Cosmic Inflation & Primordial Black Holes

in the Scalar-Tensor Theories of Gravity

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Gansukh Tumurtushaa

Nov. 28, 2021

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based on: *Eur.Phys.J.C* 79 (2019) 11, 920, arXiv: 2107.08638 with Prof. Chen (NTU) and Prof. Koh (JejuNU), &
arXiv: 2112.XXXXX with Mr. Chien (NTU)

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Content:

Part I: Motivation and A toy model

- Cosmic inflation
  - natural inflation & observational constraints

G. T, *Eur.Phys.J.C* 79 (2019) 11, 920

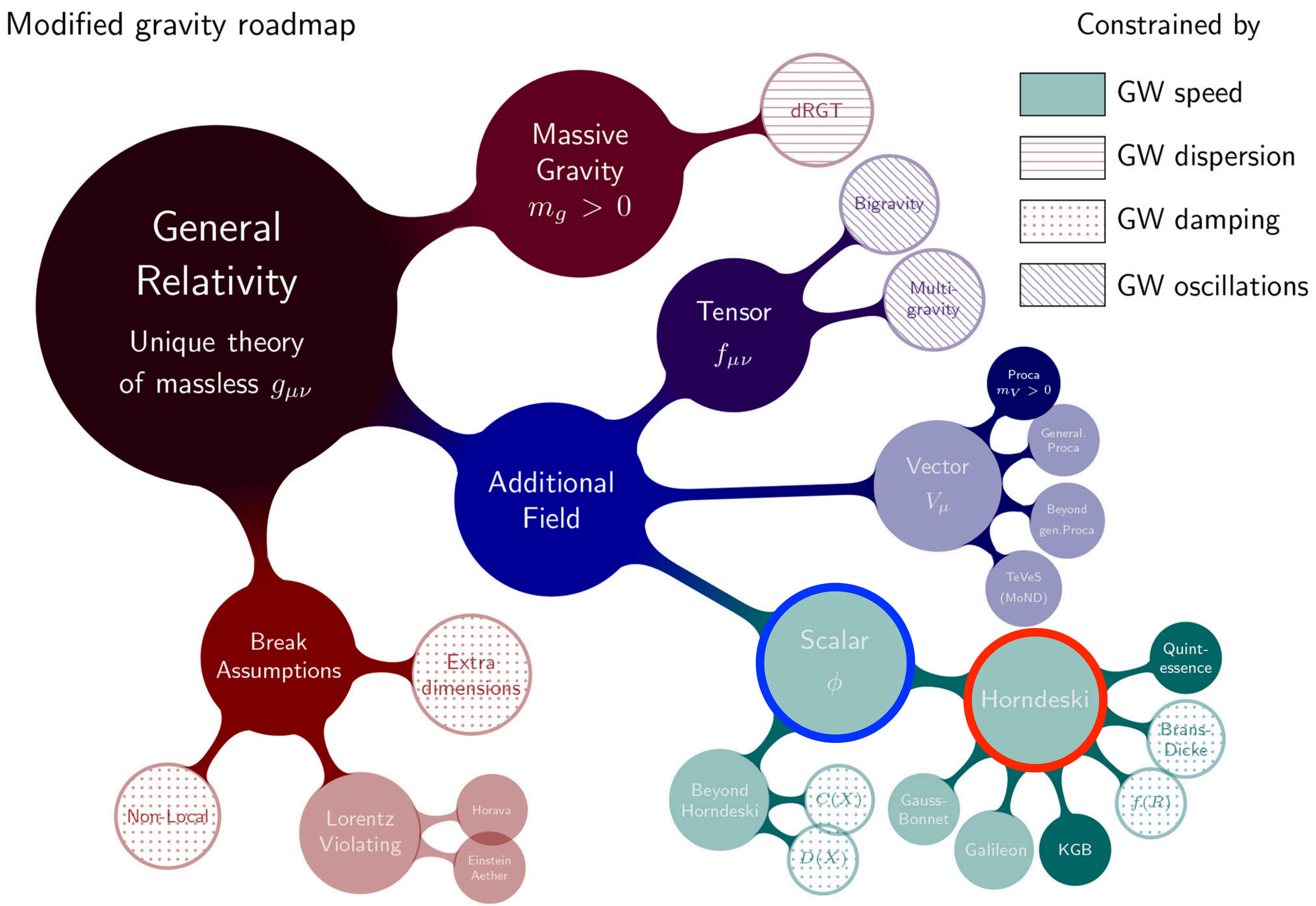
- Reheating: after natural inflation
  - temperature & constraints on natural inflation

Chen-Hsu Chien, Seoktae Koh, G. T, *a work in progress*

Part II: PBHs and GWs in the scalar-tensor theory of gravity

Pisin Chen, Seoktae Koh, G. T, arXiv:2107.08638

Conclusion



Front. Astron. Space Sci. 5:44 (2018)

- A scalar field ϕ is the simplest field by which gravity can be extended.
- Theories containing a coupling between ϕ and gravity are called "scalar-tensor theories of gravity."
- In 1974, Horndeski derived the action of the most general scalar-tensor theories with the 2^{nd} order EoM.

Generalized Galileon Theory: $S = \int d^4x\sqrt{-g}\,\left(L_2 + L_3 + L_4 + L_5\right)$

[G. Horndeski, “Second order scalar-tensor field equations in a 4D spacetime”];
C. Deffayet, X. Gao, D. A. Steer and G. Zahariade, PRD 84, 064039 (2011);
T. Kobayashi, M. Yamaguchi and J. Yokoyama, PTP 126, 511 (2011);
X. Gao, T. Kobayashi, M. Shiraishi, M. Yamaguchi, J. Yokoyama and S. Yokoyama, PTEP 2013, 053E03 (2013);

$L_2 = G_2(\phi, X)$ where $X = -\nabla_\mu\phi\,\nabla^\mu\phi/2$

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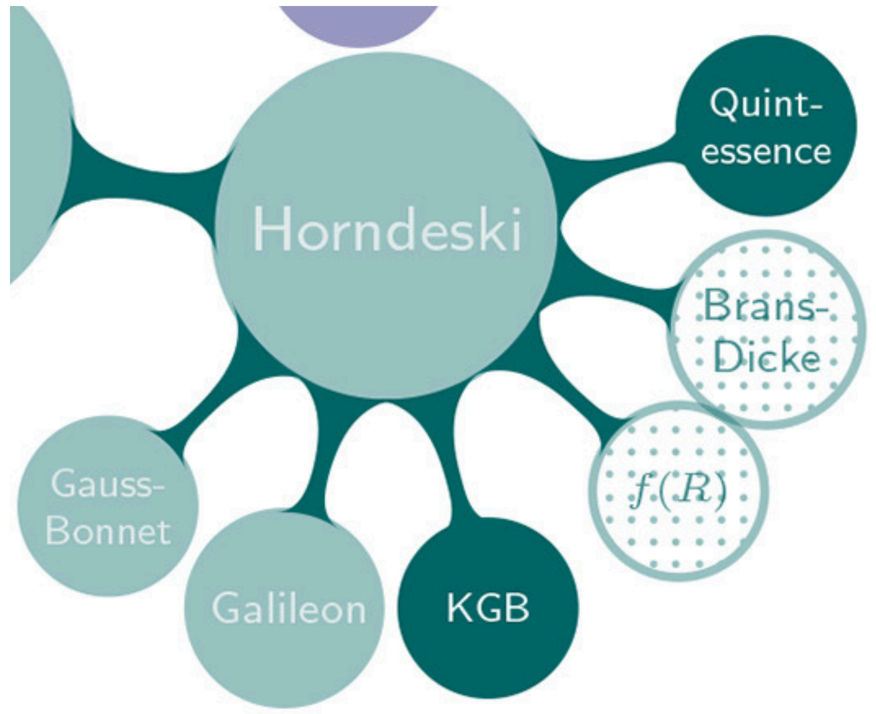
$$L_4 = G_4(\phi, X)R + G_{4,X}\left[\left(\Box\,\phi\right)^2 - \left(\nabla_\mu\nabla_\nu\phi\right)\left(\nabla^\mu\nabla^\nu\phi\right)\right]$$

$$L_5 = G_5(\phi, X)G_{\mu\nu}\left(\nabla^\mu\nabla^\nu\phi\right) - \frac{1}{6}G_{5,X}\left[(\Box\phi)^3 - 3\,\Box\,\phi\left(\nabla_\mu\nabla_\nu\phi\right)\left(\nabla^\mu\nabla^\nu\phi\right) + 2\left(\nabla^\mu\nabla_\alpha\phi\right)\left(\nabla^\alpha\nabla_\beta\phi\right)\left(\nabla^\beta\nabla_\mu\phi\right)\right]$$



by choosing a certain combinations of $G_i(\phi, X)$ functions, one can construct a broad spectrum of cosmological models describing cosmic inflation (and dark energy).

As an extended theory of gravity, Horndeski theory includes:

- **Quitessence** [Caldwell, Dave, and Steinhardt (1998)]

$$S = \int d^4\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi - V(\phi)\right],$$

- **K-essence** [Chiba, Okabe, and Yamaguchi (2000)]

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R + K(\phi, X)\right] \quad \text{where} \quad X = -\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi,$$

- **Kinetic Gravity Braiding** [Deffayet, Pujolas, Sawicki, Vikman (2010)]

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R + K(\phi, X) + G(\phi, X)\Box\phi\right],$$

- **Brans-Dicke theory** [Jordan (1959), Brans and Dicke (1961)]

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[\phi R - \frac{\omega_{BD}}{\phi}\nabla^\mu\phi\nabla_\mu\phi\right],$$

- **$F(R)$ Gravity** [Buchdahi (1970)] etc.,

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}F(R) \quad \overset{\Leftrightarrow}{\text{Equivalent}} \quad S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[\phi R - V(\phi)\right],$$

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$$G_2(\phi, X) = -\frac{R}{2\kappa^2}\left[F_{,R}(R) - F(R)\right]\,,\quad G_3 = 0\,,\quad G_4(\phi) = \frac{F(R)}{2\kappa^2}\,,\quad G_5 = 0,$$

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- **Kineti**

$S =$ … $G_5 = 0$

- **Brans**

$S = \frac{\quad}{2\kappa^2}\int$ … $\frac{\quad}{\phi}$ … $G_5 = 0$

THE CASES IN WHICH ALL $G_i(\phi, X)$ FUNCTIONS ARE PRESENT IN THE ACTION AND EQUALLY IMPORTANT DURING INFLATION HAVE *NOT* BEEN EXPLORED MUCH (SO FAR)...

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$$G_2(\phi, X) = K(\phi, X)\,, \quad G_3 = 0\,, \quad G_4 = \frac{1}{2\kappa^2}\,, \quad G_5 = 0$$

ONE SHOULD ATTEMPT A TASK TO CONSTRUCT COSMOLOGICAL MODELS

IN WHICH ALL $G_i(\phi, X)$ ARE "*PRESENT & EQUALLY IMPORTANT.*"

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[\phi R - \frac{\omega_{BD}}{\phi}\nabla^\mu\phi\nabla_\mu\phi\right],$$

$$G_2(\phi, X) = \frac{\omega_{BD}}{\kappa^2\phi}X\,, \quad G_3 = 0\,, \quad G_4(\phi) = \frac{1}{2\kappa^2}\phi\,, \quad G_5 = 0$$

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Equivalent

$$G_2(\phi, X) = -\frac{R}{2\kappa^2}\left[F_{,R}(R) - F(R)\right]\,, \quad G_3 = 0\,, \quad G_4(\phi) = \frac{F(R)}{2\kappa^2}\,, \quad G_5 = 0,$$

Action: $S = \int d^4x \sqrt{-g}\,(L_2 + L_3 + L_4 + L_5)$

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$$L_5 = G_5(\phi, X)G_{\mu\nu}(\nabla^\mu\nabla^\nu\phi) - \frac{1}{6}G_{5,X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi) + 2(\nabla^\mu\nabla_\alpha\phi)(\nabla^\alpha\nabla_\beta\phi)(\nabla^\beta\nabla_\mu\phi)\right]$$

A: When $G_2(\phi, X) \neq 0$, $G_3(\phi, X) \neq 0$, $G_4(\phi, X) = \frac{M_p^2}{2}$, **and** $G_5(\phi, X) = 0$:

G- inflation or inflation with the derivative self interaction of the scalar field.

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B: When $G_2(\phi, X) \neq 0$, $G_3(\phi, X) = 0$, $G_4 = \frac{M_p^2}{2}$, **and** $G_5(\phi, X) \neq 0$:

Inflation with the non-minimal derivative coupling between gravity and the scalar field.

A toy model: A+B

$$G_2(\phi, X) = X - V(\phi)\,,\; G_3(\phi, X) = \frac{\alpha}{M^3}\xi(\phi)X\,,\; G_4 = \frac{M_{pl}^2}{2}\,,\; G_5(\phi) = \frac{\beta}{2M^2}\phi$$

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Inflationary model:

$$G_2(\phi, X) = X - V(\phi)\,, \quad G_3(\phi, X) = \frac{\alpha}{M^3}\xi(\phi)X\,, \quad G_4 = \frac{M_{pl}^2}{2}\,, \quad G_5(\phi) = \frac{\beta}{2M^2}\phi$$

$$S = \int d^4x\sqrt{-g}\,\left(L_2 + L_3 + L_4 + L_5\right)$$

where

$$L_2 = G_2(\phi, X), \quad X = -\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi$$

$L_3 = G_3(\phi, X)\Box\phi,$ $\Longrightarrow$ "<u>Derivative self-interaction of the scalar field</u>" or "$\alpha-$term"

$$L_4 = G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - \left(\nabla_\mu\nabla_\nu\phi\right)\left(\nabla^\mu\nabla^\nu\phi\right)\right],$$

$$L_5 = G_5(\phi, X)G_{\mu\nu}\left(\nabla^\mu\nabla^\nu\phi\right) - \frac{1}{6}G_{5,X}\left[(\Box\phi)^3 - 3\Box\phi\left(\nabla_\mu\nabla_\nu\phi\right)\left(\nabla^\mu\nabla^\nu\phi\right) + 2\left(\nabla^\mu\nabla_\alpha\phi\right)\left(\nabla^\alpha\nabla_\beta\phi\right)\left(\nabla^\beta\nabla_\mu\phi\right)\right],$$

$\Longrightarrow$ "<u>Kinetic coupling between gravity and the scalar field</u>" or "$\beta-$term"

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{pl}^2}{2}R - \frac{1}{2}\left(g^{\mu\nu} - \frac{\alpha}{M^3}\xi(\phi)g^{\mu\nu}\partial_\rho\partial^\rho\phi + \frac{\beta}{M^2}G^{\mu\nu}\right)\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

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In a flat FRW universe with $ds^2 = -dt^2 + a(t)^2\delta_{ij}dx^i dx^j$:

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the background dynamical equations are obtained as

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$$3M_{pl}^2H^2 = \rho_\phi$$

$$M_{pl}^2\left(2\dot{H} + 3H^2\right) = -p_\phi\,,$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} - \frac{\alpha}{2M^3}\dot{\phi}\left[\ddot{\xi}\dot{\phi} + 3\dot{\xi}\ddot{\phi} - 6\xi\dot{\phi}\left(\dot{H} + 3H^2 + 2H\frac{\ddot{\phi}}{\dot{\phi}}\right)\right] - \frac{3\beta}{M^2}H\dot{\phi}\left(2\dot{H} + 3H^2 + H\frac{\ddot{\phi}}{\dot{\phi}}\right) = 0\,,$$

where

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V + \frac{3\alpha}{M^3}H\xi\dot{\phi}^3\left(1 - \frac{\dot{\xi}}{6H\xi}\right) - \frac{9\beta}{2M^2}\dot{\phi}^2H^2\,,$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V - \frac{\alpha}{M^3}\xi\dot{\phi}^3\left(\frac{\ddot{\phi}}{\dot{\phi}} + \frac{\dot{\xi}}{2\xi}\right) + \frac{\beta\dot{\phi}^2}{2M^2}\left(2\dot{H} + 3H^2 + 4H\frac{\ddot{\phi}}{\dot{\phi}}\right)\,.$$

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In the context of slow-roll inflation:

$$\epsilon_1 \equiv -\frac{\dot{H}}{H^2}\,,\quad \epsilon_2 \equiv -\frac{\ddot{\phi}}{H\dot{\phi}}\,,\quad \epsilon_3 \equiv \frac{\xi_{,\phi}\dot{\phi}}{\xi H}\,,\quad \epsilon_4 \equiv \frac{\xi_{,\phi\phi}\dot{\phi}^4}{V_{,\phi}}\,,\quad \epsilon_5 \equiv \frac{\dot{\phi}^2}{M_{pl}^2H^2}\,,$$

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where

⇓

$$3H\dot{\phi}\left[1 - \frac{1}{3}\epsilon_2 + \frac{\alpha}{M^3}\xi H\dot{\phi}\left(3 - \epsilon_1 - 2\epsilon_2 - \frac{2}{3}\epsilon_2\epsilon_3\right) - \frac{\beta}{M^2}H^2\left(3 - 2\epsilon_1 - \epsilon_2\right)\right] = -V_{,\phi}\left(1 - \frac{\alpha}{2M^3}\epsilon_4\right)$$

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⇓

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Since $|\epsilon_{1,2,3,4,5}| \ll 1$ during inflation

⇓

$$3H\dot{\phi}\left(1 + \mathscr{A}\right) \simeq -V_{,\phi} \quad \text{where} \quad \mathscr{A} \equiv \frac{3\alpha}{M^3}\xi H\dot{\phi} - \frac{3\beta}{M^2}H^2$$

- In the slow-roll inflation scenario ($\dot{\phi}^2 \ll V$ and $\ddot{\phi} \ll 3H\dot{\phi}$), the background EoM,

$$\Rightarrow \quad 3M_{pl}^2 H^2 = \rho_\phi \,, \qquad M_{pl}^2 \left(2\dot{H} + 3H^2\right) = -p_\phi \,,$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} + \frac{\alpha}{2M^3}\dot{\phi}\left[\ddot{\xi}\dot{\phi} + 3\dot{\xi}\ddot{\phi} - 6\xi\dot{\phi}\left(\dot{H} + 3H^2 + 2H\frac{\ddot{\phi}}{\dot{\phi}}\right)\right] - \frac{3\beta}{M^2}H\dot{\phi}\left(2\dot{H} + 3H^2 + H\frac{\ddot{\phi}}{\dot{\phi}}\right) = 0 \,,$$

can be approximated as

$$\Rightarrow \quad 3H^2 \simeq \frac{V(\phi)}{M_{pl}^2} \,,$$

$$3H\dot{\phi}\left(1 + \mathscr{A}\right) + V_{,\phi} \simeq 0 \quad \text{where} \quad \mathscr{A} \equiv \frac{3\alpha}{M^3}\xi H\dot{\phi} - \frac{3\beta}{M^2}H^2$$

$|\mathscr{A}| \ll 1$: GR limit

$|\mathscr{A}| \gtrsim 1$: a deviation from GR

- Our interest: $\alpha-$ and $\beta-$terms contribute "equally" during inflation.

G. T, *Eur.Phys.J.C* 79 (2019) 11, 920

- Thus, it is useful to introduce a new parameter: $\gamma \equiv \left|\frac{\alpha\xi H\dot{\phi}}{\beta M H^2}\right| \sim \mathcal{O}(1)$, such that $\mathscr{A} = \frac{3H^2}{M^2}\beta\left(\gamma - 1\right)$.

- $\gamma \sim \mathcal{O}(1)$ const. allows us:
  - to control the contributions of these terms
  - to determine the form of $\xi(\phi)$ for the given potential $V(\phi)$

$\gamma \to \infty$ when $\alpha -$ term dominates

$\gamma \to 0$ when $\beta -$ term dominates

- We compute the observable quantities through the linear perturbation theory

T. Kobayashi, M. Yamaguchi and J. Yokoyama, PTP 126, 511 (2011)

- Power spectra for scalar mode and its spectral tilt:

$$\mathscr{P}_S = \frac{k^3}{2\pi^2}\left|\frac{v_k}{z_S}\right|^2 \simeq \frac{\kappa^2 H^2}{8\pi^2 c_S^3 \epsilon_V}(1+\mathscr{A}) \quad \text{and} \quad n_S - 1 \equiv \left.\frac{\ln \mathscr{P}_S}{\ln k}\right|_{c_S k = aH} \simeq \frac{1}{1+\mathscr{A}}\left[2\eta_V - 2\epsilon_V\left(4 - \frac{1}{1+\mathscr{A}}\right)\right],$$

- Power spectra for tensor mode and its spectral tilt:

$$\mathscr{P}_T = \frac{k^3}{\pi^2}\sum_{\lambda=+,x}\left|\frac{u_{\lambda,k}}{z_T}\right|^2 \simeq \frac{\kappa^2 H^2}{2\pi^2 c_T^3} \quad \text{and} \quad n_T \equiv \left.\frac{\ln \mathscr{P}_T}{\ln k}\right|_{c_T k = aH} \simeq -\frac{2\epsilon_V}{1+\mathscr{A}},$$

- The tensor-to-scalar ratio:

$$r \equiv \frac{\mathscr{P}_T}{\mathscr{P}_S} \simeq \frac{16\epsilon_V}{1+\mathscr{A}},$$ $\Longrightarrow$ the ***suppression*** of r due to $\alpha-$ and $\beta-$terms

G. T, *Eur.Phys.J.C* 79 (2019) 11, 920

- In the $|\mathscr{A}| \ll 1$ limit, we obtain:

$$n_S - 1 = 2\eta_V - 6\epsilon_V\,,\ n_T = -2\epsilon_V\,, \text{and } r = 16\epsilon_V.$$

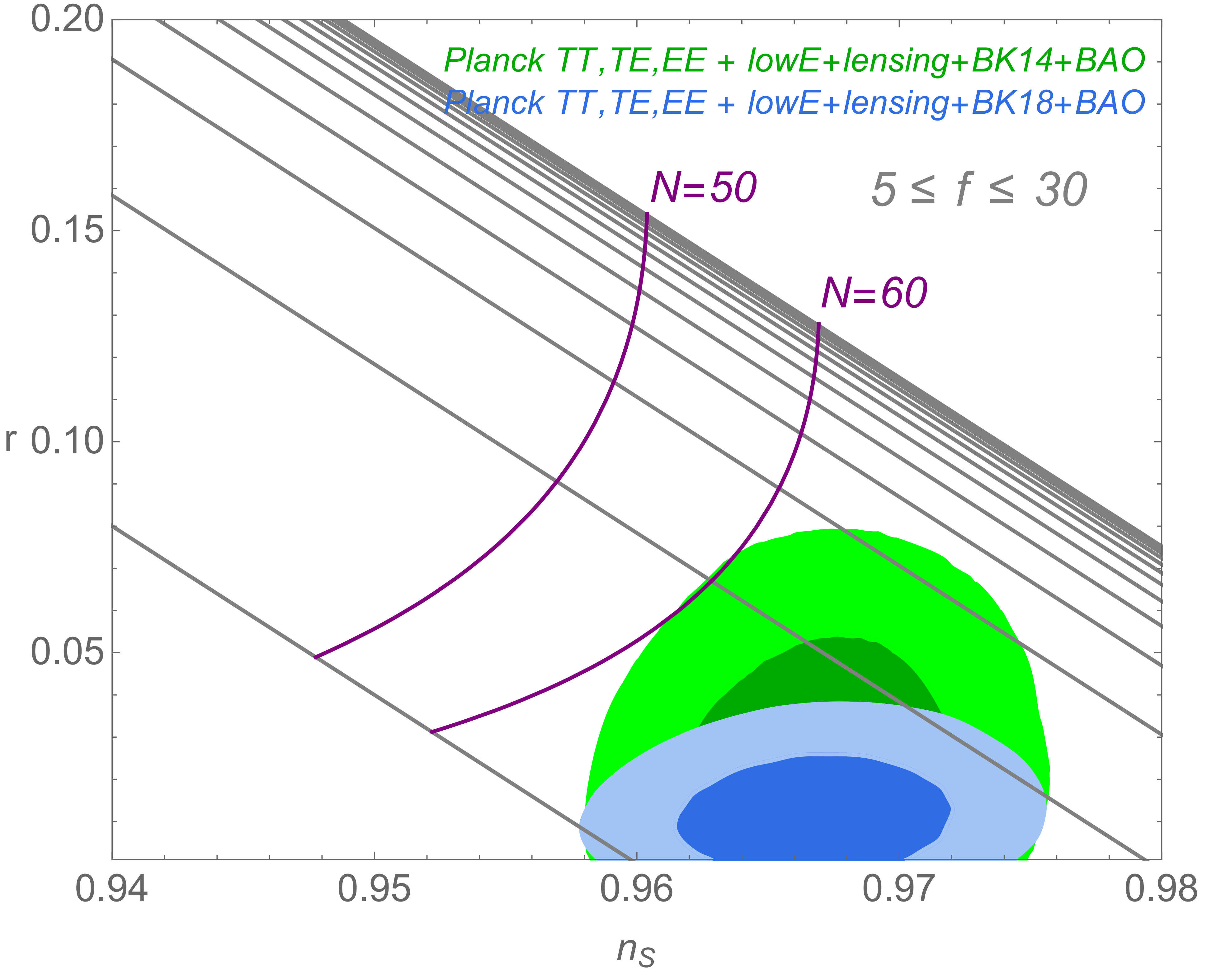
- The slow-roll parameters

$$\epsilon_1 = \frac{\epsilon_V}{1+\mathscr{A}}, \quad \epsilon_2 \simeq \frac{\eta_V - 3\epsilon_V}{1+\mathscr{A}} + \frac{2\epsilon_V}{(1+\mathscr{A})^2}, \quad \epsilon_3 \simeq \frac{\eta_V - 4\epsilon_V}{1+\mathscr{A}} + \frac{2\epsilon_V}{(1+\mathscr{A})^2}, \quad \text{where } \epsilon_V \equiv \frac{1}{2\kappa^2}\left(\frac{V_{,\phi}}{V}\right)^2, \quad \eta_V \equiv \frac{V_{,\phi\phi}}{\kappa^2 V}$$

Natural inflation: $V(\phi) = \Lambda^4 \left[1 - \cos\left(\frac{\phi}{f}\right) \right]$.

Natural inflation: $V(\phi) = \Lambda^4 \left[1 - \cos\left(\frac{\phi}{f}\right)\right]$.

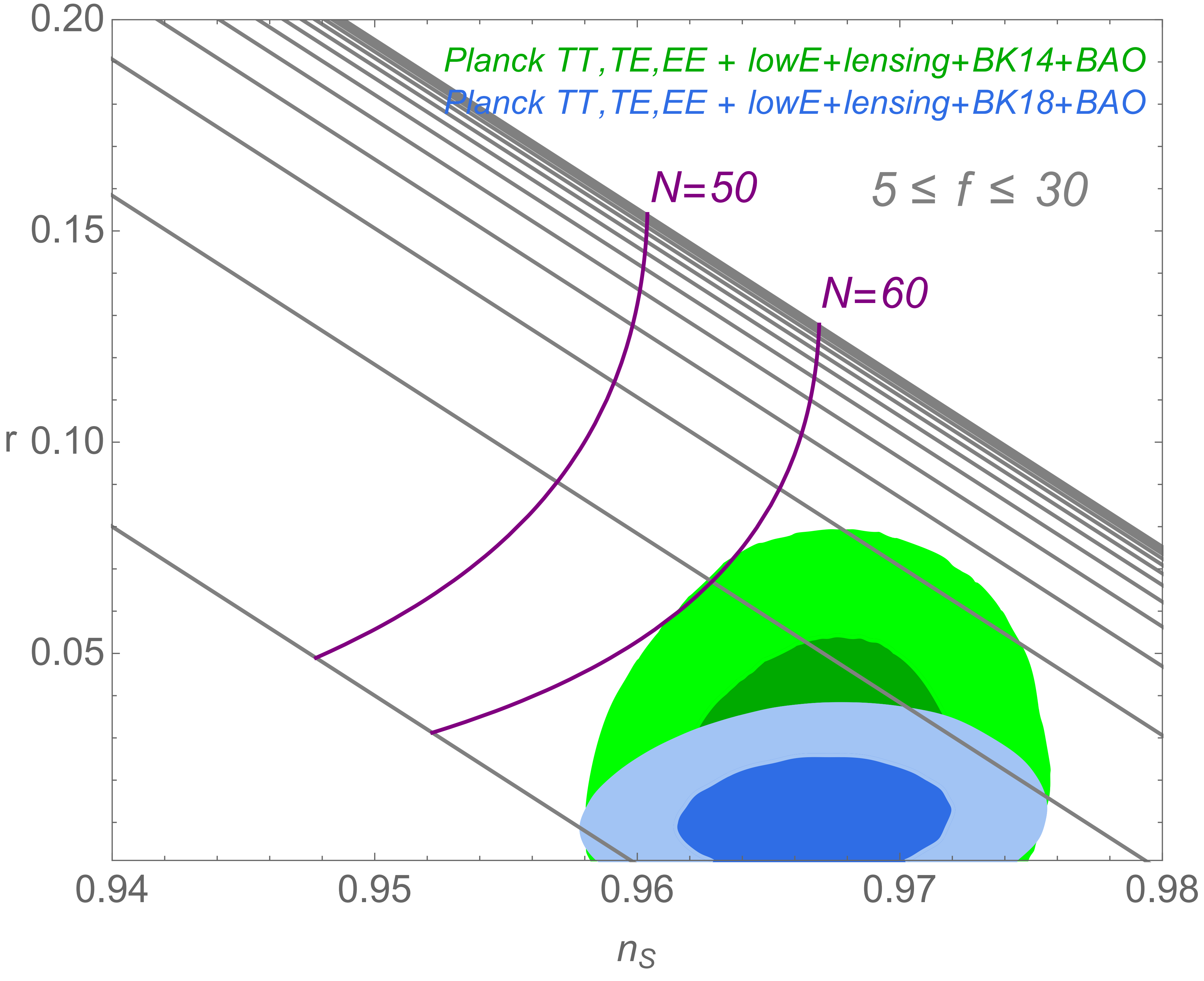
without $\alpha-$ and $\beta-$ terms



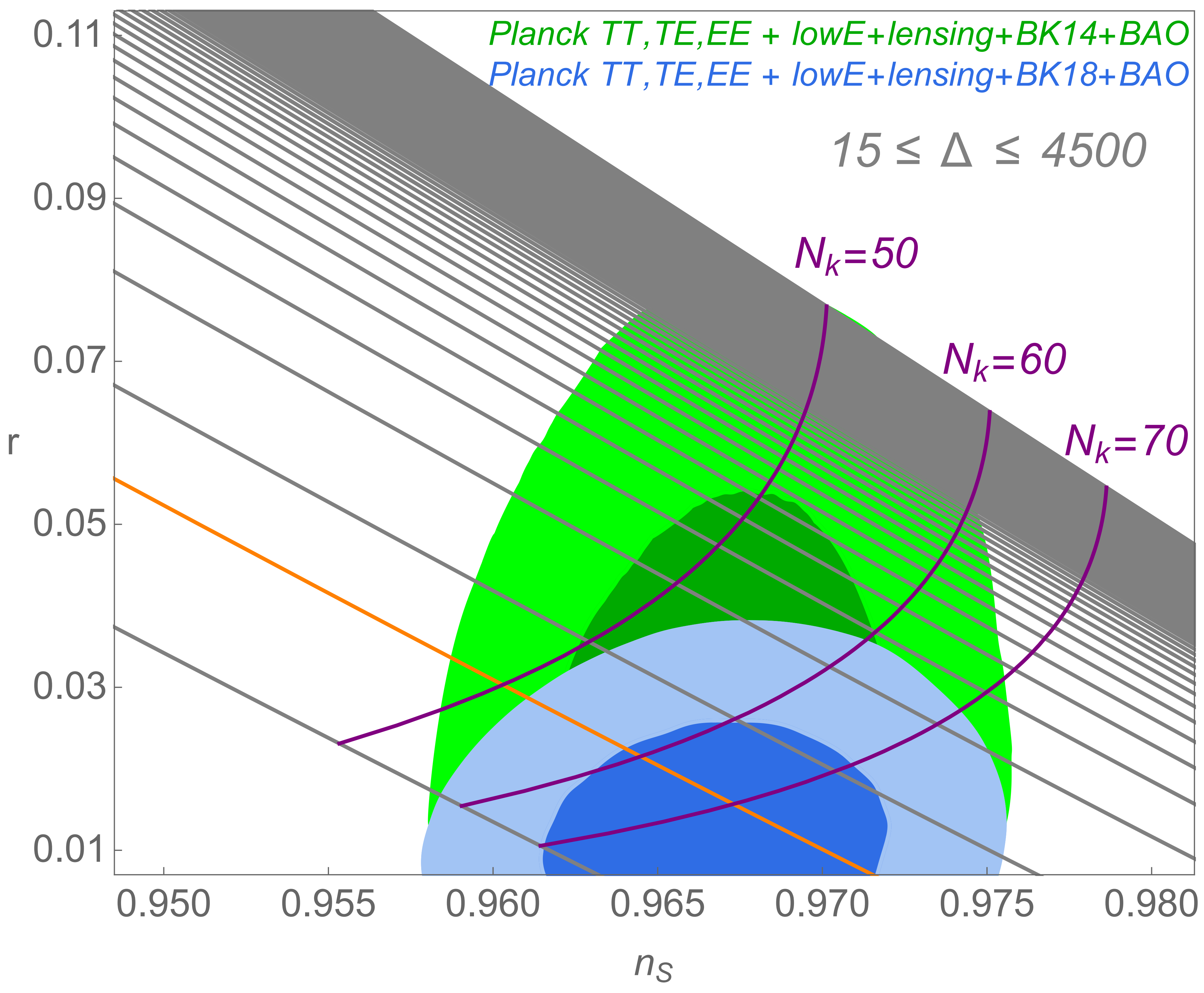
Natural inflation: $V(\phi) = \Lambda^4 \left[1 - \cos\left(\frac{\phi}{f}\right) \right]$

$$n_S = 1 - \frac{2[2-\cos(\phi/f)]}{\Delta[1+\cos(\phi/f)]^2}, \qquad r = \frac{8\,[1-\cos(\phi/f)]}{\Delta[1+\cos[\phi/f]]^2}.$$

without $\alpha-$ and $\beta-$ terms



with $\alpha-$ and $\beta-$ terms



The number of e-folds:

$$N_k = \int_{\phi}^{\phi_e} \frac{H}{\dot{\phi}} d\phi' \simeq \frac{1}{M_p^2} \int_{\phi_e}^{\phi} \frac{V}{V_{,\phi}} (1+\mathscr{A}) d\phi',$$

$$\cos\left(\frac{\phi}{f}\right) = 1 + 2\mathscr{W}\left(-e^{\frac{1}{2}\left(\mathscr{F}(\phi_e) - \frac{N_k+\Delta}{\Delta}\right)}\right)$$

and

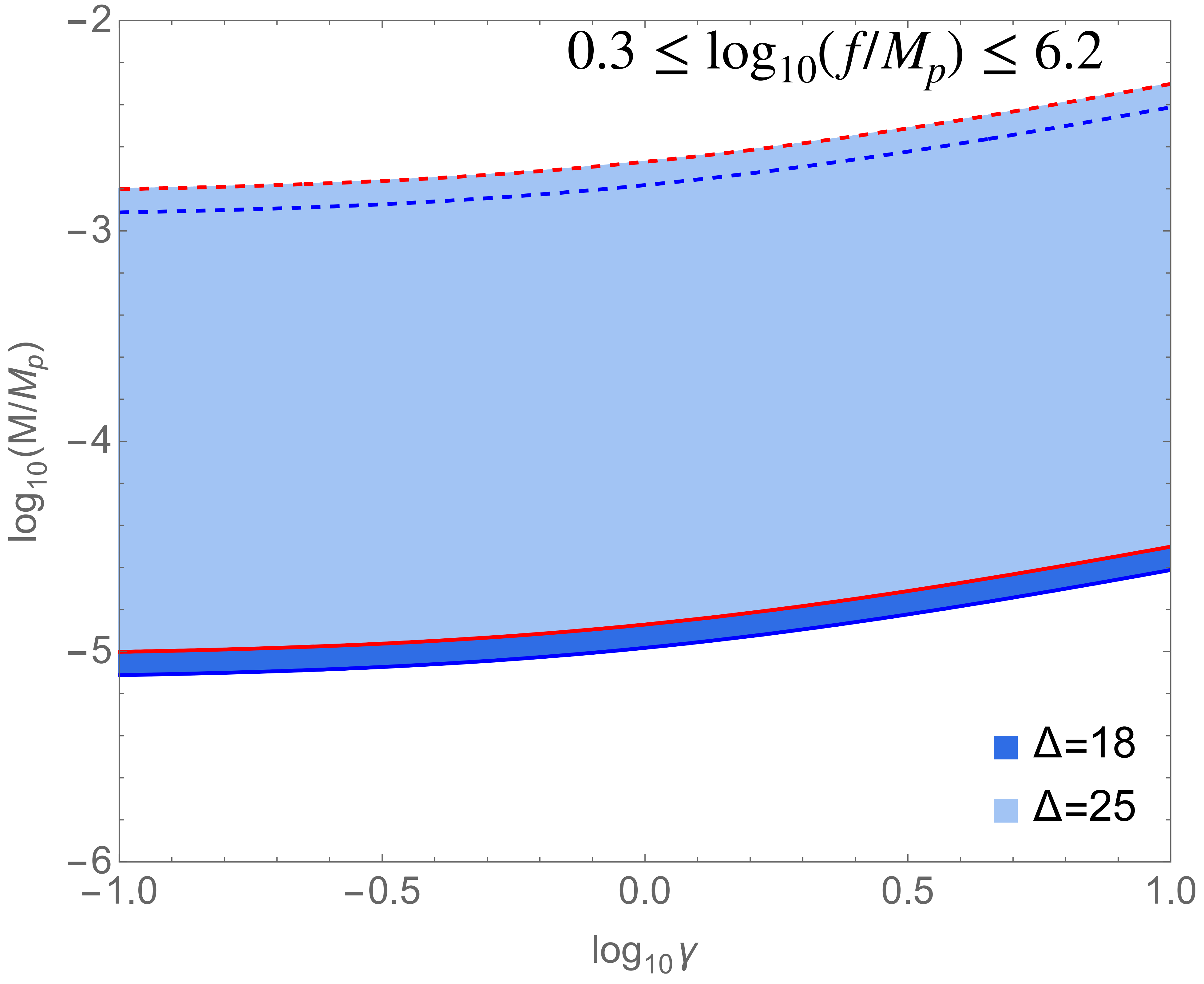
$$\Delta \equiv \beta(\gamma - 1)\frac{f^2\Lambda^4}{M^2 M_p^4}.$$

$$\mathscr{F}(\phi_e) = 2\ln\left[\frac{1+8\Delta-\sqrt{16\Delta+1}}{8\Delta}\right] - \frac{1+4\Delta-\sqrt{16\Delta+1}}{4\Delta}$$

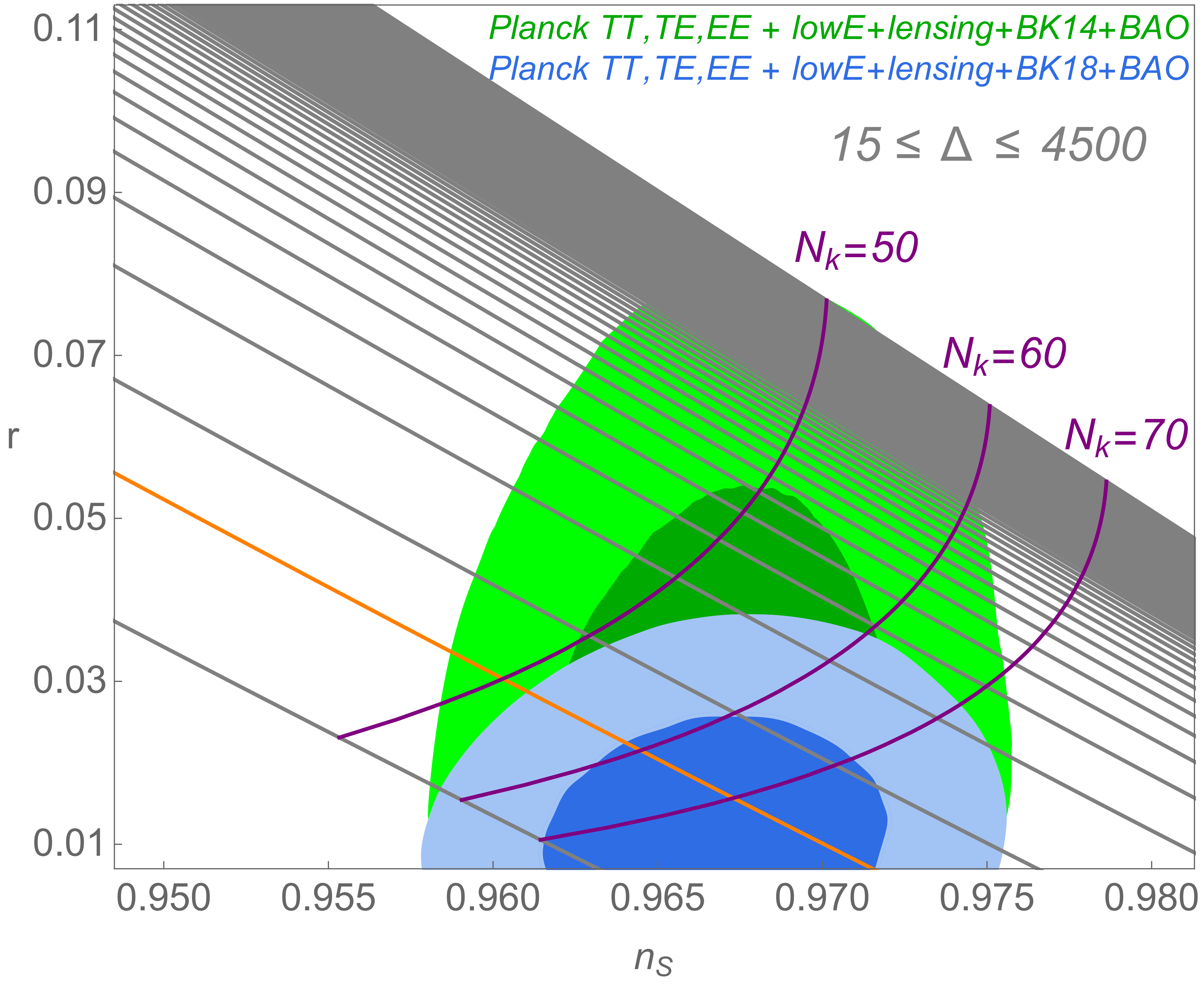
Natural inflation: $V(\phi) = \Lambda^4\left[1-\cos\left(\frac{\phi}{f}\right)\right]$.

$$n_S = 1 - \frac{2[2-\cos(\phi/f)]}{\Delta[1+\cos(\phi/f)]^2}\,, \qquad r = \frac{8\,[1-\cos(\phi/f)]}{\Delta[1+\cos[\phi/f]]^2}\,.$$

$n_S = 0.9649 \pm 0.0042\,,\ \mathscr{P}_S = 2.0989\times 10^{-9}\,(TT, TE, EE + lowE + lensing)$



with $\alpha-$ and $\beta-$ terms



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$$\cos\left(\frac{\phi}{f}\right) = 1 + 2\mathscr{W}\left(-e^{\frac{1}{2}\left(\mathscr{F}(\phi_e) - \frac{N_k+\Delta}{\Delta}\right)}\right) \quad \text{and} \quad \Delta \equiv \beta(\gamma-1)\frac{f^2\Lambda^4}{M^2M_p^4}\,.$$

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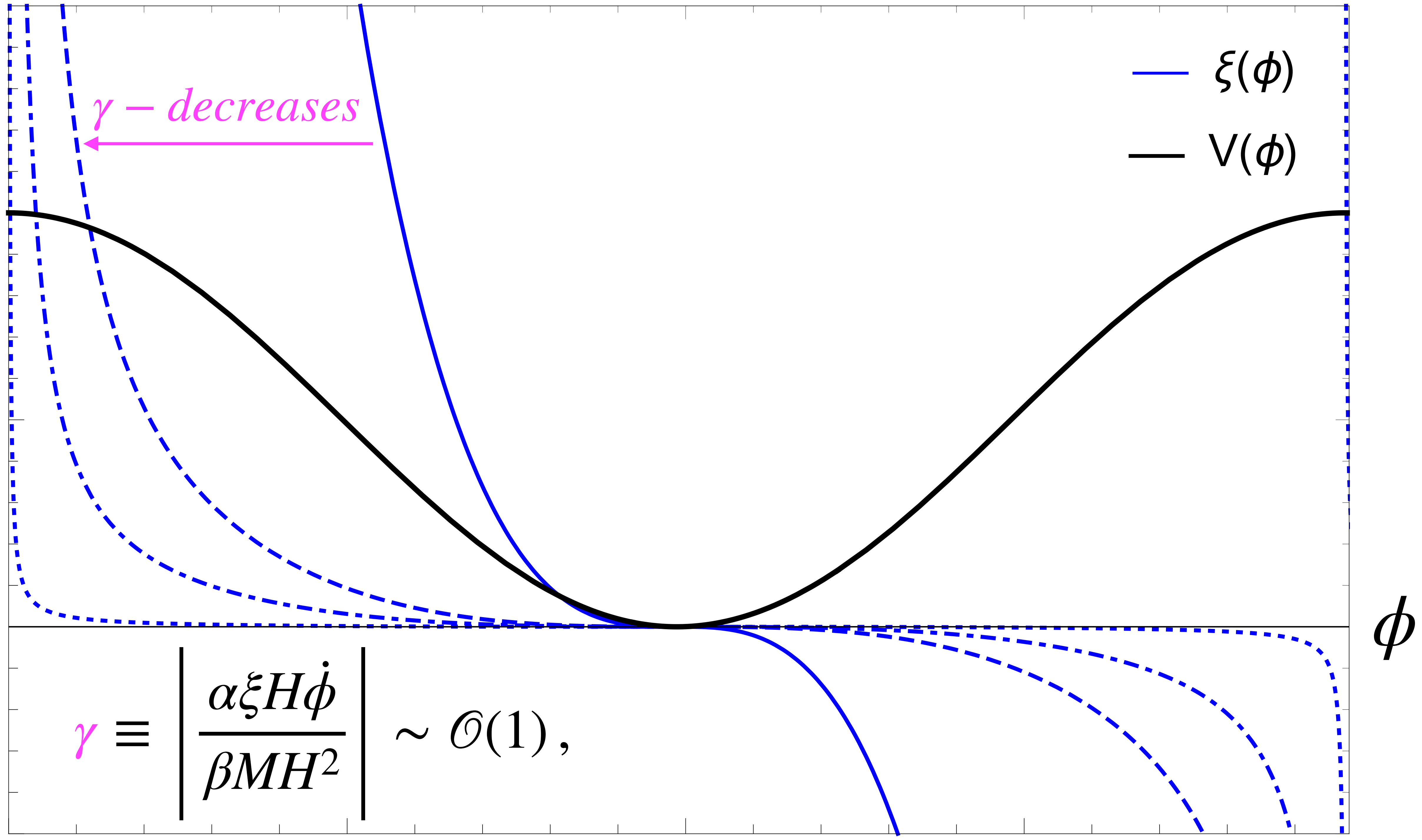
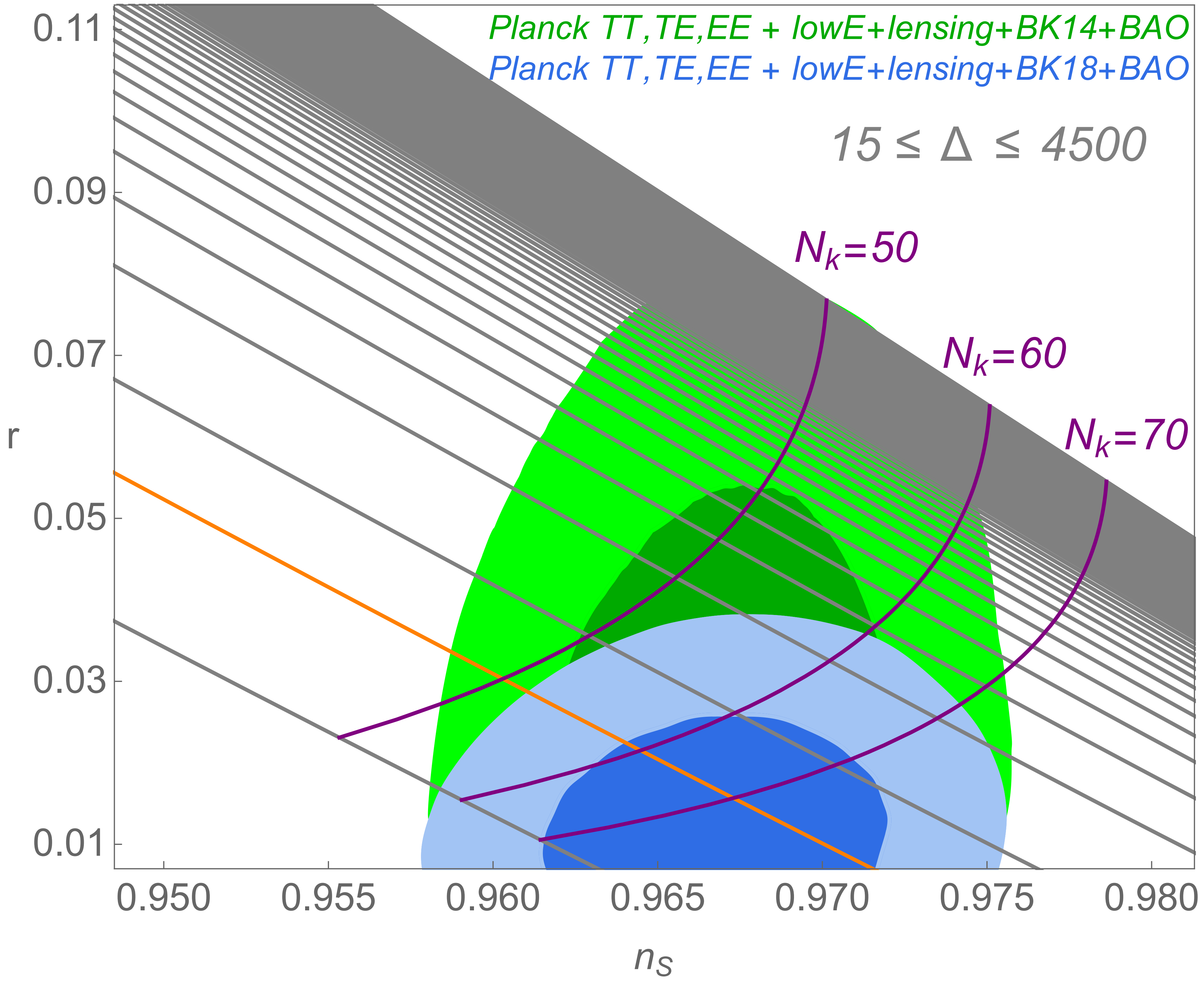
$$n_S = 1 - \frac{2[2 - \cos(\phi/f)]}{\Delta[1 + \cos(\phi/f)]^2}, \qquad r = \frac{8\,[1 - \cos(\phi/f)]}{\Delta[1 + \cos[\phi/f]]^2}.$$



$$\gamma \equiv \left|\frac{\alpha\xi H\dot{\phi}}{\beta M H^2}\right| \sim \mathcal{O}(1),$$

with $\alpha-$ and $\beta-$ terms



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$$\cos\left(\frac{\phi}{f}\right) = 1 + 2\mathcal{W}\left(-e^{\frac{1}{2}\left(\mathcal{F}(\phi_e) - \frac{N_k + \Delta}{\Delta}\right)}\right) \quad \text{and} \quad \Delta \equiv \beta(\gamma - 1)\frac{f^2\Lambda^4}{M^2 M_p^4}.$$

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Part I: Motivation and A toy model

- Cosmic inflation
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G. T, *Eur.Phys.J.C* 79 (2019) 11, 920

- Reheating: after natural inflation
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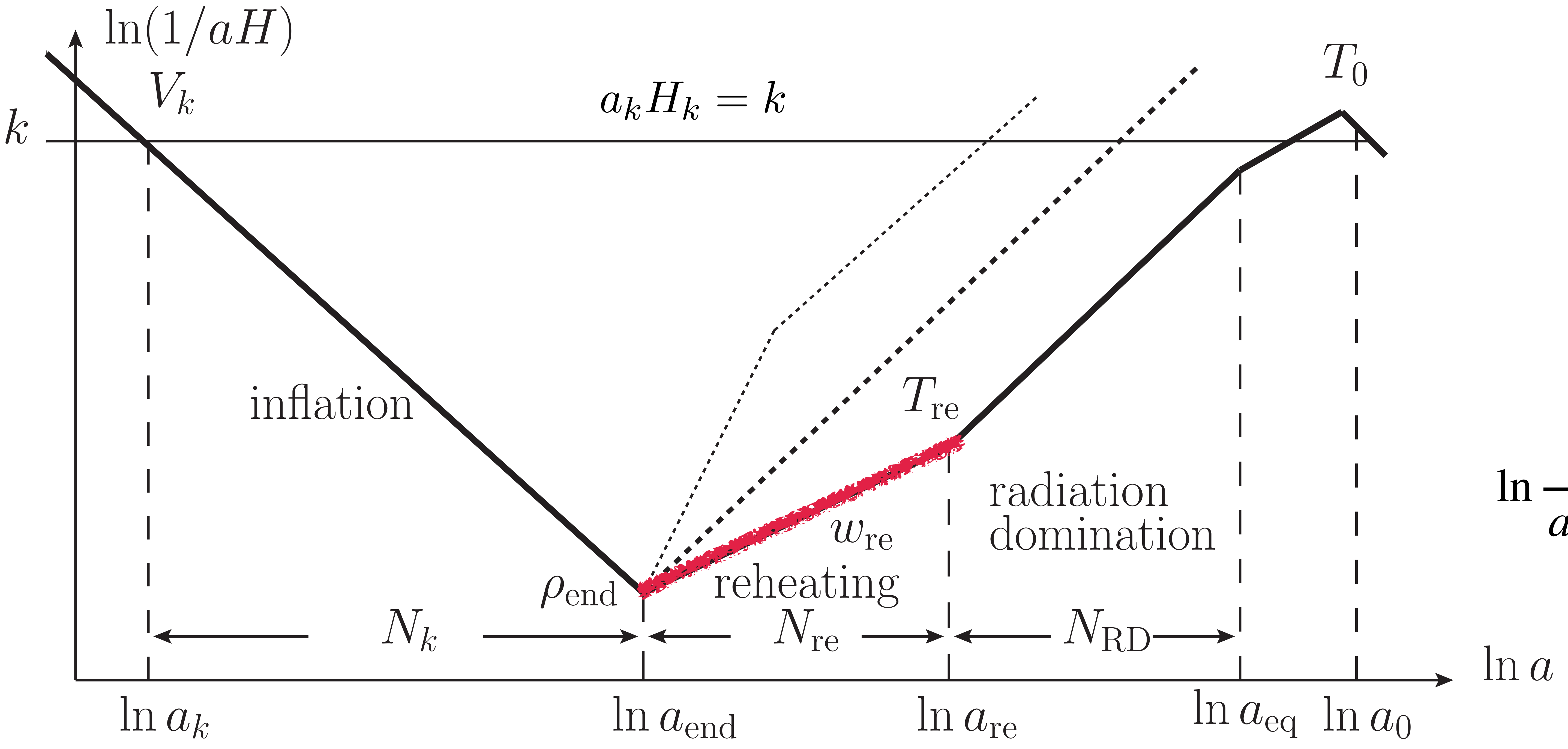
Chen-Hsu Chien, Seoktae Koh, G. T, *a work in progress*

Part II: PBHs and GWs in the scalar-tensor theory of gravity

Pisin Chen, Seoktae Koh, G. T, arXiv:2107.08638

Conclusion

- *Reheating* is a transition era, during which the energy stored in the inflaton is transferred to a plasma of relativistic particles.
- Although there are *NO* direct cosmological observables traceable this period, indirect bounds can be derived. One possibility is to consider cosmological evolution for observable CMB scales from the time of Hubble crossing to present time.



$$\frac{k}{a_0 H_0} = \frac{a_k}{a_{end}} \frac{a_{end}}{a_{re}} \frac{a_{re}}{a_{eq}} \frac{a_{eq} H_{eq}}{a_0 H_0} \frac{H_k}{Heq}$$

$$\ln \frac{k}{a_0 H_0} = -N_k - N_{re} - N_{RD} + \ln \frac{a_{eq} H_{eq}}{a_0 H_0} + \ln \frac{H_k}{H_{eq}}$$

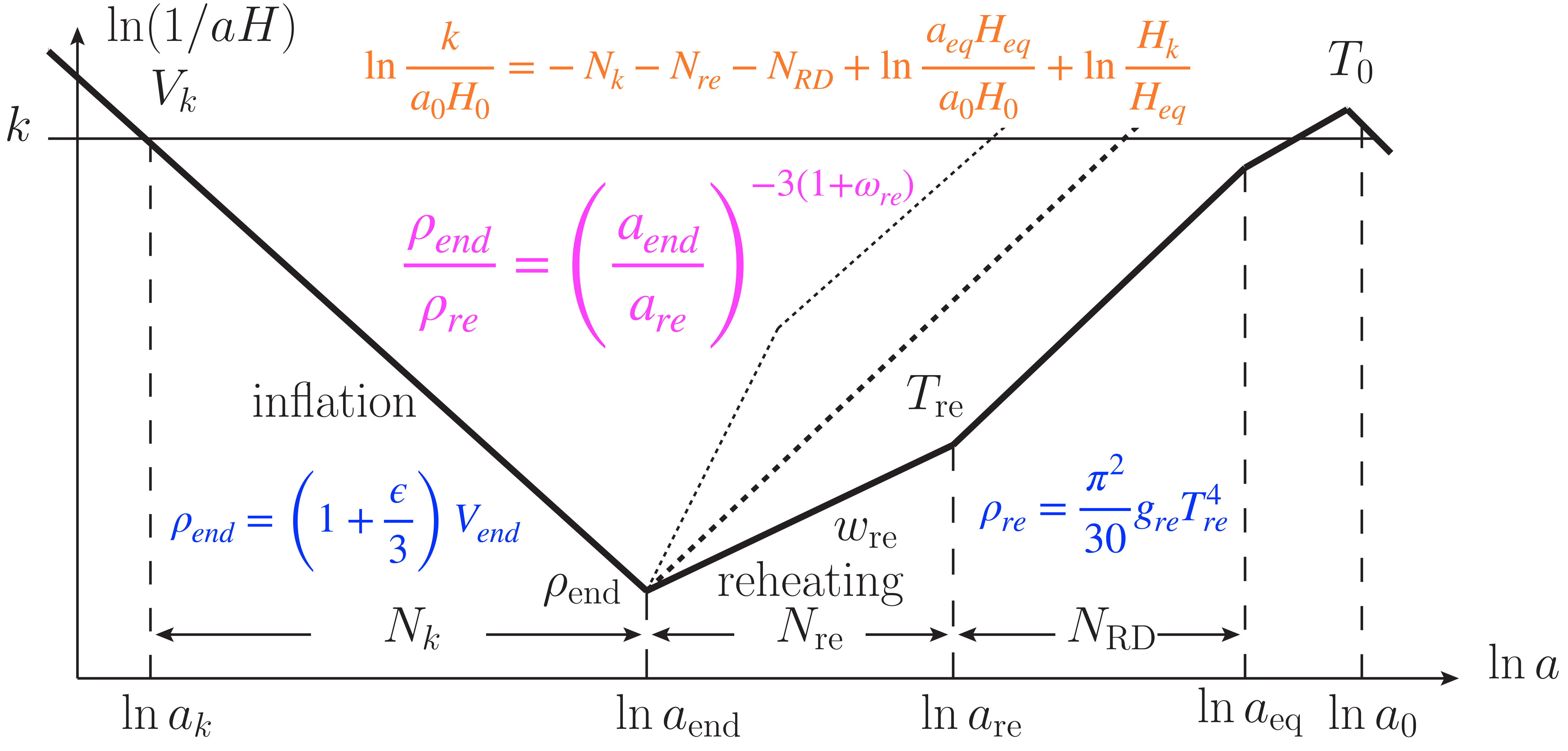
L. Dai, M. Kamionkowski, J. Wang, *PRL* 113, 041302 (2014)

- "<u>Depending on the model</u>", the duration, temperature, and equation-of-state (N_{re}, T_{re}, ω_{re}), are directly linked to inflationary observables *if we approximate reheating by a constant EoS*.
- Thus, reheating can help to break degeneracies between inflation models that otherwise overlap in their predictions of n_S and r.

Sung Mook Lee's talk (yesterday)

Calculating N_{re} and T_{re} :

If $\omega_{re} \approx$ const., the ρ_{end} at the end of inflation is related to that of reheating ρ_{re} :



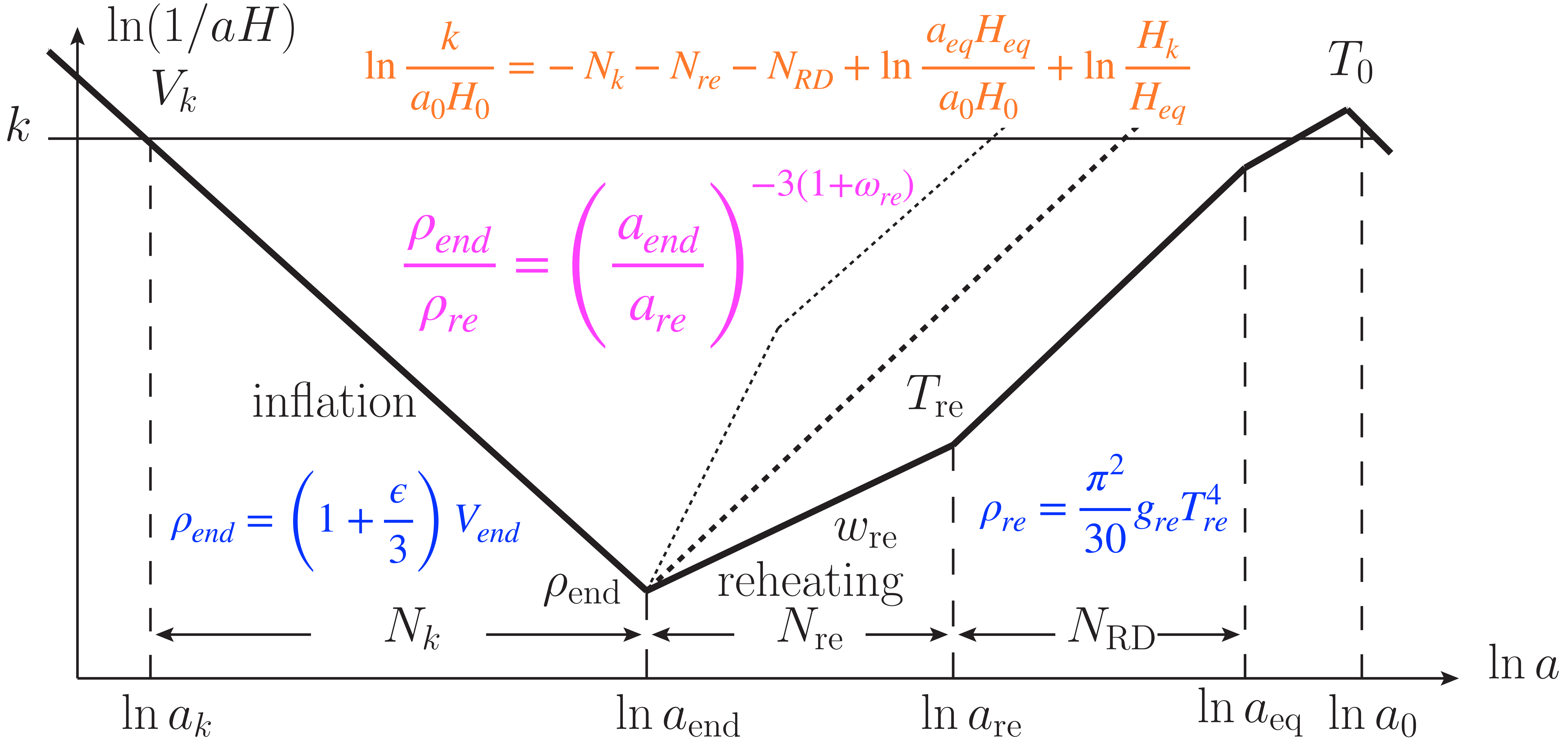
L. Dai, M. Kamionkowski, J. Wang, *PRL* 113, 041302 (2014)

$$N_{re} = \frac{4}{1 - 3\omega_{re}} \left[-N_k - \ln \frac{k}{a_0 T_0} - \frac{1}{4} \ln \frac{30}{\pi^2 g_{re}} - \frac{1}{3} \ln \frac{11 g_{s,re}}{43} - \frac{1}{4} \ln(1 + \frac{\epsilon}{3}) - \frac{1}{4} \ln V_{end} + \frac{1}{2} \ln \frac{\pi^2 M_p^2 r A_s}{2} \right] .$$

$$T_{re} = \left(\frac{30(1 + \epsilon/3)}{\pi^2 g_{re}} V_{end} \right)^{\frac{1}{4}} e^{-\frac{3}{4}(1+\omega_{re}) N_{re}} .$$

Calculating N_{re} and T_{re} :

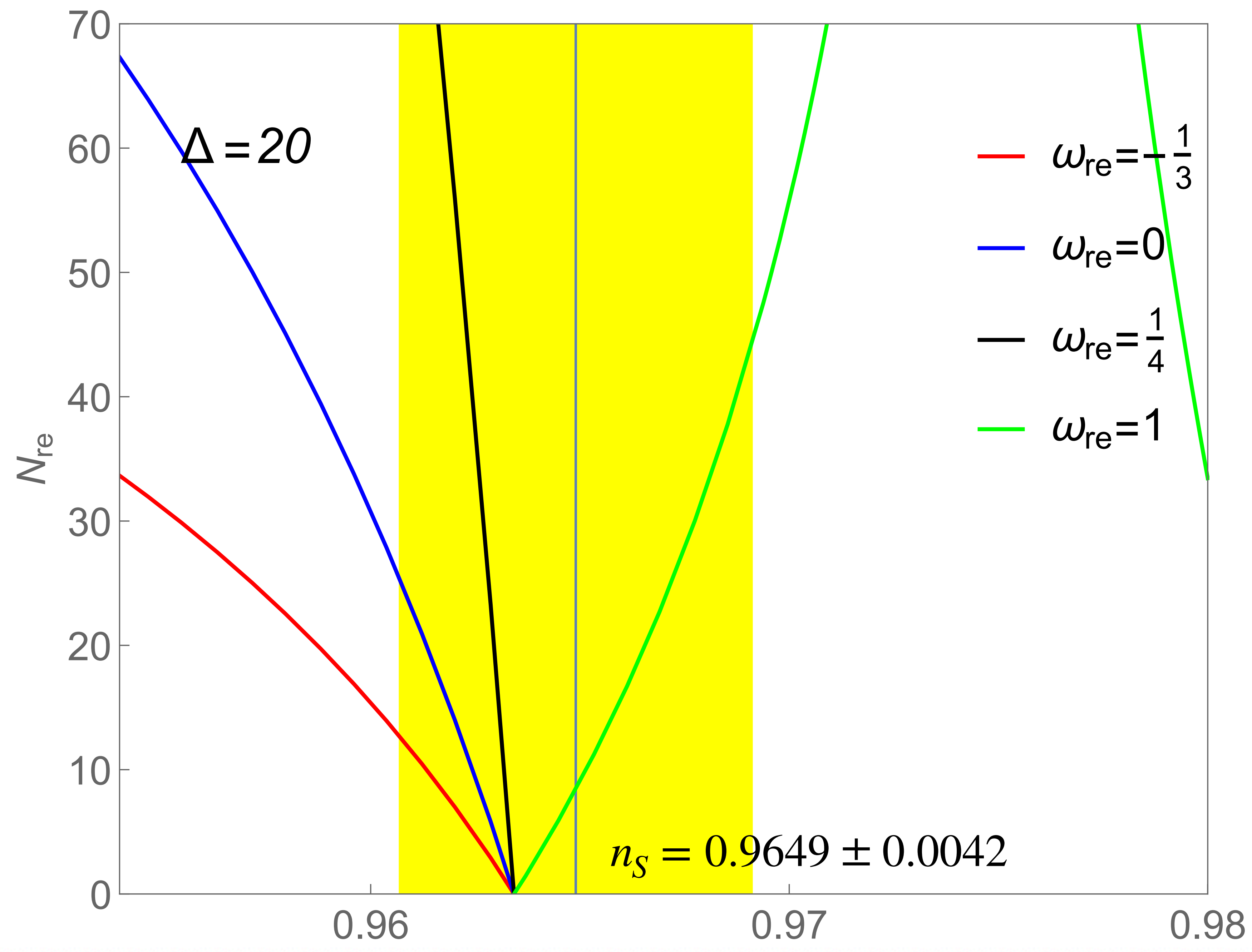
If $\omega_{re} \approx$ const., the ρ_{end} at the end of inflation is related to that of reheating ρ_{re} :

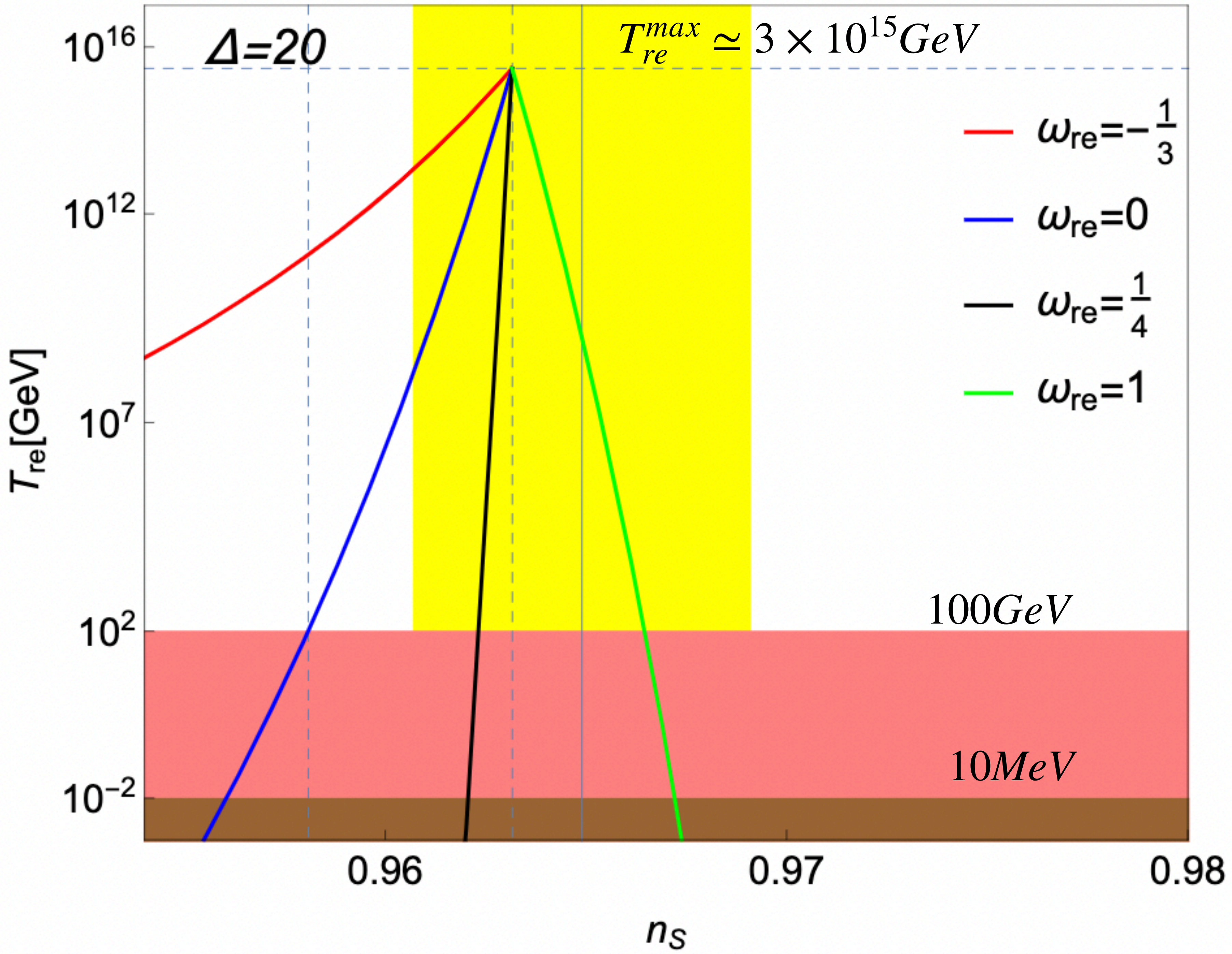


Natural inflation: $V(\phi) = \Lambda^4 \left[1 - \cos\left(\phi/f\right)\right]$ **with $\alpha-$ and $\beta-$ terms:**

$$N_{re} = \frac{4}{1-3\omega_{re}} \left[-N_k - \ln \frac{k}{a_0 T_0} - \frac{1}{4} \ln \frac{30}{\pi^2 g_{re}} - \frac{1}{3} \ln \frac{11 g_{s,re}}{43} - \frac{1}{4} \ln V(\phi_e) + \frac{1}{2} \ln \left(2\pi^2 M_p^2 r \mathscr{P}_S\right)\right]$$

$$T_{re}^4 = \left(\frac{30}{\pi^2 g_{re}}\right) V(\phi_e) e^{-3(1+\omega_{re})N_{re}} .$$





$$T_{re}^4 = \left(\frac{30}{\pi^2 g_{re}}\right) V(\phi_e) e^{-3(1+\omega_{re})N_{re}}, N_{re} = \frac{4}{1-3\omega_{re}}\left[-N_k - \ln\frac{k}{a_0 T_0} - \frac{1}{4}\ln\frac{30}{\pi^2 g_{re}} - \frac{1}{3}\ln\frac{11 g_{s,re}}{43} - \frac{1}{4}\ln V(\phi_e) + \frac{1}{2}\ln\left(2\pi^2 M_p^2 r \mathscr{P}_S\right)\right], n_S = 1 - \frac{2[2-\cos(\phi/f)]}{\Delta[1+\cos(\phi/f)]^2} \Longrightarrow r(n_S, T_{re})$$

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G. T, *Eur.Phys.J.C* 79 (2019) 11, 920

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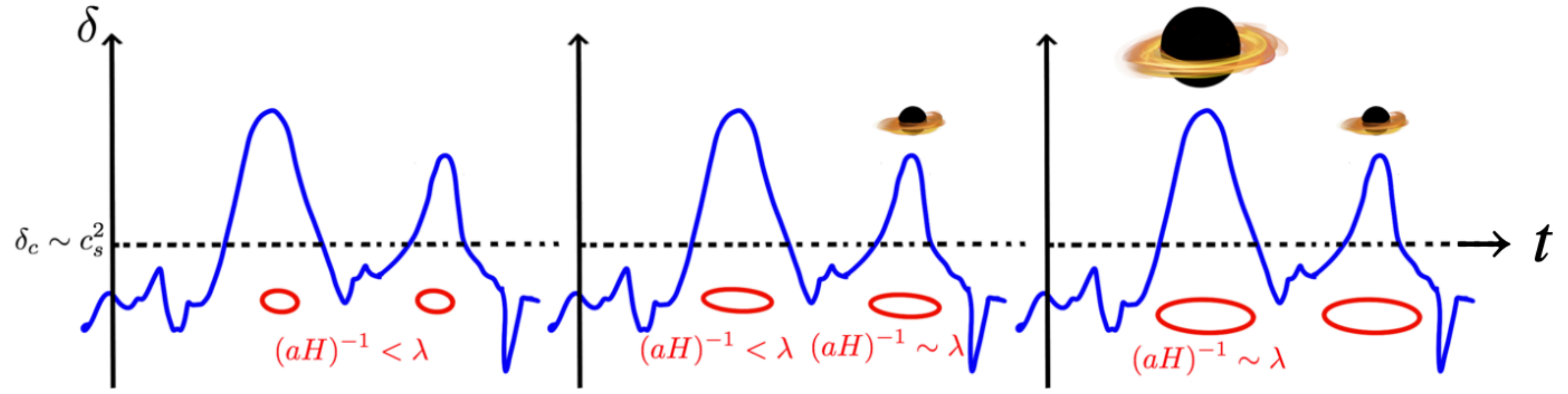
Part II: PBHs and GWs in the scalar-tensor theory of gravity

Pisin Chen, Seoktae Koh, G. T, arXiv:2107.08638

Conclusion

- **PBHs** are formed when “sufficiently large” primordial density fluctuations, generated during inflation on some small scale $k_{PBH} \gg k_*$, re-enters the Hubble radius, i.e., $aH = k$, during RD era.



P. V.-Domingo, O. Mena, S.P-Ruiz, Front. Astron. Space Sci., 28 May 2021

- At the formation, the PBH mass is related to the horizon mass, i.e., the mass within a region of size of the Hubble horizon: $M_{PBH} = \gamma M_H = \gamma \dfrac{4\pi M_p^2}{H}$

- The Hubble scale in the RD epoch is : $\dfrac{H^2}{H_0^2} = \Omega_{r,0}(1+z)^4 \left(\dfrac{g_*}{g_{*,0}}\right)^{-\frac{1}{3}} \left(\dfrac{g_*^s}{g_{*,0}}\right)^{\frac{3}{4}}$

$$\frac{M_{PBH}}{M_\odot} = 1.55 \times 10^{24} \left(\frac{\gamma}{0.2}\right) \left(\frac{g_*}{106.75}\right)^{1/6} (1+z)^{-2} \quad \Rightarrow \quad \frac{M_{PBH}}{M_\odot} = 1.13 \times 10^{15} \left(\frac{\gamma}{0.2}\right) \left(\frac{g_*}{106.75}\right)^{1/6} \left(\frac{k_{PBH}}{k_*}\right)^{-2}$$

- The solar mass PBHs are formed at $z \simeq 10^{12}$ when a mode with $k_{PBH} \simeq 10^7 k_*$ enters the horizon.

- M_{PBH} can also related to the N_{PBH}, before the end of inflation by

$$N_* - N_{PBH} = 17.33 + \frac{1}{2} \ln \frac{\gamma}{0.2} - \frac{1}{12} \ln \frac{g_*}{106.75} - \frac{M_{PBH}}{M_\odot} .$$

- This indicates that a large density fluctuation mode corresponding to solar mass PBHs must exit the Hubble radius about the 17e-fold after the exit of the k_*.

- After their formation, the PBH density redshifts just like the pressureless matter until the present epoch (ignoring the merger events and accretion).

- Thus, PBHs behaves like "*Dark Matter*" for a substantial part of cosmic history.

- The mass fraction of PBHs at formation is

$$\beta(M_{PBH}) \equiv \frac{\rho_{PBH}}{\rho_{tot}} \Longrightarrow \beta(M_{PBH}) = \Omega_{DM,0} f_{PBH}(M_{PBH}) (1+z)^3 \left(\frac{H_0}{H}\right)^2$$

- The mass function of fractional abundance of PBHs is:

$$f_{PBH}(M_{PBH}) \equiv \frac{\Omega_{PBH,0}(M_{PBH})}{\Omega_{DM,0}} = 1.68 \times 10^8 \left(\frac{\gamma}{0.2}\right)^{\frac{1}{2}} \left(\frac{g_*}{106.75}\right)^{-\frac{1}{4}} \left(\frac{M_{PBH}}{M_\odot}\right)^{-2} \beta(M_{PBH})$$

- The total PBH abundance at the present epoch is defined as

$$f_{PBH}^{tot} \equiv \int f_{PBH}(M_{PBH}) dM_{PBH}$$

- $\beta(M_{PBH})$ can be calculated from primordial power spectrum $\mathscr{P}_R$ in the "Press-Schechter" formalism

- In the Press-Schechter formalism, the mass fraction of PBH at the formation $\beta(M_{PBH})$ for a given mass is defined as

$$\beta(M_{PBH}) = \gamma \int_{\delta_{th}}^{1} P(\delta) d\delta \simeq \gamma \frac{\sigma_{M_{PBH}}}{\sqrt{2\pi}\delta_{th}} \exp\left[-\frac{\delta_{th}^2}{2\sigma_{M_{PBH}}^2}\right] .$$

- The variance of the density contrast is given by

$$\sigma_{M_{PBH}}^2 = \int \frac{dk}{k} P_\delta(k) W^2(k,R) \ \text{where}\ W(k,R) = \exp\left(-\frac{1}{2}k^2 R^2\right),$$

- The power spectrum for the density contrast is then related to the primordial power spectrum

$$P_\delta(k) = \frac{16}{81}\left(\frac{k}{aH}\right)^4 \mathscr{P}_R(k).$$

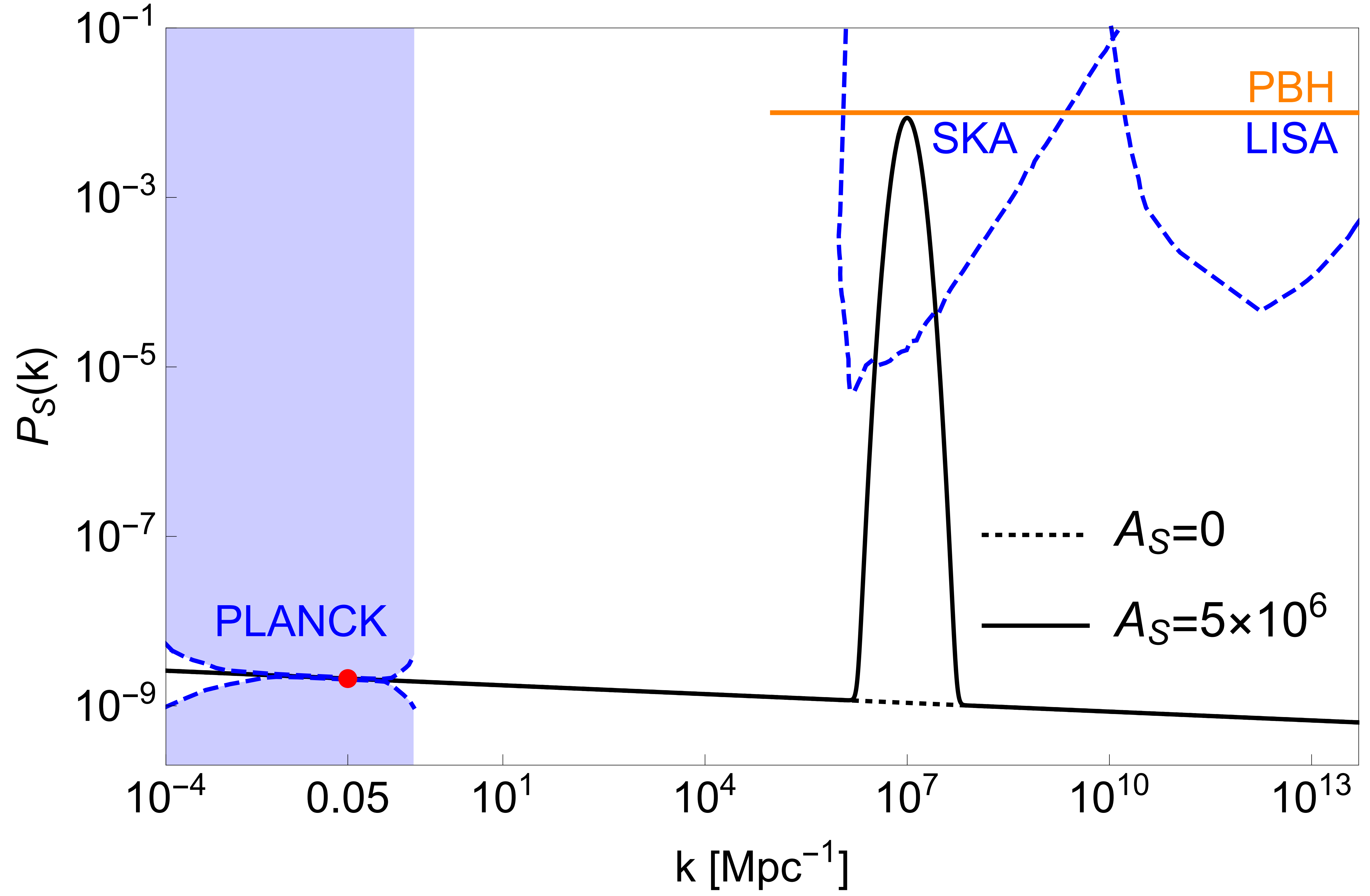
- The inflationary power spectrum for our case is: $\mathscr{P}_S = \frac{k^3}{2\pi^2}\left|\frac{v_k}{z_S}\right|^2 \simeq \frac{\kappa^2 H^2}{8\pi^2 c_S^3 \epsilon_V}(1+\mathscr{A})$

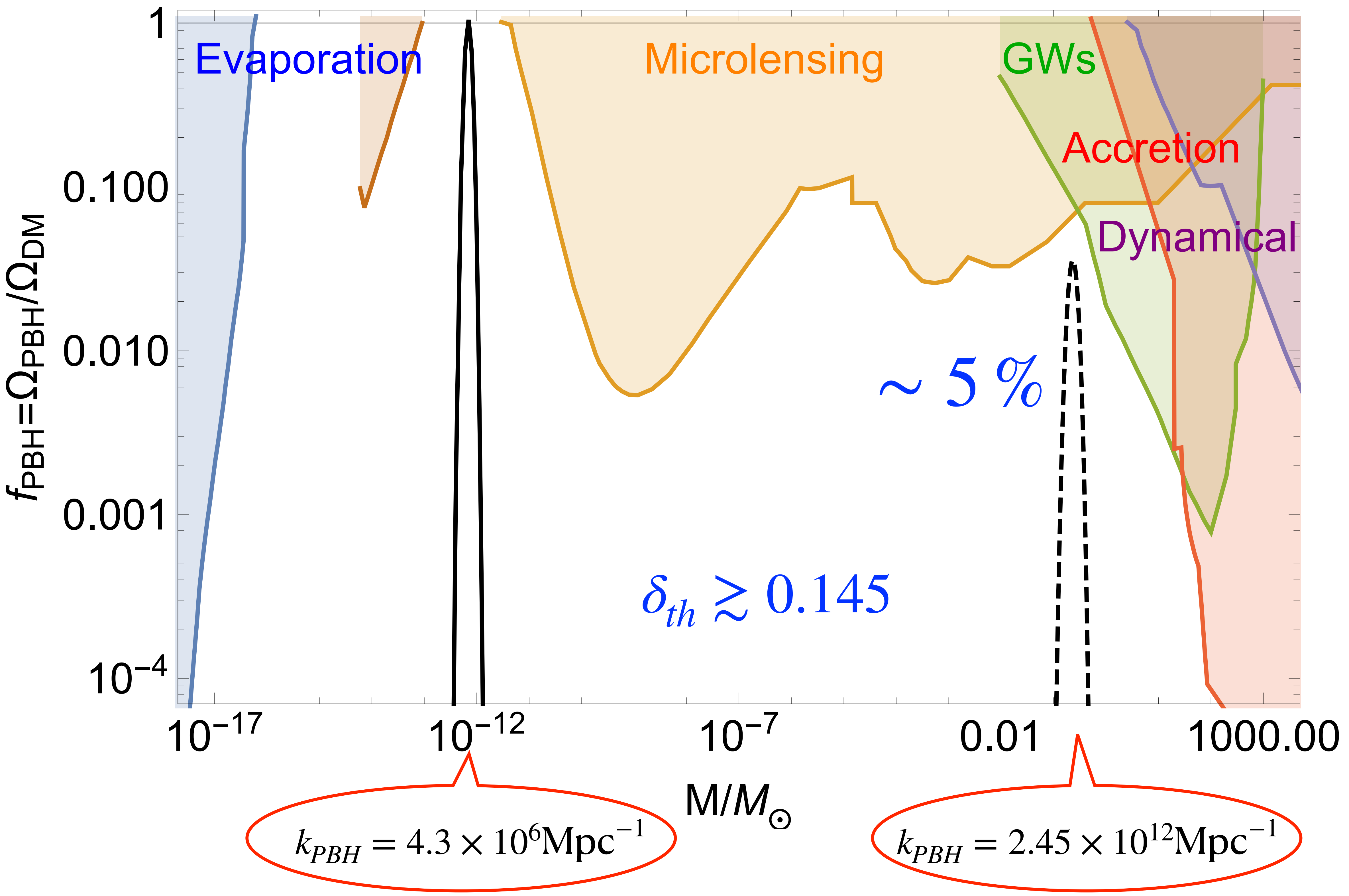
$$\mathscr{A} = \frac{3\alpha}{M^3}\xi H\dot{\phi} - \frac{3\beta}{M^2}H^2$$

- Power spectra for scalar mode:

$$\mathscr{P}_S = \frac{k^3}{2\pi^2}\left|\frac{v_k}{z_S}\right|^2 \simeq \frac{\kappa^2 H^2}{8\pi^2 c_S^3 \epsilon_V}(1+\mathscr{A})$$

$$\mathscr{P}_S(k) = \mathscr{P}_S(k_*)\left(\frac{k}{k_*}\right)^{n_S-1}\left\{1+\frac{A_S}{\sqrt{2\pi\sigma}}\exp\left[-\frac{1}{\sigma^2}\left(\ln\frac{k}{k_{PBH}}\right)^2\right]\right\}.$$





$$f_{\mathrm{PBH}}(M_{\mathrm{PBH}}) = \frac{1}{\Omega_{\mathrm{DM}}}\frac{d\Omega_{\mathrm{PBH}}}{d\ln M_{\mathrm{PBH}}} \simeq 0.28\times10^8\left(\frac{\gamma}{0.2}\right)^{\frac{3}{2}}\left(\frac{g_*}{106.75}\right)^{-\frac{1}{4}}\left(\frac{\Omega_{\mathrm{DM}}h^2}{0.12}\right)^{-1}\left(\frac{M_{\mathrm{PBH}}}{M_\odot}\right)^{-\frac{1}{2}}\beta(M_{\mathrm{PBH}}),$$

PBHs can be DM!

- Besides PBHs, the sufficiently large density fluctuations generated during inflation can simultaneously produce a substantial amount of GWs when they reenter the horizon in the RD era

- The equation of motion for the GW:

$$h_k'' + 2\mathscr{H} h_k' + k^2 h_k = 4S_k\,,$$

- The source term $S_k(\tau)$, which is a convolution of two first-order scalar perturbations at different wave numbers, is given by

$$S_k = \int \frac{d^3\tilde{k}}{(2\pi)^{3/2}} \epsilon^{ij}(k)\tilde{k}_i\tilde{k}_j \left[2\Phi_{\tilde{k}}\Phi_{k-\tilde{k}} + \frac{4}{3(1+\omega)}\left(\frac{\Phi'_{\tilde{k}}}{\mathscr{H}} + \Phi_{\tilde{k}}\right)\left(\frac{\Phi'_{k-\tilde{k}}}{\mathscr{H}} + \Phi_{k-\tilde{k}}\right)\right],$$

where the scalar part of the metric perturbation Φ_k satisfies $\Phi_k'' + \frac{4}{\tau}\Phi_k' + \frac{k^2}{3}\Phi_k = 0$, which admits a solution

$$\Phi_k(\tau) = \frac{9}{k\tau}\left[\frac{\sin(k\tau/\sqrt{3})}{k\tau/3} - \cos(k\tau/3)\right]\zeta_k\,.$$

- The fractional energy density per logarithmic wavenumber interval is

$$\Omega_{GW}(k,\tau) = \frac{1}{\rho_{tot}}\frac{d\rho_{GW}}{d\ln k} = \frac{1}{24}\left(\frac{k}{aH}\right)^2 \overline{\mathscr{P}_T(k,\tau)}$$

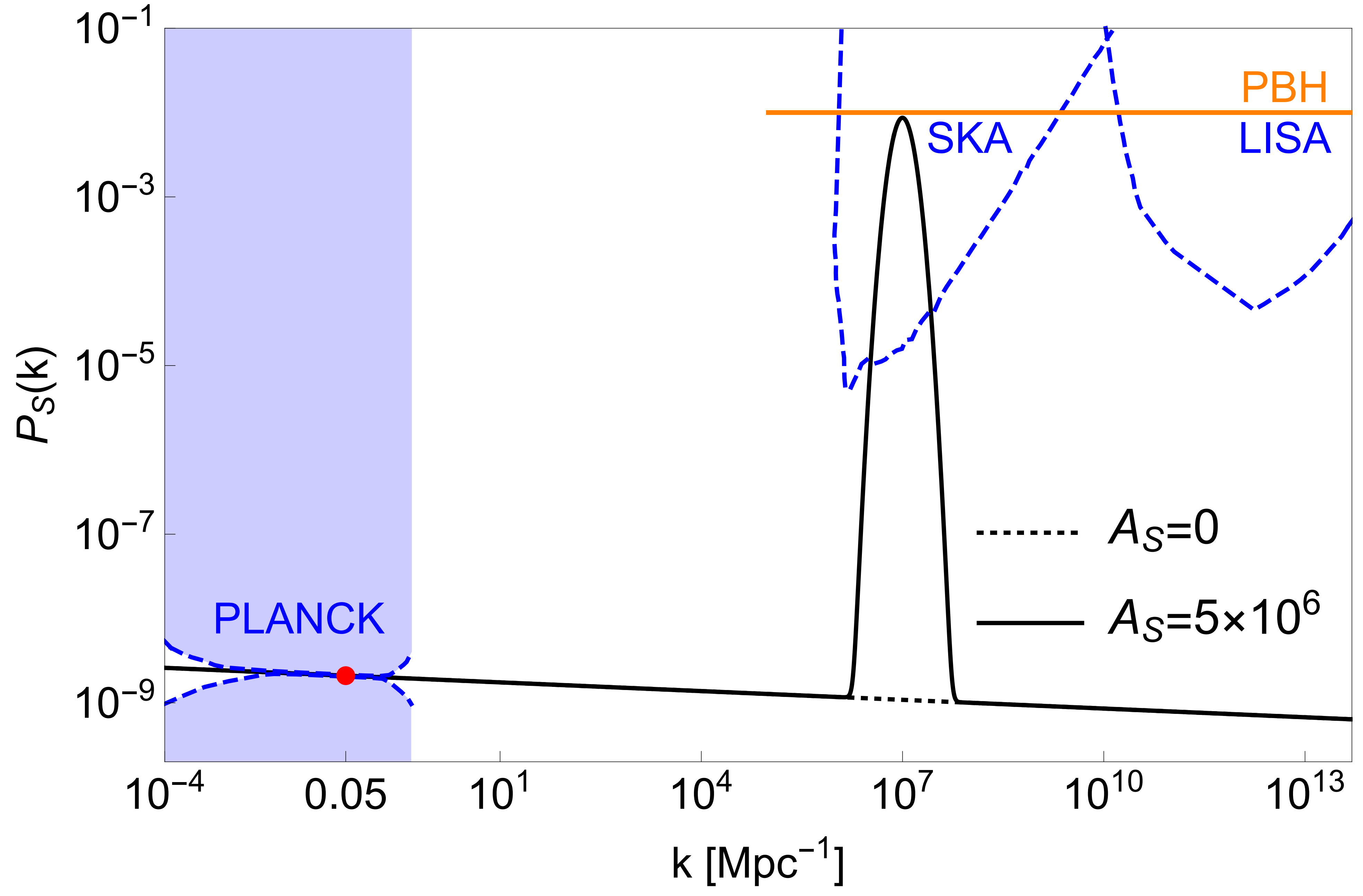
where $$\mathscr{P}_T(k,\tau) = 4\int_0^\infty dv \int_{|1-v|}^{1+v} du \left[\frac{4v^2 - (1-u^2+v^2)}{4uv}\right]^2 I_{RD}^2(u,v,x)\mathscr{P}_S(kv)\mathscr{P}_S(ku)$$

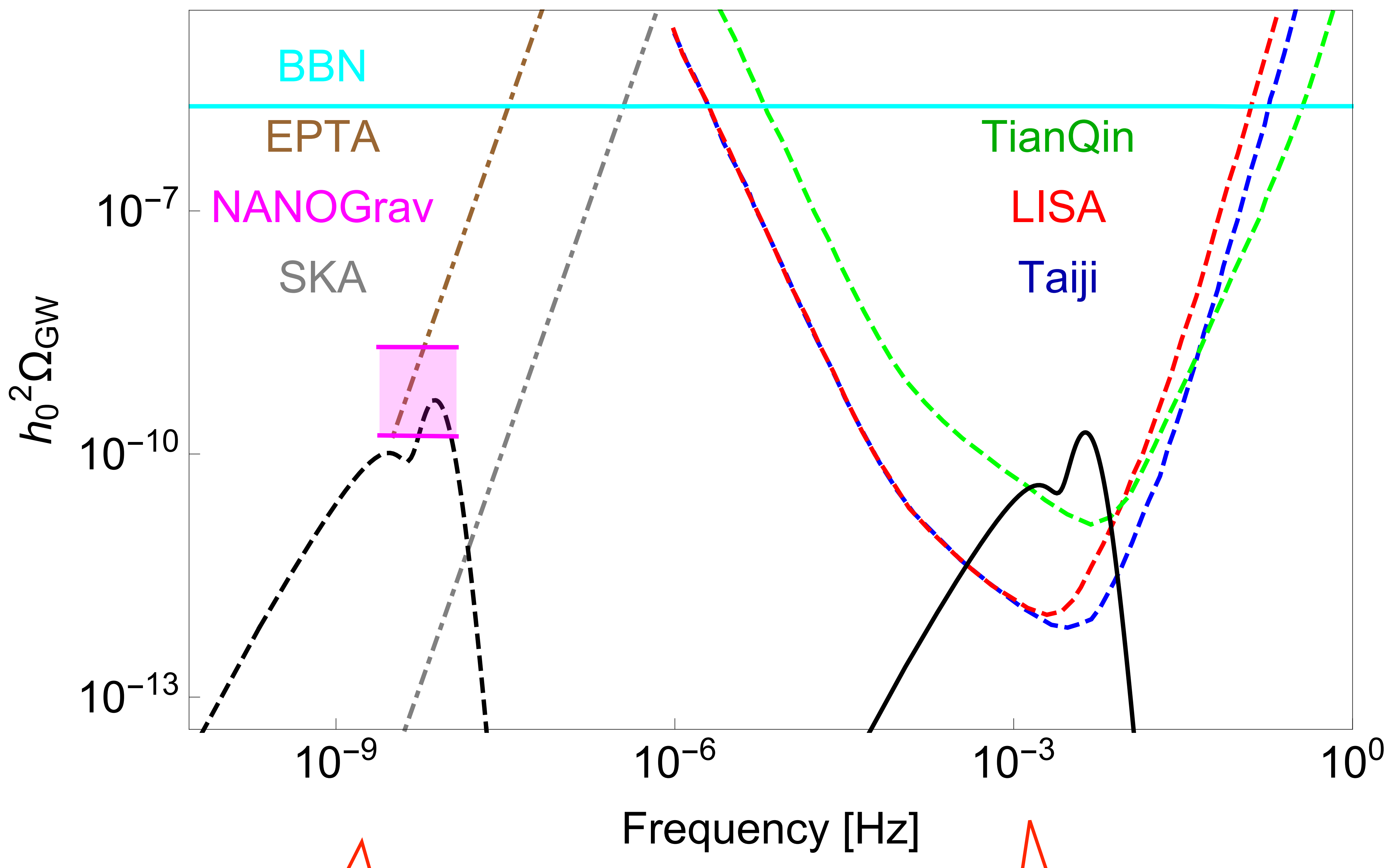
Scalar-induced GWs:

$$\Omega_{GW}(k,\tau) = \frac{1}{6}\left(\frac{k}{\mathcal{H}}\right)^2 \int_0^\infty dv \int_{|1-v|}^{1+v} du \left[\frac{4v^2-(1-u^2+v^2)}{4uv}\right]^2 \overline{I_{RD}^2(u,v,x)}\mathcal{P}_S(kv)\mathcal{P}_S(ku)\,,$$

$$\mathscr{P}_S = \frac{k^3}{2\pi^2}\left|\frac{v_k}{z_S}\right|^2 \simeq \frac{\kappa^2 H^2}{8\pi^2 c_S^3 \epsilon_V}(1+\mathscr{A})$$

$$\mathscr{P}_S(k) = \mathscr{P}_S(k_*)\left(\frac{k}{k_*}\right)^{n_S-1}\left\{1+\frac{A_S}{\sqrt{2\pi\sigma}}\exp\left[-\frac{1}{\sigma^2}\left(\ln\frac{k}{k_{PBH}}\right)^2\right]\right\}.$$





$k_{PBH} = 4.3\times10^6\mathrm{Mpc}^{-1}$

$k_{PBH} = 2.45\times10^{12}\mathrm{Mpc}^{-1}$

$$f = 1.546\times10^{-15}\left(\frac{k}{\mathrm{Mpc}^{-1}}\right)\mathrm{Hz}\,.$$

The potential and the self-coupling function:

$$\mathscr{P}_S = \frac{k^3}{2\pi^2}\left|\frac{v_k}{z_S}\right|^2 \simeq \frac{\kappa^2 H^2}{8\pi^2 c_S^3 \epsilon_V}(1+\mathscr{A}) \qquad \mathscr{P}_S(k) = \mathscr{P}_S(k_*)\left(\frac{k}{k_*}\right)^{n_S-1}\left\{1+\frac{A_S}{\sqrt{2\pi\sigma}}\exp\left[-\frac{1}{\sigma^2}\left(\ln\frac{k}{k_{PBH}}\right)^2\right]\right\}.$$

- In order to construct $V(\phi)$ and $\xi(\phi)$, we use $n_S - 1 \simeq \frac{1}{1+\mathscr{A}}\left[2\eta_V - 2\epsilon_V\left(4 - \frac{1}{1+\mathscr{A}}\right)\right]$, $\quad r \simeq \frac{16\epsilon_V}{1+\mathscr{A}}$.

  with $n_S - 1 = -\frac{2}{N_*}$, which is in good agreement with the CMB measurement for $N_* \simeq 60$.

- We rewrite : $n_S - 1 \simeq \ln\left[\frac{V_{,N_*}}{V^2}(1+\mathscr{A})\right]_{,N_*}$, $\quad r = \frac{8V_{,N_*}}{V}$ where $\mathscr{A}(N_*) = \frac{A_S}{\sqrt{2\pi\sigma^2}}e^{-\frac{\bar{N}^2}{2\sigma^2}}$ with $\bar{N} \equiv N_* - N_p$ is defined from the end of inflation.

- We first obtain the $V(N_*)$ and $\xi(N_*)$, then obtain $V(\phi)$ and $\xi(\phi)$ using $N_* \simeq \int_{\phi_e}^{\phi}\frac{V}{V_{,\phi}}(1+\mathscr{A})d\phi$.

- As a result, we obtain:

$$V(\phi) = V_0\tanh^2\left(\frac{c}{2}\frac{\phi}{M_p}\right) \text{ and } \xi(\phi) = \xi_0\sinh\left(c\frac{\phi}{M_p}\right)\sqrt{1+\mathscr{A}(\phi)} \text{ where } \mathscr{A}(\phi) = \frac{A_S}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2\sigma^2}\left[\frac{1}{c^2}\sinh^2\left(\frac{c}{2}\frac{\phi}{M_p}\right)-\phi_p\right]^2}$$

The potential and the self-coupling function:

$$\mathscr{P}_S = \frac{k^3}{2\pi^2}\left|\frac{v_k}{z_S}\right|^2 \simeq \frac{\kappa^2 H^2}{8\pi^2 c_S^3 \epsilon_V}(1+\mathscr{A})$$

$$\mathscr{P}_S(k) = \mathscr{P}_S(k_*)\left(\frac{k}{k_*}\right)^{n_S-1}\left\{1+\frac{A_S}{\sqrt{2\pi\sigma}}\exp\left[-\frac{1}{\sigma^2}\left(\ln\frac{k}{k_{PBH}}\right)^2\right]\right\}.$$

- In order to construc ... $r \simeq \dfrac{16\epsilon_V}{1+\mathscr{A}}$.

  with $n_S - 1 = -\frac{2}{N}$ ...

- We rewrite : $n_S - 1$ ... $N \equiv N_* - N_p$ is

  defined from the en...

- We first obtain the V...

- As a result, we obtai...

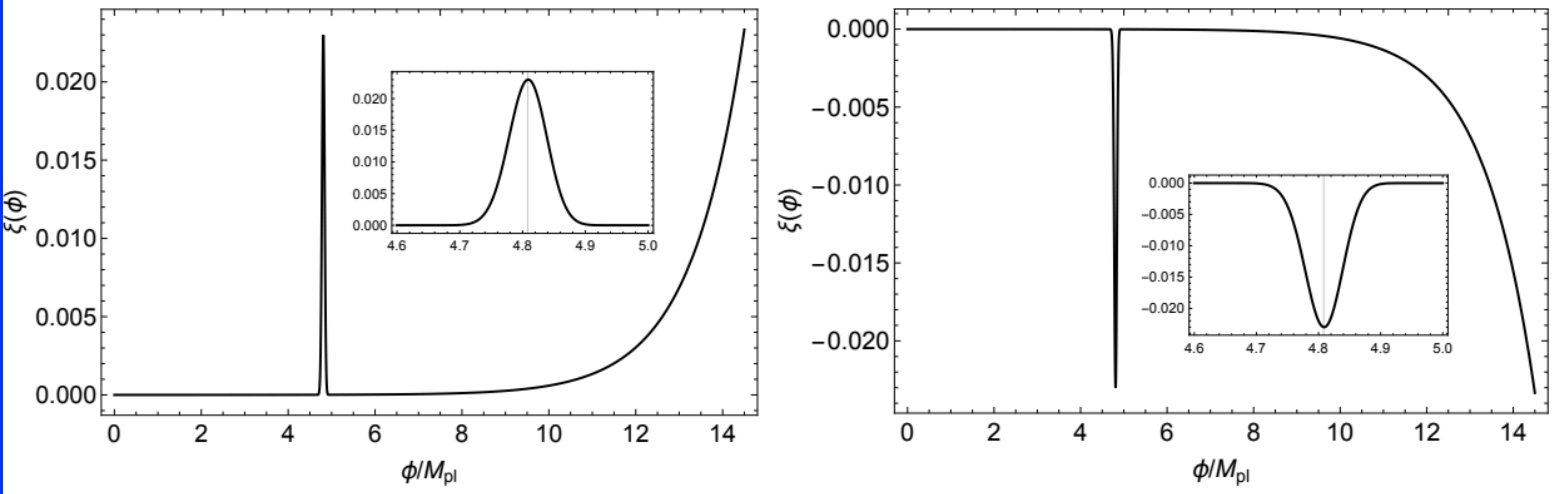


FIG. 3: The self-coupling function from Eqs. (61) for positive (left) and negative (right) values of β/α. The peak/dip position $\phi_p \simeq 4.8082$ is adjusted by the $k_{\rm PBH}$ value. Numerical inputs are $\alpha = \pm 10^3$, $\beta = -10^{-3}$, $\gamma = 0.55$, $M = M_{pl} = 1$, $\sigma = 0.3$, $A_S = 6\times 10^6$, and $k_{\rm PBH} = 4.3\times 10^6 {\rm Mpc}^{-1}$.

$$V(\phi) = V_0 \tanh^2\left(\frac{c}{2}\frac{\phi}{M_p}\right) \text{ and } \xi(\phi) = \xi_0 \sinh\left(c\frac{\phi}{M_p}\right)\sqrt{1+\mathscr{A}(\phi)} \text{ where } \mathscr{A}(\phi) = \frac{A_S}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}\left[\frac{1}{c^2}\sinh^2\left(\frac{c}{2}\frac{\phi}{M_p}\right)-\phi_p\right]^2}$$

Conclusion:

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- Inflation: natural inflation is saved!?
- The reheating consideration after natural inflation puts further constraints on the inflationary predictions!

Part II: PBHs and GWs in Horndenski theory

- PBHs can be DM and Secondary GWs are produced!
- the potential and the self-coupling functions are constructed!

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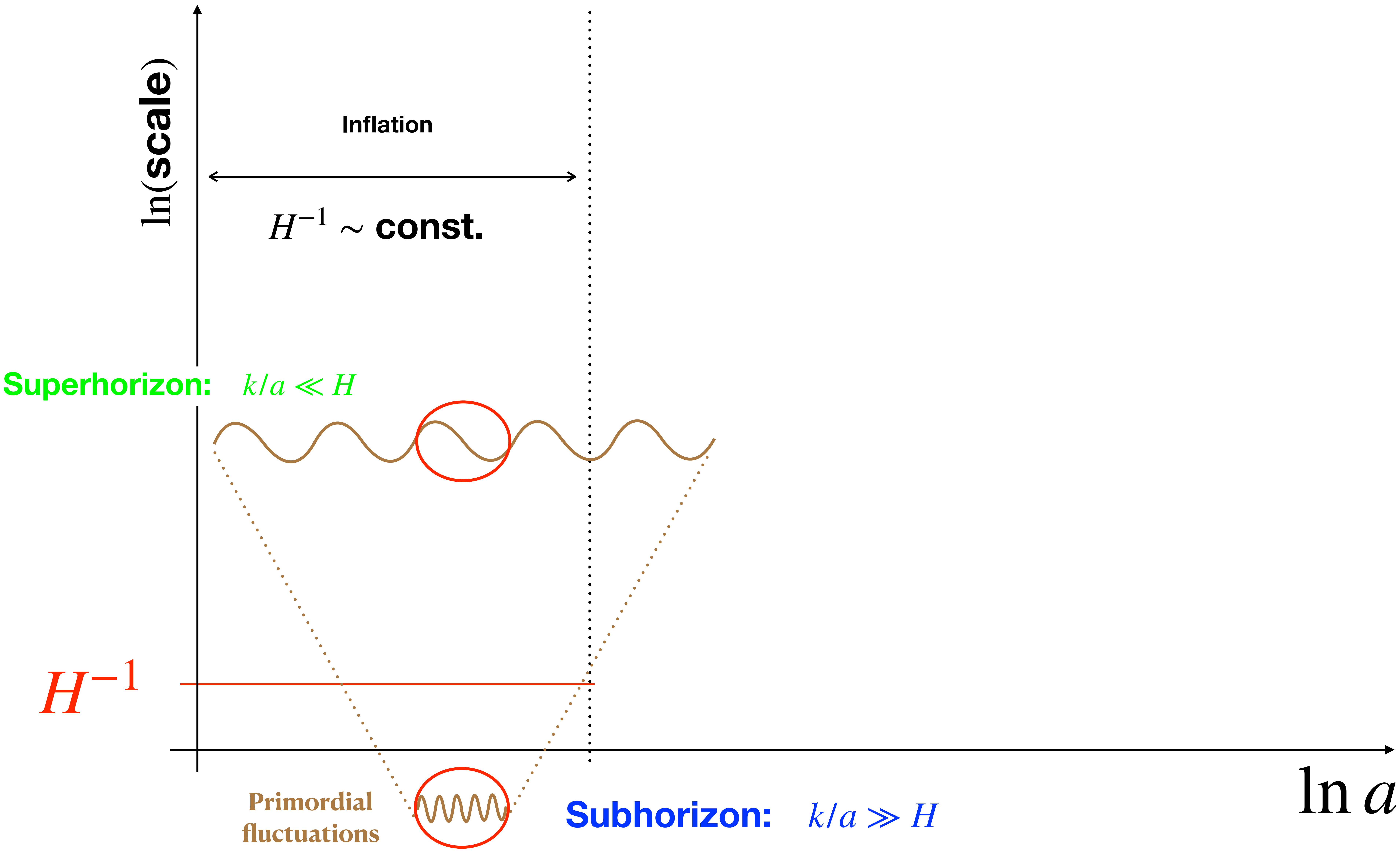
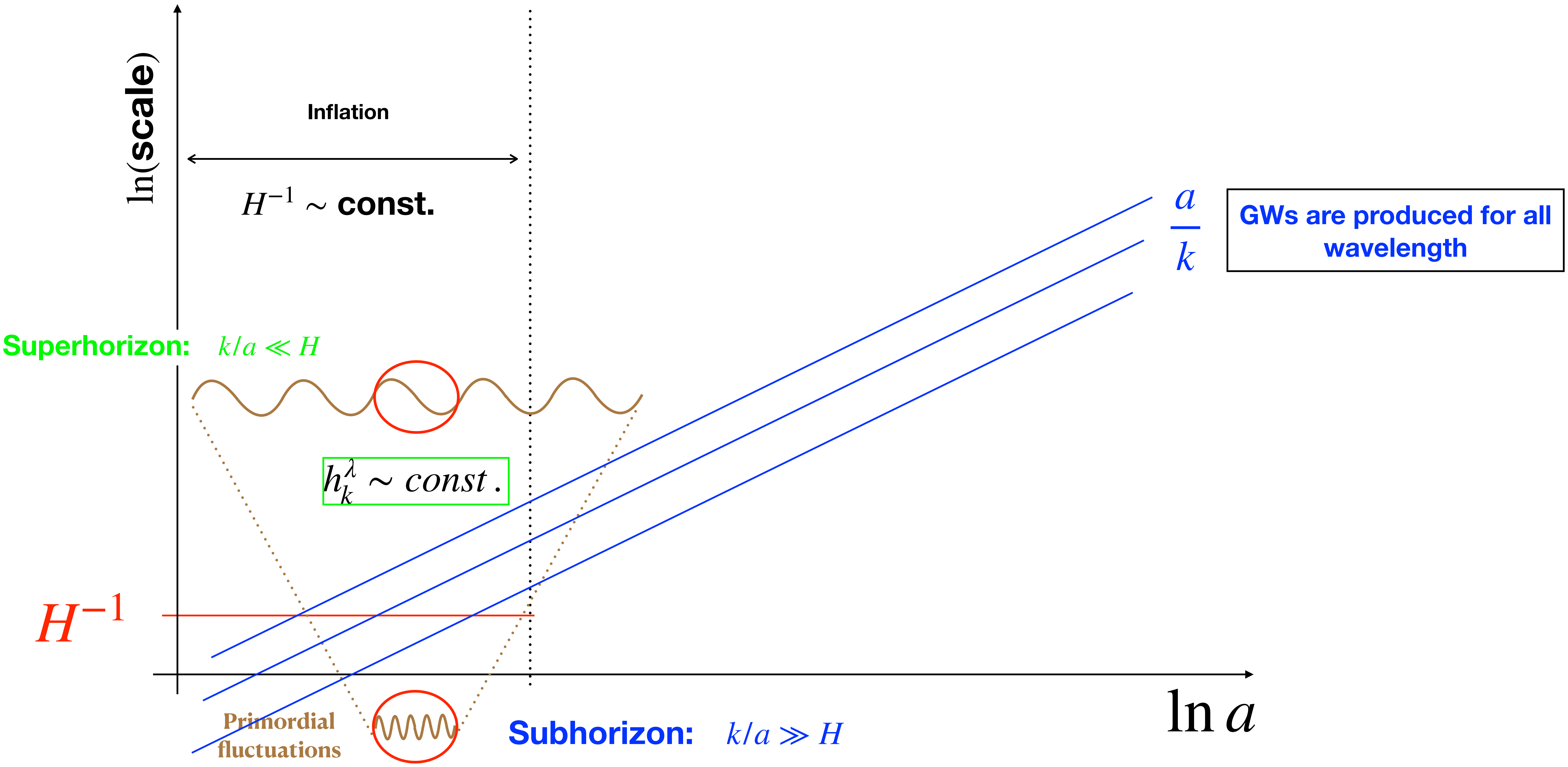
Thank you for your kind attention!

backup slides

PBHs & GWs in the early Universe from inflation:



PBHs & GWs in the early Universe from inflation:



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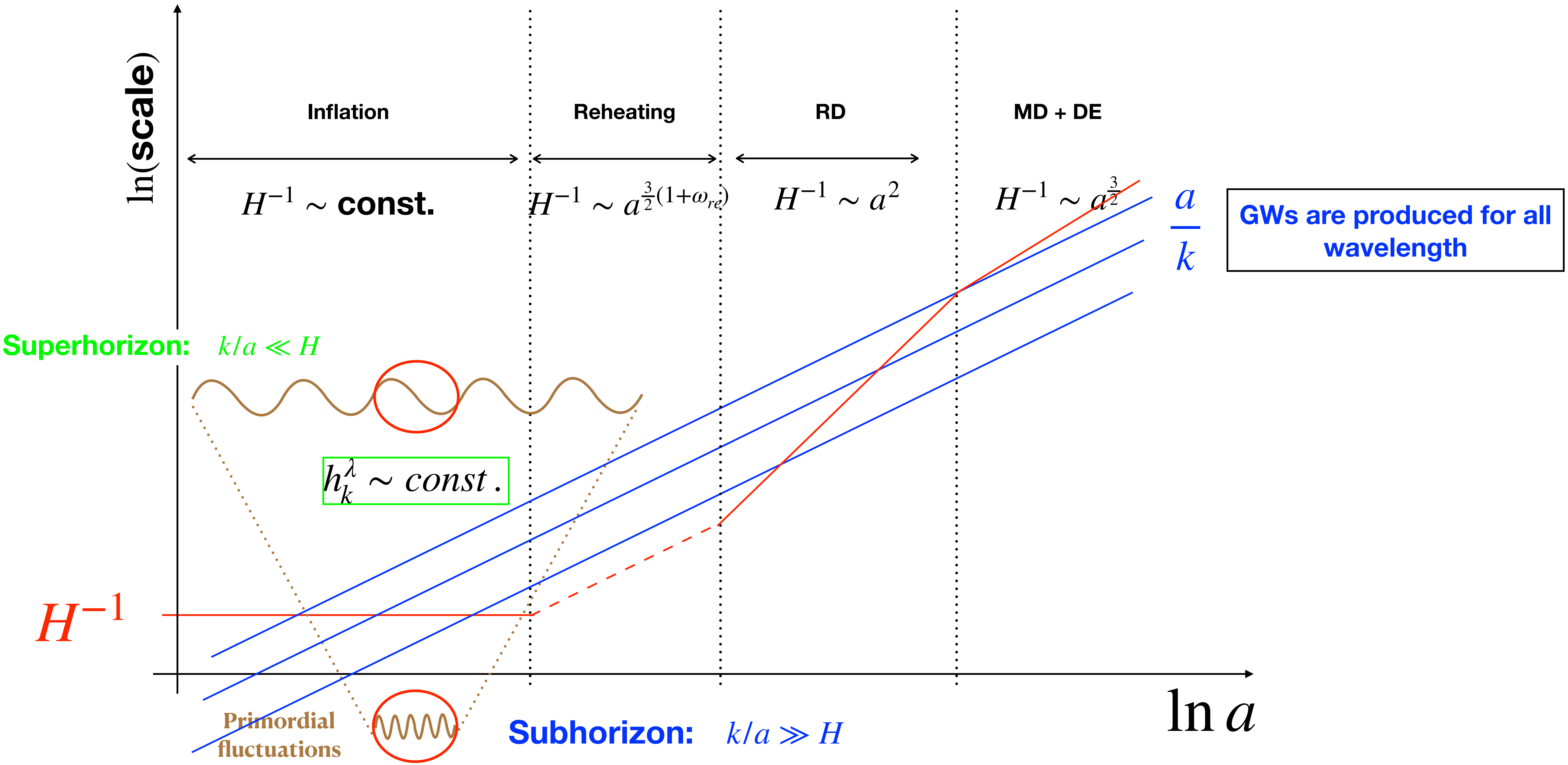

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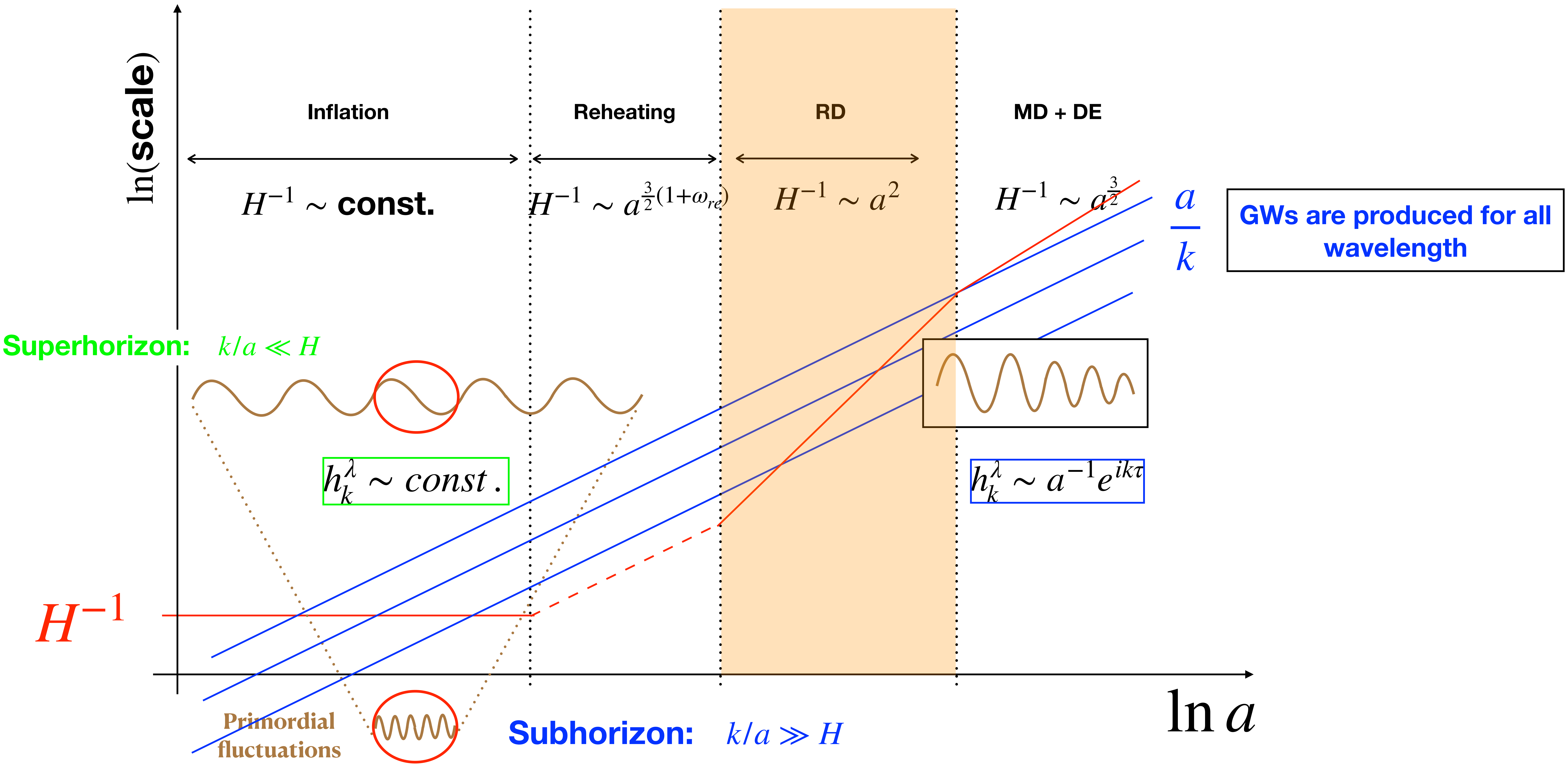

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