Multi-field Extension of Weak Gravity Conjecture

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① Weak Gravity Conjecture

(N. Arkani-Hamed, L. Motl, A. Nicolis, C. Vafa, JHEP 06 (2007) 060, hep-th/0601001)

See also T. Banks, M. Johnson, A. Shomer JHEP 09 (2006) 049 hep-th/0606277

- Preliminary: Charged Black hole in flat spacetime

$$S = \frac{1}{16\pi G}\int d^4x \sqrt{g}\, R - \frac{1}{4g^2}\int d^4x \sqrt{g}\, F_{\mu\nu}F^{\mu\nu}$$

→ Reissner-Nordström solution $ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_2^2$ $\quad f(r) = 1 - \frac{2GM}{r} + \frac{Gg^2Q^2}{r^2}$

$$A = \frac{gQ}{r}dt \quad (F = dA)$$

$f(r)=0$ at $r_\pm = GM \pm \sqrt{(GM)^2 - Gg^2Q^2}$

cosmic censorship: for the singularity ($r=0$) to be hidden behind the horizon.

$\to (GM)^2 \geq Gg^2Q^2 \iff M \geq gQ M_{Pl}$ $\quad (M_{Pl} = \frac{1}{\sqrt{G}})$

"extremal" if $M = gQM_{Pl}$

⇒ outer horizon $r_+ \in (GM, 2GM)$

$$\begin{cases} T = \dfrac{\kappa}{2\pi} = \dfrac{\sqrt{(GM)^2 - Gg^2Q^2}}{2\pi\left[2GM\left(GM + \sqrt{(GM)^2 - Gg^2Q^2}\right) - Gg^2Q^2\right]} \\ S = \dfrac{\pi r_+^2}{G} = \dfrac{\pi}{G}\left[GM + \sqrt{(GM)^2 - Gg^2Q^2}\right]^2 \end{cases}$$

* $T=0$ if BH extremal

- Extremal charged BH

$M = gQM_{Pl}$ $\quad S = \pi\frac{M^2}{M_{Pl}^2}$ $\quad$ * $r_+ = \frac{M}{M_{Pl}^2} \gg \frac{1}{M_{Pl}}$ $\quad (M \gg M_{Pl})$

consider the transition from the extremal BH (M, Q) to another extremal BH $(M+\delta M, Q+\delta Q)$ by putting δQ into BH

→ extremality: $\delta M = g M_{Pl}\,\delta Q$

⇒ the # of ways to put $\delta Q = \frac{\delta M}{gM_{Pl}}$? $(g \ll 1)$ → very close to zero

$\delta Q = Nq$

N	q
1	$\delta M/gM_{Pl}$
$\cdots$	
$\frac{\delta M}{gM_{Pl}}$	1

(with only one species ; there can be more)

i.e. the # of ways $> \frac{\delta M}{gM_{Pl}}$

but it should be smaller than $e^{S+\delta S} - e^S = \delta S\, e^S = \left(2\pi\frac{M}{M_{Pl}}\frac{\delta M}{M_{Pl}}\right)e^{\pi\frac{M^2}{M_{Pl}^2}}$

ie. $\frac{1}{g}\frac{\delta M}{M_{Pl}} < 2\pi\frac{M}{M_{Pl}}\frac{\delta M}{M_{Pl}}e^{\pi\frac{M^2}{M_{Pl}^2}}$

$\Leftrightarrow \log\frac{1}{g} < \log\left(2\pi\frac{M}{M_{Pl}}\right) + \pi\frac{M^2}{M_{Pl}^2} \simeq \pi\frac{M^2}{M_{Pl}^2}$: g cannot be arbitrarily small

BH decay by Hawking radiation $\Rightarrow M\downarrow$: for $M \sim M_{Pl}$, semi-classicality no longer valid

if BH stop decaying (remnant $\Leftarrow$ motivated by the BH information paradox)
at $M \sim M_{Pl}$

$\log\frac{1}{g} < \pi$ — Still $S \le \frac{A}{4G}$ (Bekenstein bound)

* Resolution issue: if g is close to zero,
the scattering process does not distinguish $Q \in [0, \frac{1}{g}]$

$\Rightarrow$ BH with charges $0, \cdots, \frac{1}{g}$ regarded as uncharged BH. (entropy $\sim \log\frac{1}{g} < \frac{A}{4G} \sim \pi$)
remnant

Conclusion: either 1) g cannot be arbitrarily small or
2) the charged BH cannot exist (BH completely discharged)

In fact, 1) and 2) are equivalent.

$\because$ the charged BH decay: let M: BH mass
m: charged particle mass (q) N: the # of charged particles ($Nq = Q \Rightarrow N = Q/q$)

$$M \ge Nm = \left(\frac{gQ}{gq}\right)m$$

for the extremal BH. $M = gQM_{Pl}$

$$\Rightarrow \boxed{gQ \gtrsim \frac{m}{M_{Pl}}}$$: Weak Gravity Conjecture (WGC)

* Two ways of discharge (G.W. Gibbons, Comm. Math. Phys. 44 (1975) 245)

1) the Hawking process $T \gg m$ (~~GM~~
($GM \ll \frac{1}{m}$)

$\underbrace{P(m, \pm 1)}_{\text{charge}} \sim e^{-\frac{1}{T}\left(m \mp \frac{g^2Q}{r_+}\right)}$ if $Q>0$, $P(m,+1) > P(m,-1)$

$\Rightarrow$ m having the charge >0 favored
(BH discharge)

2) the Schwinger process $T \ll m$
($GM \gg \frac{1}{m}$)

$\frac{dQ}{dt} \sim e^{-\frac{\pi m^2}{g|E|}} = e^{-\frac{\pi m^2 r_+^2}{g^2 Q}}$ $\Leftarrow$ extremal BH ($T=0$)

$\Rightarrow \frac{dQ}{dt} \sim \mathcal{O}(1)$ for $\frac{\pi r_+^2}{g^2 Q}m^2 \lesssim 1$

$\epsilon = g^2\int_V d^3x\, E^2 = \int_V d^3x \frac{g^4Q^2}{r_+^4}$ if Volume $\sim \lambda_c^3 \sim \frac{1}{m^3}$

$\simeq \frac{g^4Q^2}{r_+^4}\frac{1}{m^3}$

means that $\epsilon \gtrsim m$ (energy enough to create the particle)

② Multi-field extension

C.C. Cheung, G.N. Remmen, Phys. Rev. Lett. 113 (2014) 051601, 1402.2287

In the presence of multi-U(1),

$$\vec{z}_i = (g\vec{q}_i)\frac{M_{pl}}{m_i} \quad \text{for charged particle 'i'.}$$

particle label

vector in multi-U(1) space

In the same way, for BH

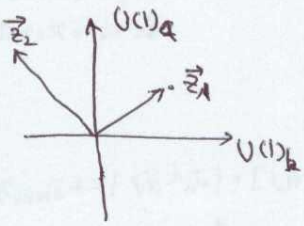
$$\vec{Z} = (g\vec{Q})\frac{M_{pl}}{M}$$

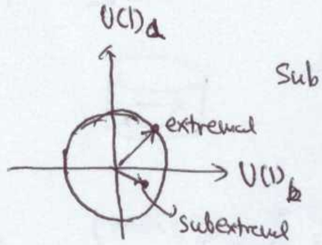
$$\Downarrow$$

Sub-extremality: $(GM)^2 \geq G(g\vec{Q})^2$

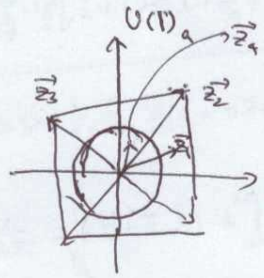
$$\Leftrightarrow |\vec{Z}| = \frac{1}{M}\sqrt{(g\vec{Q})^2} \leq 1$$

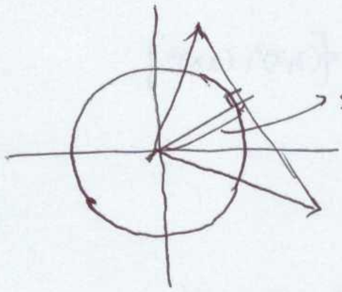
extremal BH



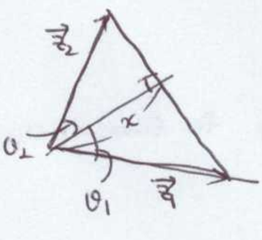


for the charged BH to completely discharge,





should be larger than 1.



$$x = z_1\cos\theta_1 = z_2\cos\theta_2$$

$$z_1\sin\theta_1 = \sqrt{z_1^2-x^2}$$

$$z_2\sin\theta_2 = \sqrt{z_2^2-x^2}$$

$$|\vec{z}_1\cdot\vec{z}_2| = -z_1z_2\cos(\theta_1+\theta_2) = -z_1z_2\cos\theta_1\cos\theta_2 + z_1z_2\sin\theta_1\sin\theta_2$$

$$= -x^2 + \sqrt{(z_1^2-x^2)(z_2^2-x^2)}$$

$$\Rightarrow (z_1^2-x^2)(z_2^2-x^2) = (x^2+|\vec{z}_1\cdot\vec{z}_2|)^2 \quad \Rightarrow \quad x^2 = \frac{z_1^2z_2^2-|\vec{z}_1\cdot\vec{z}_2|^2}{z_1^2+z_2^2+2|\vec{z}_1\cdot\vec{z}_2|} \overset{!}{>} 1$$

$$\therefore \; z_1^2+z_2^2+2|\vec{z}_1\cdot\vec{z}_2| < z_1^2z_2^2-|\vec{z}_1\cdot\vec{z}_2|^2$$

$$\Leftrightarrow \boxed{(1+|\vec{z}_1\cdot\vec{z}_2|)^2 < (z_1^2-1)(z_2^2-1)}$$

③ Application to Stringy axion

(J. Brown, W. Cottrell, G. Shiu, P. Soler, JHEP 10 (2015) 023 1503.04783
JHEP 04 (2016) 017 1504.00659)

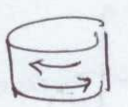
- Preliminary : D-brane and T-duality

• T-duality

under the toroidal compactification, $X^i \sim X^i + 2\pi R L \quad L \in \mathbb{Z} \qquad p_i = \frac{M}{R} \quad M \in \mathbb{Z}$

$\hookrightarrow e^{i p_i x_i}$ ~ single valued under $x_i \to x_i + 2\pi R$.

closed string $X^i(\sigma + 2\pi, \tau) = X^i(\sigma, \tau) + 2\pi R \cdot L$



$$X^i = X^i_R(\tau - \sigma) + X^i_L(\tau + \sigma)$$

$$\begin{cases} X_R^i(\tau-\sigma) = \frac{1}{2}(x^i - c) + \frac{\alpha'}{2}\left(\frac{M}{R} - \frac{R}{\alpha'}L\right)(\tau - \sigma) + i\sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{1}{n} \alpha_n^i e^{-in(\tau-\sigma)} \\ X_L^i(\tau+\sigma) = \frac{1}{2}(x^i + c) + \frac{\alpha'}{2}\left(\frac{M}{R} + \frac{R}{\alpha'}L\right)(\tau + \sigma) + i\sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{1}{n} \bar{\alpha}_n^i e^{-in(\tau+\sigma)} \end{cases}$$

$$[\alpha_m^i, \alpha_n^j] = [\bar{\alpha}_m^i, \bar{\alpha}_n^j] = m\,\delta_{m+n}\,\eta^{ij}$$

$$\uparrow$$

$$[X^\mu(\sigma,\tau), \dot{X}^\nu(\sigma',\tau)] = 2\pi i \alpha' \eta^{\mu\nu} \delta(\sigma - \sigma')$$

$$X^i(\tau, \sigma) = x^i + \alpha' \frac{M}{R}\tau + LR\sigma + i\sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{1}{n}\left(\alpha_n^i e^{-i(n-\sigma)} + \bar{\alpha}_n^i e^{-i(n+\sigma)}\right)$$

$$S = \frac{1}{2\pi\alpha'}\int_\Sigma d^2 z \underbrace{\partial X^M \bar{\partial}_M X}_{\frac{1}{2}(\dot{X}^2 - X'^2)} \to H = \int_0^{2\pi} d\sigma (\dot{X}\cdot\Pi - \mathcal{L}) = \frac{1}{2\pi\alpha'}\int_0^{2\pi} d\sigma \cdot \frac{1}{2}(\dot{X}^2 + X'^2) = \frac{1}{2\pi\alpha'}\int_0^{2\pi} d\sigma\left[(\partial_+ X)^2 + (\partial_- X)^2\right]$$

$$\Pi = \frac{1}{2\pi\alpha'}\dot{X}$$

$$\sigma^\pm = \tau \pm \sigma$$

$$= L_0 + \bar{L}_0$$

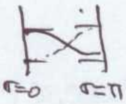
$$\begin{bmatrix} L_0 - 1 = \frac{\alpha'}{2} m_R^2 = \frac{\alpha'}{4}\left(\frac{M}{R} - \frac{R}{\alpha'}L\right)^2 + (N_R - 1) \\ \bar{L}_0 - 1 = \frac{\alpha'}{2} m_L^2 = \frac{\alpha'}{4}\left(\frac{M}{R} + \frac{R}{\alpha'}L\right)^2 + (N_L - 1) \end{bmatrix} \Rightarrow$$ the same spectrum under $R \to \frac{\alpha'}{R}$, $L \leftrightarrow M$

$$\Downarrow$$

$$(X_L, X_R) \to (X_L, -X_R)$$

open string boundary condition $\quad\to\quad$ closed string boundary condition

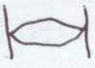
Neumann (N) : $\partial_\sigma X^i = 0 \big|_{\partial\Sigma}$



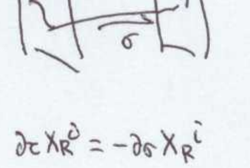
$\sigma = 0 \qquad \sigma = \pi$

$\partial_\tau X^i = 0 \big|_{\partial\Sigma}$

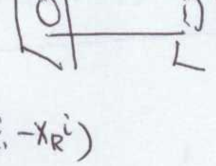
Dirichlet (D) : $\partial_\tau X^i = 0 \big|_{\partial\Sigma}$



$\partial_\sigma X^i = 0 \big|_{\partial\Sigma}$



$\tau \leftrightarrow \sigma$



Since $\partial_\tau X_R^i = -\partial_\sigma X_R^i$
$\partial_\tau X_L^i = +\partial_\sigma X_L^i$

under T: $(X_L^i, X_R^i) \to (X_L^i, -X_R^i)$

N: $\partial_\tau X^i = \partial_\tau(X_L^i + X_R^i) = \partial_\sigma(X_L^i - X_R^i) = 0 \xrightarrow{T} \partial_\sigma(X_L^i + X_R^i) = \partial_\sigma X^i = 0$: D

D: $\partial_\sigma X^i = \partial_\sigma(X_L^i + X_R^i) = \partial_\tau(X_L^i - X_R^i) = 0 \longrightarrow \partial_\tau(X_L^i + X_R^i) = \partial_\tau X^i = 0$: N

Type IIA	1-form	3-form	5-form
	D_0	D_2	D_4
Type IIB	0-form	2-form	4-form
	D_{-1}	D_1	D_3

$$S_{IIB} \supset \frac{1}{2\tilde{\kappa}_{10}^2}\int d^{10}x\sqrt{-G}\left(-\frac{1}{2}|F_{p+2}|^2\right) + \mu_p\int_{D_p} C_{p+1} \longrightarrow 2\tilde{\kappa}_{10}^2 = (2\pi)^7\alpha'^2 \;:\; \frac{m_{pl}^2}{2} = \frac{V_X}{(2\pi)^7(\alpha')^4 g_s^2}$$

$\downarrow$ eq. of motion

$$d*F_{p+2} = 2\tilde{\kappa}_{10}^2\mu_p\,\delta^{9-p}(x_\perp)\,dV_\perp \longrightarrow \int_{\mathbb{R}\times S^{8-p}} d*F_{p+2} = \int_{S^{8-p}} *F_{p+2} = 2\tilde{\kappa}_{10}^2\mu_p$$

Example: $C_2 = \sum_i a^i\omega_i$ $\quad \omega_i \in H^2(X,\mathbb{Z}) \sim \{\text{harmonic 2-forms}\} \to \underline{\text{massless}}\ (\Delta_X\omega_i = 0)$

$$\int_X \omega_i\wedge *\omega_j = \frac{V_X}{\alpha'^2}K_{ij}$$

$\Sigma_k \in H_2(X,\mathbb{Z})$ (Poincaré dual 2-cycle)

$$q_k^i = \int_{\Sigma_k}\omega^i$$

Euclidean D1-brane wrapping spacelike $\Sigma_k \to$ ~~forming w_{p+1}~~

$$S = -\frac{T_p}{g_s}\int_{w_{p+1}=\Sigma_k} d^{p+1}\xi\sqrt{\det G} + i\mu_p\int_{w_{p+1}=\Sigma_k} C_{p+1}$$

$l_s = 2\pi\sqrt{\alpha'}$

$$T_p = \mu_p = 2\pi l_s^{-(p+1)} \Rightarrow T_1 = \mu_1 = \frac{2\pi}{l_s^2} = \frac{1}{2\pi\alpha'} \text{ (String tension)}$$

$$\to i\mu_1\int_{\Sigma_k}\sum_i a^i\omega_i = i\mu_1 q^i_k a^i$$

$\downarrow$

instanton action $\quad V \supset \sum_k \Lambda^4 e^{-m_k}\left(1-\cos(\mu_1 q^i_k a^i)\right)$ $\quad \mu_1 = \frac{1}{2\pi\alpha'}$

Kinetic term $\sim \frac{m_{pl}^2}{2}\int d^4x\sqrt{-g}\left(R + \frac{1}{2}\frac{g_s^2}{(\alpha')^2}K_{ij}\,\partial a^i\partial a^j\right)$

$$\frac{1}{2\tilde{\kappa}_{10}^2}\cdot\frac{1}{2}\int_X\omega_i\wedge*\omega_j = \frac{1}{(2\pi)^7\alpha'^4}\,\frac{1}{2}\frac{V_X}{\alpha'^2} = \frac{m_{pl}^2}{2}\times\frac{1}{2}\frac{g_s^2}{(\alpha')^2}$$

Canonical quantization

$$c^i = \frac{g_s m_{pl}}{\sqrt{2}\,\alpha'}D^i{}_j a^j \quad (D^T K D = I)$$

$$\Rightarrow V \supset \sum_k \Lambda^4 e^{-m_k}\left(1-\cos\left(Q^i_k\frac{c^i}{m_{pl}}\right)\right) \qquad Q^i{}_k = \frac{1}{\sqrt{2}\pi g_s}(D^{-1})^i{}_l\, q^l{}_k$$

i.e. $f \sim \sqrt{2}\pi g_s (D^{-1}q)^{-1} m_{pl} < \underline{\underline{m_{pl}}}$

Compactifying x^3 with radius R

$$\mathrm{IIB} \xrightarrow{\quad T \quad} \mathrm{IIA} \qquad \tilde R = \frac{\alpha'}{R} = \left(\frac{l_s}{2\pi}\right)^2 \frac{1}{R}$$

$$C_2 \longrightarrow C_3$$

D1 instanton (Σ_k) $\longrightarrow$ D2 instanton $(\Sigma_k \times S^1)$

$$\frac{m_{pl}^2}{2} = \frac{V_X}{(2\pi)^7 \alpha'^4 g_s^2}$$

3-dim Plank mass coincidence:

$$1 = \frac{(\tilde m_{pl}^2)_{3\text{-dim}}}{(m_{pl}^2)_{3\text{-dim}}} = \frac{\tilde m_{pl}^2 \tilde R}{m_{pl}^2 R} = \frac{\tilde m_{pl}^2}{m_{pl}^2}\frac{\tilde R^2}{\alpha'} \qquad R = \alpha'/\tilde R$$

$$\Rightarrow \tilde m_{pl} = \frac{\alpha'^{1/2}}{\tilde R} m_{pl} = \frac{l_s}{2\pi \tilde R} m_{pl}$$

$$\frac{\tilde m_{pl}}{m_{pl}} = \frac{g_s}{\tilde g_s} \Rightarrow \tilde g_s = \frac{m_{pl}}{\tilde m_{pl}} g_s = \frac{\tilde R}{\alpha'^{1/2}} g_s = \frac{\alpha'^{1/2}}{R} g_s = \frac{l_s}{2\pi R} g_s \Rightarrow \tilde g_s = \frac{l_s}{2\pi R} g_s$$

$$\Rightarrow \cancel{m_{pl} g_s = \tilde m_{pl} \tilde g_s}$$

$$m_{pl} g_s = \tilde m_{pl} \tilde g_s$$

$$\mu_2 \int_{\Sigma_i \times S^1} C_3 = \oint A_3 \, dx^3 \Rightarrow A_3^i = \frac{a^i}{(2\pi) l_s^3}$$

$$\| $$

$$\cancel{\mu_2 \int_{\Sigma_i \times S^1} a^i \omega_i \int_{S^1}} \qquad \mu_2 = \frac{1}{2\pi l_s^3}$$

$$\mu_2 \int_{\Sigma_i} C_2 \xrightarrow{T} \mu_2 \int_{\Sigma_i \times S^1} C_2 \wedge dx^3 = \cancel{\left(\mu_2 \int_{\Sigma_k} a^i \omega_i\right) \int_{S^1} dx^3}$$

$$C_2 \wedge dx^3 = C_3 \left(\frac{\mu_3}{\mu_2}\right)$$

$$= \oint_{S^1} \left(\mu_2 \int_{\Sigma_i} C_2\right) dx^3$$

$$\frac{m_{pl}^2 g_s^2}{4\alpha'^2} K_{ij} (\partial a)^2 = \frac{\tilde m_{pl}^2 \tilde g_s^2}{4\alpha'^2} K_{ij} \left[(2\pi)^2 l_s^6 (\partial A)^2\right] = \frac{1}{4}\Big[\underbrace{(2\pi)^6 l_s^2 \tilde m_{pl}^2 \tilde g_s^2}_{= 1/g^2}\Big] (\partial A)^2$$

$$\alpha' = \frac{l_s^2}{(2\pi)^2}$$

$$\boxed{g = \frac{1}{(2\pi)^3 l_s \tilde m_{pl} \tilde g_s}}$$

$$e^{-m_k} = e^{-2\pi \tilde R \tilde M_k} \Rightarrow \boxed{\tilde M_k = \frac{m_k}{2\pi \tilde R}}$$

$$\Rightarrow g > \frac{\tilde M_k}{\tilde m_{pl}} \Leftrightarrow \frac{\tilde R}{(2\pi)^2 l_s \tilde g_s} > m_k \qquad \frac{\alpha'^{1/2}}{g_s} = \frac{l_s}{2\pi}\frac{1}{g_s}$$

$$\Leftrightarrow \frac{1}{(2\pi)^3 g_s} > m_k$$

$$\Leftrightarrow f \lesssim m_{pl} \quad ✓$$

$$f \sim \sqrt{2}\pi g_s m_{pl}$$

$$\Rightarrow \frac{1}{g_s} \sim \sqrt{2}\pi \frac{m_{pl}}{f}$$