Some thoughts on natural models for inflation

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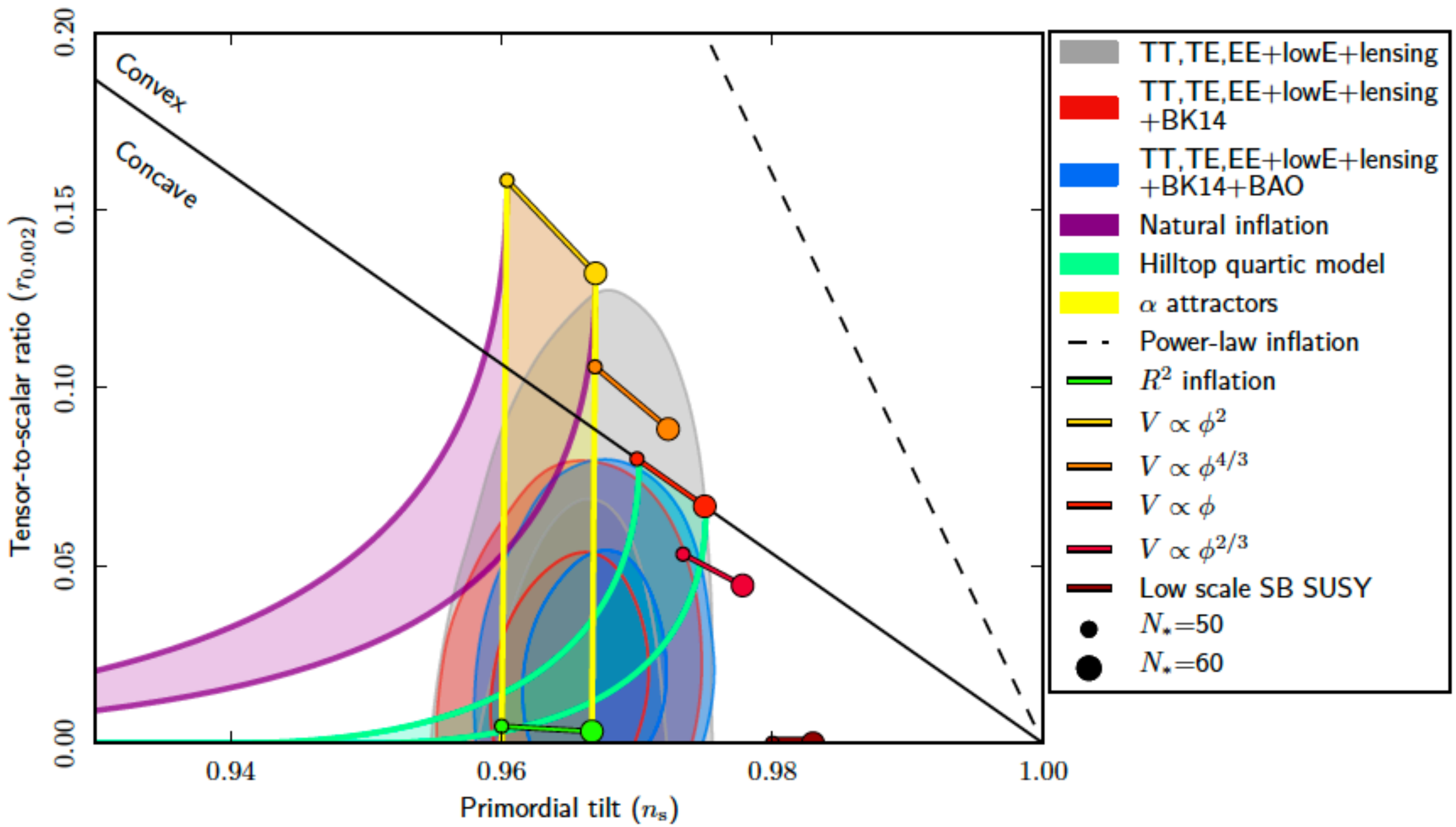
2022 Workshop on Physics of Dark Cosmos
Lavalse Hotel, Busan Oct 21, 2022

Outline

- Introduction
- Higgs inflation in sigma models
- Higgs inflation in Weyl gravity
- (Higgs inflation in supergravity)
- Natural hybrid inflation
- Conclusions

Natural inflation?



- Explains horizon, homogeneity, flatness, relics, and structures.
- Natural inflation and Higgs(R^2) inflation are motivated most and compatible with CMB.

Higgs inflation

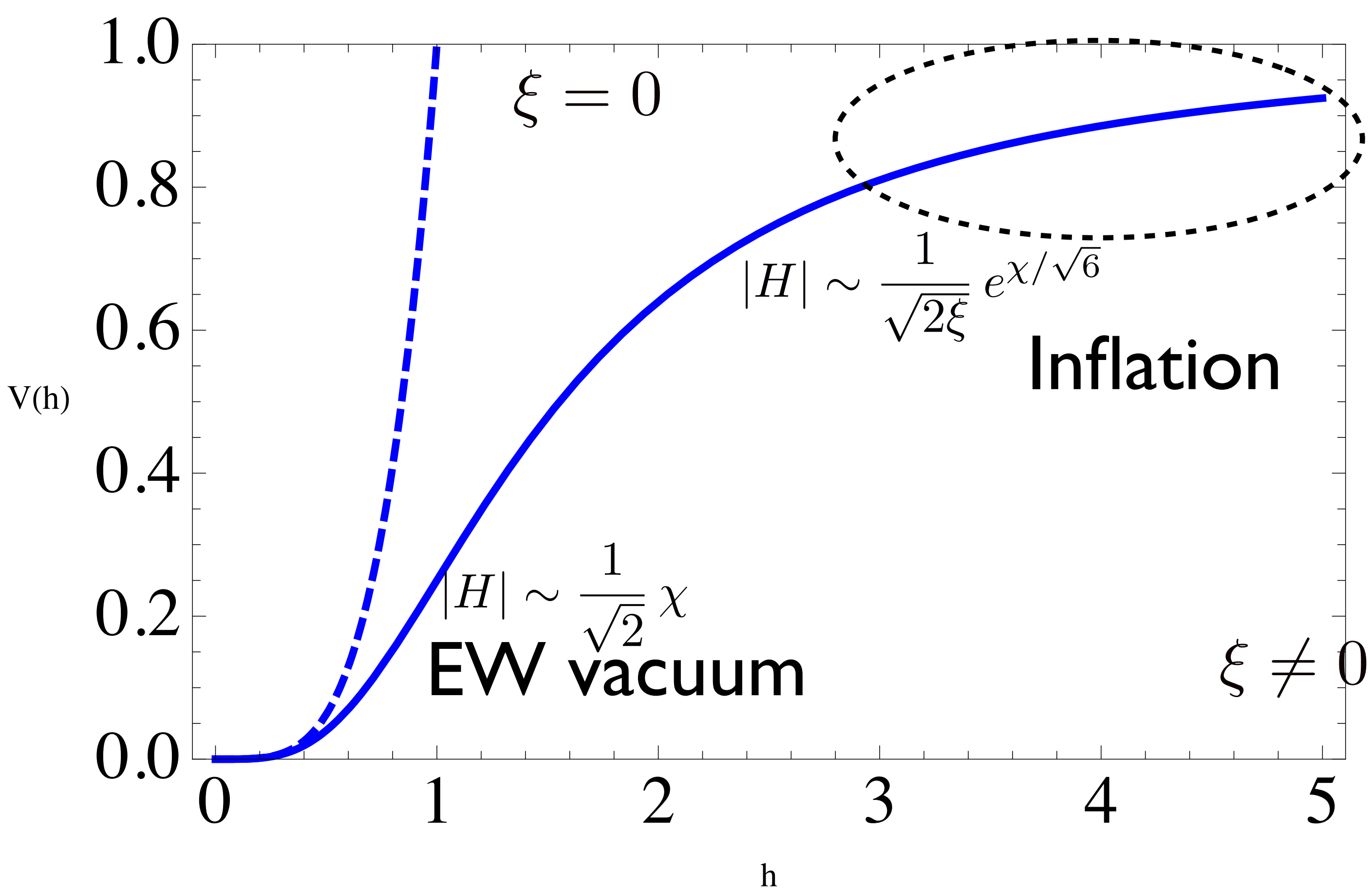
- Higgs non-minimal coupling to gravity is necessary for renormalization on curved background.

[Bezrukov, Shaposhnikov (2007)]

$$\mathcal{L} = \sqrt{-g}\Big(\frac{1}{2}\mathcal{R} + \boxed{\xi|H|^2\mathcal{R}} - |D_\mu H|^2 - \lambda_H(|H|^2 - v^2/2)^2\Big)$$

$$\Rightarrow \mathcal{L}_E = \sqrt{-g_E}\Big(\frac{1}{2}\mathcal{R}(g_E) - \boxed{\frac{3\xi^2}{\Omega^2}(\partial_\mu|H|^2)^2} - \frac{1}{\Omega}|D_\mu H|^2 - \boxed{\frac{V}{\Omega^2}}\Big)$$

$$g_{\mu\nu} = \Omega^{-1} g_{E,\mu\nu}, \quad \Omega = 1 + 2\xi|H|^2$$

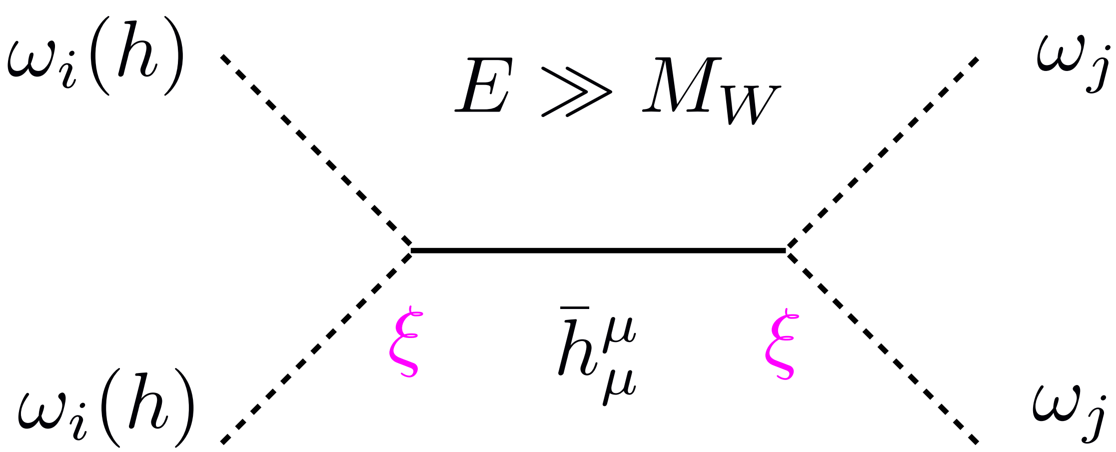


Field-dependent Planck scale drive
Higgs potential flat at large fields!

$$\frac{V}{\Omega} = \frac{\lambda_H|H|^4}{(1+2\xi|H|^2)^2} \sim \frac{\lambda_H}{4\xi^2}\left(1 - e^{-\frac{2}{\sqrt{6}}\chi}\right)$$

Troubles with large coupling

- CMB needs a large non-minimal coupling unless tuned.



$$\mathcal{M}_{\omega_i\omega_j \to \omega_i\omega_j} \sim \frac{\xi^2 E^2}{M_P^2}$$

[Burgess, HML, Trott (2009,2010); Barbon, Espinosa (2009); Hertzberg (2010)]

$$\frac{\Delta T}{T} \sim 10^{-4} \Rightarrow \frac{\xi}{\sqrt{\lambda_H}} = 5 \times 10^4$$

Perturbative unitarity is violated at low scale. $E \lesssim \frac{M_P}{\xi}$

- Unitarity scale is larger during inflation. [F. Bezrukov et al, 2010]

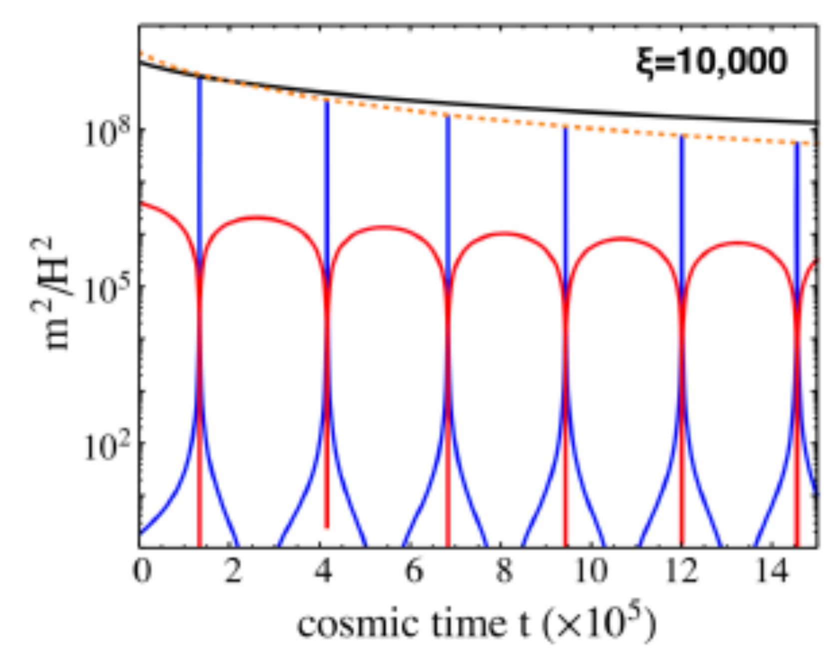
for saturated W, Z masses: $m_W^2 = \dfrac{g^2 h^2}{4(1+\xi h^2/M_P^2)} \simeq \dfrac{1}{4} g^2 \dfrac{M_P^2}{\xi}.$

- But, preheating violates unitarity.

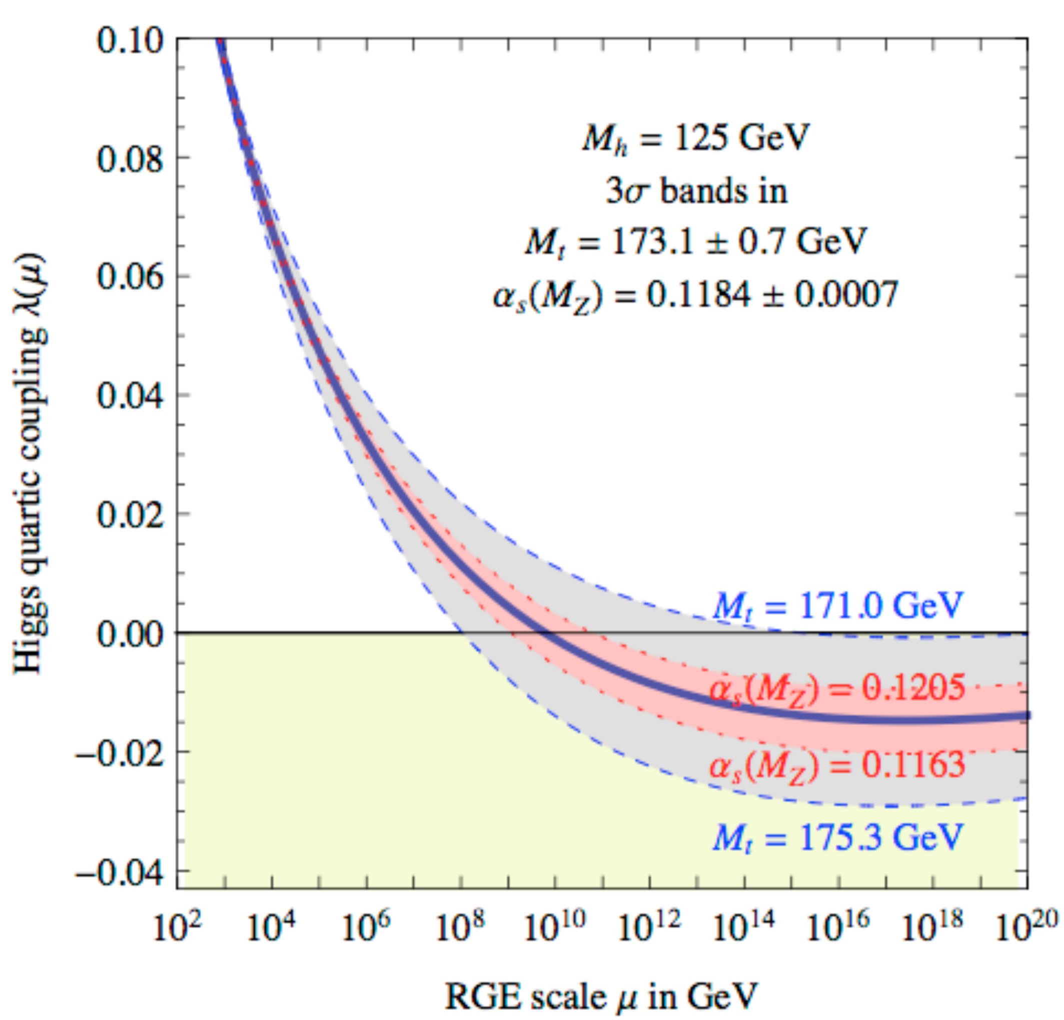
Particle momenta beyond cutoff scale:

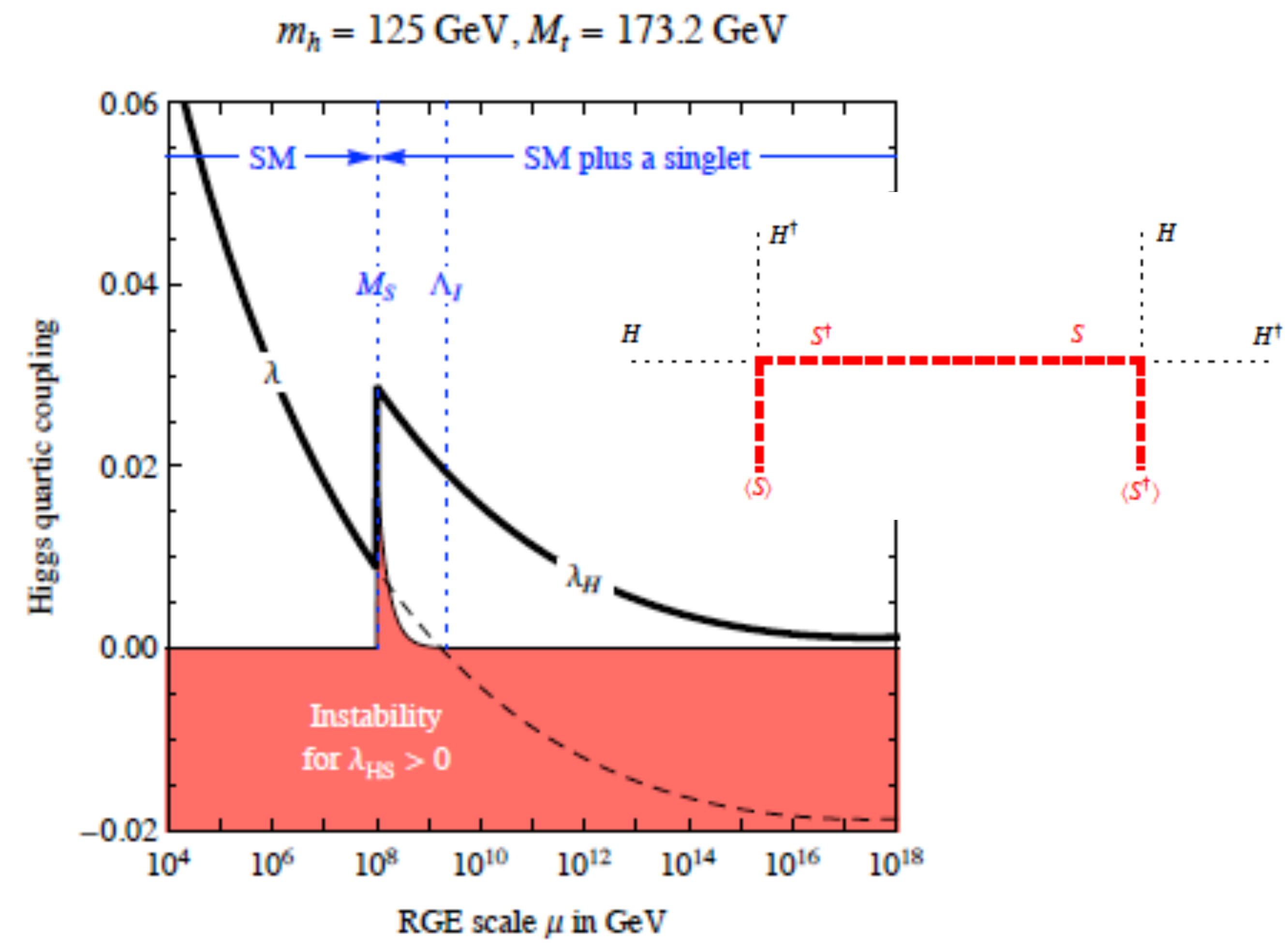
$$k_{\max} \sim \xi H_{\text{end}} \sim \sqrt{\lambda} M_{\text{Pl}}$$

[e.g. E. Sfakianakis, 2019]



Vacuum stability





[Giudice, HML, Espinosa et al (2012)]

- Higgs mass needs a small quartic coupling: $m_h^2 = 2v^2\lambda_H.$

$\lambda_H(m_H) = 0.13 \Longrightarrow$ SM vacuum would be unstable.

- Higgs mixing with singlet scalar: larger Higgs quartic coupling.

$$m_h^2 = 2v^2\left(\lambda_H - \frac{\lambda_m^2}{\lambda}\right) \Longrightarrow \lambda_H = 0.13 + \frac{\lambda_m^2}{\lambda}$$

Hybrid inflation

- Slow-roll condition is violated by another field.

$$V(\phi,\sigma) = \underbrace{\frac{1}{2}m_\phi^2\phi^2}_{\text{inflaton}} + \frac{1}{2}\lambda_{\phi\sigma}\phi^2\sigma^2 + \underbrace{\frac{1}{4}\lambda_\sigma(\sigma^2 - v^2)^2}_{\text{waterfall}}$$

[A. Linde (1994)]

Inflaton potential is sub-dominant: $\frac{1}{2}m_\phi^2\phi^2 \ll \frac{1}{4}\lambda_\sigma v^4$

Effective mass for waterfall:

$$m_{\sigma,\mathrm{eff}}^2 = \lambda_{\phi\sigma}\phi^2 - \frac{1}{2}\lambda_\sigma v^2$$

$m_{\sigma,\mathrm{eff}}^2 > 0:$ inflation

$m_{\sigma,\mathrm{eff}}^2 = 0:$ end of inflation

- Waterfall field coupling to inflaton could spoil inflation.

$$\Delta V_{\mathrm{loop}} = \frac{1}{64\pi^2}\left[2m_{\sigma,\mathrm{eff}}^2 M_*^2 - m_{\sigma,\mathrm{eff}}^4 \ln\frac{M_*^2}{m_{\sigma,\mathrm{eff}}^2}\right]$$

$$\sim \frac{\lambda_{\phi\sigma}}{32\pi^2}M_*^2\phi^2 - \frac{\lambda_{\phi\sigma}^2}{64\pi^2}\phi^4\ln\frac{M_*^2}{\lambda_{\phi\sigma}\phi^2}\cdots$$

$M_*:$ cutoff scale

Sizable corrections to inflaton potential

Higgs inflation in sigma models

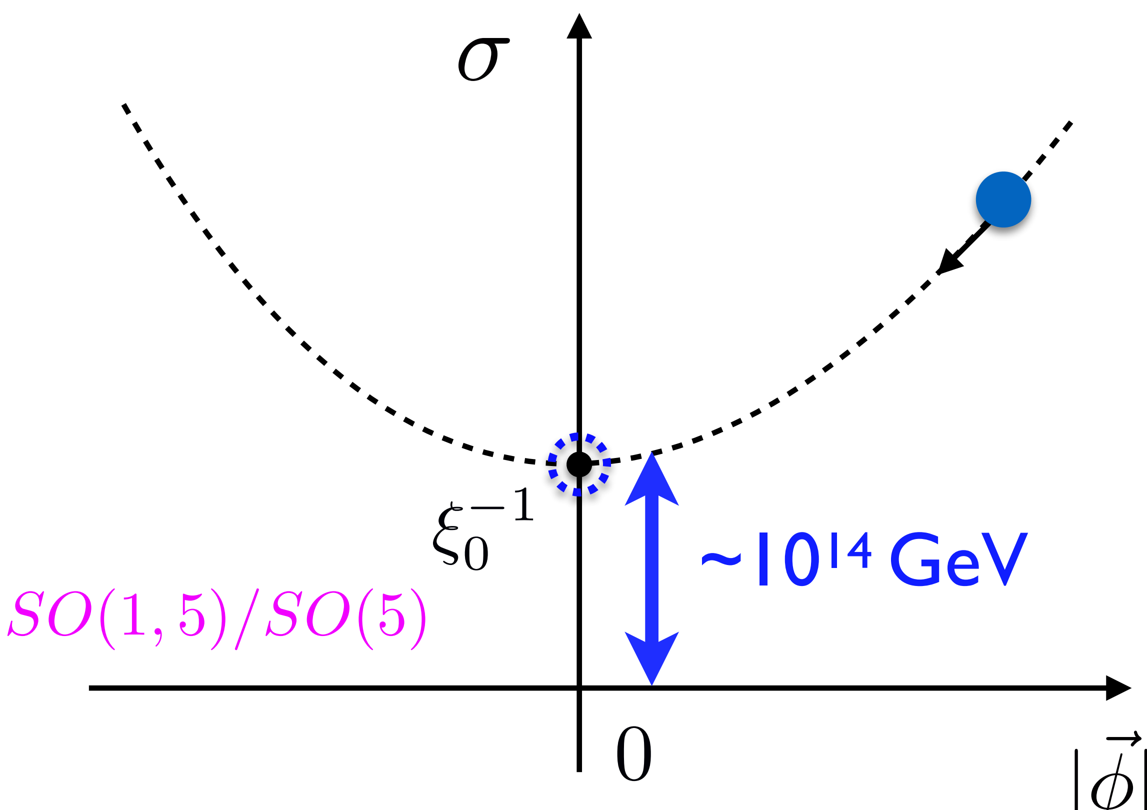
Higgs in induced gravity

- Integrating in the sigma field with "non-compact" field space, $\sigma^2 - \vec{\phi}^2 = \xi_0^{-1}$, [Giudice,HML (2010)]

$$\mathcal{L}_{\rm kin} = -\frac{1}{2\xi_0\sigma^2}\Big[(\partial_\mu\vec{\phi})^2 + 6\xi_0(\partial_\mu\sigma)^2\Big] : \ SO(1,5)/SO(5).$$

$\longleftrightarrow$ Induce gravity $\mathcal{L}_J = \sqrt{-g}\left[\xi_0\sigma^2\mathcal{R} - \frac{1}{2}(\partial_\mu\sigma)^2 - \lambda\Big(\sigma^2 - \vec{\phi}^2 - \xi_0^{-1}\Big)^2\right]$

$$\int D\psi\, e^{iS(g,\psi)} = e^{i\int d^4x\sqrt{-g}(M_P^2\mathcal{R}+\cdots)}$$ [Zee, Smolin (1979)]



1) SO(4)→SO(1,5)/SO(5)

2) Higgs inflation-like.

3) Heavy sigma saves vacuum instability.

$$m_\sigma^2 = 4\lambda H^2 \sim (10^{13}\,{\rm GeV})^2$$

Higgs in conformal frame

- Frame-independent Lagrangian for Higgs inflation.

$$\mathcal{L} = \sqrt{-\hat{g}}\Big[-\frac{1}{2}(1+\xi\hat{\phi}_i^2)\hat{R} + \frac{1}{2}g^{\mu\nu}\partial_\mu\hat{\phi}_i\partial_\nu\hat{\phi}_i - \frac{\lambda}{4}(\hat{\phi}_i^2)^2\Big]$$

Conformal mode: $\hat{g}_{\mu\nu} = e^{2\varphi}g_{\mu\nu}$

Field redefinition: $\phi_i = e^{\varphi}\hat{\phi}_i$ and $\Phi = \sqrt{6}\,e^{\varphi}$

- Fix the gauge for conformal mode: $\phi = \sqrt{6}$

$$\Rightarrow \quad \mathcal{L} = \sqrt{-g}\Big[-\frac{1}{2}\Big(1-\frac{1}{6}\phi_i^2 - \frac{1}{6}\sigma^2\Big)R + \frac{1}{2}(\partial_\mu\phi_i)^2 + \frac{1}{2}(\partial_\mu\sigma)^2 - \frac{\lambda}{4}(\phi_i^2)^2\Big]$$

Non-linear sigma model with $\underline{\Big(\sigma+\frac{\sqrt{6}}{2}\Big)^2 + 3\Big(\xi+\frac{1}{6}\Big)\phi_i^2 - \frac{3}{2} = 0}$

[Y. Ema et al (2020)]

Promote the constraint to a dynamical field with perturbativity

$\Rightarrow$ Unitarizing Higgs inflation in conformal frame.

Starobinsky in conformal frame

- Lagrangian for Starobinsky model + Higgs inflation:

$$\mathcal{L}_{R2}=\sqrt{-\hat{g}}\Big[-\frac{1}{2}(1+\xi\hat{\phi}_i^2)\hat{R}+\frac{1}{2}g^{\mu\nu}\partial_\mu\hat{\phi}_i\partial_\nu\hat{\phi}_i-\frac{\lambda}{4}(\hat{\phi}_i^2)^2+\alpha\hat{R}^2\Big]$$

Scalar-dual: $\alpha\hat{R}^2 \Longleftrightarrow -2\alpha\hat{\chi}\hat{R}-\alpha\hat{\chi}^2$

Conformal transform: $\hat{g}_{\mu\nu}=\Omega^{-2}g_{\mu\nu}$, $\hat{\phi}_i=\Omega\phi_i$ and $\hat{\chi}=\Omega^2\chi$

[Y. Ema et al (2020)] $\Omega^{-2}=\Big(1+\frac{\sigma}{\sqrt{6}}\Big)^2$

$\Rightarrow$ identical to linear sigma model: "dynamical scalar σ"

$$\frac{\mathcal{L}_{R2}}{\sqrt{-g}}=-\frac{1}{2}R\Big(1-\frac{1}{6}\phi_i^2-\frac{1}{6}\sigma^2\Big)+\frac{1}{2}(\partial_\mu\sigma)^2+\frac{1}{2}(\partial_\mu\phi_i)^2-\alpha\chi^2-\frac{\lambda}{4}\phi_i^4$$

$$\begin{cases}\text{constraint equation,} & 4\alpha\chi=\dfrac{1}{2}-\dfrac{1}{3}\Big(\sigma+\dfrac{\sqrt{6}}{2}\Big)^2-\Big(\xi+\dfrac{1}{6}\Big)\phi_i^2, \\ \text{sigma quartic coupling,} & \kappa=\dfrac{1}{36\alpha}\end{cases}$$

Generalization to F(R)

- General higher curvature terms + Higgs inflation

$$\sum_k \frac{2(-1)^{k+1}\alpha_k}{k+1}\hat{R}^{k+1} \Longleftrightarrow -2\sum_k \alpha_k \hat{\chi}_k \hat{R} \; -\sum_k 2\left(\frac{k}{k+1}\right)\alpha_k \hat{\chi}_k^{\frac{k+1}{k}}$$

A Lagrange multiplier per each term [HML, Menkara (2021)]

Conformal transform: $\hat{g}_{\mu\nu} = \Omega^{-2} g_{\mu\nu}$, $\hat{\phi}_i = \Omega\phi_i$ and $\hat{\chi}_k = \Omega^2 \chi_k$

General sigma-model Lagrangian:

$$\frac{\mathcal{L}_{\rm gen}}{\sqrt{-g}} = -\frac{1}{2}R\left(1-\frac{1}{6}\phi_i^2-\frac{1}{6}\sigma^2\right)+\frac{1}{2}(\partial_\mu\sigma)^2+\frac{1}{2}(\partial_\mu\phi_i)^2-\frac{\lambda}{4}\phi_i^4 \; -\sum_k \Omega^{-2+\frac{2}{k}}\left(\frac{2k}{k+1}\right)\alpha_k\,\chi_k^{1+\frac{1}{k}}$$

conformal-invariant

sigma potential

$$+\boxed{y(x)}\cdot\left[\sum_k 4\alpha_k\chi_k-\frac{1}{2}+\frac{1}{3}\left(\sigma+\frac{\sqrt{6}}{2}\right)^2+\left(\xi+\frac{1}{6}\right)\phi_i^2\right]$$

Lagrange multipler

F(R) on vacuum manifold

- Use equations of motion for χ_k & y. [HML, Menkara (2021)]

$$\frac{\delta\mathcal{L}_{\rm gen}}{\delta\chi_k}=0 \Longrightarrow \underline{\chi_k = 2^k\Omega^{2k-2}\,y^k}\,,\quad k=1,2,\cdots,N$$

$$\frac{\delta\mathcal{L}_{\rm gen}}{\delta y}=0 \Longrightarrow \underline{\sum_k 4\alpha_k\chi_k = \frac{1}{2}-\frac{1}{3}\Big(\sigma+\frac{\sqrt{6}}{2}\Big)^2-\Big(\xi+\frac{1}{6}\Big)\phi_i^2}$$

$$=\sum_k 4\alpha_k\,2^k\Omega^{2k-2}\,y^k$$

y : the solution to the N-th order algebraic equation,

- General sigma-model potential: a function of σ & ϕ_i

$$U(\sigma,\phi_i)=\sum_k\Omega^{-2+\frac{2}{k}}\Big(\frac{2k}{k+1}\Big)\alpha_k\,\chi_k^{1+\frac{1}{k}}=\sum_k\Big(\frac{2^{k+2}k}{k+1}\Big)\alpha_k(\Omega(\sigma))^{2k-2}(y(\sigma,\phi_i))^{k+1}$$

$$\frac{\partial U}{\partial\sigma}=0 \Longrightarrow y=0:\quad \Big(\sigma+\frac{\sqrt{6}}{2}\Big)^2+3\Big(\xi+\frac{1}{6}\Big)\phi_i^2-\frac{3}{2}=0$$

"Identical" vacuum manifold as for Starobinsky

Higgs-Starobinsky inflation

- Higgs-Starobinsky Lagrangian in Einstein frame

$$\mathcal{L}_E = \sqrt{-g_E}\Big\{-\frac{1}{2}R(g_E)+\frac{1}{2\Omega'^4}\Big[\Big(1-\frac{1}{6}\sigma^2\Big)(\partial_\mu h)^2+\Big(1-\frac{1}{6}h^2\Big)(\partial_\mu\sigma)^2+\frac{1}{3}h\,\sigma\,\partial_\mu h\partial^\mu\sigma\Big]-V(\sigma,h)\Big\}$$

$$V(\sigma,h) = \frac{1}{(1-\frac{1}{6}h^2-\frac{1}{6}\sigma^2)^2}\Big[\frac{1}{4}\kappa_1\Big(\sigma(\sigma+\sqrt{6})+3\Big(\xi+\frac{1}{6}\Big)h^2\Big)^2+\frac{1}{4}\lambda h^4\Big]$$

Sigma quartic coupling: $\kappa_1=\frac{1}{36\alpha_1}\lesssim\mathcal{O}(1) \longleftarrow \alpha_1\hat{R}^2$

Effective Higgs quartic coupling: $\lambda_{\rm eff}=\lambda+9\kappa_1\Big(\xi+\frac{1}{6}\Big)^2\lesssim\mathcal{O}(1)$

- Effective theory for inflation: [HML, Menkara (2021)]

$$\frac{\partial V}{\partial h}=0 \Rightarrow \frac{\mathcal{L}_{\rm eff}}{\sqrt{-g_E}}=-\frac{1}{2}R(g_E)+\underline{\frac{(\partial_\mu\sigma)^2}{2(1-\sigma^2/6)^2}-V_{\rm eff}(\sigma)},$$

$$V_{\rm eff}(\sigma)=9\lambda\,\kappa_1\sigma^2\Big[\lambda(\sigma-\sqrt{6})^2+\kappa_1\Big(\sigma-3\Big(\xi+\frac{1}{6}\Big)(\sigma-\sqrt{6})\Big)^2\Big]^{-1}$$

Limits of Higgs inflation

- Canonical inflaton: $\sigma = -\sqrt{6}\tanh\left(\frac{\chi}{\sqrt{6}}\right)$

$$V_{\rm eff}(\chi) = \frac{9\kappa_1}{4}\left(1-e^{-2\chi/\sqrt{6}}\right)^2\left[1+\frac{\kappa_1}{4\lambda}\left(6\xi+e^{-2\chi/\sqrt{6}}\right)^2\right]^{-1}$$

$$\approx \begin{cases} \boxed{\frac{9\kappa_1}{4}}\left(1-e^{-2\chi/\sqrt{6}}\right)^2, & 9\kappa_1\xi^2 \ll \lambda, \\ \boxed{\frac{\lambda}{4\xi^2}}\left(1-e^{-2\chi/\sqrt{6}}\right)^2, & 9\kappa_1\xi^2 \gg \lambda. \end{cases}$$

Starobinsky-like

Higgs-like

- Similar predictions as in Higgs or Starobinsky models.

CMB normalization: $\frac{\sqrt{\lambda+9\kappa_1\xi^2}}{\sqrt{\kappa_1\lambda}} = 1.5\times 10^5$

Spectral index: $n_s = 1-\frac{2}{N}-\frac{9}{2N^2}+\underline{\frac{3\kappa_1}{N^2}\frac{(-\lambda+12\lambda\xi+18\kappa_1\xi^2(1+6\xi))}{(2\lambda+3\kappa_1\xi(1+6\xi))^2}},$

small corrections $(\kappa_1\xi^2 \lesssim 1)$

Tensor-to-scalar: $r = \frac{12}{N^2}$

Reheating: Efficient for large ξ. $\longrightarrow$ S. Aoki's Talk!

Higgs inflation in Weyl gravity

Unitarizing with gauge field

- Non-canonical Higgs kinetic terms = Weyl current-current interactions

$$\frac{\mathcal{L}_{H,\mathrm{eff}}}{\sqrt{-g_E}} = \frac{3\xi_H^2}{M_P^2}\frac{(\partial_\mu|H|^2)^2}{\Omega^2} = \frac{1}{48M_P^2}\frac{K_\mu K^\mu}{\Omega^2}, \qquad K_\mu = \partial_\mu K_H \text{ with } K_H = 12\xi_H|H|^2.$$

- Introduce a Weyl gauge field in Jordan frame.

$$\frac{\mathcal{L}_J}{\sqrt{-g}} = -\frac{1}{4}w_{\mu\nu}w^{\mu\nu} + \frac{1}{2}m_w^2 w_\mu w^\mu - \frac{1}{2}g_w w_\mu K^\mu + \frac{1}{2}g_w^2 w_\mu w^\mu K_H, \qquad m_w^2 = 6g_w^2 M_P^2$$

Integrate out Weyl gauge field with $w_\mu = \frac{g_w}{2}\frac{K_\mu}{m_w^2 + g_w^2 K_H}$

$$\Rightarrow \frac{\mathcal{L}_{J,\mathrm{eff}}}{\sqrt{-g}} = -\frac{1}{48M_P^2}\frac{K_\mu K^\mu}{\Omega} \Rightarrow \boxed{\frac{\mathcal{L}_{E,\mathrm{eff}}}{\sqrt{-g_E}} = -\frac{1}{48M_P^2}\frac{K_\mu K^\mu}{\Omega^2}}$$

cancels non-canonical Higgs kinetic terms!

Higgs in Weyl gravity

- Weyl geometry

$\tilde{\nabla}_\rho g_{\mu\nu} = 0$, $\tilde{\Gamma}^\rho_{\mu\nu} = \Gamma^\rho_{\mu\nu} + g_w\Big(\delta^\rho_\mu w_\nu + \delta^\rho_\nu w_\mu - g_{\mu\nu} w^\rho\Big)$ in Weyl gravity

$\longrightarrow$ $\nabla_\rho g_{\mu\nu} = 2 g_w \omega_\rho g_{\mu\nu}$ in Einstein gravity

- Weyl invariant Lagrangian [D. Ghilencea, H. M. Lee (2018)]

Local scale transf: $g_{\mu\nu} \to e^{2\alpha} g_{\mu\nu}, \quad \phi \to e^{-\alpha}\phi, \quad H \to e^{-\alpha} H, \quad w_\mu \to w_\mu - \frac{1}{g_w}\partial_\mu \alpha$

$$\begin{aligned}\frac{\mathcal{L}}{\sqrt{-g}} &= -\frac{1}{2}(\xi_\phi \phi^2 + 2\xi_H |H|^2)\tilde{R}(\tilde{\Gamma}) + 3\xi_\phi r_\phi (D_\mu \phi)^2 + 6\xi_H r_H |D_\mu H|^2 \\ &\quad - \frac{1}{4} w_{\mu\nu} w^{\mu\nu} - V(H,\phi)\end{aligned}$$

$$D_\mu \phi_i = (\partial_\mu - g_w w_\mu)\phi_i$$

- Gauge-fixed Lagrangian: $\langle \phi^2 \rangle = M_P^2/\xi_\phi$ [S. Aoki, H. M. Lee (2022)]

$$\begin{aligned}\frac{\mathcal{L}}{\sqrt{-g}} &= -\frac{1}{2}(M_P^2 + 2\xi_H |H|^2)R + |\partial_\mu H|^2 - V(H) - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} \\ &\quad \underline{-\frac{1}{4} w_{\mu\nu} w^{\mu\nu} + \frac{1}{2} m_w^2 w_\mu w^\mu - \frac{1}{2} g_w w_\mu K^\mu + \frac{1}{2} g_w^2 w_\mu w^\mu K_H}\end{aligned}$$

$$\begin{cases} m_w^2 = 6 r_\phi g_w^2 M_P^2 \\ K_H = 12 r_H \xi_H |H|^2 \end{cases}$$

Unitarity in Weyl gravity

- Higgs kinetic terms in Einstein frame

$$\frac{\mathcal{L}_{\rm kin}}{\sqrt{-g_E}} = \underbrace{6\xi_H(r_H-1)}_{=1}\frac{|\partial_\mu H|^2}{\Omega} + \underline{\frac{3\xi_H^2}{M_P^2}\frac{(\partial_\mu|H|^2)^2}{\Omega^2}} - \underline{\frac{g_w^2 r_H^2}{8\Omega}\frac{K_\mu K^\mu}{m_w^2+g_w^2 r_H K_H}}$$

Higgs non-minimal coupling

Weyl gauge field

$$\frac{\mathcal{L}_{\rm kin}}{\sqrt{-g_E}} = \frac{1}{\Omega}|\partial_\mu H|^2 + \frac{1}{M_P^2\Omega^2}\frac{3r_\phi\xi_H^2 - 3(\xi_H+\frac{1}{6})^2 - \xi_H(\xi_H+\frac{1}{6})|H|^2/M_P^2}{r_\phi + 2(\xi_H+\frac{1}{6})|H|^2/M_P^2}(\partial_\mu|H|^2)^2$$

$$= |\partial_\mu H|^2 + \frac{1}{\Lambda^2}(\partial_\mu|H|^2)^2 + \cdots$$

[S. Aoki, H. M. Lee (2022)]

High cutoff: $\Lambda = \dfrac{M_P}{\left|\xi_H(3\xi_H+1)(1-\frac{1}{r_\phi}) - \frac{1}{12r_\phi}\right|^{1/2}} \xrightarrow{r_\phi = 1} \Lambda \sim M_P$

Inflation: $V_E = \dfrac{\lambda_H M_P^4}{4\xi_H^2}\left(1+\dfrac{M_P^2}{\xi_H h^2}\right)^{-2} \simeq \dfrac{\lambda_H M_P^4}{4\xi_H^2}\left(1+\dfrac{\xi_H\chi^2}{M_P^2}\right)^{-2}$, χ : canonical inflaton

But, Inflation would work only for small ξ_H and λ_H.

General Weyl gravity

- Extend with a pair of metrics and Weyl gauge fields:

$$\mathcal{L} = \sum_{i=1,2} \sqrt{-g_i}\Big[-\frac{1}{2}\xi_i\phi_i^2\tilde{R}(\tilde{\Gamma}_i) - \frac{1}{4}w_{i,\mu\nu}w_i^{\mu\nu}\Big] + \Delta\mathcal{L}\,,$$

$$\Delta\mathcal{L} = \sum_{i=1,2}\sqrt{-g_i}\,(-3\xi_i a_i\phi_i^2)\Big(g_{w_1}w_{1,\mu} + \kappa_i g_{w_2}w_{2,\mu}\Big)^2$$

Weyl symmetry: $g_{i,\mu\nu} \to e^{2\alpha_i}g_{i,\mu\nu}, \quad \phi_i \to e^{-\alpha_i}\phi_i, \quad w_{i,\mu} \to w_{i,\mu} - \frac{1}{g_{w_i}}\partial_\mu\alpha_i$

- Effective Weyl gravity: $g_{1,\mu\nu} = g_{2,\mu\nu} \equiv g_{\mu\nu}\,, \quad \phi_1 = \phi_2 = \phi$

$$\mathcal{L}_{\rm eff} = \sqrt{-g}\Big[-\frac{1}{2}\xi_\phi\phi^2\tilde{R}(\tilde{\Gamma}) - \frac{1}{4}w_{\mu\nu}w^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu}\Big]$$

[S. Aoki, H. M. Lee(2022)]

$$\tilde{\Gamma}^\rho_{\mu\nu} = \Gamma^\rho_{\mu\nu} + g_w\Big(\delta^\rho_\mu w_\nu + \delta^\rho_\nu w_\mu - g_{\mu\nu}w^\rho\Big), \quad g_w = \tfrac{1}{2}\sqrt{g_{w_1}^2 + g_{w_2}^2},$$

Two linear combinations of Weyl fields:

$w_\mu = (g_{w_1}w_{1,\mu} + g_{w_2}w_{2,\mu})/\sqrt{g_{w_1}^2 + g_{w_2}^2}\,, \quad X_\mu:$ orthogonal.

Higgs in general Weyl gravity

- Introduce an extra covariant kinetic term for Higgs:

$$\frac{\mathcal{L}_2}{\sqrt{-g}} = -\frac{1}{2}(\xi_\phi \phi^2 + 2\xi_H |H|^2)\tilde{R}(\tilde{\Gamma}) - \frac{1}{4}w_{\mu\nu}w^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} + \underline{|D'_\mu H|^2} - V(H,\phi)$$

Weyl symmetry: $g_{\mu\nu} \to e^{2\alpha} g_{\mu\nu}, \quad \phi \to e^{-\alpha}\phi, \quad H \to e^{-\alpha} H,$

$$D'_\mu H = (\partial_\mu - g_X X_\mu)H \qquad w_\mu \to w_\mu - \frac{1}{g_w}\partial_\mu \alpha, \quad X_\mu \to X_\mu - \frac{1}{g_X}\partial_\mu \alpha$$

Gauge-fixed Lagrangian: $m_w^2 = 6 g_w^2 M_P^2, \quad K_\mu = 12\xi_H \partial_\mu |H|^2$

$$\Rightarrow \frac{\mathcal{L}_2}{\sqrt{-g}} = -\frac{1}{2}(M_P^2 + 2\xi_H |H|^2)R + |D'_\mu H|^2 - V(H) - \frac{1}{4}X_{\mu\nu}X^{\mu\nu}$$
$$\underline{-\frac{1}{4}w_{\mu\nu}w^{\mu\nu} + \frac{1}{2}m_w^2 w_\mu w^\mu - \frac{1}{2}g_w w_\mu K^\mu + \frac{1}{2}g_w^2 w_\mu w^\mu K_H}$$

Decoupled Weyl photon w: $\frac{\mathcal{L}_E}{\sqrt{-g_E}} = -\frac{M_P^2}{2}R + \frac{|D'_\mu H|^2}{\Omega} - \frac{V(H)}{\Omega^2} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu}$

$\longleftrightarrow$ Palatini formulation, but differ by light Weyl photon X.

Metric to Palatini Higgs

- General Weyl-invariant Lagrangian:

$$\frac{\mathcal{L}_G}{\sqrt{-g}} = -\frac{1}{2}(\xi_\phi \phi^2 + \xi_H |H|^2)\tilde{R} + \underline{3\xi_\phi(r_\phi - 1)(D_\mu\phi)^2 + 6\xi_H(r_H - 1)|D_\mu H|^2}$$
$$-\frac{1}{4}w_{\mu\nu}w^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} + (1 - 6\xi_H(r_H - 1))|D'_\mu H|^2 - V(H,\phi).$$

Cutoff scale: $\Lambda_1 = \frac{M_P}{\left|3\xi_H^2\left(1 - \frac{r_H^2}{r_\phi}\right) + \xi_H\right|^{1/2}}$

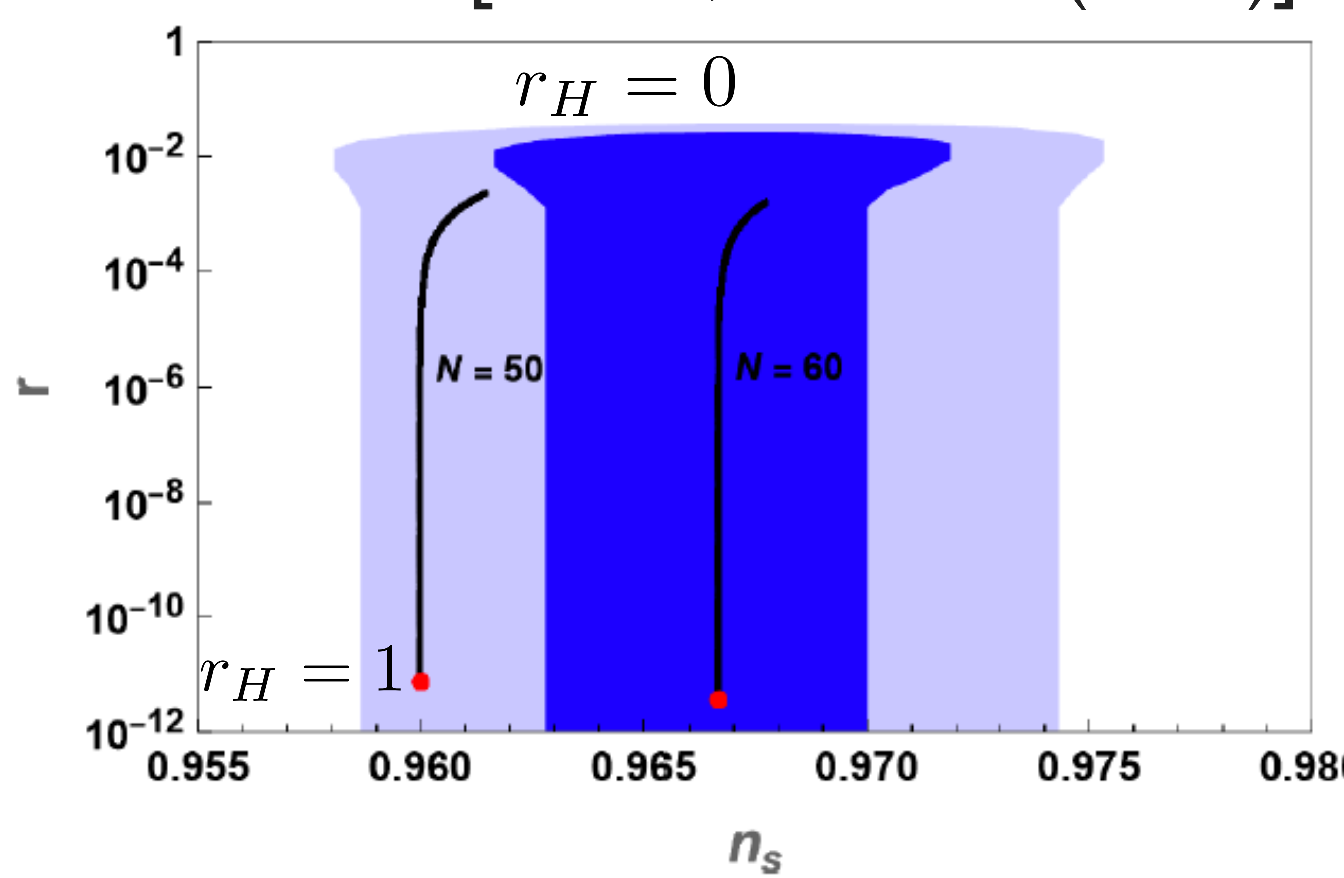
[S. Aoki, H. M. Lee (2022)]

- Higgs inflation can be interpolated from metric to Palatini cases.

  e.g. for $0 < r_\phi = r_H < 1$

  High cutoff scale

  ➡ very small r



Higgs inflation in supergravity

Higgs inflation in supergravity

- Non-minimal couplings in Jordan supergravity.

$$\mathcal{L}_J = \bar{X}^0 X^0 \Omega(z,\bar{z})|_D + (X^0)^3 W(z)|_F + \text{h.c.},$$

Frame function: $\Omega(z,\bar{z}) = -3 + \bar{z}^{\bar{I}} z^I + \underline{J(z) + \bar{J}(\bar{z})} = -3e^{-K/3}$

Superpotential: $W(z) = a_I z^I + m_{IJ} z^I z^J + d_{IJK} z^I z^J z^K$

- NMSSM: Higgs sector $\longrightarrow$ $z^\alpha = \{S, H_u, H_d\}$

$$\Omega = -3 + |S|^2 + |H_u|^2 + |H_d|^2 + \underbrace{\left(\frac{3}{2}\chi H_u \cdot H_d + \text{h.c.}\right)}_{\text{Non-minimal coupling}}$$

[D.R.T. Jones et al (2009); Ferrara et al (2010); HML (2010)]

$$W = \lambda S H_u \cdot H_d + \frac{\rho}{3} S^3 \quad \Rightarrow \quad V_H = \lambda^2 |H_u \cdot H_d|^2$$

Higgs potential

Higgs-Starobinsky in supergravity

- Frame function:

$$\Omega = \underbrace{-3 + |S|^2 + |H_u|^2 + |H_d|^2 + \left(\frac{3}{2}\chi H_u \cdot H_d + \text{h.c.}\right)}_{\text{NMSSM}} \underbrace{+ |C|^2 - (T + \bar{T})}_{\text{Dual scalars}}$$

[Aoki, HML, Menkara (2021)]

- Superpotential:

$$W = \underbrace{\lambda S H_u \cdot H_d + \frac{\rho}{3} S^3}_{\text{NMSSM}} \quad \underbrace{+ \frac{1}{\sqrt{\alpha}} T C}_{\text{Dual scalars}}$$

Jordan frame

- Jordan-frame Lagrangian: $X^0=1$ [Aoki, HML, Menkara (2021)]

$$\mathcal{L}_J/\sqrt{-g} = -\frac{1}{6}\Omega R - \Omega_{i\bar{j}}\partial_\mu z^i \partial^\mu \bar{z}^{\bar{j}} + \Omega \mathcal{A}_\mu^2 - V_J, \qquad z^i = \{S, H_u, H_d, C\}$$

$$V_J = (X^0)^4 \left[\delta^{i\bar{j}} W_i \bar{W}_{\bar{j}} + \left(W_i \delta^{i\bar{j}} \Omega_{\bar{j}} \bar{W}_{\bar{T}} - 3W_T \bar{W} + \text{c.c}\right) - \left(\Omega - \delta^{i\bar{j}}\Omega_i \Omega_{\bar{j}}\right)|W_T|^2\right]$$

$$\Rightarrow \quad \mathcal{L}_J/\sqrt{-g} = \left\{\frac{1}{2} - \frac{1}{6}|S|^2 - \frac{1}{6}|H_u|^2 - \frac{1}{6}|H_d|^2 - \frac{1}{6}|C|^2 + \underline{\left(-\frac{1}{4}\chi H_u\cdot H_d + \text{h.c.}\right)} + \frac{1}{3}\text{Re}T\right\} R$$

NMSSM

conformal symmetry, unitarity not manifest

$$- |\partial_\mu S|^2 - |\partial_\mu H_u|^2 - |\partial_\mu H_d|^2 - |\partial_\mu C|^2 + \Omega \mathcal{A}_\mu^2 - V_J$$

$$\begin{aligned} V_J =& \left|\lambda H_u\cdot H_d + \rho S^2\right|^2 + \lambda^2 |S|^2 (|H_u|^2 + |H_d|^2) + \underline{\frac{1}{\alpha}|T|^2} \\ &+ \frac{3}{2}\frac{\chi\lambda}{\sqrt{\alpha}}(S\bar{C} + \bar{S}C)(|H_u|^2 + |H_d|^2) \\ &+ \frac{1}{\alpha}|C|^2 \left\{3 + \frac{3}{2}\chi(H_u\cdot H_d + \text{c.c.}) + \frac{9}{4}\chi^2(|H_u|^2+|H_d|^2) - 2\text{Re}T\right\} \end{aligned}$$

dual-scalar mass

Sigma-model frame

- Linear sigma model Lagrangian:

[Aoki, HML, Menkara (2021)]

$$\hat{z}^i \equiv X^0 z^i, \quad \hat{T} \equiv (X^0)^2 T, \qquad z^i = \{S, H_u, H_d, C\}$$

$$|X^0|^2 \Omega(z^I, \bar{z}^{\bar{J}}) = \frac{1}{2} - \frac{1}{6}|\hat{z}^i|^2 - \frac{1}{12}\sigma^2, \qquad X^0 = 1 + \tfrac{1}{\sqrt{6}}\sigma$$

$$\mathcal{L}_{LS}/\sqrt{-g} = \frac{1}{2}\left(1 - \underline{\frac{1}{3}|\hat{S}|^2 - \frac{1}{3}|\hat{H}_u|^2 - \frac{1}{3}|\hat{H}_d|^2 - \frac{1}{3}|\hat{C}|^2 - \frac{1}{6}\sigma^2}\right) R$$

conformal-invariant, unitarity

$$- |\partial_\mu \hat{S}|^2 - |\partial_\mu \hat{H}_u|^2 - |\partial_\mu \hat{H}_d|^2 - |\partial_\mu \hat{C}|^2 - \frac{1}{2}(\partial_\mu \sigma)^2 + \Omega \mathcal{A}_\mu^2 - V_{LS},$$

$$\begin{aligned} V^F_{LS} = & |\lambda \hat{H}_u \cdot \hat{H}_d + \rho \hat{S}^2|^2 + \lambda^2 |\hat{S}|^2 (|\hat{H}_u|^2 + |\hat{H}_d|^2) \\ & + \underline{\frac{1}{4\alpha}\left(\sigma^2 + \sqrt{6}\sigma - \left(\frac{3}{2}\chi \hat{H}_u \cdot \hat{H}_d + \text{h.c.}\right)\right)^2} \\ & + \frac{1}{\alpha}(\mathrm{Im}\hat{T})^2 + \frac{3}{2}\frac{\chi\lambda}{\sqrt{\alpha}}(\hat{S}\bar{\hat{C}} + \bar{\hat{S}}\hat{C})(|\hat{H}_u|^2 + |\hat{H}_d|^2) \\ & + \frac{1}{\alpha}|\hat{C}|^2 \left\{3 + 2\sqrt{6}\sigma + \frac{3}{2}\sigma^2 + \frac{9}{4}\chi^2(|\hat{H}_u|^2 + |\hat{H}_d|^2)\right\} \end{aligned}$$

UV sigma potential

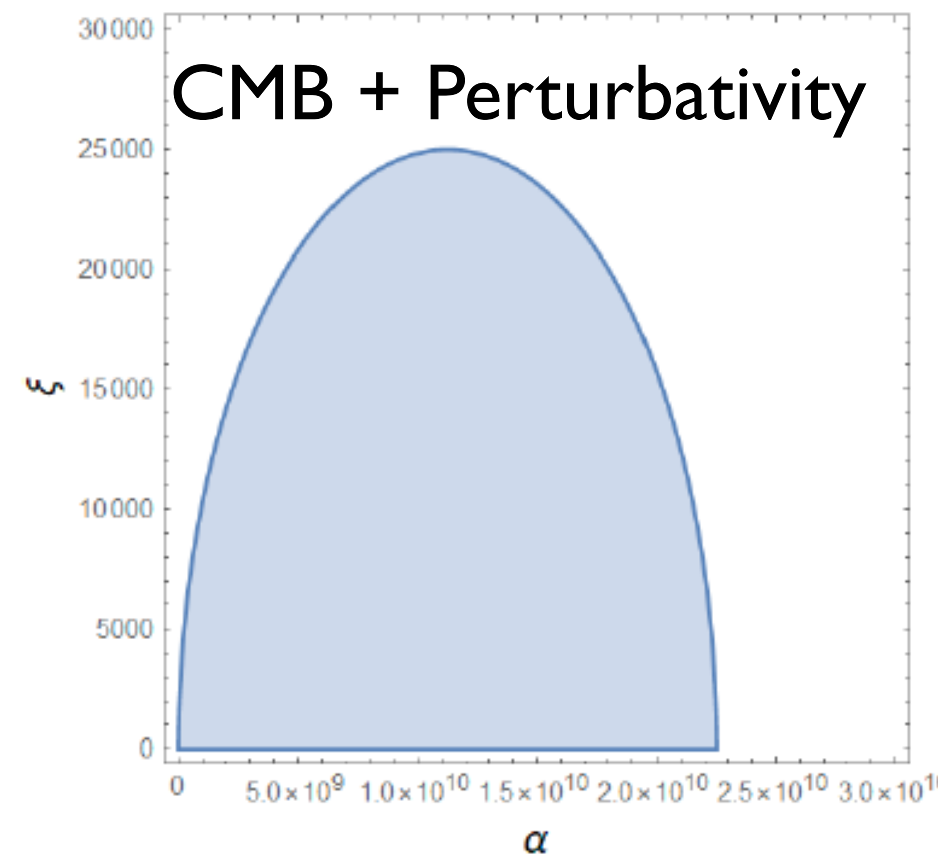
$$\chi^2/\alpha \lesssim 1$$

Stability of inflation

- Inflationary trajectory:

$$H_u^0=\frac{1}{\sqrt{2}}h\cos\beta\,e^{i\delta_1},\quad H_d^0=\frac{1}{\sqrt{2}}h\sin\beta\,e^{i\delta_2}\ \neq 0,$$

$$\mathrm{Re}T\neq 0,\qquad \text{others}=0$$



→ "non-SUSY" Higgs-Starobinsky inflation is recovered;

Effective non-minimal coupling, $\xi\equiv-\frac{1}{6}+\frac{\chi}{4}$

- All the scalar directions are decoupled;

[Aoki, HML, Menkara (2021)]

$S\,\&\,C$ fields are tachyonic but they are stabilized with

$$\Delta\Omega=-\zeta_s|S|^4-\zeta_c|C|^4-\zeta_{sc}|S|^2|C|^2\,,\quad \zeta_s,\zeta_c\gtrsim\zeta_{sc}$$

SUSY breaking

- SUSY is restored at the minimum after inflation: $z^I = 0$
- Dual superfields could lead to SUSY breaking/mediation.

a) Higher curvature terms: $\Omega = -3 + (T+\bar{T}) + |C|^2 \underline{- \gamma_c (C+\bar{C}) - \zeta_c |C|^4}$

$\langle C\rangle, \langle T\rangle \neq 0 \quad \Rightarrow \quad m_{3/2} \sim M_P/\sqrt{\alpha} \gtrsim 10^{13}\,\mathrm{GeV}$

$F_C, F_T \neq 0$

High-scale SUSY

b) O'Raifeartaigh model: Φ [Aoki, HML, Menkara (2021)]

$$\Omega = -3 - (T+\bar{T}) + |C|^2 + |\Phi|^2, \qquad W = \frac{1}{\sqrt{\alpha}} TC \underline{+ \kappa\,\Phi + g\,\Phi C^2}$$

$\langle C\rangle = \langle T\rangle = 0 \quad \Rightarrow \quad m_{3/2} = |F_\Phi|/(\sqrt{3}M_P) = \kappa/(\sqrt{3}M_P)$

$F_\Phi = \kappa$

Low-scale SUSY

Visible soft masses

- Effective Higgsino mass: $W_{\rm eff} = \mu H_u H_d$

Naturalness measure for EWSB: $$m_Z^2 = \frac{|m_{H_d}^2 - m_{H_u}^2|}{\sqrt{1-\sin^2(2\beta)}} - m_{H_u}^2 - m_{H_d}^2 - 2|\mu|^2$$

$$\mu = \lambda\langle\tilde{S}\rangle + \frac{3}{2}\chi m_{3/2} - \frac{1}{2}\chi K_{\bar{I}}\bar{F}^{\bar{I}}$$

[HML (2010); Aoki, HML, Menkara (2021)]

NMSSM

"Non-minimal coupling" contributes via Giudice-Masiero mechanism.

- Frame function must be extended by contact terms.

$$\Omega = -3 - (T+\bar{T}) + \bar{C}C + \bar{\Phi}\Phi + \bar{z}^{\bar{\alpha}} z^{\alpha} \quad \Rightarrow \quad m^2_{\bar{\alpha}\alpha} = 0$$

hidden visible

$$\Omega_{\rm contact} = C_{\bar{\alpha}\beta} X^\dagger X z^\dagger_{\bar{\alpha}} z_\beta + {\rm c.c}, \quad X = C, \Phi, \quad \Rightarrow \quad m^2_{\bar{\alpha}\beta} \sim C_{\bar{\alpha}\beta}|F_X|^2$$

soft masses

Natural hybrid inflation

Natural hybrid inflation

- Inflaton is regarded as a pseudo Goldstone boson.

$$V(\phi) = \Lambda^4 \cos\left(\frac{\phi}{f}\right)$$: protected by shift symmetry

$$\phi \to \phi + 2\pi f$$

Waterfall coupling respects shift symmetry:

$$V_W = V_0 + \frac{1}{2}m_\chi^2\chi^2 + \frac{1}{4}\lambda_\chi\chi^4 - \frac{1}{2}\mu^2 \sin\left(\frac{\phi}{2f}\right)\chi^2$$

[K. Deshpande, S. Kumar, R. Sundrum(2021); J.-O. Gong, K.S. Jeong(2021)]

$\Rightarrow$ Hybrid inflation condition: $\Lambda^4 \ll V_0$

Effective waterfall mass: $m_{\chi,\text{eff}}^2 = m_\chi^2 - \mu^2 \sin\left(\frac{\phi}{2f}\right)$

- But, the loop corrections are UV sensitive:

$$\Delta V_{\text{loop}} = \frac{1}{64\pi^2}\left[2m_{\chi,\text{eff}}^2 M_*^2 - m_{\chi,\text{eff}}^4 \ln\frac{M_*^2}{m_{\chi,\text{eff}}^2}\right] \sim \boxed{\frac{1}{32\pi^2}\mu^2 M_*^2 \sin\left(\frac{\phi}{2f}\right)}$$

Inflation with twin waterfalls

- Twin waterfall fields with Z_2 parity: $\phi \longrightarrow -\phi, \qquad \chi_1 \longleftrightarrow \chi_2$

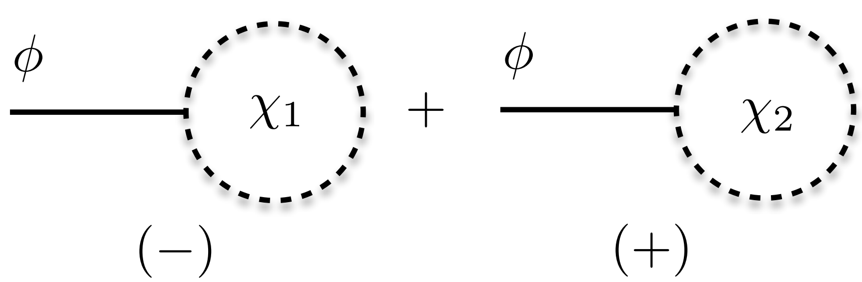
$$V_W = V_0 - \frac{1}{2}\mu^2 \sin\left(\frac{\phi}{2f}\right)(\chi_1^2 - \chi_2^2) + \frac{1}{2}m_\chi^2(\chi_1^2 + \chi_2^2) - \alpha^2\chi_1\chi_2$$
$$+\frac{1}{4}\lambda_\chi(\chi_1^4 + \chi_2^4) + \frac{1}{2}\bar{\lambda}_\chi\chi_1^2\chi_2^2$$

e.g. Z_2 invariant dark quarks in dark QCD

One combination of waterfalls becomes tachyonic!

$$m_1^2(\phi) = m_\chi^2 - \sqrt{\mu^4 \sin^2\left(\frac{\phi}{2f}\right) + \alpha^4}, \qquad m_2^2(\phi) = m_\chi^2 + \sqrt{\mu^2 \sin^2\left(\frac{\phi}{2f}\right) + \alpha^4}$$

- Inflaton potential is less sensitive to UV physics.



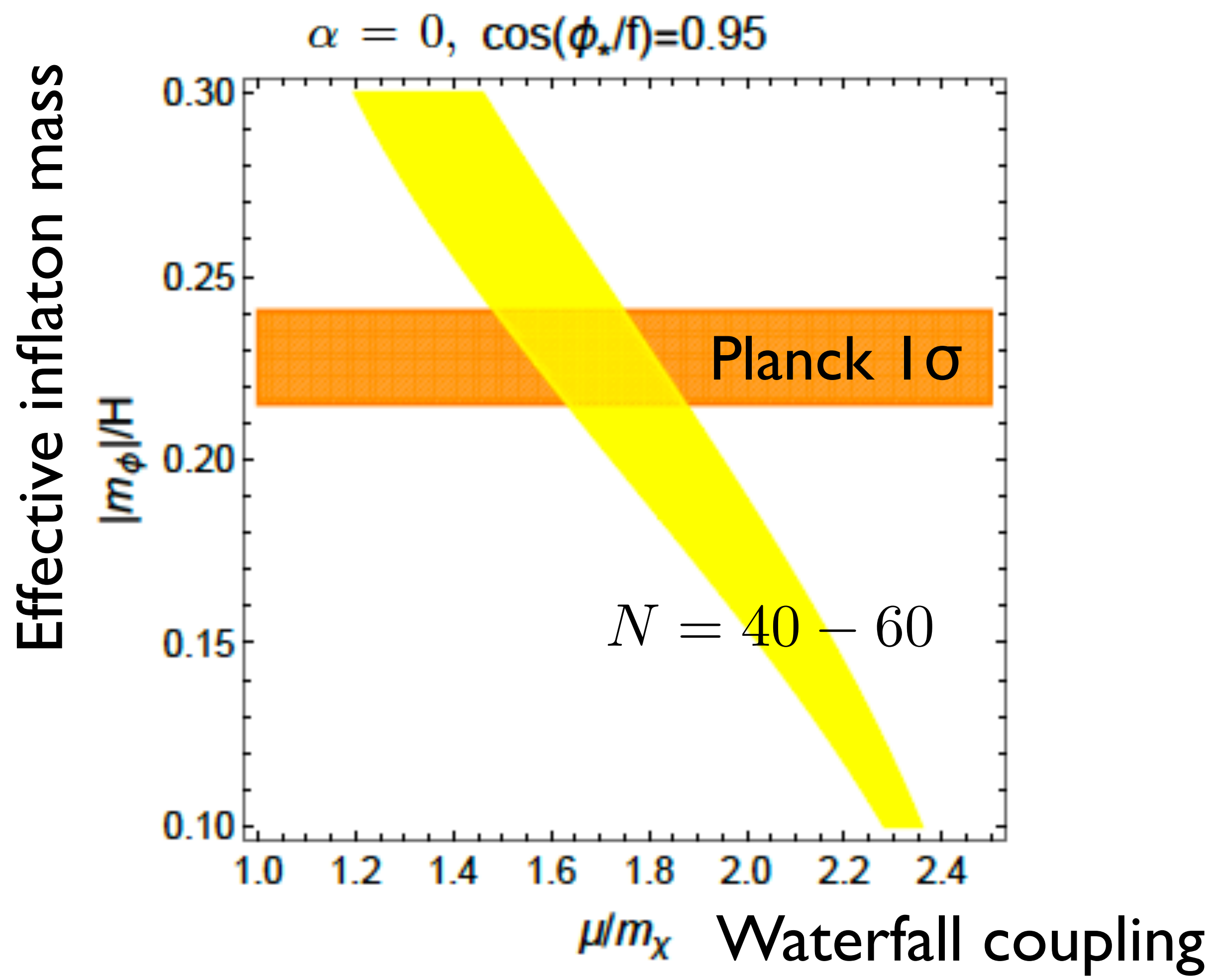
$$\Delta V_{\rm loop} = -\frac{1}{64\pi^2}\left[m_\chi^4 + \mu^4 \sin^2\left(\frac{\phi}{2f}\right) + \alpha^4\right] \ln\frac{M_*^2}{m_\chi^2}$$

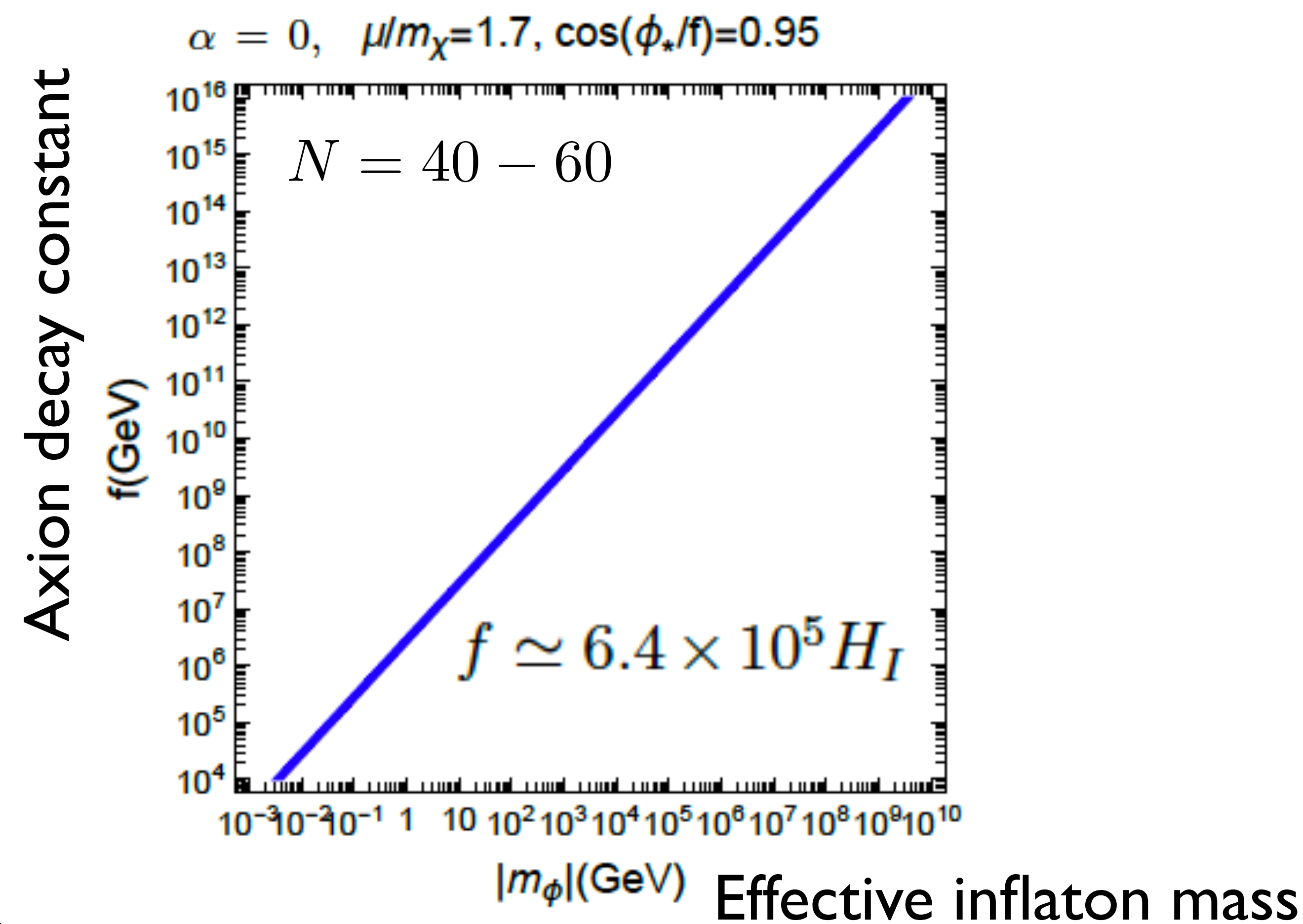
[K. Deshpande, S. Kumar, R. Sundrum(2021); H.M. Lee, A. Menkara(2022)]

Natural hierarchy of scales: $\sqrt{|m_\phi^2|} \ll H_I \ll \mu \sim m_\chi \ll \sqrt{8\pi}\Lambda \ll f$

Inflationary predictions

- CMB-compatible parameter space: [H.M. Lee, A. Menkara(2022)]





$$n_s \simeq 1 + 2\eta_* \simeq 1 - \frac{|m_\phi^2|}{3H^2}\cos(\phi_*/f), \quad |m_\phi^2| = \frac{\Lambda^4}{f^2}$$

$$N = \frac{\cos(\phi_*/f)}{|\eta_*|}\ln\left(\frac{\tan(\phi_c/(2f))}{\tan(\phi_*/(2f))}\right),$$

$$\frac{H_I}{f} = 2.9\times10^{-4}\left|\eta_*\tan(\phi_*/f)\right|, \quad \sin\left(\frac{\phi_c}{2f}\right) = \frac{\sqrt{m_\chi^4 - \alpha^4}}{\mu^2}$$

$$r = 3.2\times10^7 \cdot \frac{V_0}{M_P^4} < 0.036$$

[Planck 95%]

$$\longrightarrow H_I < 4.6\times10^{13}\,\text{GeV}$$

Reheating

- Higgs-portal coupling to twin waterfalls:

$$\mathcal{L}_H = -\kappa_1(\chi_1^2+\chi_2^2)|H|^2 - \kappa_2\chi_1\chi_2|H|^2$$

- Perturbative reheating by waterfall:

Waterfall decays after inflation $\longrightarrow$ Reheating temperature:

$$\chi_1 \to hh: \quad \Gamma_{\chi_1} = \frac{\kappa_1^2 v_\chi^2}{2\pi m_1}\sqrt{1-\frac{4m_H^2}{m_1^2}}$$

$$T_{\rm RH} \simeq \left(\frac{90}{\pi^2 g_{\rm RH}}\right)^{1/4}\left(\frac{\kappa_1^2}{4\pi\lambda_\chi}\right)^{1/2}\sqrt{M_P m_1}.$$

- Number of efoldings is sensitive to waterfall mass and T_{RH}.

$$N = 51.3 + \frac{1}{3}\ln\left(\frac{H_I}{1.6\times 10^{10}\,{\rm GeV}}\right) + \frac{1}{3}\ln\left(\frac{T_{\rm RH}}{10^{14}\,{\rm GeV}}\right)$$

$$f \lesssim 10^{16}\,{\rm GeV}\,,\ \kappa_1^2 \ll \lambda_\chi \quad \Longrightarrow \quad H_I \sim m_1 \lesssim 1.6\times 10^{10}\,{\rm GeV}, \quad T_R \ll 10^{14}\,{\rm GeV}$$

Conclusions

- Higgs inflation can be regarded as a non-linear sigma model in non-gravity theory or effective gravity theory with Weyl currents.

- We proposed a UV completion for Higgs inflation in linear sigma models and their supergravity embedding as well as in Weyl gravity.

- We also revisited natural hybrid inflation with twin waterfall fields, which is free from UV sensitivity and belongs to a different class of inflation.