Top-down effective theory of inflation

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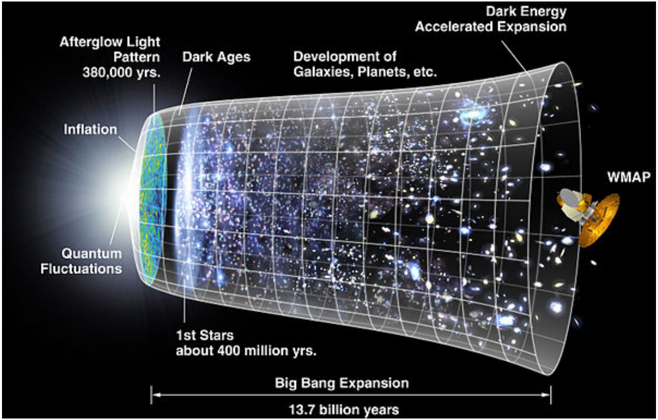
Outline

Introduction to cosmology

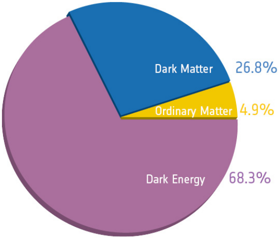
Cosmology – a study on the evolution of the universe



The universe as we understand

Standard cosmological model

- History of the universe: inflation-CMB-structure formation
- Gravity described by GR
- Matter contents described by SM
- Contents of the universe as follows:

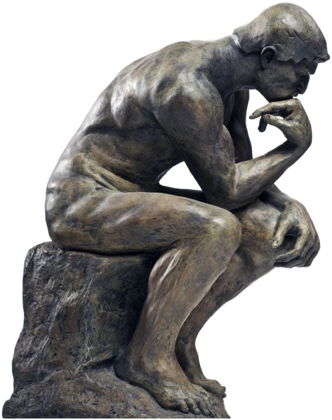


Concordant cosmological model: ΛCDM

Precision cosmology is coming

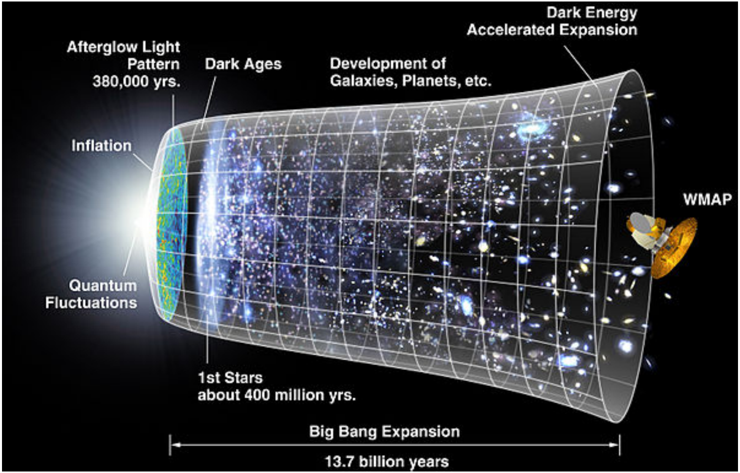
Precise data will be available soon

Observation-driven cosmology

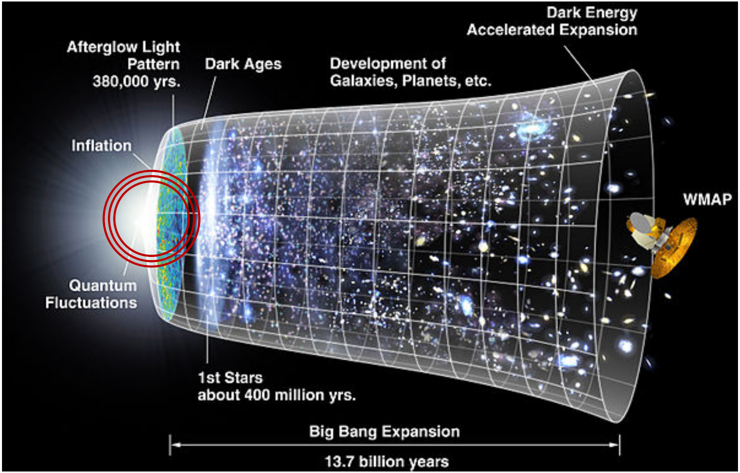


Observations can lead to breakthrough

Standard physics and then beyond?



Standard physics and then beyond?

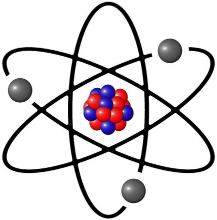


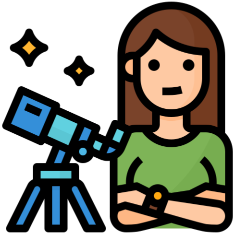
Standard physics and then beyond?

Physics of the very early universe?

We need well-defined questions verifiable with observations

Atom, sun, and us



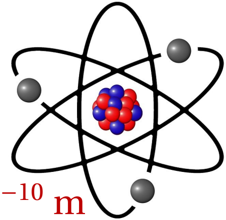


Question that you may have not heard so often

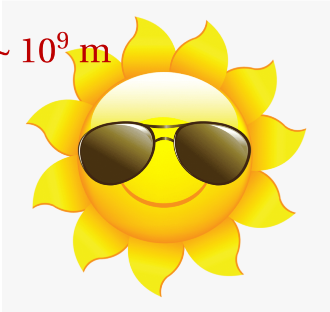
In a table-top experiment, why don't you...

- solve the Schrödinger equation?
- take into account the gravity of Sun?

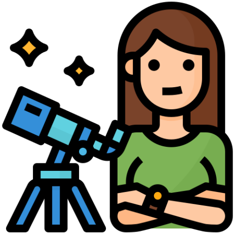
Back to atom, sun, and us: typical sizes



$\sim 10^{-10}$ m

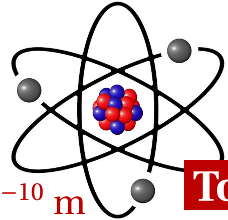


$\sim 10^{9}$ m



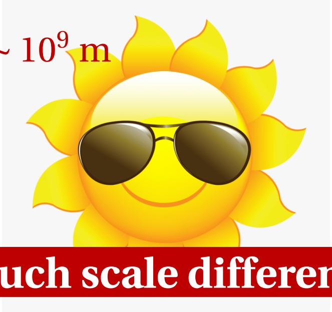
$\sim 10^{0}$ m

Back to atom, sun, and us: typical sizes

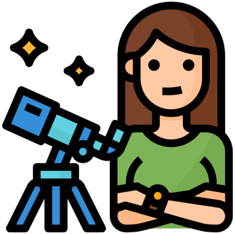


$\sim 10^{-10}$ m

$\sim 10^{9}$ m



Too much scale difference!!!



$\sim 10^{0}$ m

(Sloppy) answer why CM is enough for table-top exp

The effects of the scales too much different from relevant one are suppressed

Effective field theory

Formulating EFT 1: Top-down

If mother theory contains ϕ (relevant for obs) and Φ (tooooo heavy)

$$e^{iS_{\text{eff}}(\phi)} = \int [D\Phi] e^{iS(\phi,\Phi)}$$

Propagator of ϕ is modified with non-local / higher-deriv terms e.g.

$$\phi\left(-\Box + M^2\right)^{-1}\phi = \frac{\phi}{M^2}\left(1 + \frac{\Box}{M^2} + \cdots\right)\phi \quad (M \gg \text{our interest})$$

c.f. Reduced mass μ w/ $m_1 \gg m_2$: $\mu = \dfrac{m_1 m_2}{m_1 + m_2} \approx m_2$

Formulating EFT 2: Bottom-up

If we only know symm principles valid up to $\Lambda \gg$ our interest

$$\mathscr{L}_{\text{eff}} = \underbrace{\mathscr{L}_\phi}_{\text{already known}} + \underbrace{\sum_{n>4} c_n \frac{\mathscr{O}_n}{\Lambda^{n-4}}}_{\text{consistent w/ symmetries}}$$

$$\mathscr{O}_n \supset \phi^n , (\partial_\mu\phi)^n , , (\partial_\mu\phi)^n\phi^m , \cdots$$

EFT and inflationary cosmology

Q: Can we apply EFT to extract **universal features**?

What's wrong with big bang?

Hot big bang is the standard paradigm of modern cosmology

1. Expansion of the universe (Hubble 1929)
2. Abundance of light elements (Alpher, Bethe & Gamov 1948)
3. Cosmic microwave background (Penzias & Wilson 1965)

But the difficulties lie in the initial conditions (Misner 1968)

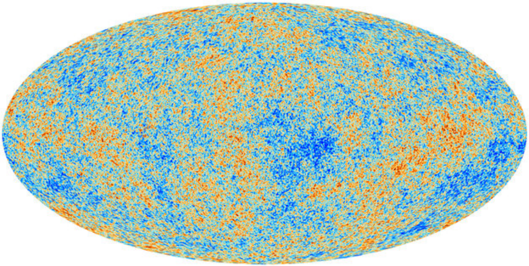
> I wish to approach relativistic cosmology from an unfamiliar point of view. Rather than taking the unique problem of relativistic cosmology to be the collection and correlation of observational data sufficient to distinguish among a small number of simple cosmological solutions of Einstein's equations, I suggest that some theoretical effort be devoted to calculations which try to "predict" the presently observable Universe. The first example of this approach would be Einstein's (1917) failure to find static cosmological solutions, which can be regarded as a lost opportunity to predict the Hubble expansion which is required (not merely permitted) if one assumes spatial isotropy in the original ($\Lambda = 0$) Einstein equations (see Einstein [1945]; the historical interaction between observation and theory on this point is reviewed by North [1965]). The difficulty in using relativistic cosmology for predictive rather than merely descriptive purposes lies in the treatment of the initial conditions. Ideally one might try to show that almost all solutions of the Einstein equations which lead to star formation also have many other properties compatible (or incompatible!) with observation. More modest but more

How to **predict** initial conditions for hot big bang?

Cosmic microwave background

- Strong observational support for hot big bang
- Snapshot of the universe of age 380,000 yrs
- Earliest universe we can observe by photons
- Almost perfect blackbody radiation of $T_0 = 2.725$ K

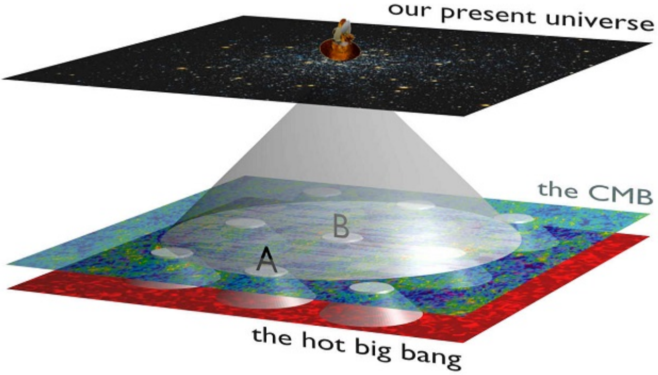
Cosmic microwave background



- Strong observational support for hot big bang
- Snapshot of the universe of age 380,000 yrs
- Earliest universe we can observe by photons
- Almost perfect blackbody radiation of $T_0 = 2.725$ K
- Genuine temperature fluctuation of $\mathcal{O}(10^{-5})$

Horizon problem

Horizon problem: why is the CMB so homogeneous and isotropic?



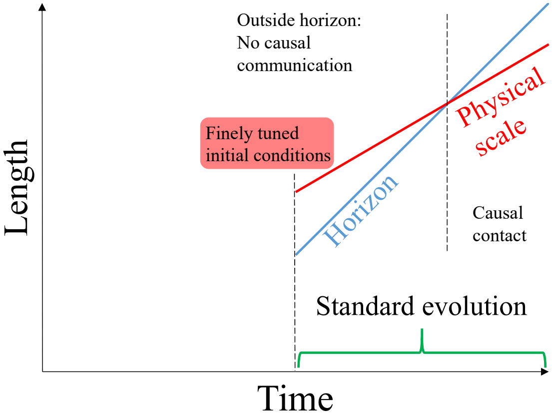
Inflation: **add-on** to hot big bang to provide these initial conditions

What is inflation?

What is inflation?

What is inflation?

Inflation = **accelerated expansion** before hot big bang evolution



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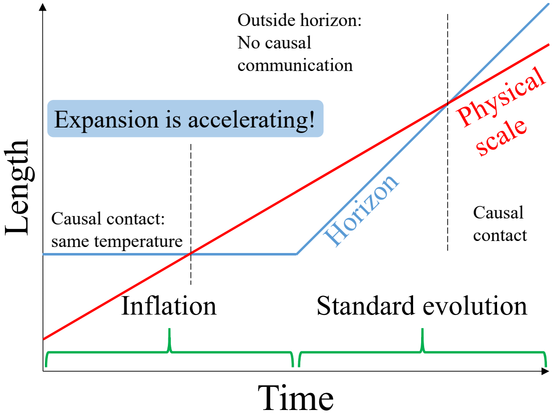

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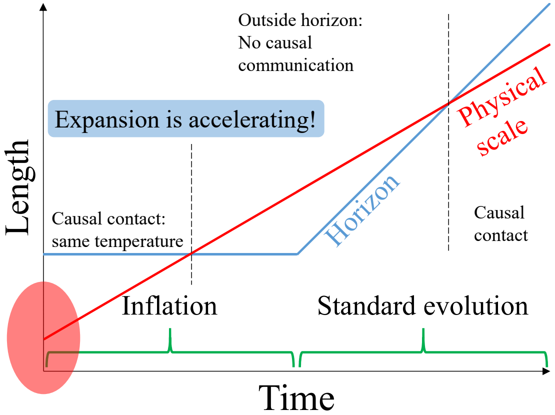


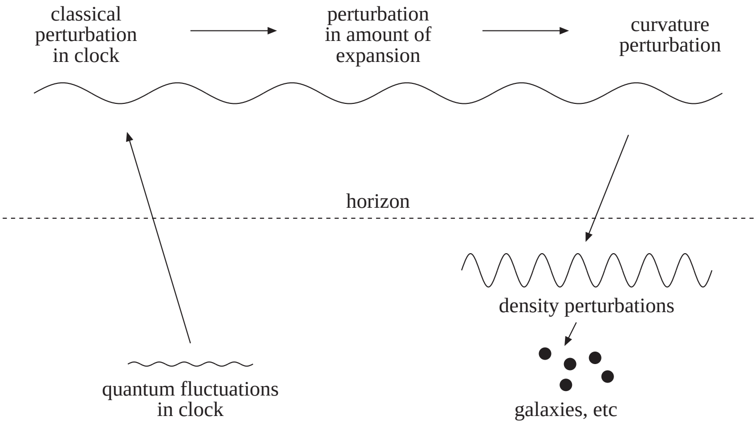
Quantum mechanics is relevant during inflation

Inflation model building

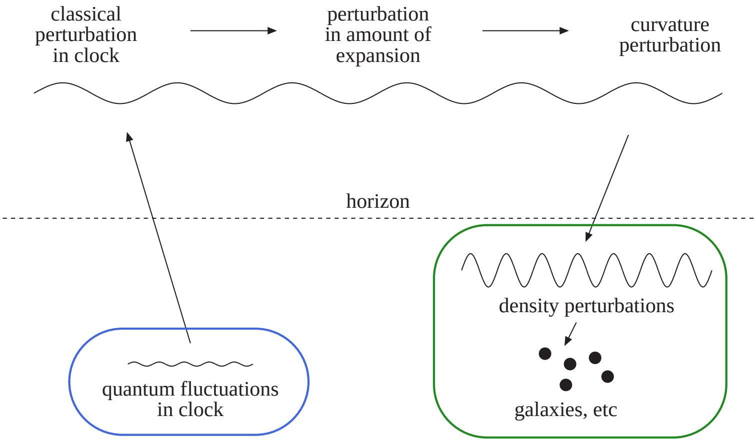
Constructing a realistic model is a challenge for particle physics

1. No candidate for inflaton in the standard model (cf. Higgs inf)
2. Beyond the standard model: SUSY, SUGRA, string theory...
3. How to protect from generic corrections?
4. **Observational predictions consistent with data?**

Generation and evolution of perturbations



Generation and evolution of perturbations



Quantum mechanical signatures on cosmic scales

Scalar and tensor perturbations

Distance between 2 points in space

$$dl^2 = a^2(t)\left\{[1+2\mathscr{R}(t,\boldsymbol{x})]\delta_{ij} + h_{ij}(t,\boldsymbol{x})\right\}dx^i dx^j$$

2 geometric perturbations constrained by observations

- $\mathscr{R}(t,\boldsymbol{x})$: curvature perturbation (scalar), relevant for $\delta T/T_0$
- $h_{ij}(t,\boldsymbol{x})$: gravitational waves (tensor), leading to B-mode pol

How primordial perturbations are constrained

Cosmological structure are all come from primordial perturbations

$$TT \text{ spectrum} \sim \int (\text{transfer fct}) \times \mathscr{P}_{\mathscr{R}}(k)$$

$$BB \text{ spectrum} \sim \int (\text{transfer fct}) \times \mathscr{P}_{h}(k)$$

Observable spectrum — Fixed by BG — Primordial spectrum

Predictions and status for perturbations

- Nearly scale-invariant power spectrum

$$\mathcal{P}_\mathcal{R} \sim \langle|\mathcal{R}|^2\rangle \propto k^{n_\mathcal{R}-1} \text{ with } n_\mathcal{R} \approx 1 \qquad \text{(no appreciable change)}$$

- Nearly Gaussian fluctuations

$$f_\text{NL} \sim \frac{\text{3-pt function}}{(\text{2-pt function})^2} \ll 1 \qquad \text{(interactions are suppressed)}$$

- Small tensor-to-scalar ratio

$$r \equiv \frac{\mathcal{P}_h}{\mathcal{P}_\mathcal{R}} \ll 1 \qquad \text{(well below quantum gravity regime)}$$

Recent CMB observations have confirmed:

$$\left.\begin{aligned} n_\mathcal{R} &= 0.9652 \pm 0.0042 \\ f_\text{NL} &= -0.9 \pm 5.1 \quad \text{(local)} \\ r &\lesssim 0.036 \quad \text{(tension with simplest model?)} \end{aligned}\right\} \quad \text{all consistent!}$$

Recipe for inflation model building

1. Write down a Lagrangian (motivated by particle physics)
2. Check whether the model supports long enough inflation (using the so-called slow-roll approximation)
3. If so, calculate the power spectrum, its spectral index, etc (again using SR approximation)
4. If consistent, a new model is born

(Lyth & Riotto 1999, Enqvist & Mazumdar 2003, Martin, Ringeval & Vennin 2014...)

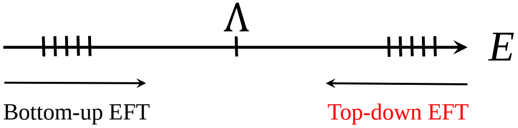
Zoology of inflation models

Already hundreds of inflation models in market (Martin, Ringeval & Vennin 2014)

Contents

Why effective field theory?

- $E_{\text{inflation}}$ ($\sim 10^{15}$ GeV?) $\gg E_{\text{LHC}} = 14$ TeV
- Hundreds of inflation models in the market
- Universality of EFT is very powerful



Universal features of heavy physics? (Achucarro, JG, Hardeman, Palma & Patil 2011)

Why bothering EFT description?

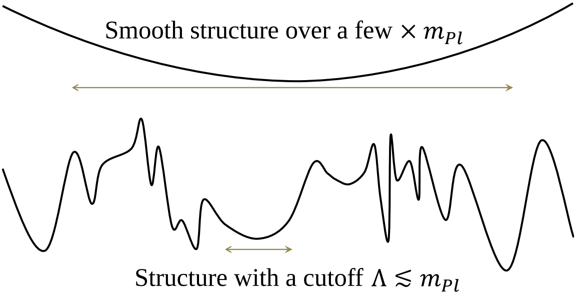
Observations prefer smooth inflation with large enough regime

$$\mathscr{L}_{\text{eff}}[\phi] = \underbrace{\mathscr{L}_0[\phi]}_{\text{smooth}}$$

Why bothering EFT description?

Observations prefer smooth inflation with large enough regime

$$\underbrace{\mathscr{L}_{\text{eff}}[\phi] = \mathscr{L}_0[\phi]}_{\text{smooth}} + \underbrace{\sum_n c_n \frac{\mathcal{O}_n[\phi]}{\Lambda^{n-4}}}_{\text{EFT allows these}}$$



EFT introduces sub-cutoff structure: **tension with observations**

Map of EFTs of inflation

Single field inflation
in Einstein gravity

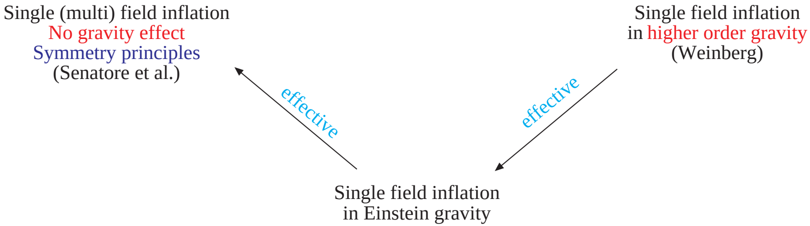
Map of EFTs of inflation

Single (multi) field inflation
No gravity effect
Symmetry principles
(Senatore et al.)

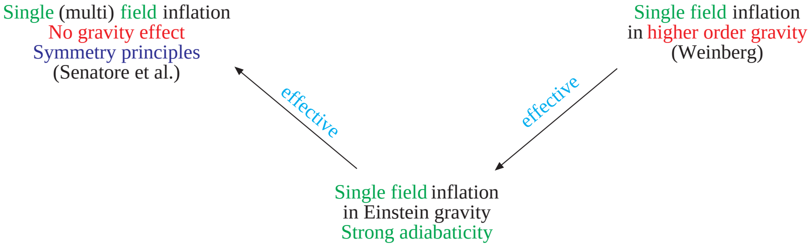
effective

Single field inflation
in Einstein gravity

Map of EFTs of inflation



Map of EFTs of inflation



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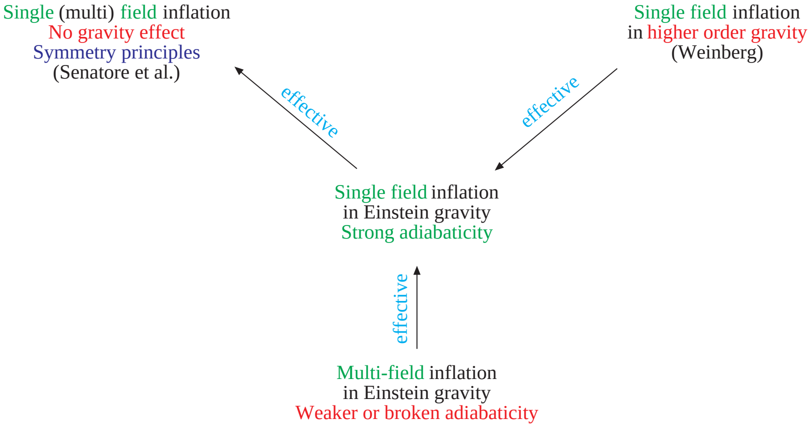

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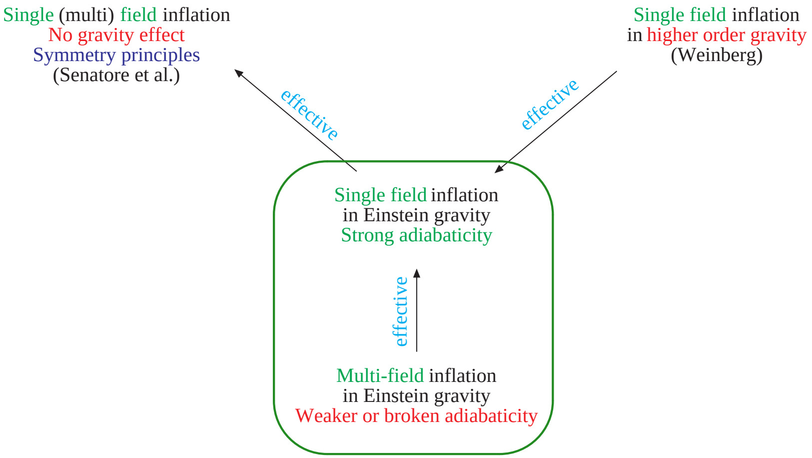
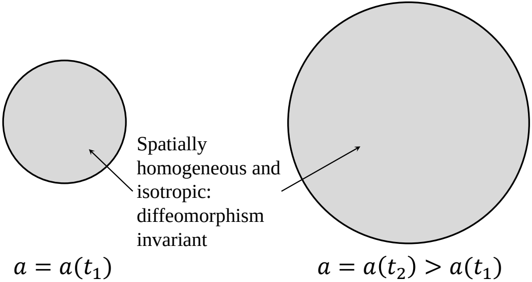


Our focus

Symmetry principles for inflation: bottom-up EFT



Broken time translational symmetry: Goldstone mode $\pi = -\mathcal{R}/H$

(Cheung et al. 2008)

$$S_\pi \supset \int d^4x\sqrt{-g}\left\{2M_2^4\left[\dot\pi^2+\dot\pi^3-\dot\pi\frac{(\nabla\pi)^2}{a^2}\right]-\frac{4}{3}M_3^4\dot\pi^3+\cdots\right\}$$

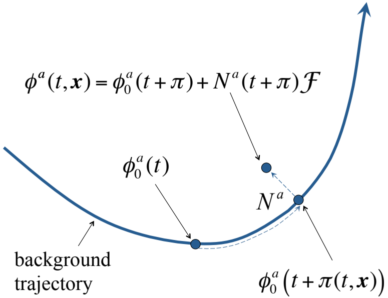
Footprint of heavy physics: top-down EFT

☺ Recipe for top-down EFT ☺

1. Write the action in terms of $\mathscr{R}$ (along traj) and $\mathscr{F}$ (off traj)
2. Integrate out $\mathscr{F}$: $e^{S_{\text{eff}}[\mathscr{R}]} = \int [D\mathscr{F}] e^{S[\mathscr{R},\mathscr{F}]}$
3. Effective single field action $S_{\text{eff}}[\mathscr{R}]$

More about recipe 1

1. Write the action in terms of $\mathscr{R}$ (along traj) and $\mathscr{F}$ (off traj)



- $\mathscr{R}$: Curvature pert, related to time translation $\mathscr{R} = -H\pi$
- $\mathscr{F}$: "Orthogonal" to the trajectory and hence to $\mathscr{R}$

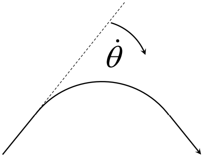
Interaction vertex

$$S = \int \sqrt{-g} d^4 x \left[\frac{m_{\rm Pl}^2}{2} R - \frac{1}{2} g^{\mu\nu} G_{ab} \partial_\mu \phi^a \partial_\nu \phi^b - V(\phi) \right]$$

(expand up to 2nd order in perturbations)

$$S_2 \supset \int d^4 x \left(-2\dot\theta \frac{\dot\phi_0}{H} \dot{\mathscr{R}} \mathscr{F} \right)$$

$\dot\theta$: angular velocity of trajectory



More about recipe 2

2. Integrate out $\mathscr{F}$: $e^{S_{\rm eff}[\mathscr{R}]} = \int [D\mathscr{F}] e^{S[\mathscr{R},\mathscr{F}]}$

Equivalent to plugging the (linear) solution of $\mathscr{F}$ back to the action

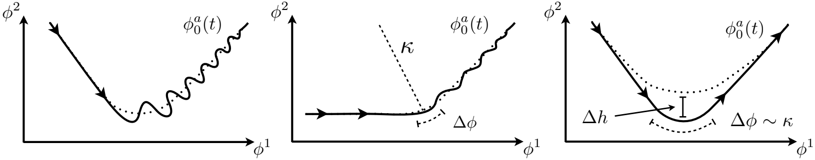
$$\underbrace{\left(-\Box + M_{\rm eff}^2\right)\mathscr{F}}_{\text{From quadratic terms purely in } \mathscr{F}} = \underbrace{-2\dot\theta\frac{\dot\phi_0}{H}\dot{\mathscr{R}}}_{\text{From the interaction with } \mathscr{R}}$$

Q: Is replacing the solution of $\mathscr{F}$ always valid?

Validity of effective theory

Truncation in $\Box/M_{\rm eff}^2$: Non-local → higher derivative theory

$$\mathscr{F} = \frac{-2\dot\theta\left(\dot\phi_0/H\right)}{-\Box + M_{\rm eff}^2}\dot{\mathscr{R}} = \frac{-2\dot\theta\left(\dot\phi_0/H\right)}{M_{\rm eff}^2}\left[1 + \frac{\Box}{M_{\rm eff}^2} + \cdots\right]\dot{\mathscr{R}}$$



Valid for "adiabatic trajectory": $M_{\rm eff}^2$ is most important, or

$$\left|\frac{\ddot\theta}{\dot\theta}\right| \ll M_{\rm eff}$$

(Achucarro, Atal, Cespedes, JG, Palma & Patil 2012)

More about recipe 3

3. Effective single field action $S_{\text{eff}}[\mathscr{R}]$

$$S_{\text{eff}}[\mathscr{R}] = \int d^4x\, a^3 \epsilon m_{\text{Pl}}^2 \left[\left(1 + \frac{4\dot{\theta}^2}{M_{\text{eff}}^2} \right) \dot{\mathscr{R}}^2 + \cdots \right]$$

Effects of heavy physics in "speed of sound"

(Achucarro, JG, Hardeman, Palma & Patil 2011, 2012)

$$c_s^{-2} \equiv 1 + \frac{4\dot{\theta}^2}{M_{\text{eff}}^2}$$

Understanding features

If the trajectory in field space is...

1. straight ($\dot{\theta} = 0$): Identical to single field case (w/ field redef)
2. curved ($\dot{\theta} \neq 0$): Deviations appear from single field case

$\mathscr{F}$ borrows kinetic energy from $\mathscr{R}$ so the sound speed c_s is reduced

Important: EFT remains valid even for **strong turn**

$$c_s^{-2} = 1 + \frac{4\dot{\theta}^2}{M_{\text{eff}}^2} \gg 1$$

Splitting quadratic action and features in $\mathcal{P}_\mathcal{R}$

EFT = canonical ($c_s = 1$) + (occasional) departure from $c_s = 1$

$$S = \underbrace{\int d^4x\, a^3 \epsilon m_{\rm Pl}^2 \left[\frac{\dot{\mathcal{R}}^2}{c_s^2} - \frac{(\nabla\mathcal{R})^2}{a^2} \right]}_{=S_2,\ \text{“free” part}} + S_3 + \cdots$$

$$= \underbrace{S_{2,\rm canonical}}_{c_s=1\ \text{part}} + \underbrace{\int d^4x\, a^3 \epsilon m_{\rm Pl}^2 \left(\frac{1}{c_s^2} - 1 \right) \dot{\mathcal{R}}^2}_{\equiv S_{2,\rm int}\ \text{due to non-trivial heavy physics}} + S_3 + \cdots$$

Features in the power spectrum $\Delta\mathcal{P}_\mathcal{R}$ is due to the interaction $S_{2,\rm int}$

$$\frac{\Delta\mathcal{P}_\mathcal{R}}{\mathcal{P}_\mathcal{R}}(k) = k \int_{-\infty}^{0} d\tau \left(1 - \frac{1}{c_s^2} \right) \sin(2k\tau)$$

Inverting this to write c_s in terms of $\Delta\mathcal{P}_\mathcal{R}$

Manifest correlation between 2- & 3-point functions

3-pt fct is completely fixed by 2-pt fct and its first 2 derivs

(Achucarro, JG, Palma & Patil 2013; JG, Schalm & Shiu 2014)

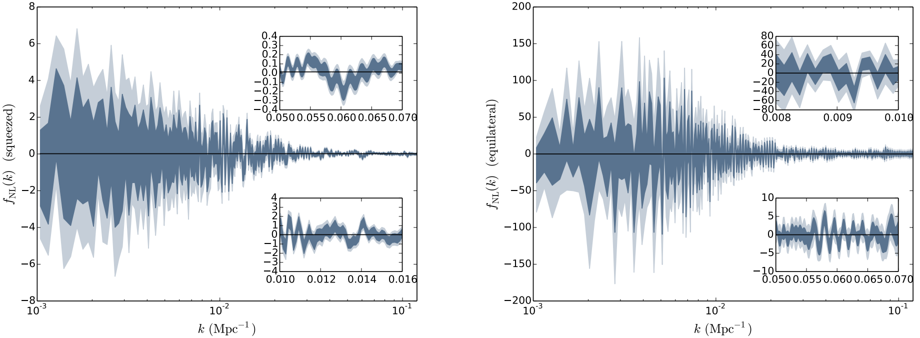
$$f_{\rm NL}(k_1,k_2,k_3) = c_0^\Delta(\boldsymbol{k}) \log P_{\mathscr{R}} + c_1^\Delta(\boldsymbol{k}) \frac{d\log P_{\mathscr{R}}}{d\log k} + c_2^\Delta(\boldsymbol{k}) \frac{d^2\log P_{\mathscr{R}}}{d\log k^2}$$

Applicable to any configuration by projection

(JG and Yamaguchi 2017; JG, Palma & Sypsas 2017)

We only need precise measurement on $\mathscr{P}_{\mathscr{R}}(k)$

Reconstructed 3-pt fct directly from CMB data



Marginal 2σ hints in the 3-pt fct at $k \sim 0.014\ \mathrm{Mpc}^{-1}$ and $0.06\ \mathrm{Mpc}^{-1}$

(Appleby, JG, Hazra, Shafieloo & Sypsas 2016; JG, Palma & Sypsas 2017)

Conclusions

- Cosmology in the era of precision
  - Concordance model but new physics is called upon
  - Questions verifiable with observations are necessary
  - EFT: a powerful tool even for cosmology
- Cosmic inflation in the very early universe
  - Accelerated expansion of the universe
  - Initial conditions are “predicted”
- EFT of inflation
  - Bottom-up and top-down approaches
  - Universal features described in the speed of sound
  - Correlation of correlation functions