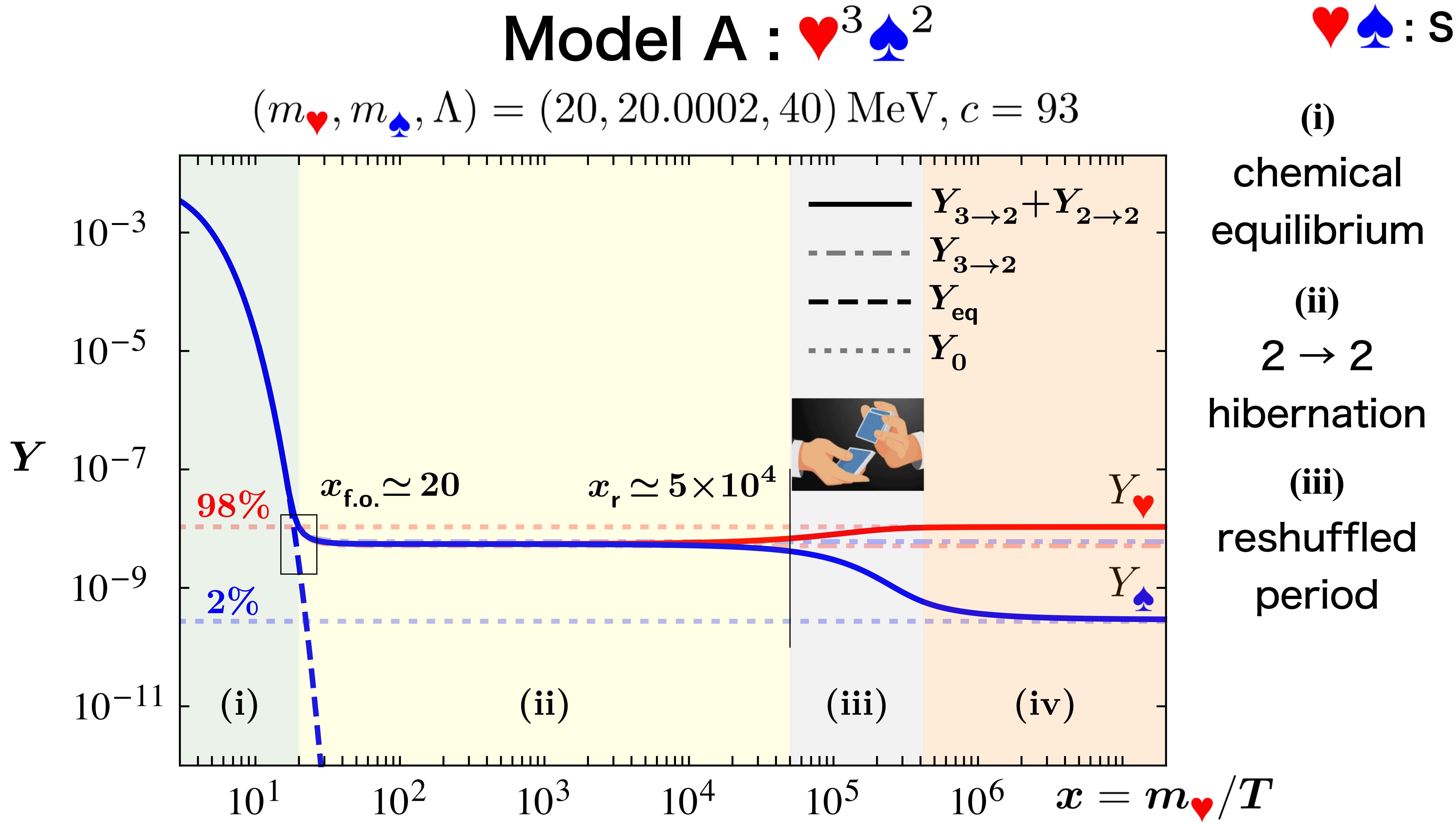
Model A : ♥³♠²
♥♠ : S
(m♥, m♠, Λ) = (20, 20.0002, 40) MeV, c = 93
(i)
chemical
equilibrium
Y₃→₂+Y₂→₂
Y₃→₂
Y_eq
Y₀
chemical
freeze-out
Γ₃→₂ ≃ H
x_f.o. ≃ 20
98%
2%
Y♥
Y♠
Y
10−3
10−5
10−7
10−9
10−11
(i)
(ii)
(iii)
(iv)
10¹
10²
10³
10⁴
10⁵
10⁶
x = m♥/T



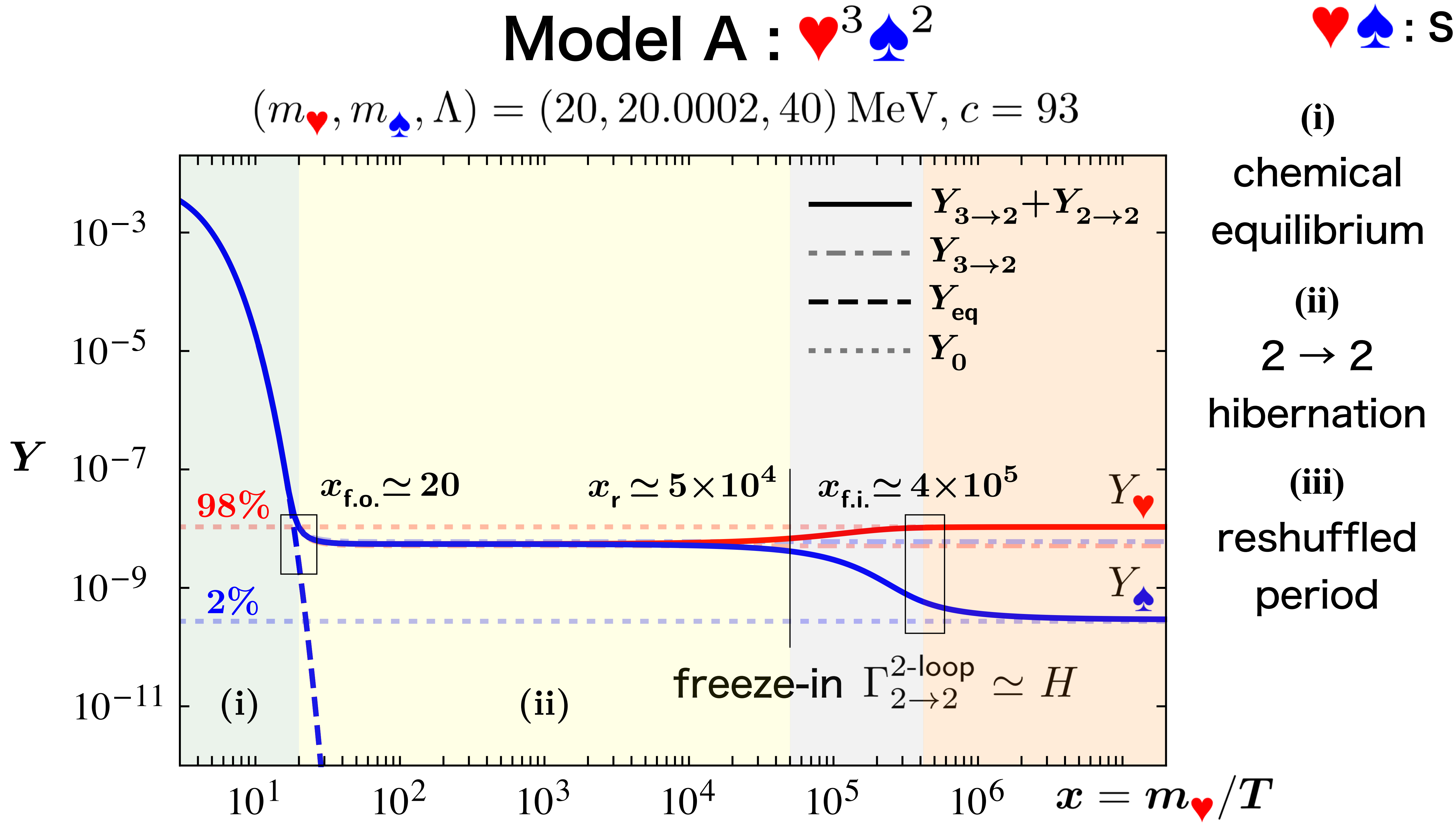
Model A : ♥³♠²
♥♠ : S
(m♥, m♠, Λ) = (20, 20.0002, 40) MeV, c = 93
(i)
chemical
equilibrium
(ii)
2 → 2
hibernation
Y₃→₂+Y₂→₂
Y₃→₂
Y_eq
Y₀
x_f.o. ≃ 20
x_r ≃ 5×10⁴
98%
2%
Y♥
Y♠
(i)
(ii)
(iii)
(iv)
Y
10⁻³
10⁻⁵
10⁻⁷
10⁻⁹
10⁻¹¹
10¹
10²
10³
10⁴
10⁵
10⁶
x = m♥/T



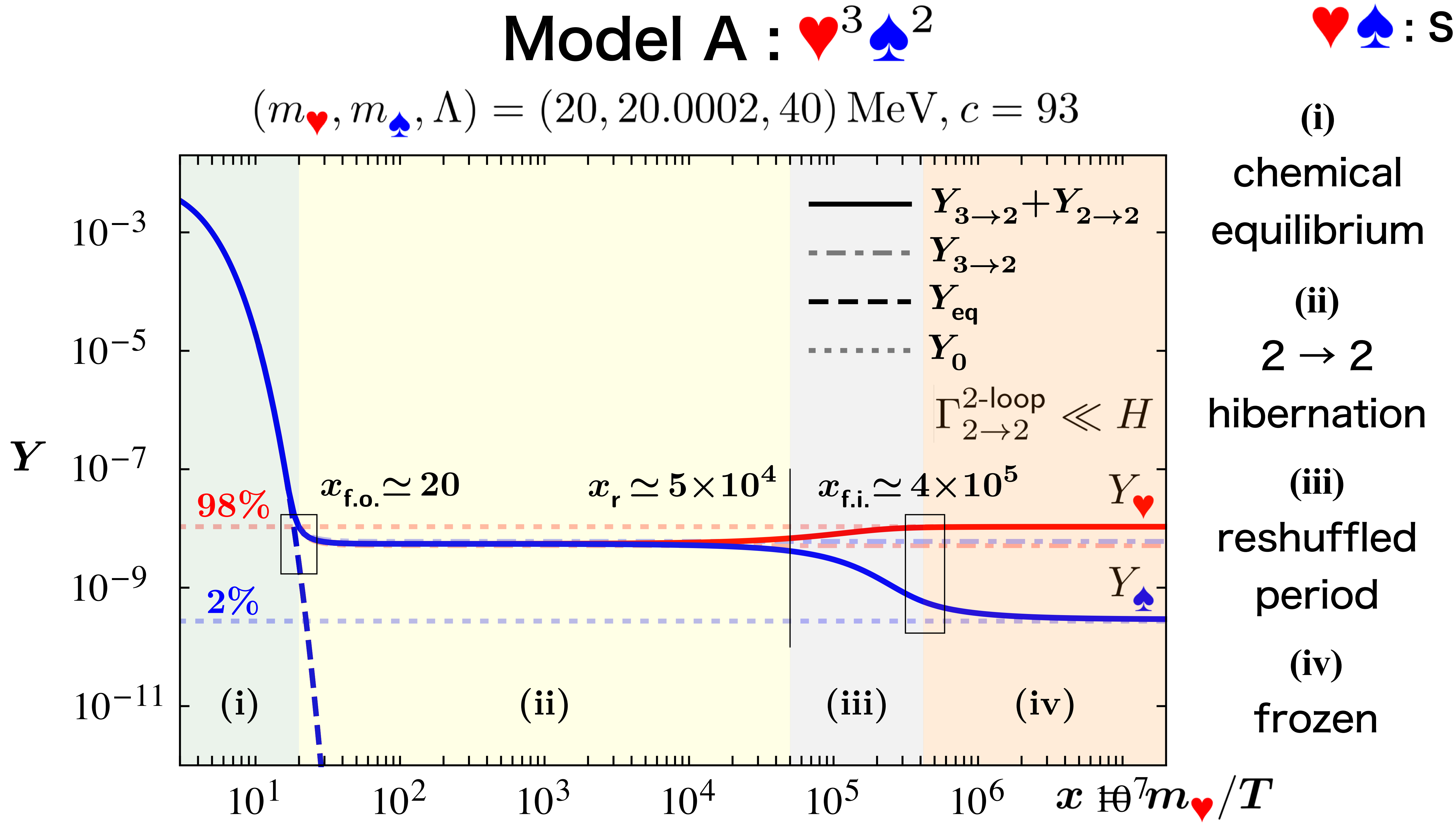
Model A : ♥³♠²
♥♠ : S
$(m_{♥}, m_{♠}, \Lambda) = (20, 20.0002, 40)$ MeV, $c = 93$
(i) chemical equilibrium
(ii) 2 → 2 hibernation
$\langle\sigma v\rangle_{♠\bar{♠}\to♥\bar{♥}}\left[Y_{♠}^2 - Y_{♥}^2 r_{♠}^3 e^{-2(r_{♠}-1)x}\right]$
≃ 1
$x_{\rm r} \equiv \frac{1}{2|r_{♠}-1|}$
$Y_{3\to2}+Y_{2\to2}$
$Y_{3\to2}$
$Y_{\rm eq}$
Y_0
$x_{\rm f.o.} \simeq 20$
$x_{\rm r} \simeq 5\times10^4$
98%
2%
$Y_{♥}$
$Y_{♠}$
(i)
(ii)
(iii)
(iv)
Y
$x = m_{♥}/T$



Model A : ♥^{3}♠^{2}
♥♠ : S
$(m_{♥}, m_{♠}, \Lambda) = (20, 20.0002, 40)$ MeV, $c = 93$
(i) chemical equilibrium
(ii) 2 → 2 hibernation
(iii) reshuffled period
$Y_{3\to2}+Y_{2\to2}$
$Y_{3\to2}$
$Y_{\rm eq}$
Y_0
$x_{\rm f.o.} \simeq 20$
$x_{\rm r} \simeq 5\times10^4$
98%
2%
$Y_{♥}$
$Y_{♠}$
(i)
(ii)
(iii)
(iv)
Y
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^1
10^2
10^3
10^4
10^5
10^6
$x = m_{♥}/T$



Model A : ♥³♠²
♥♠ : S
$(m_{\heartsuit}, m_{\spadesuit}, \Lambda) = (20, 20.0002, 40)$ MeV, $c = 93$
(i) chemical equilibrium
(ii) 2 → 2 hibernation
(iii) reshuffled period
$Y_{3\to2}+Y_{2\to2}$
$Y_{3\to2}$
$Y_{\rm eq}$
Y_0
Y
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
98%
2%
$x_{\rm f.o.} \simeq 20$
$x_{\rm r} \simeq 5\times10^4$
$x_{\rm f.i.} \simeq 4\times10^5$
$Y_{\heartsuit}$
$Y_{\spadesuit}$
(i)
(ii)
freeze-in $\Gamma^{\text{2-loop}}_{2\to2} \simeq H$
10^1
10^2
10^3
10^4
10^5
10^6
$x = m_{\heartsuit}/T$



Model A : ♥³♠²
♥♠ : S
(m♥, m♠, Λ) = (20, 20.0002, 40) MeV, c = 93
(i)
chemical
equilibrium
(ii)
2 → 2
hibernation
(iii)
reshuffled
period
(iv)
frozen
Y₃→₂+Y₂→₂
Y₃→₂
Y_eq
Y₀
Γ^2-loop_2→2 ≪ H
x_f.o. ≃ 20
x_r ≃ 5×10⁴
x_f.i. ≃ 4×10⁵
98%
2%
Y♥
Y♠
Y
(i)
(ii)
(iii)
(iv)
10⁻³
10⁻⁵
10⁻⁷
10⁻⁹
10⁻¹¹
10¹
10²
10³
10⁴
10⁵
10⁶
10⁷
x ≡ m♥/T

Model B : $\heartsuit^3\,\overline{\diamondsuit^c}\,\diamondsuit$

$\heartsuit$: S $\diamondsuit$: F

The 3 to 2 annihilation cross-sections w/ degenerate masses

$$\langle \sigma v^2 \rangle_{\heartsuit\heartsuit\heartsuit \to \bar{\diamondsuit}\bar{\diamondsuit}} = \frac{\sqrt{5}\,c^2 (m_\heartsuit/\Lambda)^4}{4608\pi m_\heartsuit^5}\left(5 + \frac{18}{x} + \frac{12}{x^2}\right)$$

SO(9) inv. form

$$\eta \equiv \tfrac{1}{2}(v_1^2 + v_2^2 + v_3^2)$$

S. M. Choi, H. M. Lee & M. S. Seo (2017)

$$\langle \sigma v^2 \rangle_{\heartsuit\heartsuit\diamondsuit \to \bar{\heartsuit}\bar{\diamondsuit}} = \frac{\sqrt{5}\,c^2 (m_\heartsuit/\Lambda)^4}{768\pi m_\heartsuit^5}\left(5 + \frac{6}{x} + \frac{4}{x^2}\right)$$

$$\langle \sigma v^2 \rangle_{\heartsuit\diamondsuit\diamondsuit \to \bar{\heartsuit}\bar{\heartsuit}} = \frac{\sqrt{5}\,c^2 (m_\heartsuit/\Lambda)^4}{768\pi m_\heartsuit^5}\left(\frac{3}{x} + \frac{2}{x^2}\right)$$

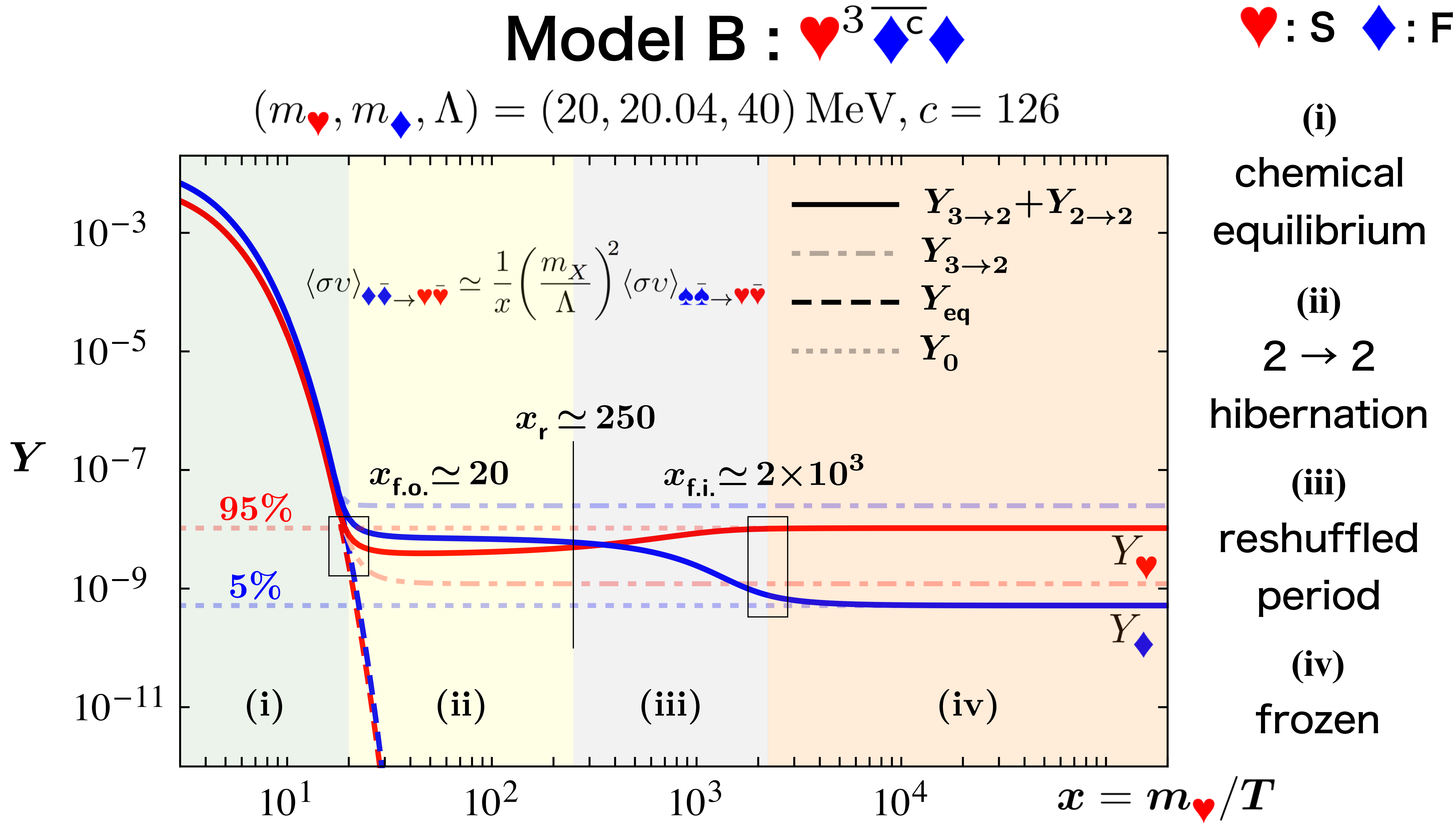
The 2 to 2 annihilation cross-sections

p-wave dominate

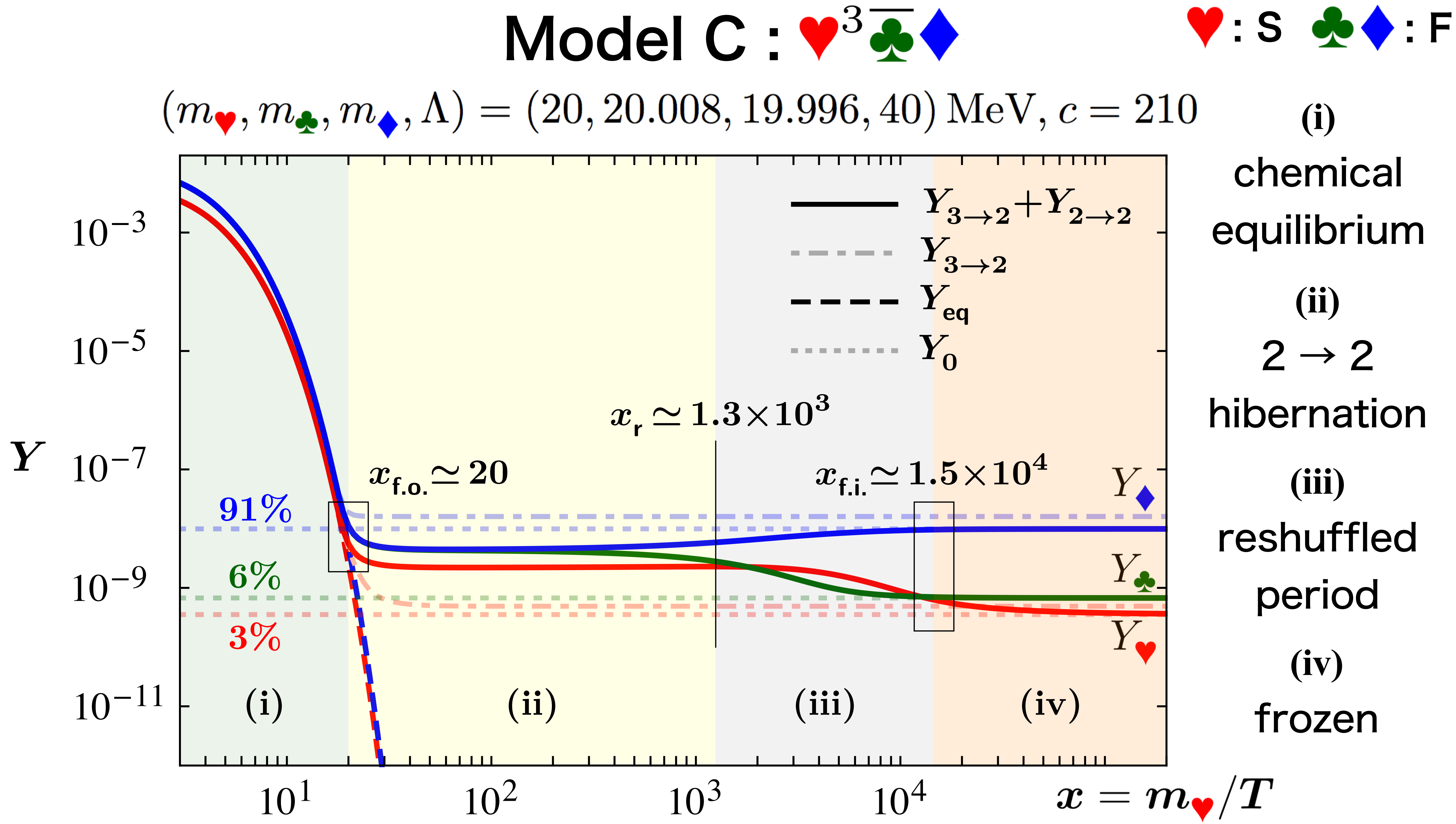
$$\langle \sigma v \rangle_{\heartsuit\bar{\heartsuit} \to \diamondsuit\bar{\diamondsuit}} = \frac{c^4 (m_\heartsuit/\Lambda)^4}{16\pi (4\pi)^8 m_\heartsuit^2}\,\mathcal{I}^2\!\left(\frac{\Lambda}{m_\heartsuit}\right)\sqrt{1 - r_\diamondsuit^2} \times\left[1 - r_\diamondsuit^2 + \frac{3}{4x}(5r_\diamondsuit^2 - 2)\right]$$

$$\langle \sigma v \rangle_{\diamondsuit\bar{\diamondsuit} \to \heartsuit\bar{\heartsuit}} = \frac{3c^4 (m_\heartsuit/\Lambda)^4}{32\pi (4\pi)^8 x\, m_\heartsuit^2}\,\mathcal{I}^2\!\left(\frac{\Lambda}{m_\heartsuit}\right)\frac{\sqrt{r_\diamondsuit^2 - 1}}{r_\diamondsuit}$$

fermion-number-violating Feynman rules A. Denner, et al. 92’

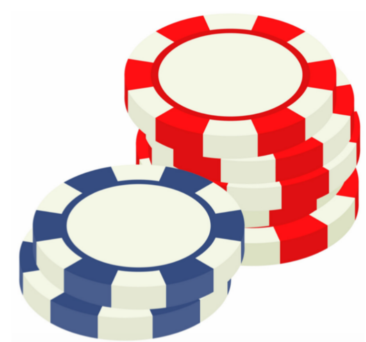


Model B : ♥³ $\overline{♦^c}$ ♦
♥ : S ♦ : F
$(m_♥, m_♦, \Lambda) = (20, 20.04, 40)$ MeV, $c = 126$
(i) chemical equilibrium
(ii) 2 → 2 hibernation
(iii) reshuffled period
(iv) frozen
$Y_{3\to2}+Y_{2\to2}$
$Y_{3\to2}$
Y_{eq}
Y_0
$\langle\sigma v\rangle_{♦\bar{♦}\to♥\bar{♥}} \simeq \frac{1}{x}\left(\frac{m_X}{\Lambda}\right)^2 \langle\sigma v\rangle_{♠\bar{♠}\to♥\bar{♥}}$
$x_r \simeq 250$
$x_{f.o.} \simeq 20$
$x_{f.i.} \simeq 2\times10^3$
95%
5%
$Y_♥$
$Y_♦$
Y
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^1
10^2
10^3
10^4
$x = m_♥/T$
(i) (ii) (iii) (iv)



Model C : ♥³ $\overline{♣}$ ♦
♥ : S ♣♦ : F
$(m_♥, m_♣, m_♦, \Lambda) = (20, 20.008, 19.996, 40)$ MeV, $c = 210$
(i)
chemical
equilibrium
(ii)
2 → 2
hibernation
(iii)
reshuffled
period
(iv)
frozen
$Y_{3\to2}+Y_{2\to2}$
$Y_{3\to2}$
$Y_{\rm eq}$
Y_0
$x_{\rm r} \simeq 1.3\times10^3$
$x_{\rm f.o.} \simeq 20$
$x_{\rm f.i.} \simeq 1.5\times10^4$
91%
6%
3%
$Y_♦$
$Y_♣$
$Y_♥$
Y
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^1
10^2
10^3
10^4
$x = m_♥/T$
(i)
(ii)
(iii)
(iv)

IV. Kinetic equilibrium



Kinetic equilibrium

♥ : S

- In the SIMP scenario, the DM particles should keep kinetic equilibrium with SM particles until the f.o. temperature

kinetic equilibrium condition

$$\gamma_e(x_{\text{f.o.}}) \gtrsim H(x_{\text{f.o.}})\, x_{\text{f.o.}}^2$$

momentum relaxation rate

S. M. Choi, *et al.* (2019)

$$\gamma(T) = \sum_i \frac{g_i}{6 m_\chi T} \int_0^\infty \frac{d^3\mathbf{p}}{(2\pi)^3} f_i (1 \pm f_i) \frac{p}{\sqrt{p^2 + m_i^2}} \times \int_{-4p^2}^{0} dt(-t) \frac{d\sigma_{\chi+i\to\chi+i}}{dt}.$$

P. Gondolo, *et al.* (2012)

$$\frac{d\sigma_{\chi+i\to\chi+i}}{dt} = \frac{1}{64\pi m_\chi^2 p^2} \overline{|\mathcal{M}_{\chi+i\to\chi+i}|^2}$$

Kinetic equilibrium

♥ : S

- 🎲 In the SIMP scenario, the DM particles should keep kinetic equilibrium with SM particles until the f.o. temperature

kinetic equilibrium condition

$$\gamma_e(x_{\text{f.o.}}) \gtrsim H(x_{\text{f.o.}})\, x_{\text{f.o.}}^2$$

momentum relaxation rate

S. M. Choi, *et al.* (2019)

$$\gamma_e(x) = \frac{31\pi^3}{189\, x^6} \frac{m_{♥}^5}{\Lambda_{Z'}^4} \sum_{j=\text{SIMP}} c_{je}^2$$

$$\mathcal{O}_{\phi e}^{(6)} = \frac{c_{\phi e}}{\Lambda_{Z'}^2} \left(i\phi^\dagger \overleftrightarrow{\partial_\mu} \phi \right) \left(\bar{e} \gamma^\mu e \right) , \quad \phi = ♥, ♠$$

$$\mathcal{O}_{\psi e}^{(6)} = \frac{c_{\psi e}}{\Lambda_{Z'}^2} \left(\overline{\psi} \gamma_\mu \psi \right) \left(\bar{e} \gamma^\mu e \right) , \quad \psi = ♣, ♦$$

Kinetic equilibrium

♥ : S

- In the SIMP scenario, the DM particles should keep kinetic equilibrium with SM particles until the f.o. temperature

kinetic equilibrium condition

$$\gamma_e(x_{\text{f.o.}}) \gtrsim H(x_{\text{f.o.}})\, x_{\text{f.o.}}^2$$

momentum relaxation rate

S. M. Choi, *et al.* (2019)

$$\gamma_e(x) = \frac{31\pi^3}{189\, x^6} \frac{m_{\heartsuit}^5}{\Lambda_{Z'}^4} \sum_{j=\text{SIMP}} c_{je}^2$$

$$\sum_{j=\text{SIMP}} c_{je}^2 \gtrsim 10^{-9} \left(\frac{\Lambda_{Z'}}{200\ \text{MeV}}\right)^4 \left(\frac{m_{\heartsuit}}{20\ \text{MeV}}\right)^{-3}$$

Kinetic equilibrium

♥ : S

- In the SIMP scenario, the DM particles should keep kinetic equilibrium with SM particles until the f.o. temperature

kinetic decoupling condition

$$\gamma_e(x_{\text{k.d.}}) \simeq 2H(x_{\text{k.d.}})$$

$$\gamma_e(x) = \frac{31\pi^3}{189\,x^6}\frac{m_{♥}^5}{\Lambda_{Z'}^4}\sum_{j=\text{SIMP}} c_{je}^2$$

Kinetic equilibrium

♥ : S

- In the SIMP scenario, the DM particles should keep kinetic equilibrium with SM particles until the f.o. temperature

kinetic decoupling condition

$$\gamma_e(x_{\text{k.d.}}) \simeq 2H(x_{\text{k.d.}})$$

$$\gamma_e(x_{\text{f.o.}}) \simeq H(x_{\text{f.o.}})x_{\text{f.o.}}^2$$

$$\gamma_e(x) = \frac{31\pi^3}{189x^6}\frac{m_\heartsuit^5}{\Lambda_{Z'}^4}\sum_{j=\text{SIMP}} c_{je}^2$$

Kinetic equilibrium

♥ : S

- In the SIMP scenario, the DM particles should keep kinetic equilibrium with SM particles until the f.o. temperature

kinetic decoupling condition

$$\gamma_e(x_{\rm k.d.}) \simeq 2H(x_{\rm k.d.})$$

$$\gamma_e(x_{\rm f.o.}) \simeq H(x_{\rm f.o.})x_{\rm f.o.}^2$$

$$\gamma_e(x) = \frac{31\pi^3}{189\,x^6}\frac{m_\heartsuit^5}{\Lambda_{Z'}^4}\sum_{j=\rm SIMP} c_{je}^2$$

$$\rightarrow\; x_{\rm k.d.} \simeq x_{\rm f.o.}^{3/2}/\sqrt[4]{2} \simeq 75 < x_{\rm f.i.}$$

$$\rightarrow\; \Gamma_{2\to2}^{\text{2-loop}} > \Gamma_{\rm el}$$

V. UV complete model

rSIMP model

🎲 Particle content and charge assignment

	H	N	X	S	Φ
SU(2)	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\mathrm{U(1)_Y}$	$-1/2$	0	0	0	0
$\mathrm{U(1)_D}$	0	$-1/8$	$+1/12$	$+1/4$	$-1/2$
$\mathbb{Z}_4$	$+1$	$\pm i$	-1	-1	$+1$

- ✓ N and X are SIMP DM candidates in our setup.
- ✓ S is a mediator connecting N and X.
- ✓ Φ develops vev that breaks the dark symmetry.

rSIMP model

- Lagrangian : Scalar sector

$$\mathcal{L}_{\text{scalar}} = \left(\mathcal{D}^\rho H\right)^\dagger \mathcal{D}_\rho H + \left(\mathcal{D}^\rho X\right)^\dagger \mathcal{D}_\rho X + \left(\mathcal{D}^\rho S\right)^\dagger \mathcal{D}_\rho S + \left(\mathcal{D}^\rho \Phi\right)^\dagger \mathcal{D}_\rho \Phi - \mathcal{V}$$

$$\begin{aligned}
\mathcal{V} = \; & \mu_h^2 H^\dagger H + \mu_X^2 X^* X + \mu_S^2 S^* S + \mu_\phi^2 \Phi^* \Phi \\
& + \lambda_h \left(H^\dagger H\right)^2 + \lambda_X \left(X^* X\right)^2 + \lambda_S \left(S^* S\right)^2 + \lambda_\phi \left(\Phi^* \Phi\right)^2 \\
& + \lambda_{hX} \left(H^\dagger H\right)\left(X^* X\right) + \lambda_{hS} \left(H^\dagger H\right)\left(S^* S\right) + \lambda_{h\phi} \left(H^\dagger H\right)\left(\Phi^* \Phi\right) \\
& + \lambda_{XS} \left(X^* X\right)\left(S^* S\right) + \lambda_{X\phi} \left(X^* X\right)\left(\Phi^* \Phi\right) + \lambda_{S\phi} \left(S^* S\right)\left(\Phi^* \Phi\right) \\
& + \left(\lambda_3 X^3 S^* + \tfrac{1}{\sqrt{2}} \kappa v_\phi S^2 \Phi + \text{h.c.}\right)
\end{aligned}$$

rSIMP model

Lagrangian : Scalar sector

$$\mathcal{L}_{\text{scalar}} = (\mathcal{D}^\rho H)^\dagger \mathcal{D}_\rho H + (\mathcal{D}^\rho X)^\dagger \mathcal{D}_\rho X + (\mathcal{D}^\rho S)^\dagger \mathcal{D}_\rho S + (\mathcal{D}^\rho \Phi)^\dagger \mathcal{D}_\rho \Phi - \mathcal{V}$$

$$\begin{aligned}\mathcal{V} = &\mu_h^2 H^\dagger H + \mu_X^2 X^* X + \mu_S^2 S^* S + \mu_\phi^2 \Phi^* \Phi \\ &+ \lambda_h (H^\dagger H)^2 + \lambda_X (X^* X)^2 + \lambda_S (S^* S)^2 + \lambda_\phi (\Phi^* \Phi)^2 \\ &+ \lambda_{hX} (H^\dagger H)(X^* X) + \lambda_{hS} (H^\dagger H)(S^* S) + \lambda_{h\phi} (H^\dagger H)(\Phi^* \Phi) \\ &+ \lambda_{XS} (X^* X)(S^* S) + \lambda_{X\phi} (X^* X)(\Phi^* \Phi) + \lambda_{S\phi} (S^* S)(\Phi^* \Phi) \\ &+ \left(\lambda_3 X^3 S^* + \frac{1}{\sqrt{2}} \kappa v_\phi S^2 \Phi + \text{h.c.}\right)\end{aligned}$$

Vacuum stability & self-interacting of DM

Higgs invisible decay

3 to 2 process

$U(1)_D$ symmetry breaking term

rSIMP model

- Lagrangian : Yukawa sector

$$\mathcal{L}_N = \overline{N}\left(i\gamma^\rho \mathcal{D}_\rho - m_N\right)N - \frac{1}{2}\left(y_N \overline{N^{\mathsf{c}}} N S + \mathrm{h.c.}\right)$$

$m_S > 2m_N$ or $m_S > 3m_X$ $\quad$ S is unstable

rSIMP model

- Lagrangian : Yukawa sector

$$\mathcal{L}_N = \overline{N}\left(i\gamma^\rho \mathcal{D}_\rho - m_N\right)N - \frac{1}{2}\left(y_N \overline{N^{\mathsf{c}}} N S + \mathrm{h.c.}\right)$$

$m_S > 2m_N$ or $m_S > 3m_X$ S is unstable

- Lagrangian : Gauge sector

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}W^{3\rho\sigma}W^3_{\rho\sigma} - \frac{1}{4}B^{\rho\sigma}B_{\rho\sigma} - \frac{1}{4}C^{\rho\sigma}C_{\rho\sigma} - \frac{1}{2}s_\epsilon B^{\rho\sigma}C_{\rho\sigma} - \frac{1}{2}m_C^2 C^\rho C_\rho$$

rSIMP model

- Lagrangian : Yukawa sector

$$\mathcal{L}_N = \overline{N}\left(i\gamma^\rho \mathcal{D}_\rho - m_N\right)N - \frac{1}{2}\left(y_N \overline{N^{\mathsf{c}}} N S + \text{h.c.}\right)$$

$m_S > 2m_N$ or $m_S > 3m_X$ S is unstable

- Lagrangian : Gauge sector

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}W^{3\rho\sigma}W^3_{\rho\sigma} - \frac{1}{4}B^{\rho\sigma}B_{\rho\sigma} - \frac{1}{4}C^{\rho\sigma}C_{\rho\sigma} - \frac{1}{2}s_\epsilon B^{\rho\sigma}C_{\rho\sigma} - \frac{1}{2}m_C^2 C^\rho C_\rho$$

$$\mathcal{D}_\rho \supset i\left(g_{\mathsf{D}}\mathcal{Q}_{\mathsf{D}} - g_e c_{\mathsf{W}}\epsilon \mathcal{Q}_e\right)Z'_\rho$$

$$\epsilon \ll 1 \qquad m_Z^2 \gg m_{Z'}^2$$

rSIMP model

- Lagrangian : Yukawa sector

$$\mathcal{L}_N = \overline{N}\left(i\gamma^\rho \mathcal{D}_\rho - m_N\right)N - \frac{1}{2}\left(y_N \overline{N^{\mathsf{c}}} N S + \text{h.c.}\right)$$

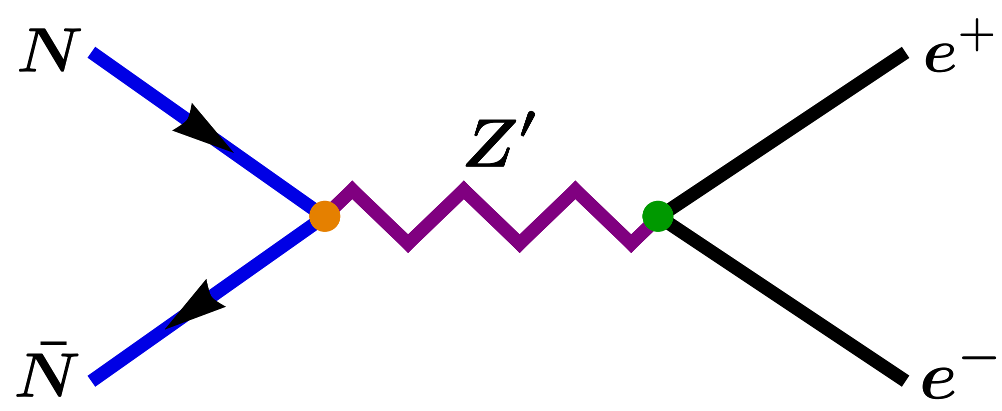
$m_S > 2m_N$ or $m_S > 3m_X$ S is unstable

- Lagrangian : Gauge sector

$$\mathcal{L}_{\mathsf{gauge}} = -\frac{1}{4}W^{3\rho\sigma}W^3_{\rho\sigma} - \frac{1}{4}B^{\rho\sigma}B_{\rho\sigma} - \frac{1}{4}C^{\rho\sigma}C_{\rho\sigma} - \frac{1}{2}s_\epsilon B^{\rho\sigma}C_{\rho\sigma} - \frac{1}{2}m_C^2 C^\rho C_\rho$$

$$\mathcal{D}_\rho \supset i\left(g_{\mathsf{D}}\mathcal{Q}_{\mathsf{D}} - g_e c_{\mathsf{W}}\epsilon \mathcal{Q}_e\right)Z'_\rho$$

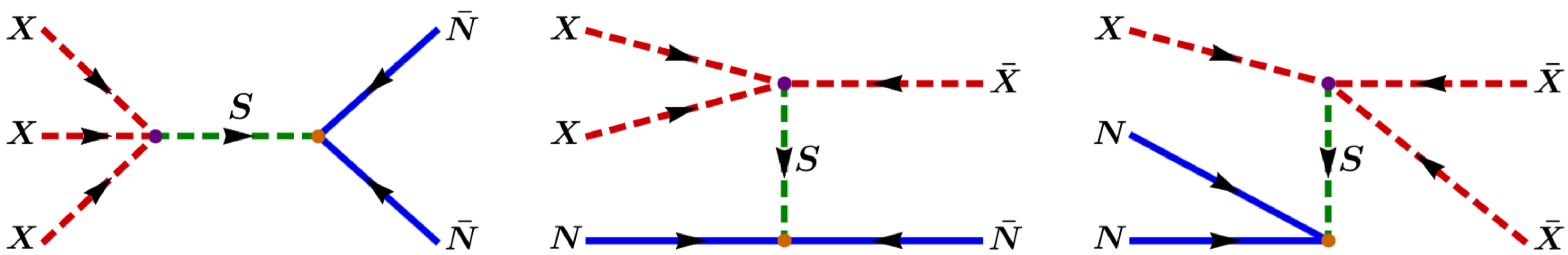
$$\epsilon \ll 1 \qquad m_Z^2 \gg m_{Z'}^2$$



Annihilation process

- Interactions for the annihilations

$$\mathcal{L}_{\text{ann}} = -\lambda_3\left[X^3 S^* + (X^*)^3 S\right] - \frac{1}{2} y_N \left(\overline{N^{\mathsf{c}}} N S + \overline{N} N^{\mathsf{c}} S^* \right)$$



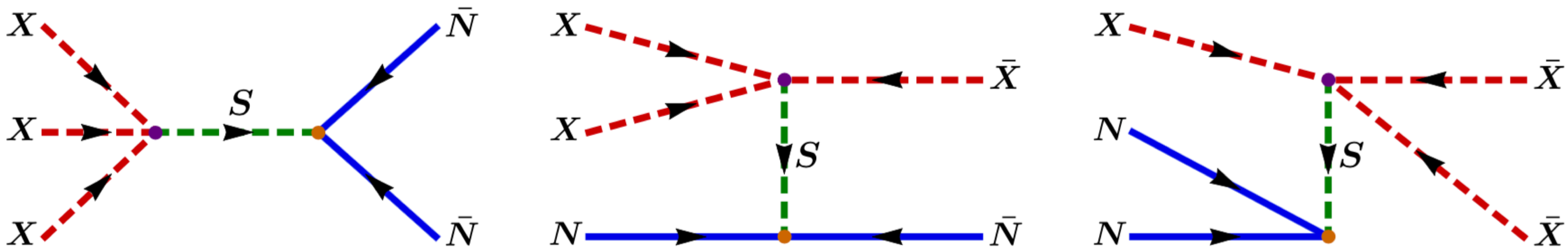
$$3m_X > 2m_N > m_X$$

Annihilation process

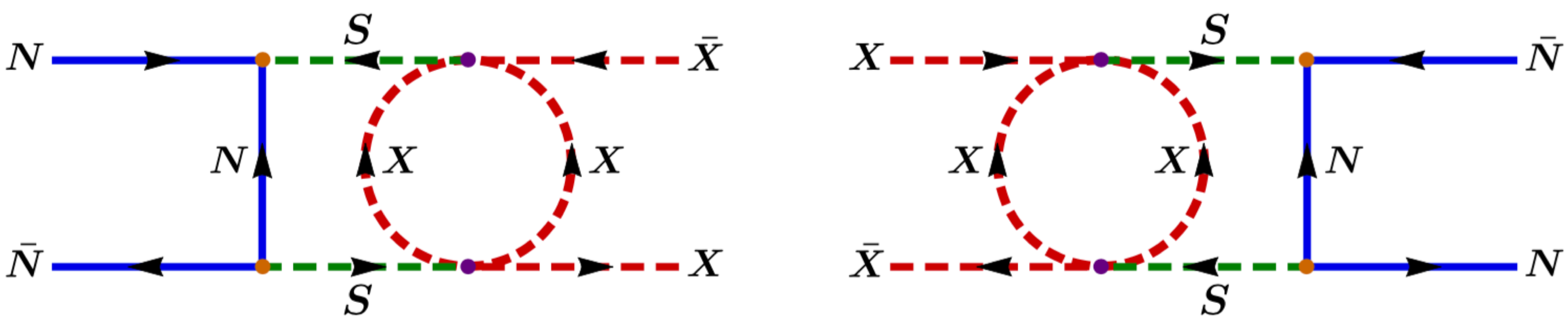
- Interactions for the annihilations

−1 −1 i i −1

$$\mathcal{L}_{\text{ann}} = -\lambda_3\left[X^3 S^* + (X^*)^3 S\right] - \frac{1}{2} y_N \left(\overline{N^{c}} N S + \overline{N} N^{c} S^* \right)$$

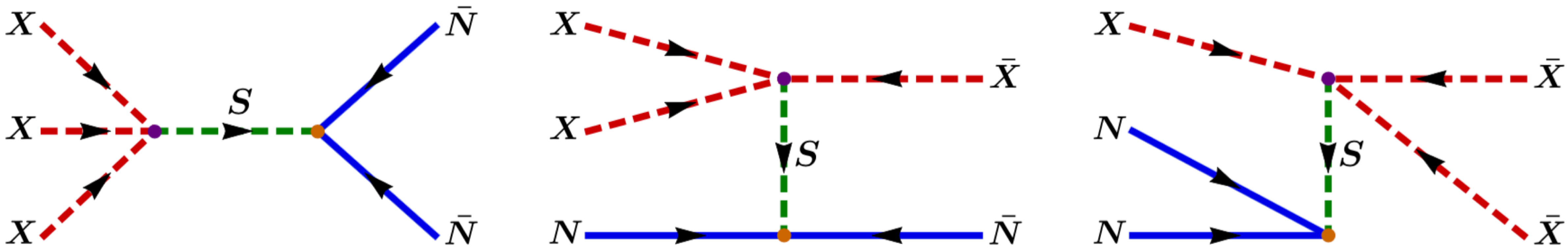


$$3m_X > 2m_N > m_X$$



Annihilation cross section

- 3 to 2 processes :



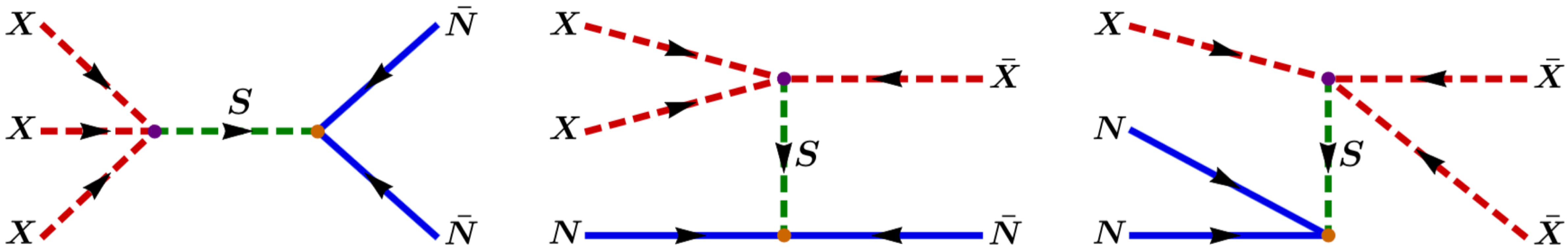
$$(\sigma v^2)_{XXX\to\bar{N}\bar{N}} = \frac{\lambda_3^2 y_N^2}{128\pi m_X^5}\frac{\left(9-4r_N^2\right)^{3/2}}{\left(9-r_S^2\right)^2}$$

$$(\sigma v^2)_{XXN\to\bar{X}\bar{N}} = \frac{9\sqrt{3}\,\lambda_3^2 y_N^2}{32\pi m_X^5}\frac{\left(1+r_N\right)\left(1+2r_N+2r_N^2\right)\sqrt{3+8r_N+4r_N^2}}{\left(2+r_N\right)^2\left[r_S^2\left(1+r_N\right)+2r_N\right]^2}$$

$$(\sigma v^2)_{XNN\to\bar{X}\bar{X}} = \mathcal{O}(v^2) \qquad r_{N,S} \equiv m_{N,S}/m_X$$

Annihilation cross section

- 3 to 2 processes :



$$(\sigma v^2)_{XXX\to\bar{N}\bar{N}} = \frac{\lambda_3^2 y_N^2}{128\pi m_X^5}\frac{\left(9-4r_N^2\right)^{3/2}}{\left(9-r_S^2\right)^2}$$

resonant SIMP

S. M. Choi, *et al.*, 2016

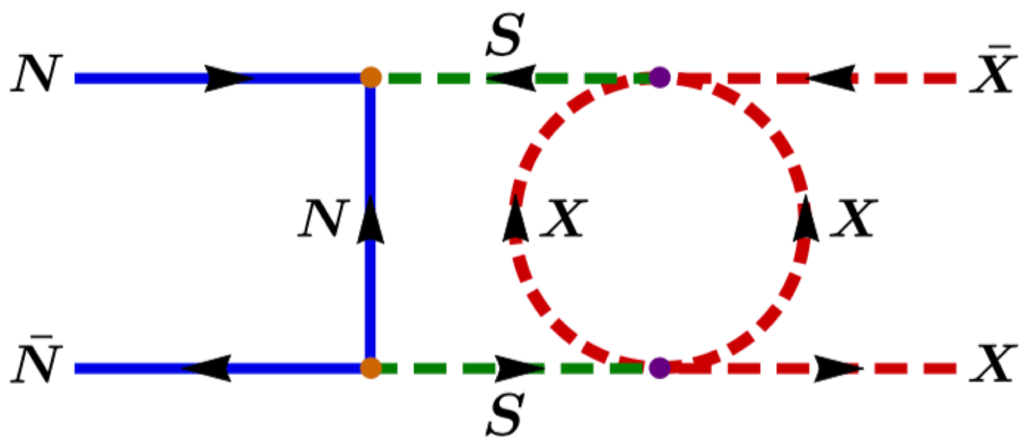
$$(\sigma v^2)_{XXN\to\bar{X}\bar{N}} = \frac{9\sqrt{3}\,\lambda_3^2 y_N^2}{32\pi m_X^5}\frac{\left(1+r_N\right)\left(1+2r_N+2r_N^2\right)\sqrt{3+8r_N+4r_N^2}}{\left(2+r_N\right)^2\left[r_S^2\left(1+r_N\right)+2r_N\right]^2}$$

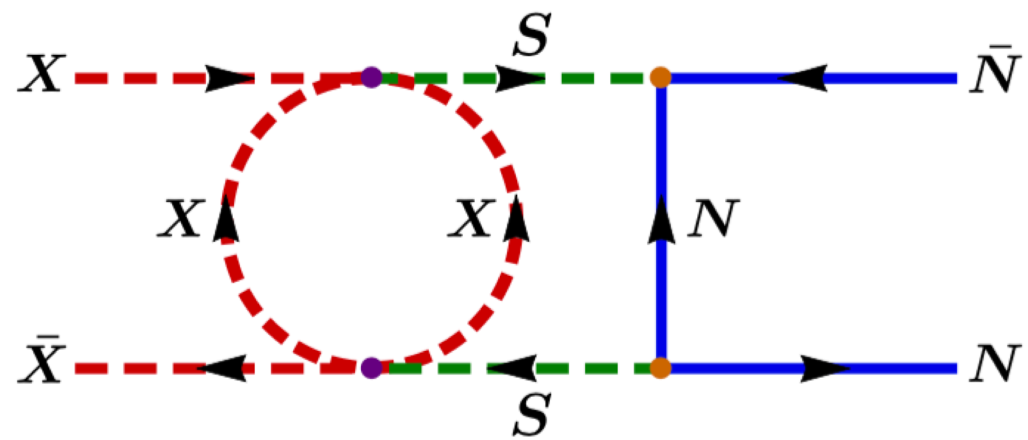
$$(\sigma v^2)_{XNN\to\bar{X}\bar{X}} = \mathcal{O}(v^2)$$

$$r_{N,S} \equiv m_{N,S}/m_X$$

Annihilation cross section

🎲 2 to 2 processes :





$$\langle \sigma v \rangle^{\text{2-loop}}_{N\bar{N}\to X\bar{X}} = \frac{81\lambda_3^4 y_N^4 \sqrt{r_N^2 - 1}}{\pi(4\pi)^8 r_S^4 m_X^2 r_N} \left[\left(r_N^2 - 1\right)|\mathcal{I}_1|^2 + \frac{\left(11 - 2r_N^2\right)|\mathcal{I}_1|^2 + 6r_N^2|\mathcal{I}_2|^2}{4x} \right]$$

$$\langle \sigma v \rangle^{\text{2-loop}}_{X\bar{X}\to N\bar{N}} = \frac{81\lambda_3^4 y_N^4 r_N^2 \sqrt{1 - r_N^2}}{\pi(4\pi)^8 r_S^4 m_X^2} \left[\left(1 - r_N^2\right)|\mathcal{I}_2|^2 + \frac{2\left(1 + 2r_N^{-2}\right)|\mathcal{I}_1|^2 + 3\left(5r_N^2 - 2\right)|\mathcal{I}_2|^2}{4x} \right]$$

Annihilation cross section

- Two-loop functions :

$$\mathcal{I}_{1,2}(r_N, r_S) = \int_0^1 \mathrm{d}z_1 \int_0^1 \mathrm{d}z_2 \int_0^{1-z_2} \mathrm{d}z_3 \int_0^{z_1(1-z_1)} \mathrm{d}z_4 \int_0^1 \mathrm{d}z_5 \, \mathcal{F}_{1,2}(r_N, r_S)$$

$$\mathcal{F}_1(r_N, r_S) = \frac{r_S^2 z_5^2 \left[2P^2 z_5^3 - \left(P^2 + 3Q^2\right) z_5^2 + \left(2Q^2 + 3\right) z_5 - 2\right]}{2\left(P^2 z_5^2 - Q^2 z_5 + 1\right)^2}$$

$$\mathcal{F}_2(r_N, r_S) = \frac{r_S^2 (1 - z_2 - z_3) z_5^3 \left(2P^2 z_5^2 - 3Q^2 z_5 + 3\right)}{2\left(P^2 z_5^2 - Q^2 z_5 + 1\right)^2}$$

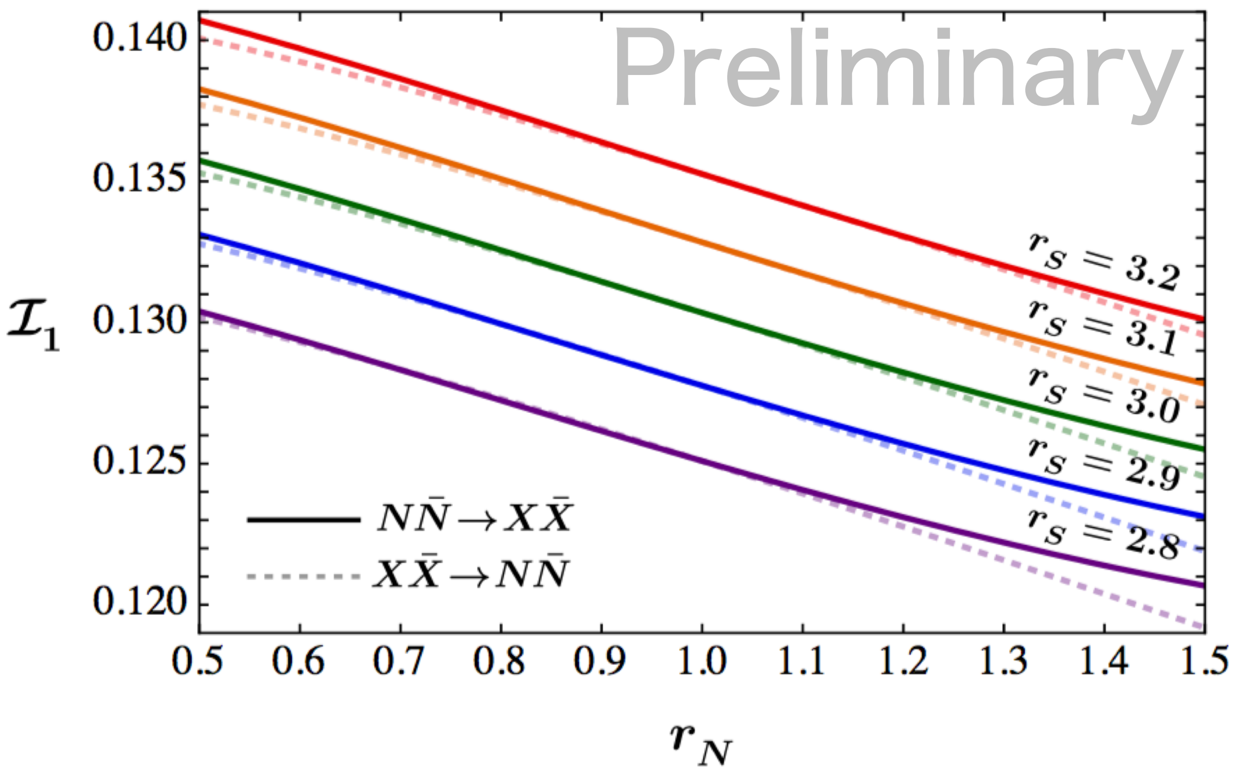
$$P^2 = \begin{cases} z_4 \left[r_N^2 (z_2 - z_3 + 1)(z_2 - z_3 - 1) + 1\right] & \text{for} \quad N\bar{N} \to X\bar{X} \\ z_4 \left[r_N^2 (z_2 + z_3 - 1)^2 - (2z_2 - 1)(2z_3 - 1)\right] & \text{for} \quad X\bar{X} \to N\bar{N} \end{cases}$$

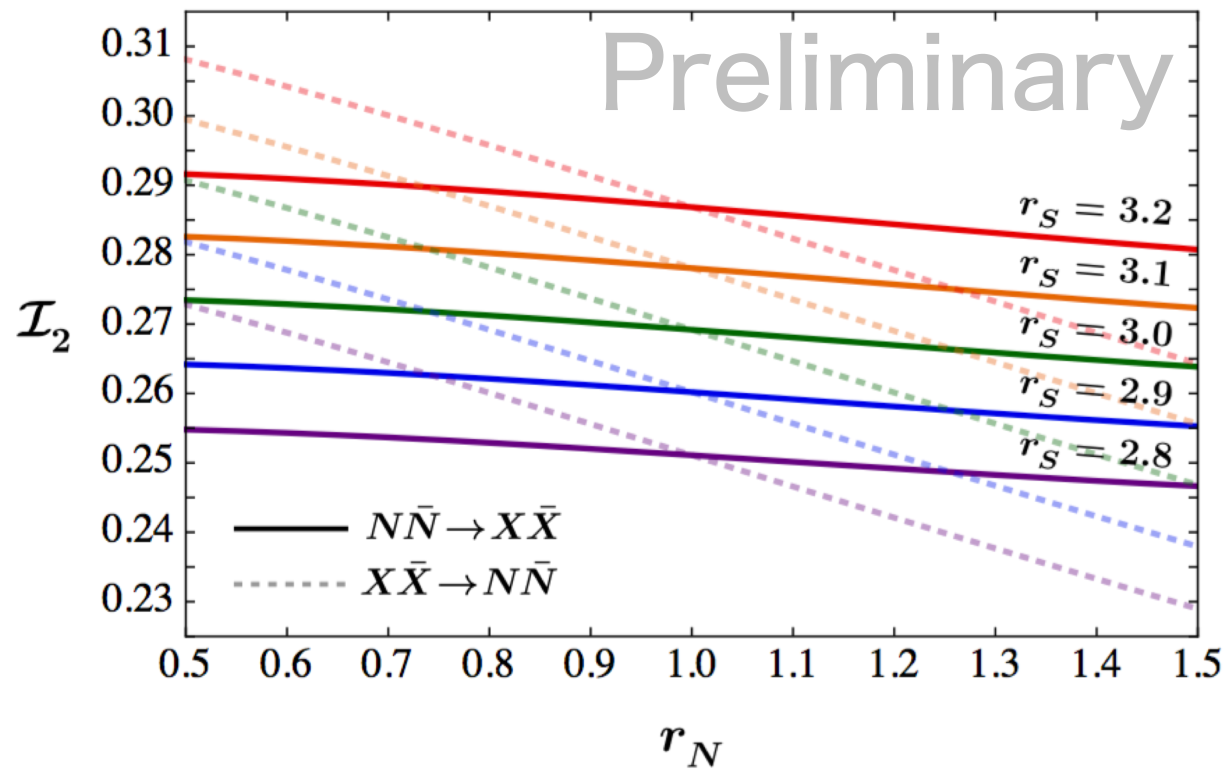
$$Q^2 = \begin{cases} 1 + z_4 \left[2r_N^2 (z_2 + z_3 - 1) - r_S^2 (z_2 + z_3) + 1\right] & \text{for} \quad N\bar{N} \to X\bar{X} \\ 1 + z_4 \left[\left(2 - r_S^2\right)(z_2 + z_3) - 1\right] & \text{for} \quad X\bar{X} \to N\bar{N} \end{cases}$$

Annihilation cross section

- Two-loop functions :

$$\mathcal{I}_{1,2}(r_N, r_S) = \int_0^1 \mathrm{d}z_1 \int_0^1 \mathrm{d}z_2 \int_0^{1-z_2} \mathrm{d}z_3 \int_0^{z_1(1-z_1)} \mathrm{d}z_4 \int_0^1 \mathrm{d}z_5 \, \mathcal{F}_{1,2}(r_N, r_S)$$





UV model & EFT theory

- UV model

$$\langle \sigma v \rangle^{\text{UV}}_{N\bar{N}\to X\bar{X}} \approx \frac{243\,\lambda_3^4\, y_N^4 \sqrt{r_N^2 - 1}}{2\pi (4\pi)^8 x m_X^2} \left(\frac{m_X}{m_S}\right)^4 |\mathcal{I}_2|^2$$

$$\mathcal{I}_2 = \mathcal{I}_2(r_N = 1, r_S = 3) \simeq 0.27$$

$r_N \simeq 1$

$\Lambda \sim m_S \simeq 3 m_X$

- EFT theory

$$\langle \sigma v \rangle^{\text{EFT}}_{N\bar{N}\to X\bar{X}} \approx \frac{243\, c^4 \sqrt{r_N^2 - 1}}{2\pi (4\pi)^8 x m_X^2} \left(\frac{m_X}{\Lambda}\right)^4 |\mathcal{I}_\Lambda|^2$$

$$\mathcal{I}_\Lambda = \mathcal{I}_\Lambda(r_S = 3) \simeq 0.45$$

Constraints

- Perturbativity

L. Allwicher, *et al.*, 2021
S. M. Choi, *et al.*, 2021

$$\lambda_k < 4\pi \qquad y_N < \sqrt{8\pi} \qquad g_{\mathrm{D}} < 4\pi$$

- Unitarity

M.H. Namjoo, *et al.*, 2019

$$\langle \sigma v^2 \rangle_{XXX\to \bar{N}\bar{N}} \leqslant \frac{192\sqrt{3}\pi^2 x^2}{m_X^5} \qquad \langle \sigma v^2 \rangle_{XXN\to \bar{X}\bar{N}} \leqslant \frac{16\pi^2 x^2}{m_X^5}\left(1+\frac{2}{r_N}\right)^{3/2}$$

$$\langle \sigma v^2 \rangle_{XNN\to \bar{X}\bar{X}} \leqslant \frac{4\pi^2 x^2}{m_X^5}\left(\frac{1}{r_N^2}+\frac{2}{r_N}\right)^{3/2}$$

$$\langle \sigma v \rangle_{N\bar{N}\to X\bar{X}} \leqslant \frac{4\sqrt{\pi x}}{m_X^2 r_N^{3/2}} \qquad \langle \sigma v \rangle_{X\bar{X}\to N\bar{N}} \leqslant \frac{64\sqrt{\pi x}}{m_X^2}$$

Constraints

- Vacuum stability

$$\mathcal{V}_{XS} = \lambda_X (X^*X)^2 + \lambda_S (S^*S)^2 + \lambda_{XS}(X^*X)(S^*S) + \lambda_3 \left[X^3 S^* + (X^*)^3 S\right]$$

$$\lambda_{X,S} > 0 \qquad \lambda_{XS} + 2\sqrt{\lambda_X \lambda_S} > 0$$

S. M. Choi, *et al.*, 2016

$$|\lambda_3| < \sqrt{\frac{(12\lambda_X\lambda_S + \lambda_{XS})^{3/2} + 36\lambda_X\lambda_S\lambda_{XS} - \lambda_{XS}^3}{54\lambda_S}}$$

$$\lambda_{XS} \to 0 \quad \longrightarrow \quad \lambda_{X,S} > 0 \text{ and } |\lambda_3| < \left(16\lambda_X^3\lambda_S/27\right)^{1/4}$$

Constraints

🎲 Effective number of neutrino species

C. Boehm, M. J. Dolan & C. McCabe (2013)

$$N_{\text{eff}}(T_{\text{CMB}}) = \left[1 + \frac{4}{11}\sum_{j=X,N} g_{\star s}^{\text{DM}}(m_j, T_{\nu\text{d}})\right]^{-4/3} N_{\text{eff}}^{\text{SM}}(T_{\text{CMB}})$$

$$g_{\star s}^{\text{DM}}(m_j, x) = \frac{15 g_j}{4\pi^4}\int_{r_j x}^{\infty} du\, \frac{\left(4u^2 - r_j^2 x^2\right)\left(u^2 - r_j^2 x^2\right)^{1/2}}{e^u \pm 1}$$

B. V. Lehmann & S. Profumo (2020)

$$N_{\text{eff}} = 2.99^{+0.34}_{-0.33}\ (95\%\ \text{C.L.}) \longrightarrow m_{X,N} \gtrsim 12\,\text{MeV}$$

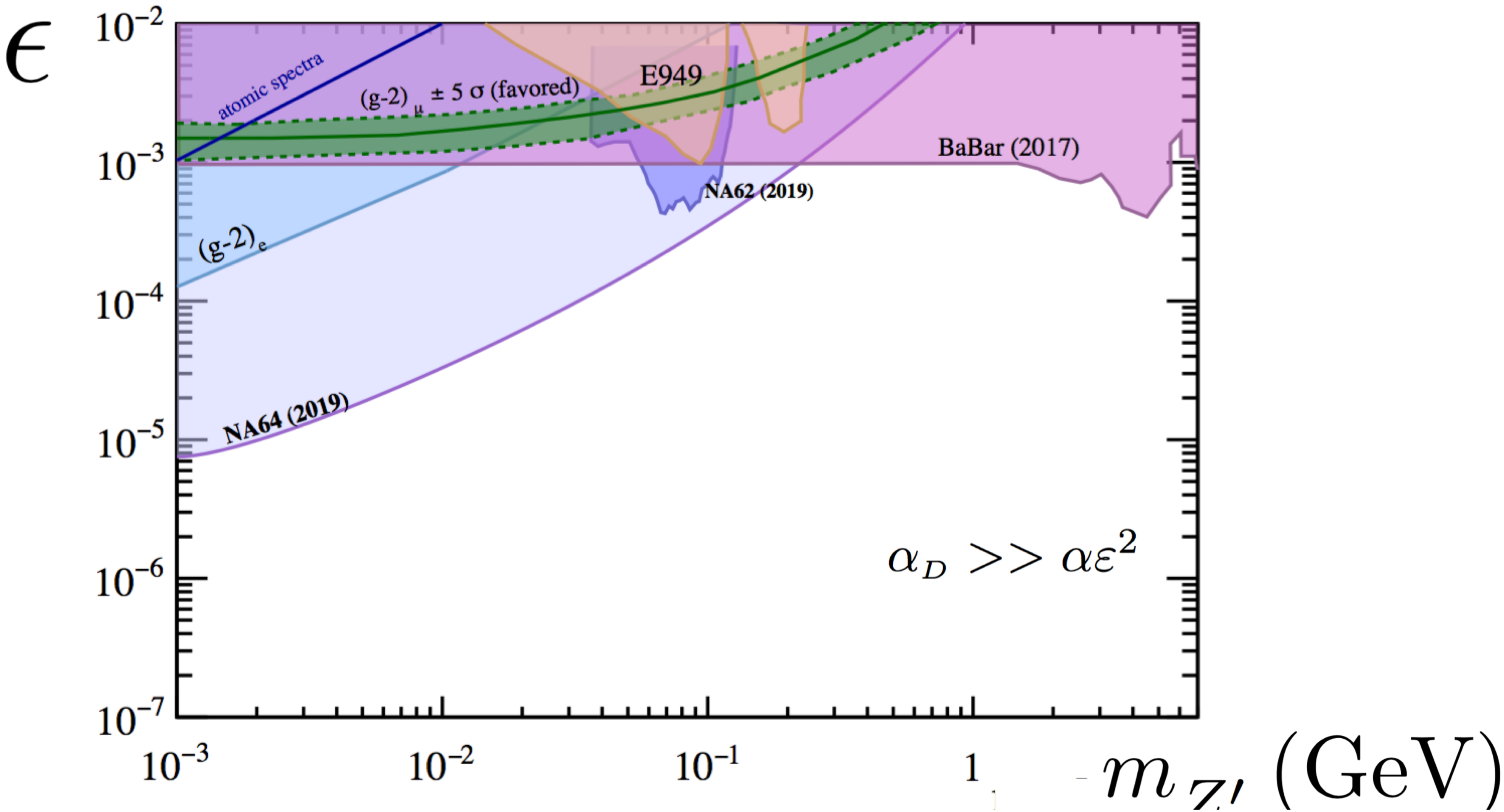
Planck 2018

$$r_N \simeq 1$$

Constraints

- Kinetic mixing & dark gauge boson mass

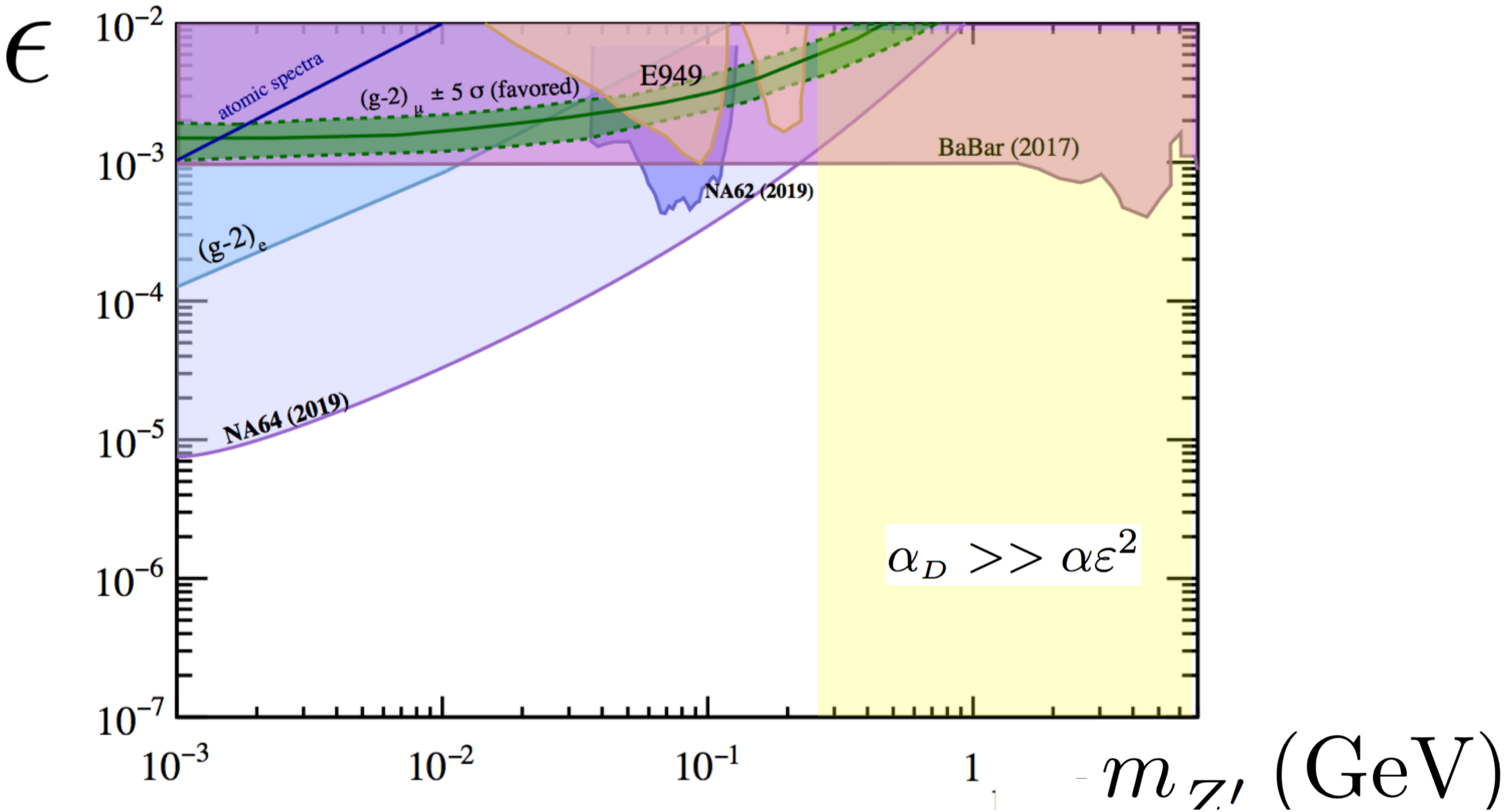
M. Fabbrichesi, *et al.*, 2020



Constraints

🎲 Kinetic mixing & dark gauge boson mass

M. Fabbrichesi, *et al.*, 2020



Cosmological evolution of DM

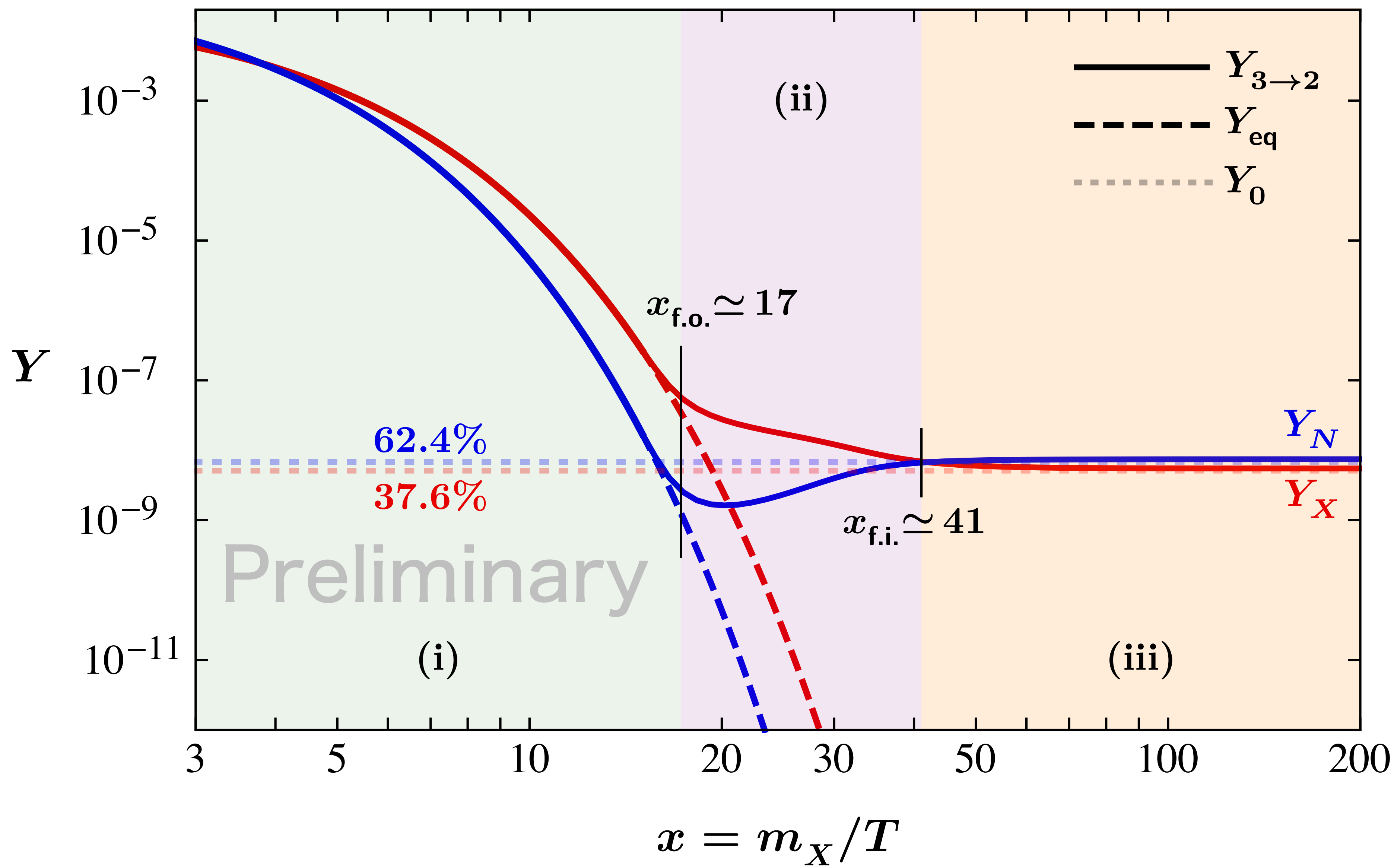
- Boltzmann equation

$$\frac{\mathrm{d}Y_X}{\mathrm{d}x} = -\frac{s^2}{Hx}\left\{12\langle\sigma v^2\rangle_{XXX\to\bar{N}\bar{N}}\left[Y_X^3 - Y_N^2\frac{(Y_X^{\mathrm{eq}})^3}{(Y_N^{\mathrm{eq}})^2}\right] + 2\langle\sigma v^2\rangle_{XXN\to\bar{X}\bar{N}}Y_XY_N\left(Y_X - Y_X^{\mathrm{eq}}\right)\right.$$
$$\left. -\langle\sigma v^2\rangle_{XNN\to\bar{X}\bar{X}}Y_X\left[Y_N^2 - Y_X\frac{(Y_N^{\mathrm{eq}})^2}{Y_X^{\mathrm{eq}}}\right]\right\}$$
$$-\frac{s}{Hx}\left\{4\langle\sigma v\rangle_{X\bar{X}\to N\bar{N}}\left[Y_X^2 - Y_N^2\frac{(Y_X^{\mathrm{eq}})^2}{(Y_N^{\mathrm{eq}})^2}\right] - \langle\sigma v\rangle_{N\bar{N}\to X\bar{X}}\left[Y_N^2 - Y_X^2\frac{(Y_N^{\mathrm{eq}})^2}{(Y_X^{\mathrm{eq}})^2}\right]\right\}$$

$$\frac{\mathrm{d}Y_N}{\mathrm{d}x} = -\frac{s^2}{Hx}\left\{2\langle\sigma v^2\rangle_{XNN\to\bar{X}\bar{X}}Y_X\left[Y_N^2 - Y_X\frac{(Y_N^{\mathrm{eq}})^2}{Y_X^{\mathrm{eq}}}\right] - 8\langle\sigma v^2\rangle_{XXX\to\bar{N}\bar{N}}\left[Y_X^3 - Y_N^2\frac{(Y_X^{\mathrm{eq}})^3}{(Y_N^{\mathrm{eq}})^2}\right]\right\}$$
$$-\frac{s}{Hx}\left\{\langle\sigma v\rangle_{N\bar{N}\to X\bar{X}}\left[Y_N^2 - Y_X^2\frac{(Y_N^{\mathrm{eq}})^2}{(Y_X^{\mathrm{eq}})^2}\right] - 4\langle\sigma v\rangle_{X\bar{X}\to N\bar{N}}\left[Y_X^2 - Y_N^2\frac{(Y_X^{\mathrm{eq}})^2}{(Y_N^{\mathrm{eq}})^2}\right]\right\}$$

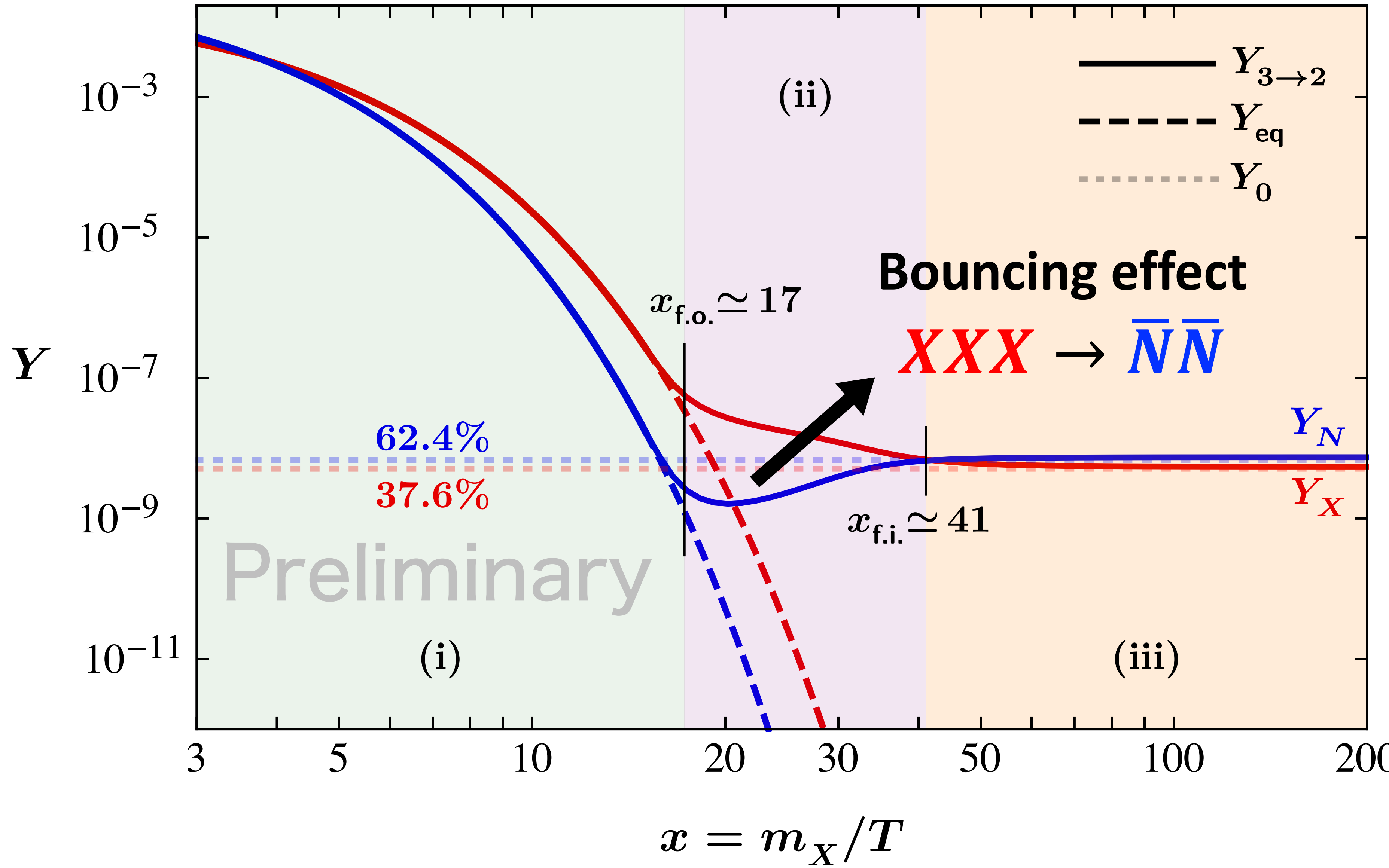
Cosmological evolution w/o 2 to 2 process

$(m_X, m_N, m_S) = (16, 20, 43)\,\mathrm{MeV}, \lambda_3 = 10, y_N = 3$



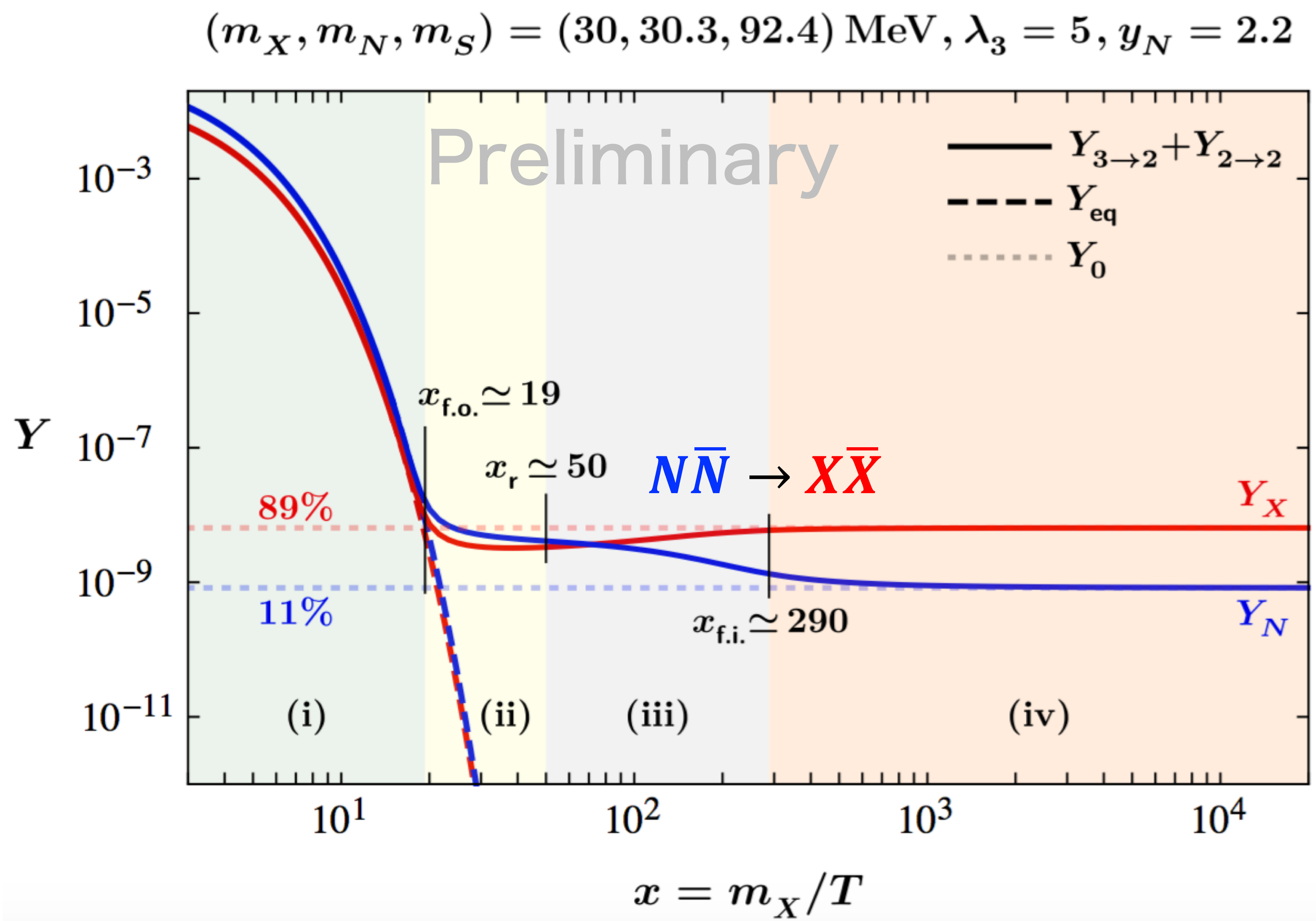
Cosmological evolution w/o 2 to 2 process

$(m_X, m_N, m_S) = (16, 20, 43)\,\mathrm{MeV}, \lambda_3 = 10, y_N = 3$



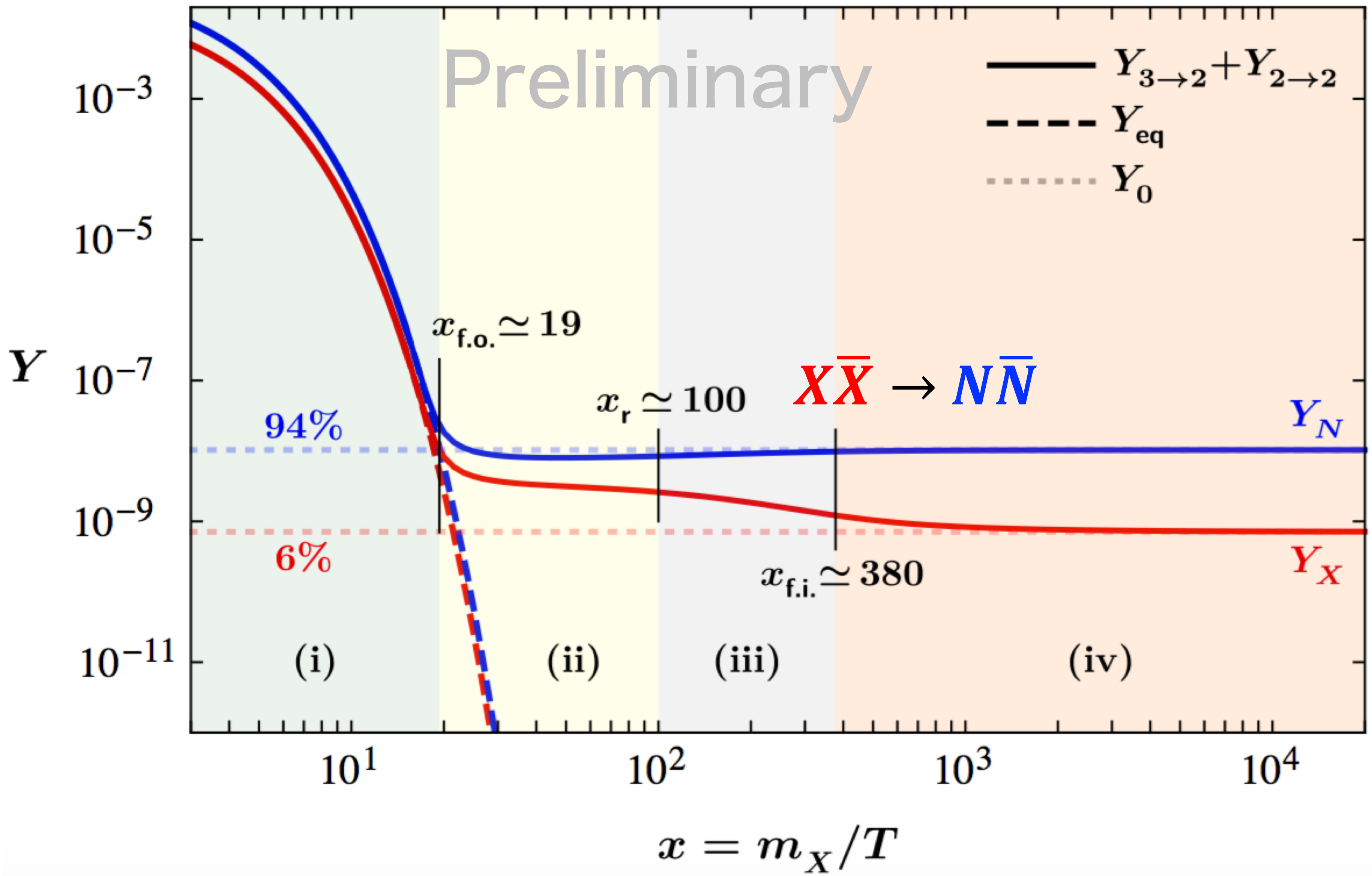
J. T. Ruderman
PASCOS 2021

Cosmological evolution w/ 2 to 2 process



Cosmological evolution w/ 2 to 2 process

$(m_X, m_N, m_S) = (20, 19.9, 61)\,\text{MeV}\,, \lambda_3 = 3.9\,, y_N = 2$



SIMP condition

- Thermalization

kinetic equilibrium condition $\gamma_e(x_{\text{f.o.}}) \gtrsim H(x_{\text{f.o.}})x_{\text{f.o.}}^2$

$$\longrightarrow g_{\text{D}} \gtrsim \frac{0.2}{\sqrt{\mathcal{Q}_X^2 + \mathcal{Q}_N^2}} \left(\frac{\epsilon}{10^{-3}}\right)^{-1} \left(\frac{m_{Z'}}{250\,\text{MeV}}\right)^2 \left(\frac{m_X}{20\,\text{MeV}}\right)^{-3/2}$$

- Suppression of WIMP annihilation

$$\Gamma_{\text{ann}}(x_{\text{f.o.}}) \ll H(x_{\text{f.o.}}) \simeq \Gamma_{3\to 2}$$

$$\longrightarrow g_{\text{D}} \ll \frac{3}{|\mathcal{Q}_N|} \left(\frac{\epsilon}{10^{-3}}\right)^{-1} \left(\frac{m_{Z'}}{250\,\text{MeV}}\right)^2 \left(\frac{m_X}{20\,\text{MeV}}\right)^{-3/2}$$

Self-interacting DM

See H.M Lee
or Seongsik Kim's
Talks

- Resolving the small-scale problems

$$0.1\,\frac{\mathrm{cm}^2}{\mathrm{g}} < \frac{\sigma_{\mathsf{self}}}{m_{\mathsf{DM}}} < 10\,\frac{\mathrm{cm}^2}{\mathrm{g}}$$

X. Chu, *et al.*, 2019
S. Tulin & H.B. Yu, 2018

- The Bullet Cluster

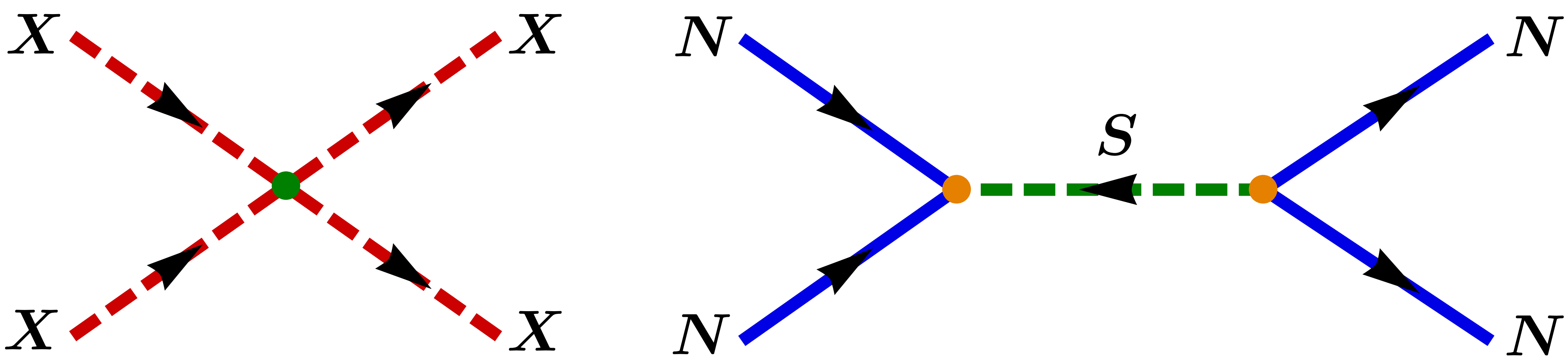
$$\frac{\sigma_{\mathsf{self}}}{m_{\mathsf{DM}}} < 1\,\frac{\mathrm{cm}^2}{\mathrm{g}}$$

M. Markevitch *et al.*, 2004
D. Clowe, *et al.*, 2004

Self-interacting DM

🎲 Self-interacting cross section

$$\frac{\sigma_{\text{self}}}{m_{\text{DM}}} = \mathcal{R}_X^2 \frac{\sigma_X}{m_X} + \mathcal{R}_N^2 \frac{\sigma_N}{m_N}$$



Self-interacting DM

🎲 Self-interacting cross section

$$\frac{\sigma_{\text{self}}}{m_{\text{DM}}} = \mathcal{R}_X^2 \frac{\sigma_X}{m_X} + \mathcal{R}_N^2 \frac{\sigma_N}{m_N}$$

$$\mathcal{R}_X = \frac{\Omega_X}{\Omega_X + \Omega_N} \qquad \mathcal{R}_N = \frac{\Omega_N}{\Omega_X + \Omega_N}$$

$$\sigma_X = \tfrac{1}{4}\left(\sigma_{XX\to XX} + \sigma_{X\bar{X}\to X\bar{X}} + \sigma_{\bar{X}\bar{X}\to \bar{X}\bar{X}}\right) = \frac{\lambda_X^2}{8\pi m_X^2}$$

$$\sigma_N = \tfrac{1}{4}\left(\sigma_{NN\to NN} + \sigma_{N\bar{N}\to N\bar{N}} + \sigma_{\bar{N}\bar{N}\to \bar{N}\bar{N}}\right) = \frac{y_N^4}{16\pi m_X^2}\frac{r_N^2}{r_S^4}$$

Benchmark points

$N\bar{N} \to X\bar{X}$

λ_X	λ_S	λ_3	y_N	$(m_X, m_N, m_S)/\mathrm{MeV}$	$\mathcal{R}_X$	$\mathcal{R}_N$	$\sigma_{\mathsf{self}}/m_{\mathsf{DM}}\ (\mathrm{cm}^2/\mathrm{g})$
4.4	10	4.7	3	(20, 20.02, 59.6)	0.56	0.44	6.70
4.5	8	4.5	2	(25, 25.1, 76)	0.66	0.34	4.92
4	10	4.3	2.5	(25, 25.2, 77)	0.86	0.14	6.66
5	9	5	2.2	(30, 30.3, 92.4)	0.89	0.11	6.31

$X\bar{X} \to N\bar{N}$

λ_X	λ_S	λ_3	y_N	$(m_X, m_N, m_S)/\mathrm{MeV}$	$\mathcal{R}_X$	$\mathcal{R}_N$	$\sigma_{\mathsf{self}}/m_{\mathsf{DM}}\ (\mathrm{cm}^2/\mathrm{g})$
5	4	3.9	2	(20, 19.9, 61)	0.06	0.94	0.20
				(, ,)			

Take home message

- We have built a UV complete DM model to realize the rSIMP scenario with the following condition

$$\Gamma_{2\to 2}^{\text{2-loop}} > \Gamma_{\text{el}} \gtrsim \Gamma_{3\to 2} \gg \Gamma_{\text{ann}} > H_{\text{DM}}$$

- In the rSIMP scenario, there is an inevitable two-loop induced 2 to 2 process which would redistribute the DM yields after the chemical freeze-out of DM.

- The rSIMP masses must be degenerate and $\mathcal{O}(20\,\mathrm{MeV})$ to contribute sizable densities to the observed DM abundance.

- Self-interacting of DM provides the observational signature to test the reshuffled effect in this model.