General Higgs-sigma models for inflation and its supergravity embedding

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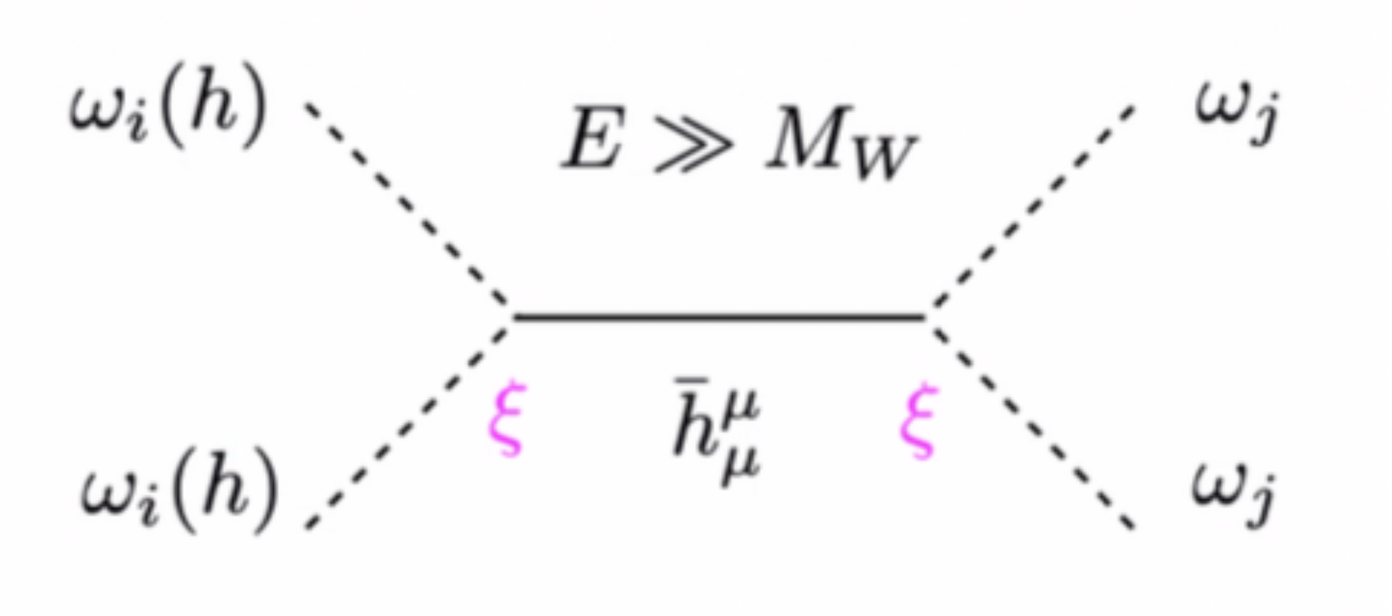
Higgs Inflation

$$\mathscr{L} = \sqrt{-\bar{g}}\left[-\frac{1}{2}(1+\xi\bar{\phi}_i^2)\bar{R} + \frac{1}{2}g^{\mu\nu}\partial_\mu\bar{\phi}_i\partial_\nu\bar{\phi}_i - \frac{\lambda}{4}(\bar{\phi}_i^2)^2\right]$$

[Bezrukov, Shaposhnikov - 0710.3755]



- A non-minimal coupling of the Higgs to gravity is required.
- $\xi \simeq 10^4$ large non minimal coupling is needed to be consistent with Planck data.
- This leads to unitarity violation even at small values of the field. **[Burgess, HML,Trott(2009,2010); Barbon, Espinosa (2009); Hertzberg(2010)]**

$$\Lambda_{\text{cutoff}} = \frac{M_P}{\xi}$$

- Including higher order operators spoils the slow-roll regime
- A new scalar field instead can restore unitarity up to the Plank scale. **[Giudice, Lee - 1010.1417]**

Introducing a scalar through a conformal transformation

$$\mathscr{L} = \sqrt{-\bar{g}}\left[-\frac{1}{2}(1+\xi\bar{\phi}_i^2)\bar{R} + \frac{1}{2}g^{\mu\nu}\partial_\mu\bar{\phi}_i\partial_\nu\bar{\phi}_i - \frac{\lambda}{4}(\bar{\phi}_i^2)^2\right] \xrightarrow{\bar{g}_{\mu\nu} = e^{2\varphi}g_{\mu\nu}}$$

$$\mathscr{L} = \sqrt{-g}\left\{-\frac{1}{2}\left(1-\frac{1}{6}\phi_i^2-\frac{1}{6}\sigma^2\right)R + \frac{1}{2}(\partial_\mu\phi_i)^2 + \frac{1}{2}(\partial_\mu\sigma)^2 - \frac{\lambda}{4}(\phi_i^2)^2 - \frac{\kappa}{4}\left[\left(\sigma+\frac{\sqrt{6}}{2}\right)^2 + 3\left(\xi+\frac{1}{6}\right)\phi_i^2 - \frac{3}{2}\right]^2\right\}.$$

- Constraint equation: $\sigma = \frac{1}{2}\left(\sqrt{\phi^2 - 12\left(\xi+\frac{1}{6}\right)\phi_i^2} - \phi\right)$

- If $\kappa(x)$ is a constant, the constraint equation is set dynamically by the minimum of the potential.

- If instead it is dynamical we have a UV completion.

- From the potential we can directly see the perturbativity conditions:

$$\kappa \lesssim 1, \qquad \lambda + 9\kappa\left(\xi+\frac{1}{6}\right)^2 \lesssim 1, \qquad 6\kappa\left(\xi+\frac{1}{6}\right) \lesssim 1$$

General UV completions from σ models

We add higher curvature terms

$$\mathscr{L}_{\rm gen} = \sqrt{-\hat{g}}\left[-\frac{1}{2}(1+\xi\hat{\phi}_i^2)\hat{R} + \frac{1}{2}g^{\mu\nu}\partial_\mu\hat{\phi}_i\partial_\nu\hat{\phi}_i - \frac{\lambda}{4}(\hat{\phi}_i^2)^2 + \sum_k \frac{2(-1)^{k+1}\alpha_k}{k+1}\hat{R}^{k+1}\right] \longrightarrow \frac{\mathscr{L}_{\rm gen}}{\sqrt{-g}} = -\frac{1}{2}R\left(1-\frac{1}{6}\phi_i^2-\frac{1}{6}\sigma^2\right) + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\phi_i)^2$$

$$-\sum_k \Omega^{-2+\frac{2}{k}}\left(\frac{2k}{k+1}\right)\alpha_k\,\chi_k^{1+\frac{1}{k}} - \frac{\lambda}{4}\phi_i^4$$

Constraint equation $\quad \sum_k 4\alpha_k\chi_k = \frac{1}{2} - \frac{1}{3}\left(\sigma+\frac{\sqrt{6}}{2}\right)^2 - \left(\xi+\frac{1}{6}\right)\phi_i^2$

Example. Only one curvature term $\quad \mathscr{L}_{\rm gen} = \sqrt{-\hat{g}}\left[-\frac{1}{2}(1+\xi\hat{\phi}_i^2)\hat{R} + \frac{1}{2}g^{\mu\nu}\partial_\mu\hat{\phi}_i\partial_\nu\hat{\phi}_i - \frac{\lambda}{4}(\hat{\phi}_i^2)^2 + \frac{2(-1)^{p+1}\alpha_p}{p+1}\hat{R}^{p+1}\right]$

$$\chi_p = \frac{1}{4\alpha_p}\left[\frac{1}{2} - \frac{1}{3}\left(\sigma+\frac{\sqrt{6}}{2}\right)^2 - \left(\xi+\frac{1}{6}\right)\phi_i^2\right]$$

$$\mathscr{L}_{\rm eff} = -\frac{1}{4}\kappa_n(-1)^{2n}\sigma^{2n}(\sigma+\sqrt{6})^{2(2-n)}\sum_{l=0}^{\infty}\binom{2n}{l}\frac{3^l(\xi+\frac{1}{6})^l(\phi_i^2)^l}{\sigma^l(\sigma+\sqrt{6})^l}$$

$$n = \frac{1}{2}\left(1+\frac{1}{p}\right)$$

$$U(\sigma,\phi_i) \equiv \frac{1}{4}\kappa_n(\sigma+\sqrt{6})^{4(1-n)}\left[-\sigma(\sigma+\sqrt{6}) - 3\left(\xi+\frac{1}{6}\right)\phi_i^2\right]^{2n}$$

For 2n non integer, infinite series $\quad \kappa_n(\xi+\frac{1}{6})^2 \lesssim 1$

For 2n integer $\quad \kappa_n(\xi+\frac{1}{6})^{2n} \lesssim 1$

Inflation

$$\mathscr{L}_E = \sqrt{-g_E}\left\{ -\frac{1}{2}R(g_E) + \frac{3}{4\Omega'^4}(\partial_\mu\Omega'^2)^2 + \frac{1}{2\Omega'^2}(\partial_\mu h)^2 + \frac{1}{2\Omega'^2}(\partial_\mu \sigma)^2 - V(\sigma,h)\right\}$$

$$V(\sigma,h) = \frac{1}{\left(1-\frac{1}{6}h^2-\frac{1}{6}\sigma^2\right)^2}\left[\frac{1}{4}\kappa_1\left(\sigma(\sigma+\sqrt{6}) + 3\left(\xi+\frac{1}{6}\right)h^2\right)^2 + \frac{1}{4}\lambda h^4\right]$$

Integrating out the Higgs field

$$h^2 = \frac{\kappa_1\sigma(\sigma+\sqrt{6})\left(\sigma-3\left(\xi+\frac{1}{6}\right)(\sigma-\sqrt{6})\right)}{\lambda(\sigma-\sqrt{6})-3\kappa_1\left(\xi+\frac{1}{6}\right)\left(\sigma-3\left(\xi+\frac{1}{6}\right)(\sigma-\sqrt{6})\right)}$$

Canonical field

$$\sigma = -\sqrt{6}\tanh\left(\frac{\chi}{\sqrt{6}}\right)$$

$$V_{\rm eff}(\chi) = \frac{9\kappa_1}{4}\left(1-e^{-2\chi/\sqrt{6}}\right)^2\left[1+\frac{\kappa_1}{4\lambda}\left(6\xi+e^{-2\chi/\sqrt{6}}\right)^2\right]^{-1}$$

$$n_s = 1 - \frac{2}{N} - \frac{9}{2N^2} + \frac{3\kappa_1}{N^2}\frac{(-\lambda+12\lambda\xi+18\kappa_1\xi^2(1+6\xi))}{(2\lambda+3\kappa_1\xi(1+6\xi))^2}$$

$$r = 16\epsilon_* = \frac{12}{N^2}$$

The Higgs quartic coupling changes predictions but this contribution is small $\lambda \approx \kappa_1\xi^2 \leq 1$

Future measurements of n_s (CMB-S4, LiteBIRD) could see the difference due to the $1/N^2$ terms.

Reheating Temperature

$$\mathscr{L} \supset -3\frac{\sqrt{6}}{2}\kappa(\xi + \frac{1}{6})\sigma h^2 - \frac{\kappa}{2}\left(3\left(\xi + 1/6\right) + 1\right)\sigma^2 h^2$$

$$\Gamma(\sigma \to 2h) \approx \frac{27M_P^2}{16\pi m_\sigma}\kappa^2(\xi + 1/6)^2 \qquad m_\sigma >> m_h$$

T_{reh} **depends on the non-minimal coupling**

$$\Gamma = H_{rh} = \sqrt{\frac{\rho_{rn}}{3M_P}} \qquad \rho_{rh} = \frac{\pi^2 g_*}{30}T_{rh}^4 \qquad T_{reh} = \kappa^{3/4}\left(\xi + \frac{1}{6}\right) \times 1.4 \times 10^{18} \quad \text{GeV} \qquad T_{reh} \sim 10^{13} \quad \text{GeV}$$

Reheating is more efficient than in the Starobinsky case

Dark Energy from R^{p+1}

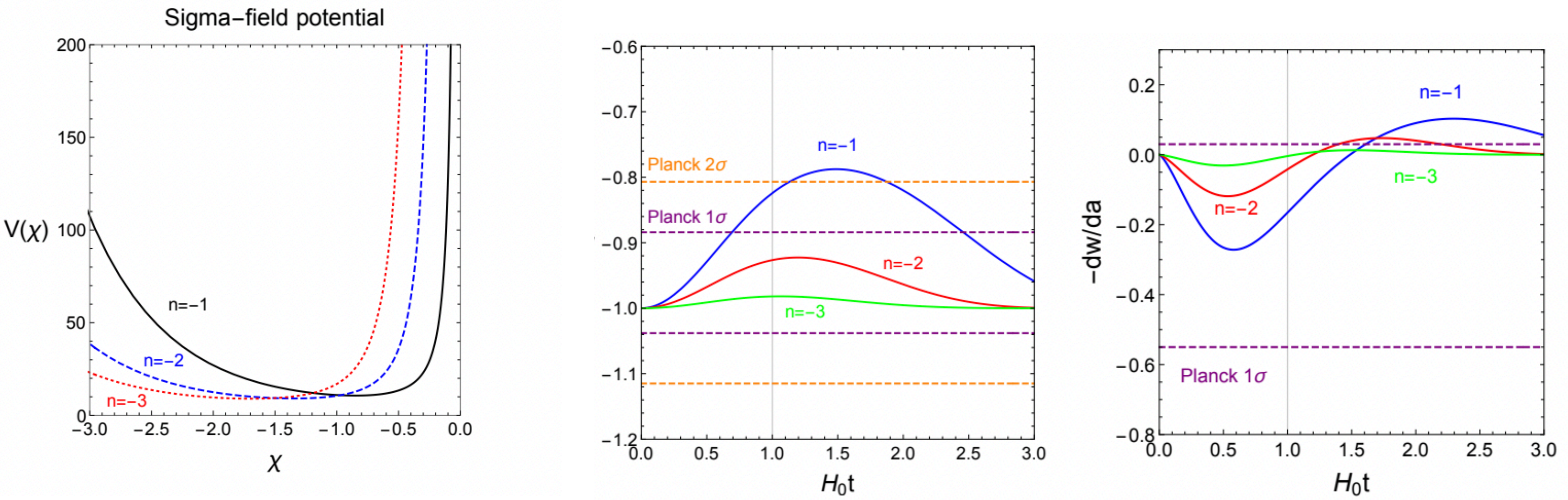
$$V(\chi) = \frac{9\kappa_n}{4^n} \cdot e^{-2(1-\frac{1}{p})\chi/\sqrt{6}}\left(1 - e^{-2\chi/\sqrt{6}}\right)^{1+\frac{1}{p}}$$

$$w' = (w-1)\left[3(1+w) - c\sqrt{3(1+w)\Omega_\chi}\right]$$

$$\Omega_\chi' = -3(w - w_m)\Omega_\chi(1-\Omega_\chi)$$

$$c' = -\sqrt{3(1+w)\Omega_\chi}\,(\Gamma - 1)c^2$$

$$\Gamma = V\frac{d^2V}{d\chi^2}\left(\frac{dV}{d\chi}\right)^{-2}$$



Supergravity set-up

$$S = \Big[\,|X^0|\,\tilde{\Omega}(z^\alpha, \bar{z}^{\bar{\beta}})\Big]_{\mathrm{D}} + \big[(X^0)^3\tilde{\Omega}(z^\alpha)\big]_{\mathrm{F}} + \big[f_{AB}(z^\alpha)\bar{\mathcal{W}}^A\mathcal{W}^B\big]_{\mathrm{F}} + \big[\alpha\bar{R}R\big]_D$$

- X^0 is a chiral compensator multiplet. It is unphysical and eliminated through gauge conditions.
- z^α and $\bar{z}^{\bar{\beta}}$ are chiral and anti-chiral matter multiplets (0,0).
- $\tilde{\Omega}$ is a real frame function related to the Kähler potential $\quad \bar{K} = -3\log\left(-\frac{\tilde{\Omega}}{3}\right)$
- W is the superpotential and f the gauge kinetic function.
- $\big[\alpha\bar{R}R\big]_D$ is a curvature multiplet with $R = \left(X^0\right)^{-1}\Sigma(\bar{X}^{\bar{0}})$

Bosonic Lagrangian

$$\mathcal{L}/\sqrt{-g} = -\tilde{\Omega}_{\alpha\bar{\beta}}\partial_\mu z^\alpha \partial^\mu \bar{z}^{\bar{\beta}} + (-i\tilde{\Omega}_\alpha \partial_\mu z^\alpha \mathcal{A}^\mu + \mathrm{c.c.}) + \tilde{\Omega}(-\mathcal{A}^2 + \left|F^0\right|^2) + (3F^0\tilde{W} + \mathrm{c.c.})$$

$$+\left(-\frac{\tilde{\Omega}}{6} + \frac{\alpha}{6}|F^0|^2 + \frac{\alpha}{3}\mathcal{A}^2\right)R + \frac{\alpha}{36}R^2 \quad +\alpha\left(\mathcal{A}^2 + \left|F^0\right|^2\right)^2 + \alpha(\nabla_\mu \mathcal{A}^\mu)^2$$

Non-minimal couplings to matter

$$-\alpha\left|\partial_\mu F^0 - 3i\mathcal{A}_\mu F^0\right|^2 - \tilde{\Omega}^{\alpha\bar{\beta}}(\tilde{\Omega}_\alpha \bar{F}^{\bar{0}} + \tilde{W}_\alpha)(\tilde{\Omega}_{\bar{\beta}} F^0 + \bar{\tilde{W}}_{\bar{\beta}}) \quad -\frac{1}{2}(\mathrm{Re}f)^{-1AB}\tilde{\Omega}_\alpha k_A^\alpha \tilde{\Omega}_{\bar{\beta}} k_B^{\bar{\beta}}$$

F^0 is dynamical

Killing vectors

The Higgs sector is composed of $z^\alpha = \{H, H_u, H_d\}$ $\quad H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix} \quad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$

$$\tilde{\Omega}(z^\alpha, \bar{z}^{\bar{\beta}}) = -3 + |S|^2 + |H_u|^2 + |H_d|^2 + \left(\frac{3}{2}\chi H_u \cdot H_d + \mathrm{h.c.}\right)$$

$$\tilde{W}(z^\alpha) = \lambda S H_u \cdot H_d + \frac{\rho}{3} S^3$$

Kinetic terms of the NMSSM

$$\mathscr{L}/\sqrt{-g} = \left\{\frac{1}{2} - \frac{1}{6}|S|^2 - \frac{1}{6}|H_u|^2 - \frac{1}{6}|H_d|^2 + \left(-\frac{1}{4}\chi H_u \cdot H_d + \mathrm{h.c.}\right)\right\} R$$

$$-|\partial_\mu S|^2 - |\partial_\mu H_u|^2 - |\partial_\mu H_d|^2 + \frac{\alpha}{36} R^2 + \cdots$$

Extra scalar fields appear in comparison to the non-supersymmetric case

Dual-scalar Lagrangian

$$[\alpha \bar{\mathscr{R}} \mathscr{R}]_D = [\alpha \bar{C} C]_D + [T(C - \mathscr{R})]_F$$

The EOM for T leads to $C = R$

$$S = [\,|X^0|^2(\tilde{\Omega} + \alpha \bar{C} C - (T + \bar{T}))]_D + [(X^0)^3(\tilde{W} + TC)]_F + [f_{AB}(z^\alpha)\bar{\mathscr{W}}^A \mathscr{W}^B]_F$$ **Total action**

$$S = [\,|X^0|^2 \Omega(z^I, \bar{z}^{\bar{J}})]_D + [(X^0)^3 W(z^I)]_F + [f_{AB}(z^I)\bar{\mathscr{W}}^A \mathscr{W}^B]_F$$ **Standard supergravity**

$$\Omega(z^I, \bar{z}^{\bar{J}}) \equiv \tilde{\Omega}(z^\alpha, \bar{z}^{\bar{\beta}}) + |C|^2 - (T + \bar{T}) = -3 + |S|^2 + |H_u|^2 + |H_d|^2 + \left(\frac{3}{2}\chi H_u \cdot H_d + \mathrm{h.c.}\right) + |C|^2 - (T + \bar{T})$$

$$W(z^I) \equiv \tilde{W}(z^\alpha) + \frac{1}{\sqrt{\alpha}} TC = \lambda S H_u \cdot H_d + \frac{\rho}{3} S^3 + \frac{1}{\sqrt{\alpha}} TC$$

Higher derivatives have been moved to T and C

Conclusions

- General linear sigma models can be regarded as UV completions of Higgs inflation.

- A particular family of models coming from R^2 leads to successful inflation.
- Higher curvature terms R^{p+1} can explain Dark Energy with tracker behavior.
- Predictions for the time-varying equation of state can be consistent with observations within 1σ [Plank 18 -1807.06209]
- We provided a supersymmetry embedding of general Higgs-sigma models.

- In the next talk by Shuntaro Aoki:

- Inflation in supersymmetric models and SUSY breaking at low energies

- On-going and future work:

- Dark Matter production