Lepton sector in a modular flavor symmetry

Hiroshi Okada
(Asia Pacific Center for Theoretical Physics)

26th-28th Nov. 2021 at …Jeju Hidden Cliff

Content: 1. Introduction…A4

2. Modular group and its application (Zee model)

3. Summary

apctp

1 Introduction

Q.
Predictions for quark and lepton masses and mixings as well as reproducing experimental results are very important issue to understand flavor physics.

The CKM mixing angles and CP violating phase of quarks have been precisely measured in the SM.

Neutrino sector may be more attractive since it would include new physics(NP).

Precise measurement for the neutrino oscillation observations and CP violations will be done by

T2K and Nova experiments T2HK, DUNE.

What is the principle to control flavors of leptons ? => We expect a flavor symmetry.

Alternative group with four objects (A4) are frequently applied to lepton sector to get predictions!

A4: **Minimum non-Abelian discrete group with triplet irreducible representation.**

Related to Three families ?

Why A4???

Tri-bimaximal Mixing of Neutrino flavors.

$\sin^2\theta_{12} = 1/3,\ \sin^2\theta_{23} = 1/2,\ \sin^2\theta_{13} = 0,$

$$U_{\rm tri-bimaximal} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

Harrison, Perkins, Scott (2002) proposed

Tri-bimaximal Mixing (TBM) is realized by the mass matrix

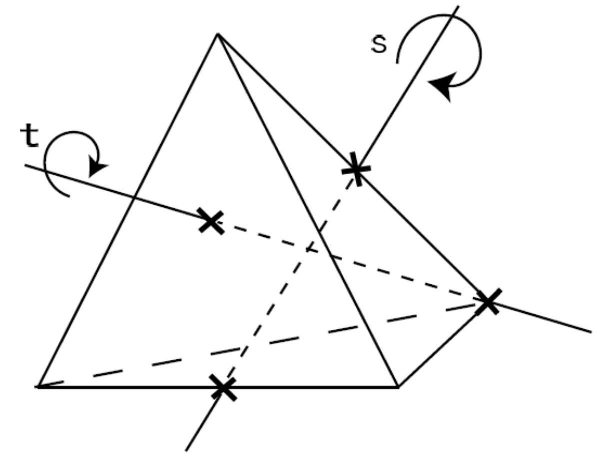
$$m_{TBM} = \frac{m_1+m_3}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{m_2-m_1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_1-m_3}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

in the diagonal basis of charged leptons.

A_4 symmetric

E. Ma, G. Rajasekaran 2001

A_4 group



Symmetry of tetrahedron

Even permutation group of four objects (1234)
12 elements (order 12) are generated by
S and T: $S^2=T^3=(ST)^3=1$: S=(14)(23), T=(123)

4 conjugacy classes

C1: 1 h=1
C3: S, T^2ST, TST^2 h=2
C4: T, ST, TS, STS h=3
C4': T^2, ST^2, T^2S, ST^2S h=3

	h	χ_1	$\chi_{1'}$	$\chi_{1''}$	χ_3
C_1	1	1	1	1	3
C_3	2	1	1	1	-1
C_4	3	1	ω	ω^2	0
$C_{4'}$	3	1	ω^2	ω	0

Irreducible representations: 1, 1', 1", 3
The minimum group containing triplet that is identified as 3 flavor.

Multiplication rule of A_4 group

Irreducible representations: 1, 1', 1", 3

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}; \quad \omega = e^{2\pi i/3}$$ **for triplet**

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{\mathbf{3}} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{\mathbf{3}} = (a_1b_1 + a_2b_3 + a_3b_2)_{\mathbf{1}} \oplus (a_3b_3 + a_1b_2 + a_2b_1)_{\mathbf{1'}}$$

$$\oplus (a_2b_2 + a_1b_3 + a_3b_1)_{\mathbf{1''}}$$

$$\oplus \frac{1}{3}\begin{pmatrix} 2a_1b_1 - a_2b_3 - a_3b_2 \\ 2a_3b_3 - a_1b_2 - a_2b_1 \\ 2a_2b_2 - a_1b_3 - a_3b_1 \end{pmatrix}_{\mathbf{3}} \oplus \frac{1}{2}\begin{pmatrix} a_2b_3 - a_3b_2 \\ a_1b_2 - a_2b_1 \\ a_3b_1 - a_1b_3 \end{pmatrix}_{\mathbf{3}}$$

A_4 invariant Majorana neutrino mass term

$$(LL)_1 = L_1L_1 + L_2L_3 + L_3L_2$$

3 x 3

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

A_4 invariant

Concrete realization

Neutrino sector

G. Altarelli, F. Feruglio, Nucl.Phys. B720 (2005) 64

$\mathbf{3_L \times 3_L \times 1_{flavon} \rightarrow 1}$

$$m_{\nu LL} \sim y_1 \begin{pmatrix} 2\langle\phi_{\nu 1}\rangle & -\langle\phi_{\nu 3}\rangle & -\langle\phi_{\nu 2}\rangle \\ -\langle\phi_{\nu 3}\rangle & 2\langle\phi_{\nu 2}\rangle & -\langle\phi_{\nu 1}\rangle \\ -\langle\phi_{\nu 2}\rangle & -\langle\phi_{\nu 1}\rangle & 2\langle\phi_{\nu 3}\rangle \end{pmatrix} + y_2\langle\xi\rangle \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$\langle\phi_{\nu 1}\rangle = \langle\phi_{\nu 2}\rangle = \langle\phi_{\nu 3}\rangle$, which preserves S symmetry.

$$m_{\nu LL} = 3a \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - a \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Charged-lepton sector

$\mathbf{3_L \times 1_R (1_R', 1_R'') \times 3_{flavon} \rightarrow 1}$

$$m_E \sim \begin{pmatrix} y_e\langle\phi_{E1}\rangle & y_e\langle\phi_{E3}\rangle & y_e\langle\phi_{E2}\rangle \\ y_\mu\langle\phi_{E2}\rangle & y_\mu\langle\phi_{E1}\rangle & y_\mu\langle\phi_{E3}\rangle \\ y_\tau\langle\phi_{E3}\rangle & y_\tau\langle\phi_{E2}\rangle & y_\tau\langle\phi_{E1}\rangle \end{pmatrix}$$

If the following conditions, diagonal mass matrix if obtained! $\langle\phi_{E2}\rangle = \langle\phi_{E3}\rangle = 0$

In 2012, θ_{13} was measured by Daya Bay, RENO, Double Chooz, T2K, MINOS,

Tri-bimaximal mixing was ruled out !

$$\theta_{13} \simeq 9^\circ \simeq \theta_c/\sqrt{2}$$

Additional Matrix

$$M_\nu = a\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} + b\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + c\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix} + d\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}$$

More flavors are needed!

Neutrino mixing matrix

$\nu_\alpha = (U_{PMNS})_{\alpha i} \nu_i$

$\alpha = e, \mu, \tau$ $\qquad$ $i = 1, 2, 3$

flavor eigenstates $\qquad$ mass eigenstates

$$U_{\rm PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\rm CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\rm CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\rm CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\rm CP}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

c_{ij} and s_{ij} denote $\cos\theta_{ij}$ and $\sin\theta_{ij}$, respectively.

$m_1 < m_2 < m_3$ $\qquad$ $m_3 < m_1 < m_2$

without SK atmospheric data	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.7$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.44^{+0.78}_{-0.75}$	$31.27 \to 35.86$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.570^{+0.018}_{-0.024}$	$0.407 \to 0.618$	$0.575^{+0.017}_{-0.021}$	$0.411 \to 0.621$
$\theta_{23}/^\circ$	$49.0^{+1.1}_{-1.4}$	$39.6 \to 51.8$	$49.3^{+1.0}_{-1.2}$	$39.9 \to 52.0$
$\sin^2\theta_{13}$	$0.02221^{+0.00068}_{-0.00062}$	$0.02034 \to 0.02430$	$0.02240^{+0.00062}_{-0.00062}$	$0.02053 \to 0.02436$
$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	$8.20 \to 8.97$	$8.61^{+0.12}_{-0.12}$	$8.24 \to 8.98$
$\delta_{\rm CP}/^\circ$	195^{+51}_{-25}	$107 \to 403$	286^{+27}_{-32}	$192 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ {\rm eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ {\rm eV}^2}$	$+2.514^{+0.028}_{-0.027}$	$+2.431 \to +2.598$	$-2.497^{+0.028}_{-0.028}$	$-2.583 \to -2.412$

NuFIT5.0 (2020) $\qquad$ $\Delta m^2_{atm} = m_3^2 - m_1^2$, $\Delta m^2_{sol} = m_2^2 - m_1^2$

Unsatisfactory points for traditional flavor models

1. A large number of scalars(flavons) are needed.

2. Vacuum alignment for bosons are imposed.

Modular flavor symmetries can resolve these issues!!!

2 Modular Group and its application

It is well known that the superstring theory on certain compactifications lead to non-Abelian finite groups.

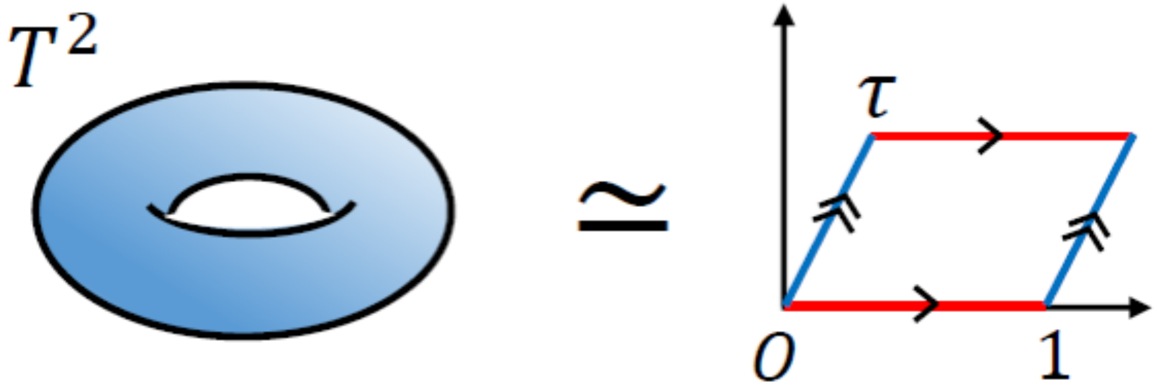
Indeed, two dimensional torus compactification leads to Modular symmetry, which includes S_3, A_4, S_4, A_5 as its congruence subgroup.

Advantages:

1. Additional **scalar bosons contributing to mass matrix are not needed, because Yukawa coupling has its structure originated from a modular group; More predictions without assumptions!**

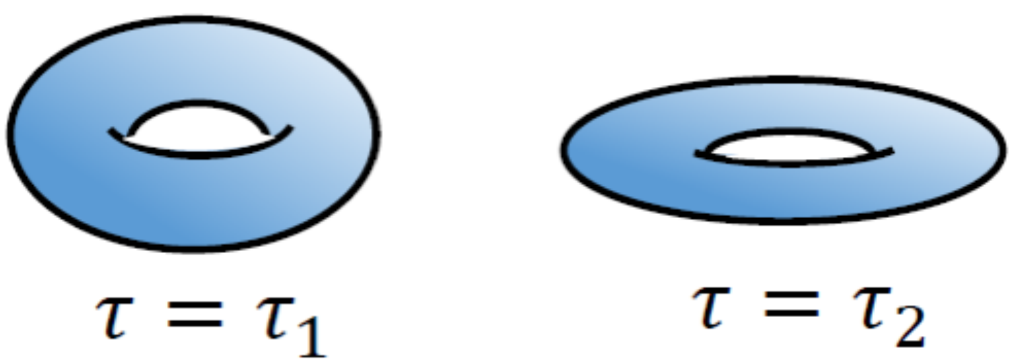
2. DM stability can be assured by modular number that is required by a modular group; Additional symmetry (Z_2) is not needed!

$2D$ torus (T^2) is equivalent to parallelogram with identification of confronted sides.



Two-dimensional torus T^2 is obtained as $T^2 = \mathbb{R}^2 / \Lambda$
Λ is two-dimensional lattice

The shape of torus is represented by a modulus $\tau \in \mathbb{C}$.



The different value of τ realize the different shape of T^2

$\mathcal{L}_{\rm eff}$ depends on τ.

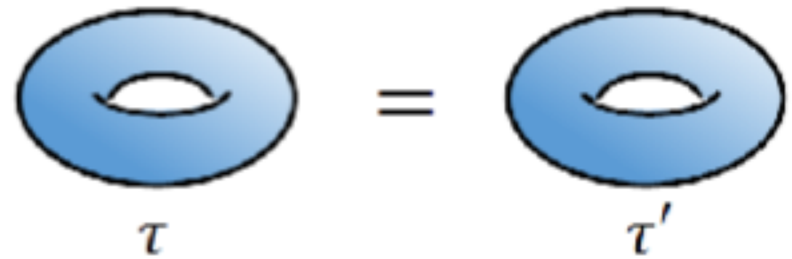
e.g.) $\mathcal{L}_{\rm eff} \supset Y(\tau)_{ij} \phi \overline{\psi_i} \psi_j + \cdots$

- $4D$ effective theory depends on a modulus τ

However,
there are specific transformations of τ which don't change T^2

➡ Modular transformation

$\tau \rightarrow \tau'$



$$\tau \longrightarrow \tau' = \frac{a\tau + b}{c\tau + d}$$

Modular transformation

Modular transf. does not change the lattice (torus)

⬇

$4D$ effective theory (depends on τ)
must be invariant under modular transf.

The modular transformation is generated by S and T .

$$\tau \longrightarrow \tau' = \frac{a\tau + b}{c\tau + d}$$

translation

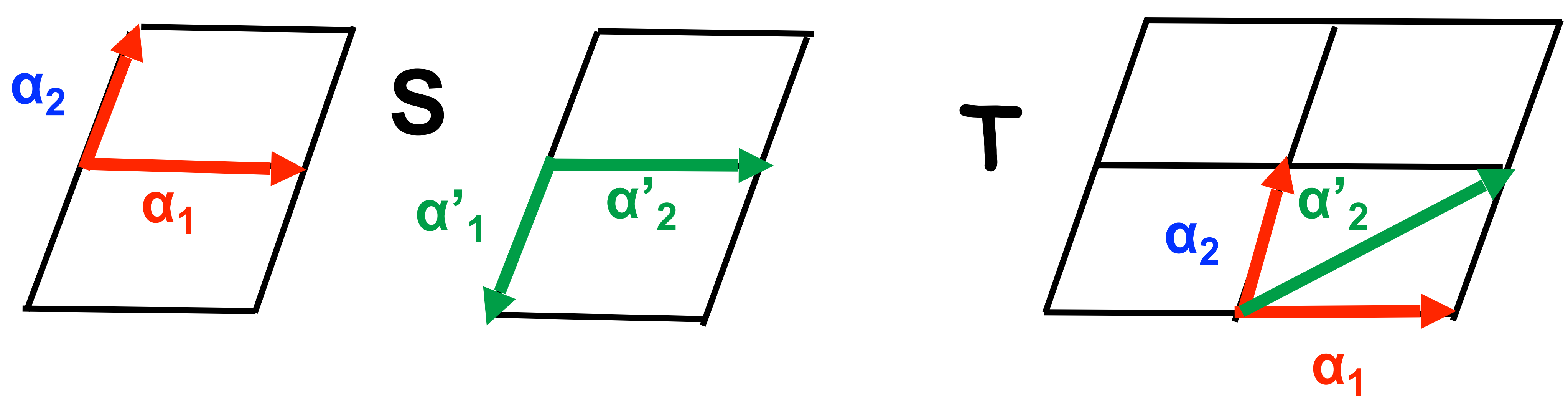
$$S : \tau \longrightarrow -\frac{1}{\tau}$$

$$T : \tau \longrightarrow \tau + 1$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

Matrix Form



Schematical Form

$\tau = \alpha_2 / \alpha_1$

$$S : \tau \longrightarrow -\frac{1}{\tau},$$
$$T : \tau \longrightarrow \tau + 1.$$

$$S^2 = 1, \qquad (ST)^3 = 1.$$

generate infinite discrete group

Modular group

4D effective theory

- **depends on a modulus τ**
- **is independent under modular transformation**

An example

$$\mathcal{L}_1 = f(\tau)\phi_1\phi_2 \cdots \phi_n$$

- $f(\tau)$: coupling constant
- ϕ_i: scalar fields

$f(\tau) \to (c\tau + d)^k f(\tau)$ ← Modular form with weight k

$\phi_i \to (c\tau + d)^{-k_i}\phi_i$

When $k = \sum_i k_i$, $\mathcal{L}_1$ is modular invariant.

Modular group has interesting subgroups

Modular group

$$\Gamma \simeq \{S,T|S^2=\mathbb{I},(ST)^3=\mathbb{I}\}$$

Infinite discrete group

Impose $T^N=1$ congruence condition

$$\overline{\Gamma}(N) \simeq \{S,T|S^2=\mathbb{I},(ST)^3=\mathbb{I},T^N=\mathbb{I}\}$$

$$\Gamma(N) \equiv \Gamma / \overline{\Gamma}(N)$$

$$\Gamma(2) \simeq S_3,\ \Gamma(3) \simeq A_4,\ \Gamma(4) \simeq S_4, \text{ and } \Gamma(5) \simeq A_5$$

Concrete forms of Yukawa couplings of A4

$$\mathbf{Y_3}^{(2)} = \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix}$$

$$\begin{aligned}
Y_1(\tau) &= \frac{i}{2\pi}\left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)} - \frac{27\eta'(3\tau)}{\eta(3\tau)}\right), \\
Y_2(\tau) &= \frac{-i}{\pi}\left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \omega^2\frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \omega\frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)}\right), \\
Y_3(\tau) &= \frac{-i}{\pi}\left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \omega\frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \omega^2\frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)}\right),
\end{aligned}$$

$$\eta(\tau) = q^{1/24}\prod_{n=1}^{\infty}(1-q^n)$$

Dedekind eta-function

$$\eta(-1/\tau) = \sqrt{-i\tau}\,\eta(\tau), \qquad \eta(\tau+1) = e^{i\pi/12}\eta(\tau)$$

$$\begin{aligned}
Y_1(\tau) &= 1 + 12q + 36q^2 + 12q^3 + \cdots, \\
Y_2(\tau) &= -6q^{1/3}(1 + 7q + 8q^2 + \cdots), \\
Y_3(\tau) &= -18q^{2/3}(1 + 2q + 5q^2 + \cdots).
\end{aligned}$$

$$q = e^{2\pi i\tau}$$

$|q| \ll 1$

$$Y_2^2 + 2Y_1Y_3 = 0$$

- Yukawa couplings with higher orders are constructed by multiplication rules of A4 symmetry!

$$\mathbf{Y}_1^{(4)} = Y_1^2 + 2Y_2Y_3, \quad \mathbf{Y}_{1'}^{(4)} = Y_3^2 + 2Y_1Y_2, \quad \mathbf{Y}_{1''}^{(4)} = Y_2^2 + 2Y_1Y_3 = 0,$$

$$\mathbf{Y}_3^{(4)} = \begin{pmatrix} Y_1^2 - Y_2Y_3 \\ Y_3^2 - Y_1Y_2 \\ Y_2^2 - Y_1Y_3 \end{pmatrix},$$

$$\mathbf{Y}_1^{(6)} = Y_1^3 + Y_2^3 + Y_3^3 - 3Y_1Y_2Y_3,$$

$$\mathbf{Y}_3^{(6)} \equiv \begin{pmatrix} Y_1^{(6)} \\ Y_2^{(6)} \\ Y_3^{(6)} \end{pmatrix} = \begin{pmatrix} Y_1^3 + 2Y_1Y_2Y_3 \\ Y_1^2Y_2 + 2Y_2^2Y_3 \\ Y_1^2Y_3 + 2Y_3^2Y_2 \end{pmatrix}, \qquad \mathbf{Y}_{3'}^{(6)} \equiv \begin{pmatrix} Y_1'^{(6)} \\ Y_2'^{(6)} \\ Y_3'^{(6)} \end{pmatrix} = \begin{pmatrix} Y_3^3 + 2Y_1Y_2Y_3 \\ Y_3^2Y_1 + 2Y_1^2Y_2 \\ Y_3^2Y_2 + 2Y_2^2Y_1 \end{pmatrix}$$

- Singlet Yukawa starts at k=4.

A concrete model

A Theory of Lepton Number Violation, Neutrino Majorana Mass, and Oscillation
A. Zee(Pennsylvania U.)

Feb, 1980
1 page
Published in:
Phys.Lett.B 93 (1980) 389, Phys.Lett.B 95 (1980) 461 (erratum)

Published: 1980

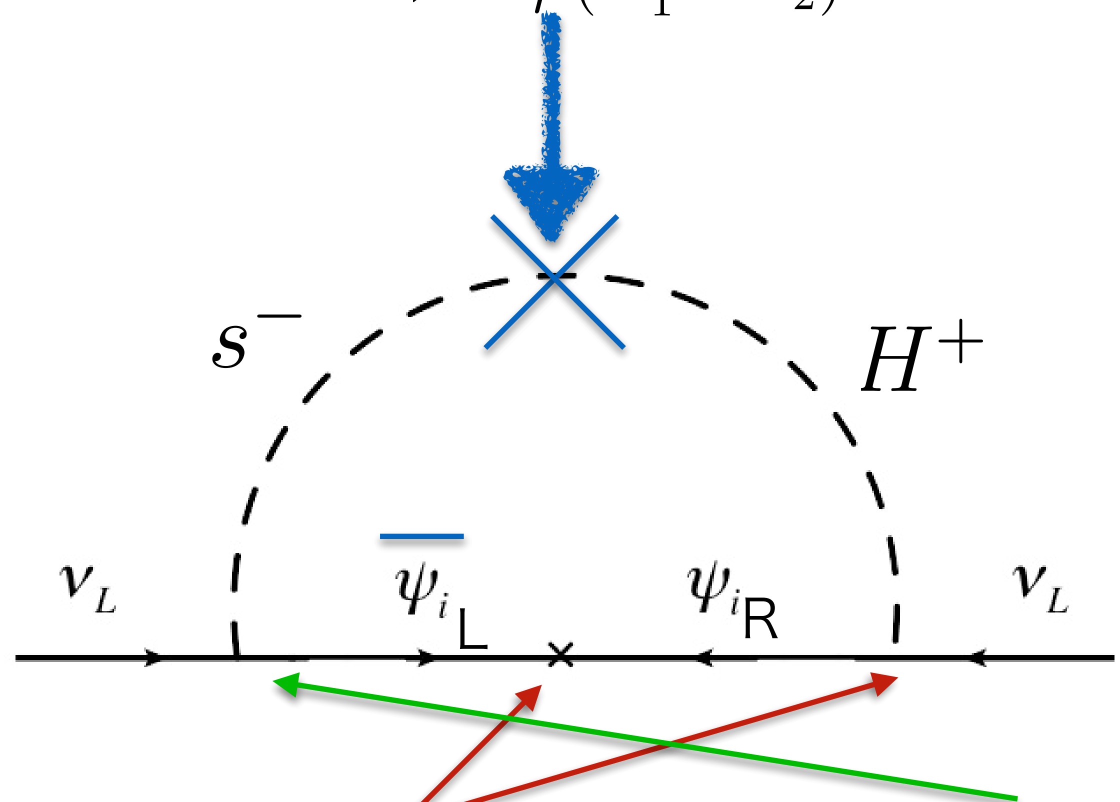
Zee model

One of the simplest neutrino model

New particles: h^-,H2

No additional symmetries are needed!

$$\mathcal{V} = \mu(H_1^T \cdot H_2)s^-$$



$$-\mathcal{L}_Y = y_{ij}^{\ell}\bar{L}_{L_i}H_1 e_{R_j} + y_{ij}^{\prime\ell}\bar{L}_{L_i}H_2 e_{R_j} + f_{ab}\bar{L}_{L_a}(i\sigma_2)L_{L_b}^C s^- + \text{h.c.},$$

Anti-symmetric!

Zee model in a modular A_4 symmetry
T. Nomura, HO, Yong-hui Qi (2111.10944)

Field contents and their assignments

Fermions:

A_4 singlets Le_bar 1; Lμ_bar 1" ; Lτ_bar 1'…with [-2-2-4] modular weight.

A_4 triplet 3 (e_{Re}, $e_{R\mu}$, $e_{R\tau}$)…with zero modular weight.

Bosons: all the bosons are trivial singlets with different modular weights!

H_1, H_2… doublet bosons with [0-2] modular weight,
s-, φ … singlet bosons with [-4-2] modular weight.

	Leptons	
	$[\bar{L}_{L_e}, \bar{L}_{L_\mu}, \bar{L}_{L_\tau}]$	$[e_R, \mu_R, \tau_R]$
$SU(2)_L$	$\mathbf{2}$	$\mathbf{1}$
$U(1)_Y$	$\frac{1}{2}$	-1
A_4	$[1, 1'', 1']$	3
$-k_I$	$[-2, -2, -4]$	0

	H_1	H_2	s^-	φ
$SU(2)_L$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$
$U(1)_Y$	$\frac{1}{2}$	$\frac{1}{2}$	-1	0
A_4	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$-k_I$	0	-2	-4	-2

$$\begin{aligned} -\mathcal{L}_Y = & a_\ell \bar{L}_{L_e} H_1 (y_1 e_R + y_2 \tau_R + y_3 \mu_R) + b_\ell \bar{L}_{L_\mu} H_1 (y_3 \tau_R + y_1 \mu_R + y_2 e_R) \\ & + c_\ell \bar{L}_{L_\tau} H_1 (y_2^{(4)} \mu_R + y_1^{(4)} \tau_R + y_3^{(4)} e_R) \\ & + a'_\ell \bar{L}_{L_e} H_2 (y_1^{(4)} e_R + y_2^{(4)} \tau_R + y_3^{(4)} \mu_R) + b'_\ell \bar{L}_{L_\mu} H_2 (y_3^{(4)} \tau_R + y_1^{(4)} \mu_R + y_2^{(4)} e_R) \\ & + c'_\ell \bar{L}_{L_\tau} H_2 (y_2^{(6)} \mu_R + y_1^{(6)} \tau_R + y_3^{(6)} e_R) + d'_\ell \bar{L}_{L_\tau} H_2 (y_2'^{(6)} \mu_R + y_1'^{(6)} \tau_R + y_3'^{(6)} e_R) \\ & + a \bar{L}_{L_e} (i\sigma_2) L^C_{L_\mu} s^- + b \bar{L}_{L_e} (i\sigma_2) L^C_{L_\tau} s^- + c \bar{L}_{L_\mu} (i\sigma_2) L^C_{L_\tau} s^- + \text{h.c.}, \end{aligned}$$

$\rightarrow y^\ell_{ij}$

$\rightarrow y'^\ell_{ij}$

$\rightarrow f_{ab}$

$$\mathcal{V} = \mu (H_1^T \cdot H_2) s^- + \mu' \varphi H_1^\dagger H_2 + \text{h.c.},$$

Singlets have non-zero coupling in case of 4=k.

Higgs sector:

$$O_h M^2_{even} O_h^T \;=\; \mathrm{diag}[m_h^2, m_H^2],$$

$$O_z M^2_{odd} O_z^T \;=\; \mathrm{diag}[m_z^2(=\;0), m_A^2],$$

$$O_C M^2_C O_C^T \;=\; \mathrm{diag}[m^2_{w^+}(=\;0), m^2_{h^+}, m^2_{H^+}].$$

$$[v_1/v_H, v_2/v_H, 0]$$

$$\frac{M_C}{\mathrm{GeV}} \approx \begin{pmatrix} 49.3 & 110i & 77.8 \\ 110i & 246 & 174i \\ 77.8 & 174i & 181i \end{pmatrix}, \quad O_C \approx \begin{pmatrix} 0.981 & 0.196 & 0 \\ -0.0553i & 0.277i & 0.959i \\ 0.188 & -0.941 & 0.282 \end{pmatrix}.$$

$$v_1/v_2 = 5$$

Charged-lepton mass matrix:

$$M_e = \frac{v_1}{\sqrt{2}}\tilde{M}_e + \frac{v_2}{\sqrt{2}}\tilde{M}'_e,$$

$$\tilde{M}_e = \begin{pmatrix} |a_\ell| & 0 & 0 \\ 0 & |b_\ell| & 0 \\ 0 & 0 & |c_\ell| \end{pmatrix} \begin{pmatrix} y_1 & y_3 & y_2 \\ y_2 & y_1 & y_3 \\ y_3^{(4)} & y_2^{(4)} & y_1^{(4)} \end{pmatrix}, \qquad \text{where } e'_\ell \equiv \frac{d'_\ell}{c'_\ell}.$$

$$\tilde{M}'_e = \begin{pmatrix} a'_\ell & 0 & 0 \\ 0 & b'_\ell & 0 \\ 0 & 0 & c'_\ell \end{pmatrix} \begin{pmatrix} y_1^{(4)} & y_3^{(4)} & y_2^{(4)} \\ y_2^{(4)} & y_1^{(4)} & y_3^{(4)} \\ y_3^{(6)} + e'_\ell y_3'^{(6)} & y_2^{(6)} + e'_\ell y_2'^{(6)} & y_1^{(6)} + e'_\ell y_1'^{(6)} \end{pmatrix},$$

$$D_e \equiv \mathrm{diag}(m_e, m_\mu, m_\tau) = V_{eL}^\dagger M_e V_{eR}.$$

$$\mathrm{Tr}[M_e M_e^\dagger] = |m_e|^2 + |m_\mu|^2 + |m_\tau|^2,$$

$$\mathrm{Det}[M_e M_e^\dagger] = |m_e|^2 |m_\mu|^2 |m_\tau|^2,$$

$$(\mathrm{Tr}[M_e M_e^\dagger])^2 - \mathrm{Tr}[(M_e M_e^\dagger)^2] = 2(|m_e|^2|m_\nu|^2 + |m_\mu|^2|m_\tau|^2 + |m_e|^2|m_\tau|^2),$$

→ al, bl cl are fixed giving numerical values of a'l,b'l,c'l and charged-lepton masses.

Neutrino sector:

$$-\mathcal{L}_\nu = \bar{\nu}_{L_i}(\tilde{M}_e)_{ij}(V_{eR})_{ja}(\ell_R)_a(O_C^T)_{1\alpha}(h_m^+)_\alpha + \bar{\nu}_{L_i}(\tilde{M}'_e)_{ij}(V_{eR})_{ja}(\ell_R)_a(O_C^T)_{2\alpha}(\vec{h}_m^+)_\alpha$$
$$+ \left[-\bar{\ell}_{L_a}(V_{eL}^\dagger)_{ai} f_{ij}\nu_{L_j}^C + \bar{\nu}_{L_i} f_{ij}^T (V_{eL}^*)_{jb}\ell_{L_b}^C\right](O_C^T)_{3\alpha}(h_m^+)_\alpha + \text{h.c.},$$

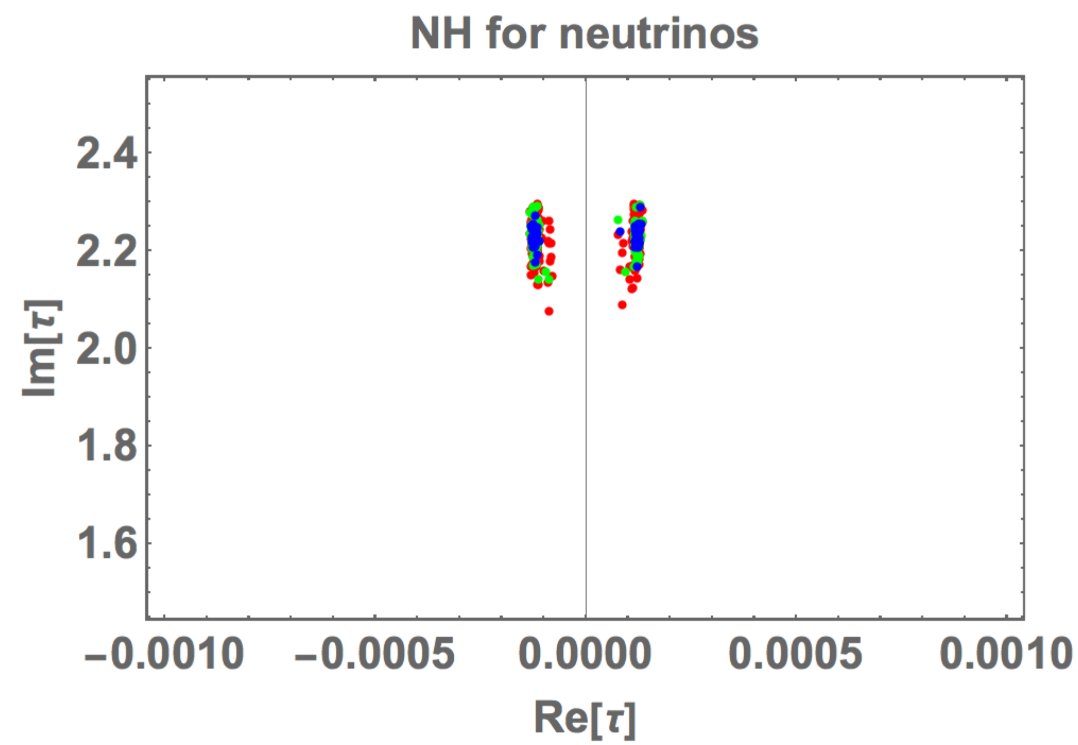
$$f = \begin{pmatrix} 0 & a & b \\ -a & 0 & |c| \\ -b & -|c| & 0 \end{pmatrix} = |c| \begin{pmatrix} 0 & \epsilon' & \epsilon \\ -\epsilon' & 0 & 1 \\ -\epsilon & -1 & 0 \end{pmatrix} \equiv |c|\tilde{f},$$

$$(m_\nu)_{ij} = |c|\left[(\tilde{m}_\nu^{(I)})_{ij} + (\tilde{m}_\nu^{(II)})_{ij} + (\tilde{m}_\nu^{(I)})_{ij}^T + (\tilde{m}_\nu^{(II)})_{ij}^T\right],$$
$$(\tilde{m}_\nu^{(I)})_{ij} \simeq \frac{1}{(4\pi)^2}(\tilde{M}'_e)_{ij'}(V_{eR})_{j'a}D_{e_a}(V_{eL}^\dagger)_{ai'}\tilde{f}_{i'j}\left[(O_C^T)_{2\alpha}\underline{(O_C)_{\alpha 3}}\right] I(D_{e_a}, m_{h_{m\alpha}^\pm}),$$
$$(\tilde{m}_\nu^{(II)})_{ij} \simeq -\frac{1}{(4\pi)^2}\tilde{f}_{ij'}^T(V_{eL}^*)_{j'a}D_{e_a}(V_{eR}^T)_{ai'}(\tilde{M}_e^T)_{i'j}\left[\underline{(O_C^T)_{3\alpha}}(O_C)_{\alpha 1}\right] I(D_{e_a}, m_{h_{m\alpha}^\pm}),$$
$$I(D_{e_a}, m_{h_{m\alpha}^\pm}) = \int_0^1 \ln\left[xD_{e_a}^2 + (1-x)m_{h_{m\alpha}^\pm}^2\right],$$

Charged-NGB does not contribute to the neutrino mass since 1-3 component of Oc is zero.

Numerical results:

※IH is not favored up to 5 chi-square analysis.

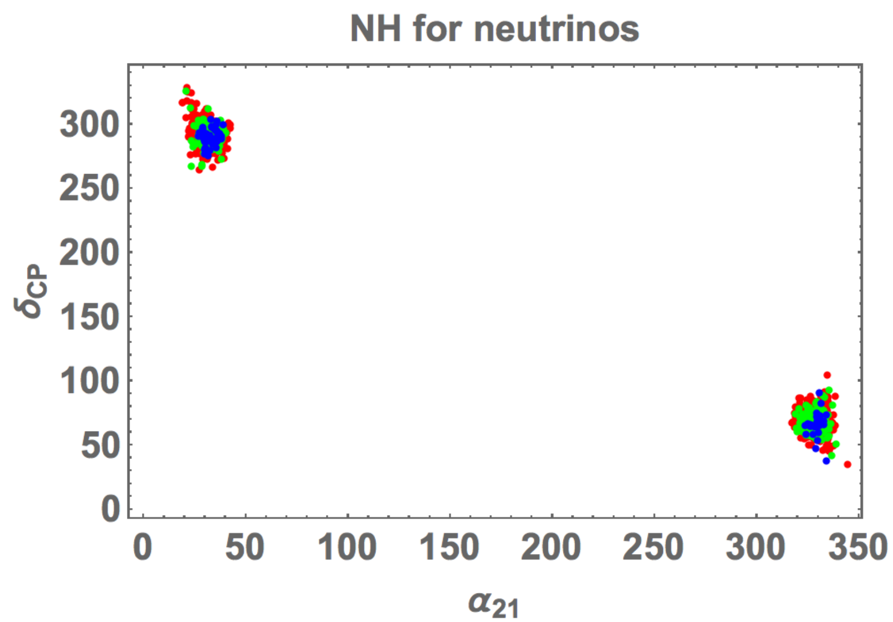


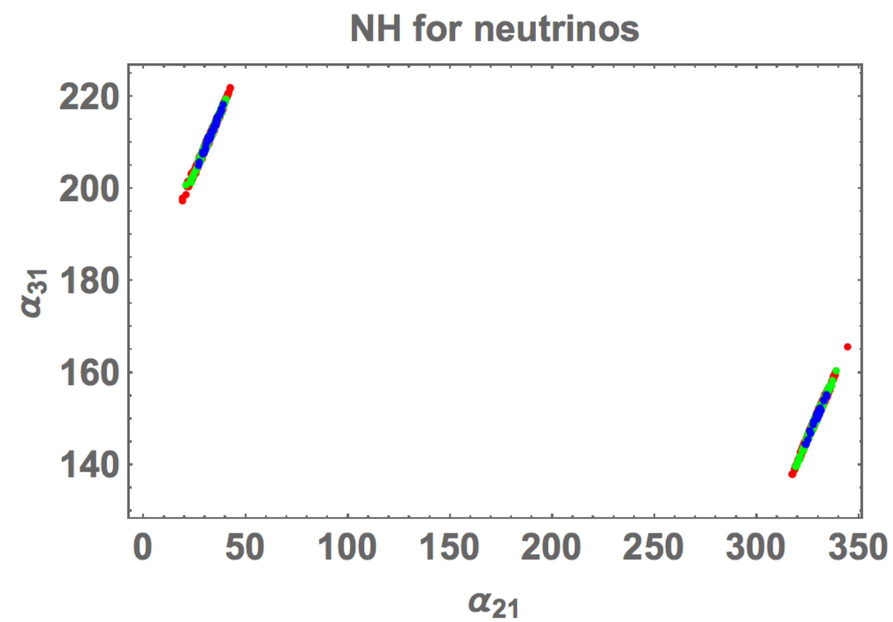
τ=2.1i~2.3i is favored.

$$Y_3^{(2,4,6)} \sim [1,0,0]^T,\ \ Y_3^{(6')} \sim [0,0,0]^T$$

The mass matrix is simplified!=> later!

Numerical results:



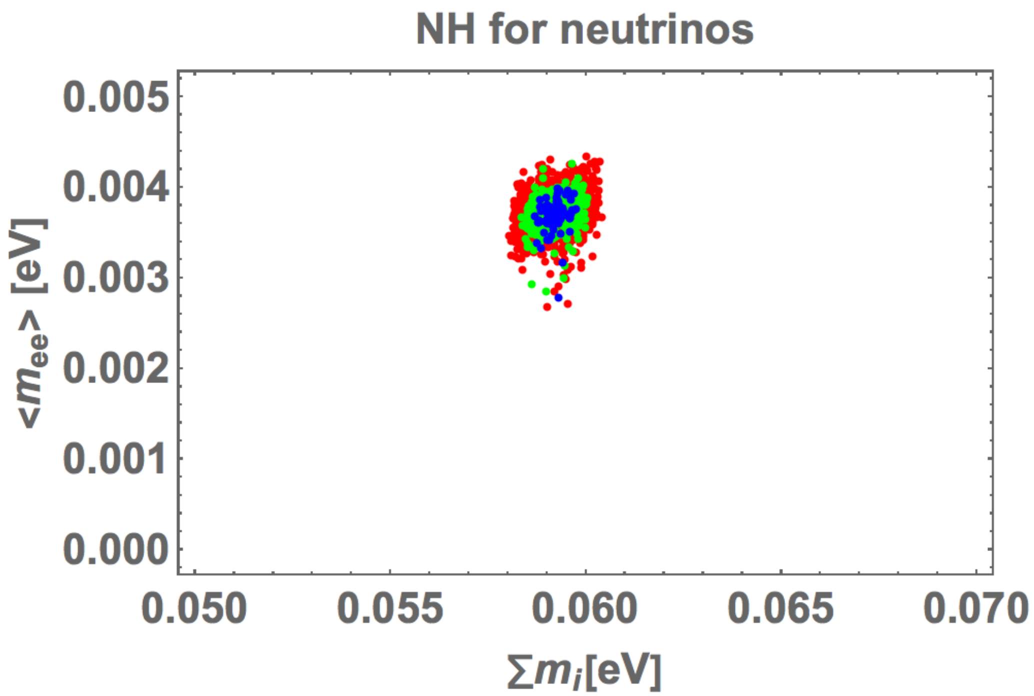


$$\alpha_{21} = [20 - 50,\ 320 - 360]\ [\mathrm{deg}],$$

$$\alpha_{31} = [135 - 165,\ 195 - 225]\ [\mathrm{deg}]$$

$$\delta_{CP} = [30 - 100,\ 260 - 330]\ [\mathrm{deg}].$$

Numerical results:



$$\langle m_{ee} \rangle = [2.6 - 4.4]\ [\text{meV}].$$

$$\sum m_i = [58 - 61]\,[\text{meV}]$$

$m_1 << m_2 << m_3$, because $\sum m_i$ is close to $\sqrt{\Delta m^2_{atm}}$.

Neutrino mass matrix in the limit of τ=i∞.

$$Y_3^{(2,4,6)} \sim [1,0,0]^T,\ Y_3^{(6')} \sim [0,0,0]^T$$

The neutrino mass matrix is given by B1 form, since the loop function does not depend on the charged-lepton masses; me, mμ,mτ << mh±.

$$\mathrm{diag}[m_{w^+}, m_{h^+}, m_{H^+}] \approx \mathrm{diag}[0,\ 204,\ 269]\ \mathrm{GeV}.$$

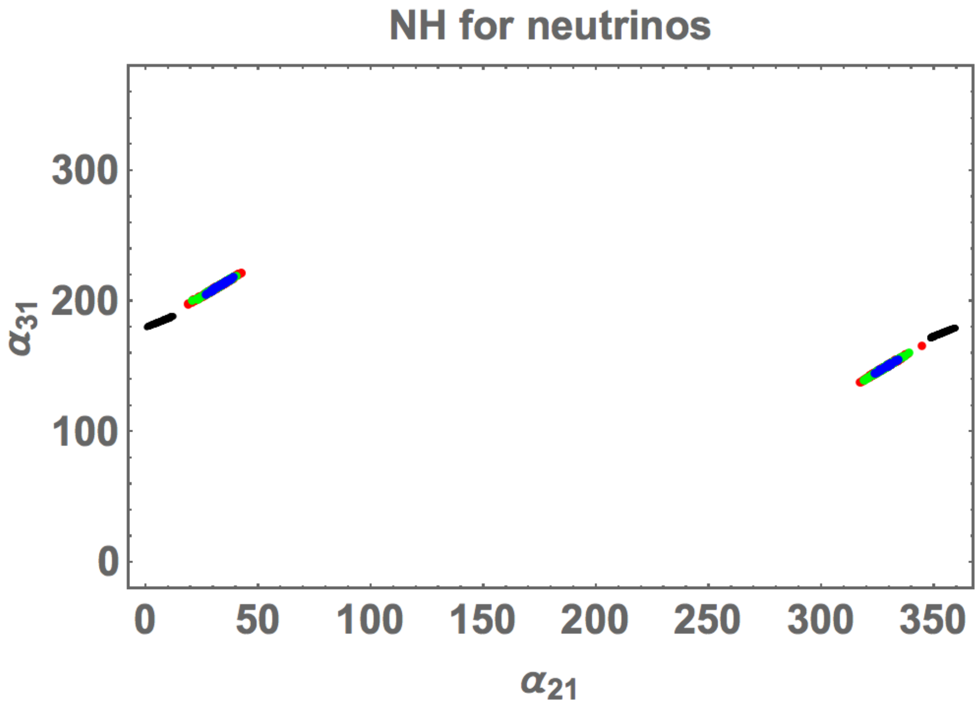
$$m_\nu \sim \begin{pmatrix} \times & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}.$$

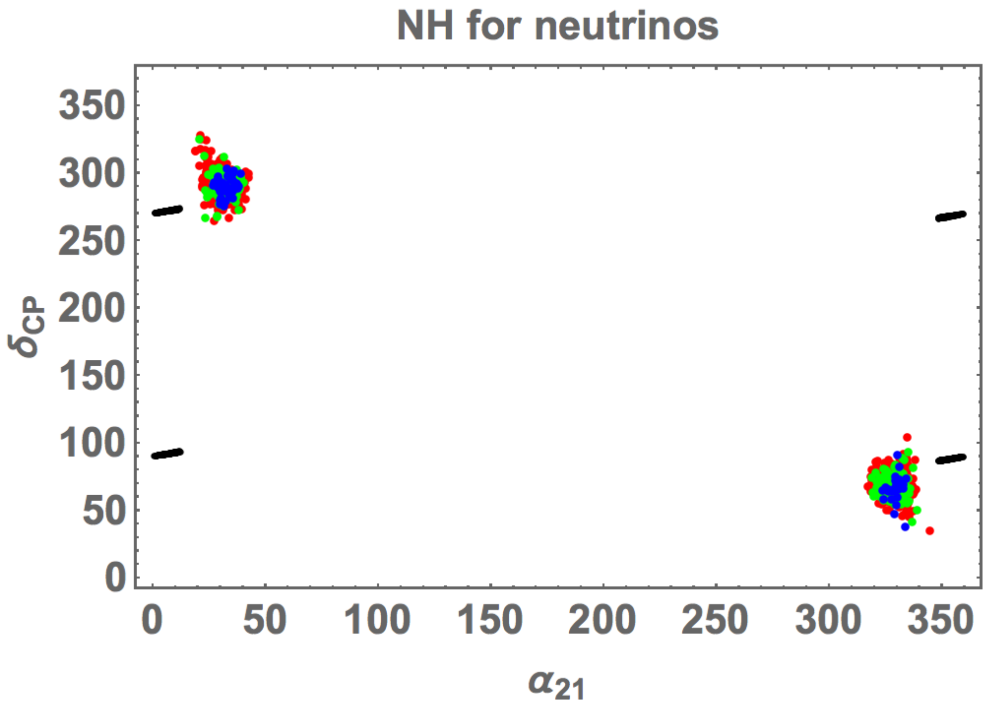
→

NH is favored.

$$\alpha_{21} \simeq -\alpha_{31} \simeq \delta_{CP} - \frac{\pi}{2}.$$

Numerical results:





Our predictions for phases are similar to the predictions for B1 texture!

3 Summary

- Modular non-Abelian discrete flavor symmetries provide several predictions without introducing so many Higgs fields.

- Neutrino mass matrix of A_4 Zee-model favors $\tau \sim 2i$;
  $Y \sim [1,0,0]$.

- It leads to B1 two zero texture and similar predictions are obtained for phases.

Sum of neutrino mass is about 60 meV that is equal to square root of atmospheric neutrino mass squared difference; $m1 \ll m2 \ll m3$.

Thanks!