Dark Radiation (KIAS, Apr. 26 & 28, 2022 & Yonsei University, Jun 13, 2022)

1. Definitions

radiation in cosmological context = relativistic particles ($p = \rho/3$)

expansion → redshift → radiation can become (non-relativistic) matter ($p = 0$) eventually ⇝ equivalent to hot DM

BBN, CMB: $T \sim 0.1\,\mathrm{MeV} \,..\, 0.1\,\mathrm{eV}$

⇝ radiation in SM: γ, ν

dark: not in SM

⇝ dark radiation (DR): relativistic particles $\neq \gamma$, SM ν

examples: (light) sterile neutrino (fermion)

dark photon (vector)

⇝ total radiation energy density:

$$\rho_{rad} = \rho_\gamma + \underbrace{\rho_\nu}_{\text{SM } \nu} + \rho_{DR}$$

$$\rho_\gamma = \frac{\pi^2}{15} T^4 \quad (T \equiv T_\gamma)$$

$$\rho_\nu = N_{eff}^{SM} \frac{7}{8} \frac{\pi^2}{15} T_\nu^4$$

$\frac{7}{8}$: fermions

N_{eff}^{SM}: effective number of ν species in SM, ≈ 3

Neutrino temperature:

weak interactions keeping ν in thermal (chemical) equilibrium:

$\nu\nu \leftrightarrow e^+e^-$, rate $\propto G_F^2 T^5$

expansion rate $H \propto T^2$ decreases more slowly with decreasing T

$\rightsquigarrow$ ν decouple from thermal bath at $T \approx 1.4$ MeV

Later: $T < m_e \rightsquigarrow e^+e^- \to 2\gamma$ but not $e^+e^- \not\to 2\nu$

$\rightsquigarrow$ T increases (or decreases more slowly than $\propto a^{-1}$)

but not T_ν

$\rightsquigarrow T_\nu < T$

Derivation: comoving entropy density $sa^3 = $ const. in sectors in thermal equilibrium

$$sa^3 = \frac{2\pi^2}{45} g_* T^3 a^3 \quad , \quad g_* = \sum_{\text{bosons}} g_i + \frac{7}{8} \sum_{\text{fermions}} g_i$$

for particles with common temperature $\gg$ masses

consider $T^{(h)} \gg m_e$ (but $T^{(h)} \ll m_\mu$), $T \ll m_e$

$$\gamma, e^-, e^+ : \frac{2\pi^2}{45}\left(2 + \frac{7}{8}\cdot 4\right) T^{(h)^3} a^{(h)^3} = \frac{2\pi^2}{45} 2 T^3 a^3$$

$$\nu : \quad \frac{2\pi^2}{45} g_{*\nu} \underbrace{T_\nu^{(h)^3}}_{= T^{(h)^3}} a^{(h)^3} = \frac{2\pi^2}{45} g_{*\nu} T_\nu^3 a^3$$

(since $T_\nu = T$ even after ν decoupling as long as $T \gg m_e$)

$$\rightsquigarrow \frac{11}{2} T_\nu^3 a^3 = 2 T^3 a^3$$

$$\rightsquigarrow T_\nu = \left(\frac{4}{11}\right)^{1/3} T$$

$$\rho_{rad} = \left[1 + N_{eff}^{SM} \frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\right]\rho_\gamma + \rho_{DR}$$

Corrections to ν temperature in SM:

- $m_e \not\ll 1.4\,\mathrm{MeV}$ $\rightsquigarrow$ $e^+e^- \to 2\nu$ not completely negligible (and e-ν scattering)
  $\rightsquigarrow$ ν temperature $> \left(\frac{4}{11}\right)^{1/3} T$
- $m_e > 0$ $\rightsquigarrow$ finite-T effects relevant
  (QED e.o.s. deviates from ideal gas, corrections to e-ν interactions)

convention: keep $T_\nu = \left(\frac{4}{11}\right)^{1/3} T$ (i.e., T_ν is not the precise ν temperature), absorb corrections into N_{eff}^{SM}

latest calculation: $N_{eff}^{SM} = 3.0440 \pm 0.0002$

[Bennett et al., 2012.02726]

convention: use $N_{eff} \equiv N_{eff}^{SM} + \Delta N_{eff}$ to parametrize DR density

$$\rho_{rad} = \left[1 + N_{eff} \frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\right]\rho_\gamma$$

$$\Delta N_{eff} = \underbrace{\frac{8}{7}\left(\frac{11}{4}\right)^{4/3}}_{\simeq 4.4} \frac{\rho_{DR}}{\rho_\gamma}$$

$$\rho_{DR} = 0.13\, \Delta N_{eff} \underbrace{(\rho_\gamma + \rho_\nu)}_{= \rho_{rad}^{SM}}$$ (as long as ν are relativistic)

2. Effects

Friedmann equation: $H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G \rho$ (flat Universe)

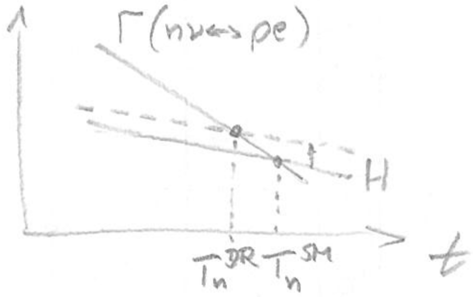
DR $\rightsquigarrow \rho \uparrow \rightsquigarrow H \uparrow \rightsquigarrow$ faster expansion

2.1 Big Bang Nucleosynthesis (BBN)

$T > T_n^{SM} \approx 0.75$ MeV : n, p in chemical equilibrium

via $n \nu_e \leftrightarrow p e^-$

$H \uparrow \rightsquigarrow T_n \uparrow$



$\rightsquigarrow$ neutron-to-proton ratio

$$\frac{n_n}{n_p} = e^{-\frac{m_n - m_p}{T_n}} \uparrow$$

$\rightsquigarrow$ primordial abundances of D, ^{4}He $\uparrow$

(but ^{7}Li abundance $\downarrow$)

result [Yeh et al., 2011.13874]:

$\Delta N_{eff} < 0.124$ @ 95% C.L.

Caveats:

- Input from CMB needed to constrain baryon density
  $\rightsquigarrow$ non-trivial effects if ΔN_{eff} changes between BBN and CMB
- Li problem

2.2 Cosmic Microwave Background (CMB)

CMB power spectrum determined by $\mathcal{O}(10)$ parameters

$\rightsquigarrow$ not appropriate to consider N_{eff} separately

$\rightsquigarrow$ vary <u>all</u> parameters to find largest allowed ΔN_{eff}

[Hou et al., 1104.2333 ; Hinshaw et al., 1212.5226]

- matter-radiation equality: $1 + z_{eq} = \frac{\rho_m}{\rho_{rad}}$

  $\rightsquigarrow$ $\rho_{rad} \uparrow$ $\rightsquigarrow$ $z_{eq} \downarrow$ $\rightsquigarrow$ $t_{eq} \uparrow$

  $\rightsquigarrow$ early integrated Sachs-Wolfe effect $\uparrow$ $\rightsquigarrow$ 1^{st} peak $\uparrow$

  (evolution of amplitude of density fluctuations depends on ρ_m/ρ_{rad} when Fourier mode reenters Hubble horizon)

  $\rightsquigarrow$ keep z_{eq} constant by $\rho_m \uparrow$ (remember for later!)

- plasma e.o.s. ($\rightsquigarrow$ sound speed, gravity-pressure equilibrium point)

  change of ρ_b/ρ_γ $\rightsquigarrow$ change of $\frac{\text{even peak heights}}{\text{odd peak heights}}$

  $\rightsquigarrow$ ρ_b fixed (ρ_γ fixed by CMB spectrum)

  $\rightsquigarrow$ $\rho_m \uparrow$ has to come from $\rho_{DM} \uparrow$

- sound horizon: $r_s = \int_0^{t_{rec}} c_s \frac{dt}{a} = \int_0^{a_{rec}} \frac{c_s}{a^2} \frac{da}{H}$

  $\rightsquigarrow$ $\rho_{rad} \uparrow$, $\rho_m \uparrow$ $\rightsquigarrow$ $r_s \downarrow$

  measured: angular size $\Theta_s = \frac{r_s}{D_A}$

  $D_A = \int_{t_{LS}}^{t_0} c \frac{dt}{a}$ distance to last-scattering surface

  $\rho_{rad} \uparrow$, $\rho_m \uparrow$ $\rightsquigarrow$ $D_A \downarrow$, but <u>less</u> than r_s, because

  H dominated by ρ_Λ at late times net effect: $\Theta_s \downarrow$ ⇝ peak positions shifted

⇝ keep Θ_s constant by $\rho_\Lambda \uparrow$

- anisotropic stress: present for free-streaming (non-interacting) radiation, dampens density fluctuations during radiation domination

  ⇝ $N_{eff} \uparrow$ ⇝ power spectrum $\downarrow$ for $\ell > 130$
  (small-scale fluctuations reenter horizon during radiation domination)

  similar effect by changing amplitude and spectral index of primordial fluctuations ⇝ no strong bound on N_{eff}

- Silk damping: photons diffuse ⇝ dampen density fluctuations on scales < diffusion length $r_D \sim \frac{1}{\sqrt{H}}$
  (random walk during time $t \sim \frac{1}{H}$)

  ⇝ $\rho_{rad} \uparrow$ ⇝ $r_D \downarrow$

  angular size: $\Theta_D = \frac{r_D}{D_A}$

  $\rho_{rad} \uparrow, \rho_m \uparrow, \rho_\Lambda \uparrow$ ⇝ $D_A \sim \frac{1}{H} \downarrow$ more than r_D

  ⇝ $\Theta_D \uparrow$ ⇝ more Silk damping (starts at smaller ℓ)

result from Planck (+BAO to partially break degeneracies) [1807.06209]:

$\Delta N_{eff} < 0.29$ @ 95% CL. (< 0.30 if constrained to $\Delta N_{eff} > 0$)

Caveats:

- Silk damping ↓ if He abundance Y_p ↓
  (Y_p ↓ ⇝ density of free e^- ↑ because He recombines earlier than H ⇝ photon mean free path ↓)
  ⇝ change of Y_p can compensate effect of DR
  ⇝ non-trivial interplay between BBN and CMB

- inconsistencies between high-ℓ and low-ℓ data, lensing anomaly
  ⇝ underestimated systematic errors?

2.3 Hubble Tension

Planck [1807.06209]: $H_0 = (67.4 \pm 0.5)\,\frac{km}{s\,Mpc}$

"Local" measurement (Cepheids, SN Ia) [Riess et al., 2112.04510]

$$H_0 = (73.04 \pm 1.04)\,\frac{km}{s\,Mpc}$$

⇝ 5σ discrepancy

N_{eff} ↑ ⇝ $\rho_{rad}, \rho_m, \rho_\Lambda$ ↑ ⇝ H_0 ↑ (as measured by CMB)

⇝ problem solved? (Planck: tension "somewhat eased")

Need to take into account another observable:
σ_8 = amplitude of matter fluctuations on scales of $8/h$ Mpc
$\left(h = \frac{H_0}{100\,\frac{km}{s\,Mpc}}\right)$

N_{eff} ↑, H_0 ↑ ⇝ σ_8 ↑

galaxy surveys measure σ_8 (or $S_8 = \sigma_8 \sqrt{\frac{\Omega_m}{0.3}}$)
via weak lensing, tend to find smaller S_8 (2..3σ tension)
⇝ larger N_{eff} makes this tension worse
finite DR mass could help (free-streaming ⇝ $\sigma_8 \downarrow$) [1307.7715, 1308.3255, 1308.5870, 1309.3192]

3 Model Building

3.1 SIDM + R

DM self-interactions proposed to solve small-scale problems of ΛCDM

DM-DR interactions in addition ⇝ DM stays in kinetic equilibrium with DR for a relatively long time ⇝ fewer small dwarf galaxies formed ⇝ missing satellites problem solved

E.g., Bringmann et al., 1312.4947,
Dasgupta & Kopp, 1310.6337v3:

DM, mass ~ TeV or GeV
dark photon, mass ~ MeV
dark neutrino, mass ~ eV (DR)
dark Higgs, mass ~ MeV
dark sector thermalized at $T > T_x^{dec}$ via Higgs portal
entropy conservation ⇝ $\Delta N_{eff}(T) = \left(\frac{T_x}{T_\nu}\right)^4 = \left[\frac{g_*^\nu(T)}{g_*^x(T)} \; \frac{g_*^x(T_x^{dec})}{g_*^\nu(T_x^{dec})}\right]^{4/3}$

$T_x^{dec} \gg m_t$: $\Delta N_{eff}(T_{BBN}) \approx 0.33$ ☹

Variations:

- lighter dark photon or dark Higgs ⇝ relativistic during BBN ⇝ $g_*^X(T_{BBN}) \uparrow$ ⇝ $\Delta N_{eff} \downarrow$

- additional light dark particles
  ⇝ $g_*^X(T_{BBN}) \uparrow$ and $g_*^X(T_X^{dec}) \uparrow$ but $\frac{g_*^X(T_X^{dec})}{g_*^X(T_{BBN})} \downarrow$ ⇝ $\Delta N_{eff} \downarrow$
  Ko & Tang, 1404.0236

- $\nu_{DM} - \nu_d$ oscillations after BBN ⇝ $\Delta N_{eff}(T_{CMB}) < 0$ possible
  ($T_\nu \downarrow$, ν_d becomes non-relativistic)
  Mirizzi et al., 1410.1385
  Tang, 1501.00059
  Chu et al., 1505.02795

3.2 DR from Decays

Long-lived X, lifetime τ

decay after BBN, before recombination ⇝ only CMB affected, strong ΔN_{eff} bound from BBN avoided

light decay products ("daughters") form DR while relativistic

$\Delta N_{eff} = \Delta N_{eff}(\Omega_X, \frac{m_2}{m_X}, \tau)$ (m_2: mass of heavier daughter)

single decay mode: heavier daughter is WDM (or $\Delta N_{eff} \ll 1$)
⇝ can only be a small part of DM

2 decay modes: produce DR + DM with adjustable free-streaming length ⇝ address missing satellites, H_0 / S_8 ?

Hasenkamp & Kersten, 1212.4160

Ex.: saxion → 2axion or 2axino
modulus → 2gravitino or 2axion