Reheating and Dark Matter Freeze-in in the Higgs-R2 Inflation Model

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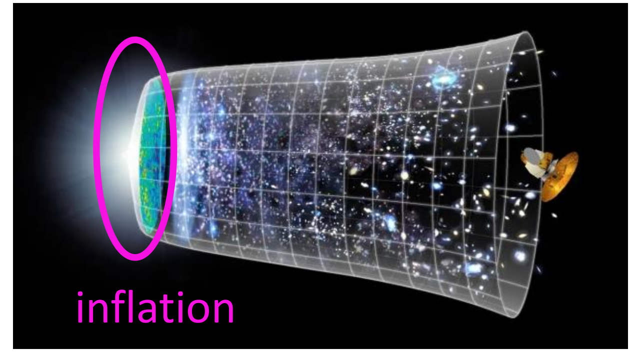
with Hyun Min Lee, Adriana G. Menkara, and Kimiko Yamashita

Based on 2202.13063

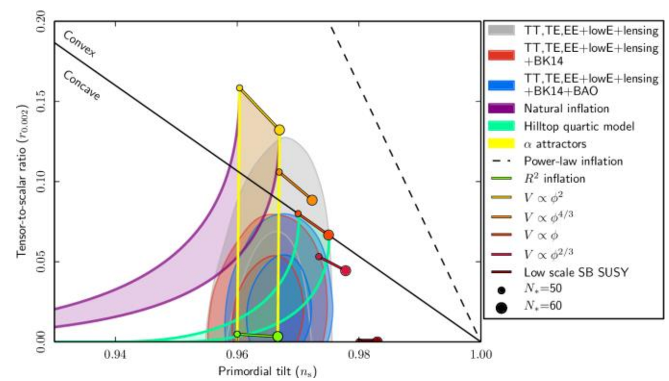
Workshop on Physics of Dark Cosmos
2022/10/21

Introduction

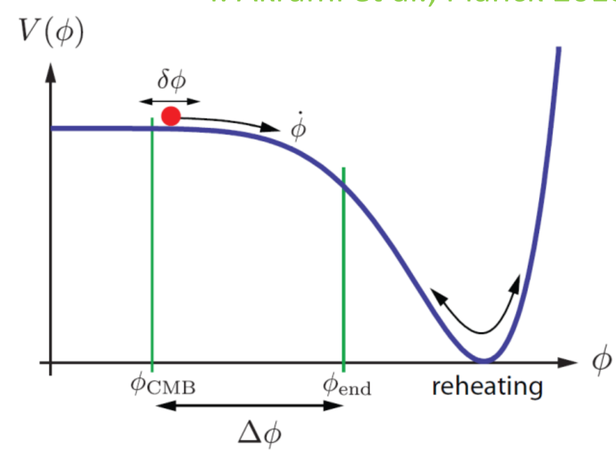


Inflation

- Rapid expansion of early universe
- theoretically & observationally established
- Slow roll inflation (scalar field : inflaton)



Y. Akrami et al., Planck 2018



arXiv:0907.5424

Natural idea = Higgs inflation

Original Higgs inflation

$$\mathcal{L} = \sqrt{-g}\Big(-\frac{1}{2}(M_P^2 + \underline{2\xi_H |H|^2})R + |D_\mu H|^2 - V(H)\Big)$$

F. L. Bezrukov, M. Shaposhnikov, '08

successful but requires a large $\xi(\sim 10^4)$
Low cutoff scale$\sim M_{pl}/\xi$ ⇒ unitarity?

Burgess, Lee, Trott, '09
Barbon, Espinosa, '09

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Recent development

- introduce a new d.o.f
- other formulations of GR (Palatini, Einstein-Cartan, …)
- Critical Higgs inflation

Higgs-R^2 inflation model

Salvio & Mazumdar '15, Ema '17, Gorbunov & Tokareva '18, Gundhi & Steinwachs '18, D. Y. Cheong & S. M. Lee & S. C. Park '21, SA, H. M. Lee, A. G. Menkara,'21...

$$\mathcal{L}/\sqrt{-g_J} = \frac{1}{2}(M_{\rm Pl}^2 + \xi \hat{h}^2)R_J - \frac{1}{2}(\partial_\mu \hat{h})^2 - \frac{\lambda}{4}\hat{h}^4 + \alpha R_J^2$$

- $R^2 \supset$ higher derivative of metric $\Rightarrow$ a new d.o.f (scalaron $= \sigma$-field)
- σ-field push up the cutoff to $\sim M_{pl}$ (solve unitarity problem)
- keep successful inflation (Higgs inflation + Starobinsky inflation)
- predictive

Short summary of Higgs-R^2 inflation

✓ Inflatons = $\{\sigma, h\}$.
But for large (positive) ξ, Higgs can be integrated out with non-trivial VEV.

✓ n_s & r : Perfect agreement with observation

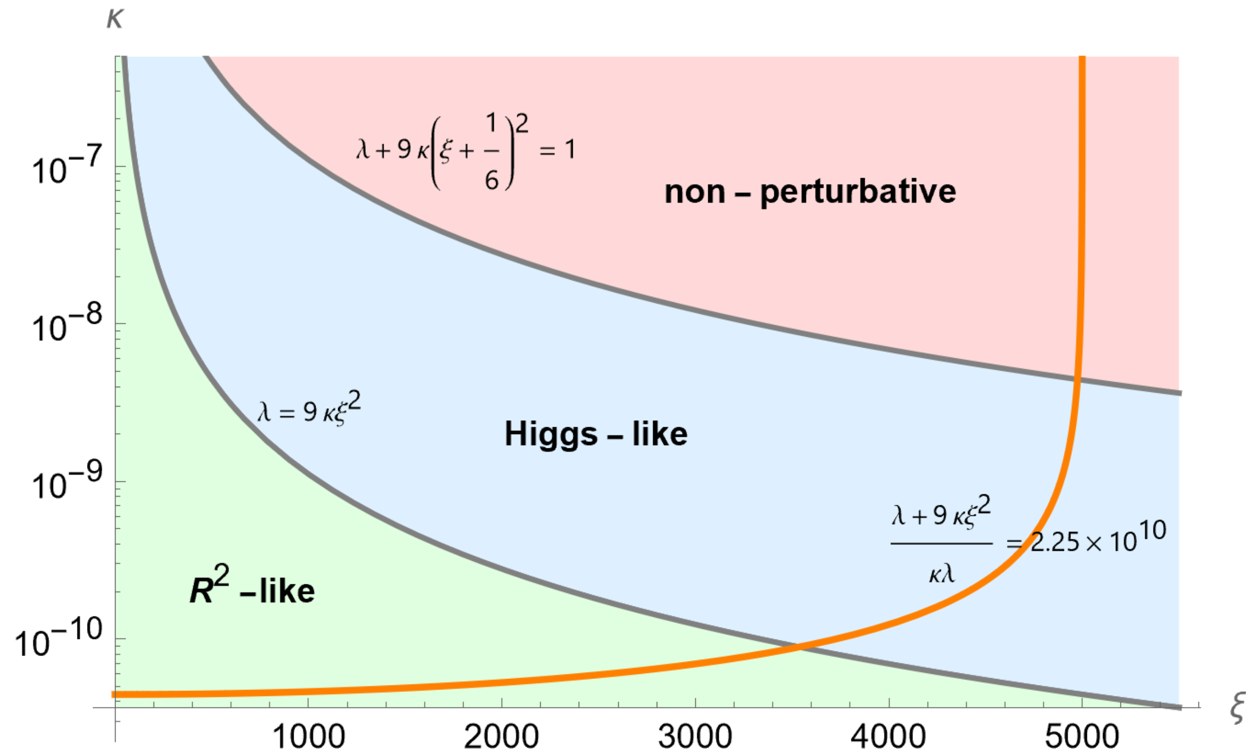
✓ CMB normalization:

$$\frac{\lambda + 9\kappa\left(\xi + \frac{1}{6}\right)^2}{\kappa\lambda} = 2.25 \times 10^{10}. \qquad \kappa \equiv 1/(36\alpha)$$

✓ interpolate Higgs and R^2 inflation (λ vs $\kappa\xi^2$)

Parameter space

$\lambda = 0.01$ fixed



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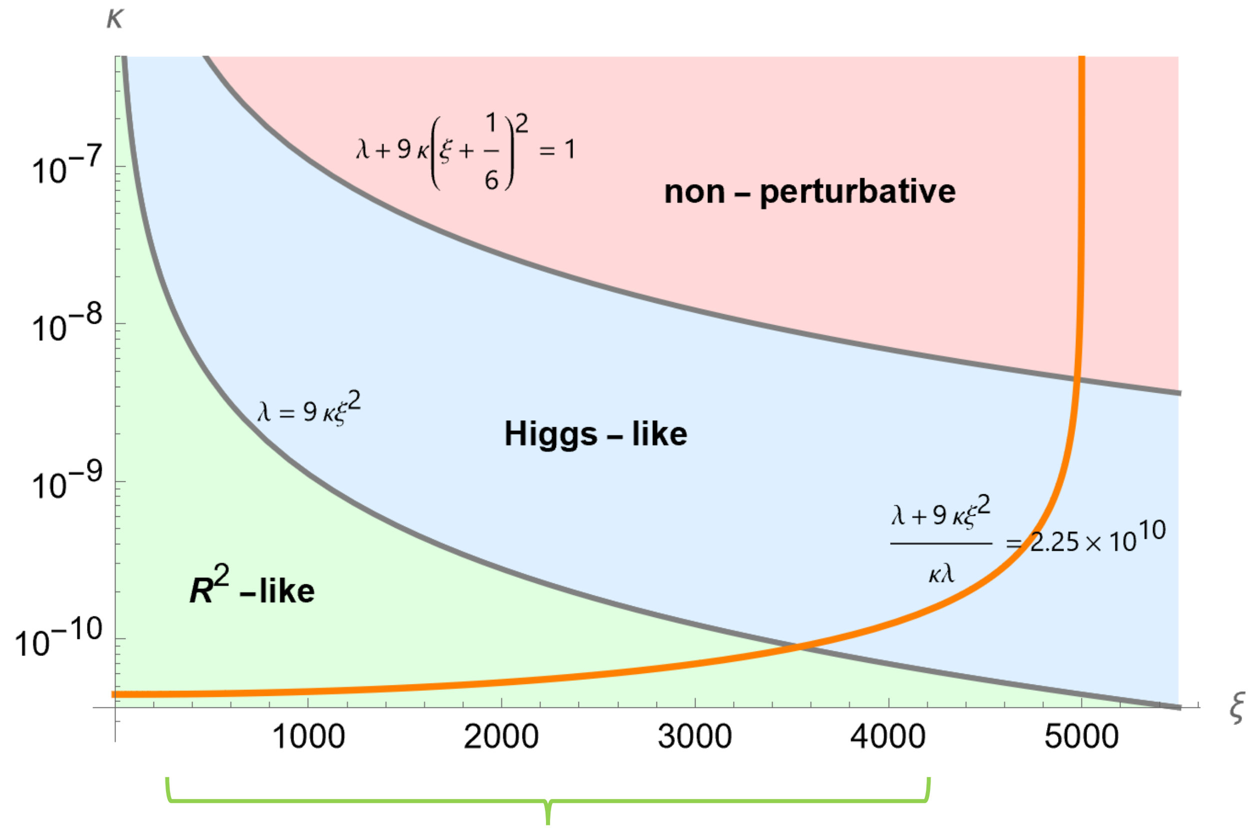


$100 \leq \xi \leq 4000$

Today

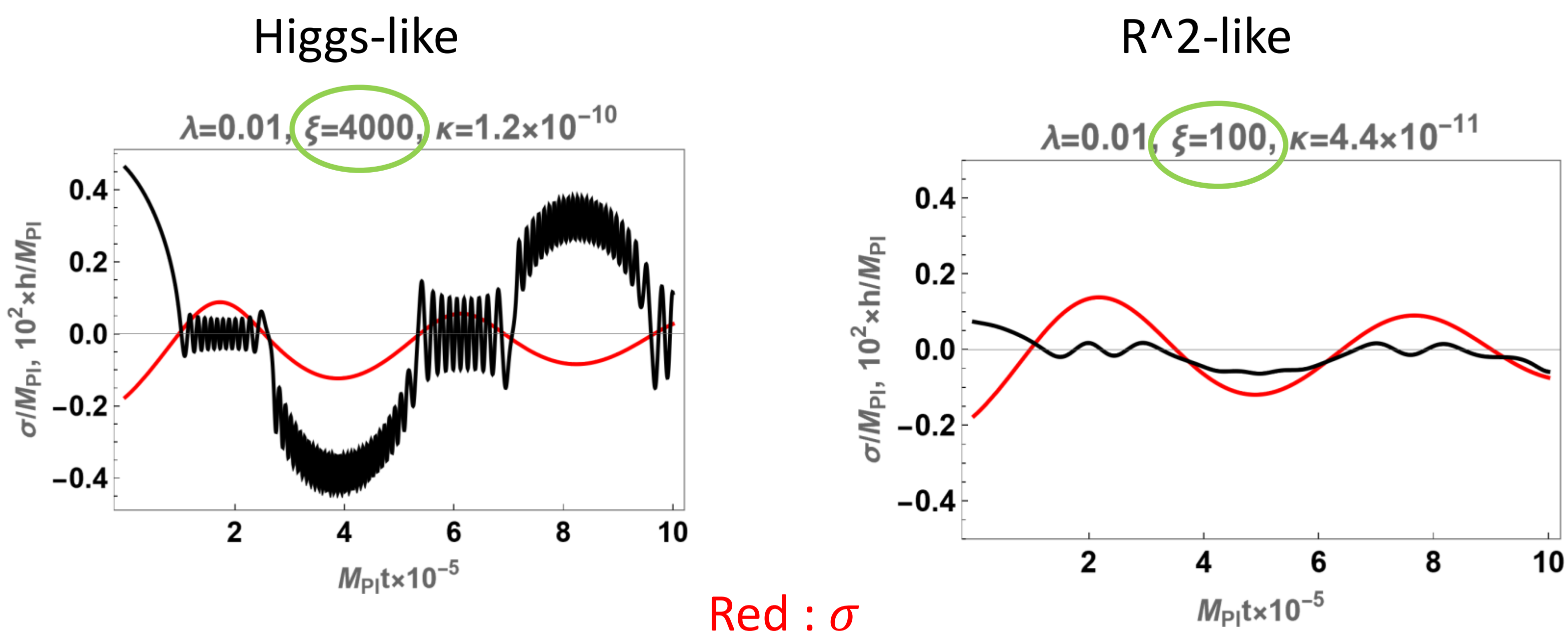
Dynamics after inflation = Reheating

Dark matter production during/after reheating

Reheating

Background dynamics after inflation

After inflation, h and σ start to oscillate



Red : σ

Black : h

Asymmetric for $\sigma > 0$ and $\sigma < 0$

Reheating

M. He, 2010.11717
SA, A.G.Menkara, H.M.Lee, K.Yamashita, 2202.13063

Decay process

✓ $h \to t + \bar{t}$ (open only for $\sigma < 0$) : dominant channel
✓ σ decay is almost blocked at early time

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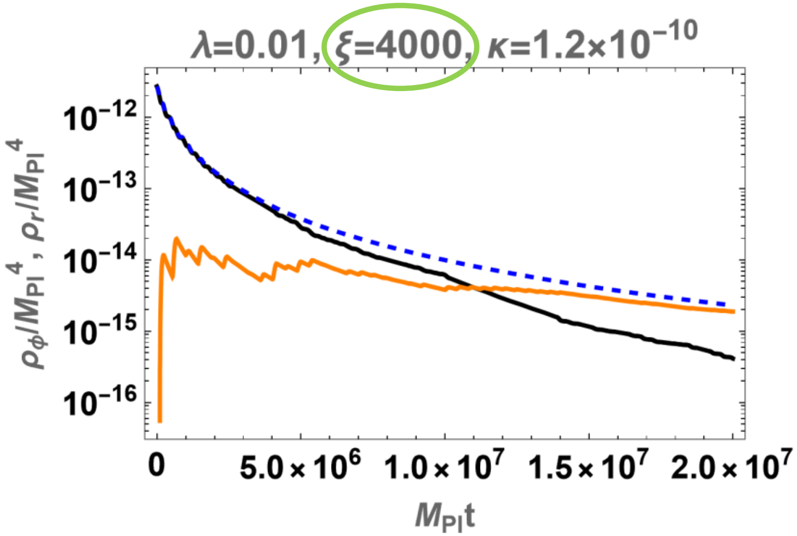
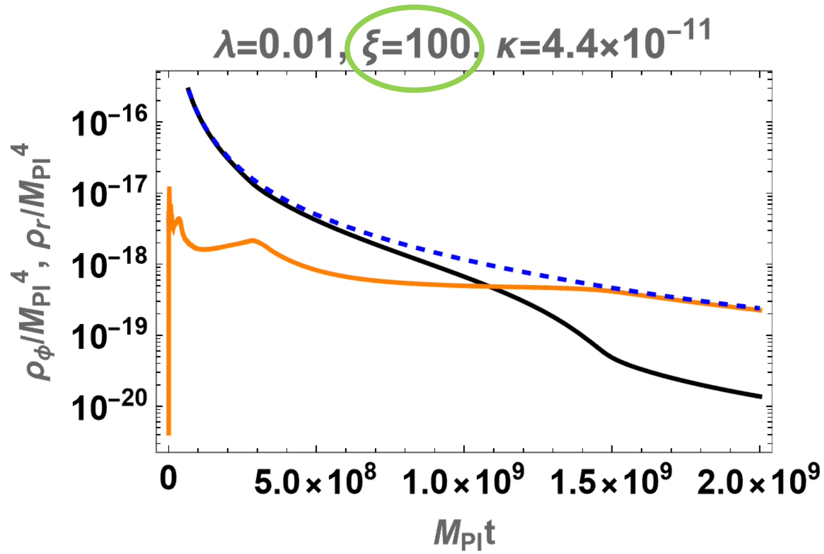
Numerical simulation

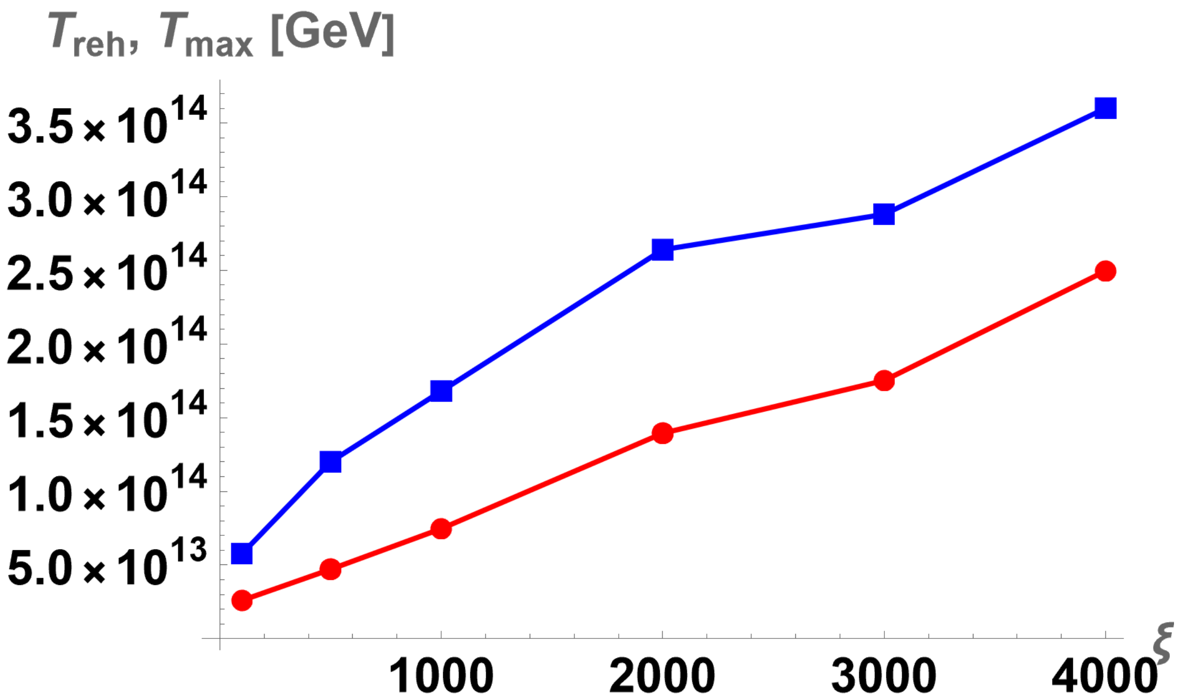




Black : $\rho_{\sigma+h}$
Orange : ρ_r
Blue dashed : Total

$\rho_r = \rho_{\sigma+h} \Rightarrow$ reheating completion
Small $\xi \Rightarrow$ delay reheating

Reheating and maximum temperature



Blue : $T_{\rm max}$
Red : $T_{\rm reh}$

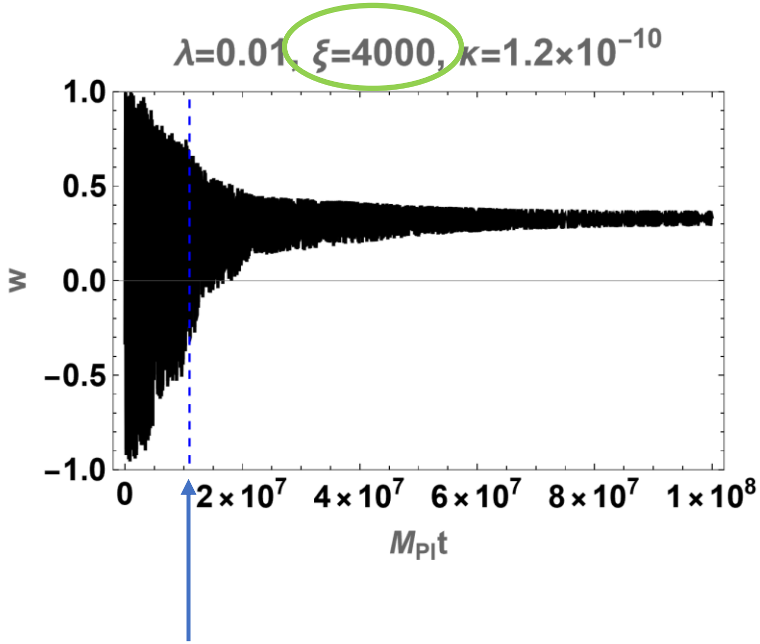
$\xi \downarrow \Rightarrow T_{\rm reh} \downarrow$

No significant difference in $T_{\rm reh}$ and $T_{\rm max}$ $\sim 10^{13-14}$ (GeV) for relatively large ξ

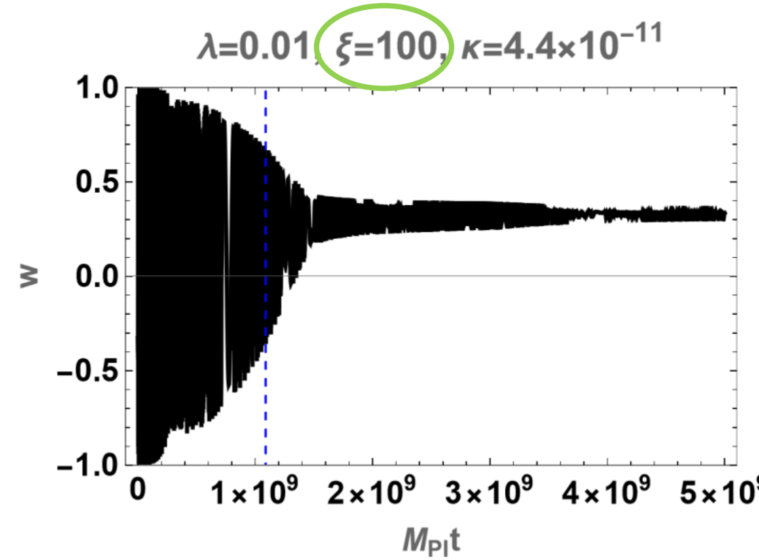
Equation of state

$$w = \frac{p_{\sigma+h} + p_r}{\rho_{\sigma+h} + \rho_r}$$

Higgs-like



R^2-like



reheating completion

$\langle\omega\rangle \sim 0$ during reheating (matter like)

$\omega \sim 1/3$ after reheating

Freeze-in Dark Matter

Motivation

- In addition to hot thermal plasma,
  Dark matter (DM) is also necessary ingredient

- FIMP (= feebly interacting massive particle) ⇔ WIMP

  Freeze-in ⇔ Freeze-out

  Reheating dynamics matters for the relic abundance

- Other mechanisms for DM, e.g., PBH → D.Y. Cheong's talk

Model (X : scalar DM with Z_2)

Jordan frame action :

$$\mathcal{L}/\sqrt{-g_J} = \frac{1}{2}(M_{\rm Pl}^2 + \xi \hat{h}^2 + \boxed{\eta \hat{X}^2}) R_J - \frac{1}{2}(\partial_\mu \hat{h})^2 - \frac{1}{2}(\partial_\mu \hat{X})^2 - \tilde{V}(\hat{h}, \hat{X}) + \alpha R_J^2 + \mathcal{L}_{\rm SM}$$

Non-minimal coupling

$$\tilde{V}(\hat{h}, \hat{X}) = \frac{\lambda}{4}\hat{h}^4 + \frac{m_X^2}{2}\hat{X}^2 + \frac{\lambda_X}{4}\hat{X}^4 + \boxed{\frac{\lambda_{hX}}{4}\hat{h}^2\hat{X}^2}$$

Higgs portal

$|\tilde{\eta}|$, $|\lambda_{hX}| \sim 0$ (feeble interactions)

$$\tilde{\eta} \equiv \eta + \frac{1}{6}.$$

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Higgs portal

$|\tilde{\eta}|$, $|\lambda_{hX}| \sim 0$ (feeble interactions)

$$\tilde{\eta} \equiv \eta + \frac{1}{6}.$$

Move to Einstein frame

Many Planck suppressed interactions (feeble interactions)

What to do

Solve Boltzmann equation for FIMP

$$\dot{n}_X + 3Hn_X = R(T),$$ n_X : DM number density

To solve this,

specify

- time (temperature) evolution of H
- $R(T)$: Reaction rate

Temperature evolution of H

universe evolves differently during and after RH

During RH

$$T \propto a^{-3/8}$$

$$\boxed{H \propto T^4}$$

After RH

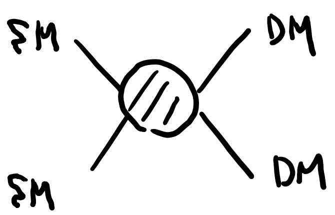
$$T \propto a^{-1}$$

$$\boxed{H \propto T^2}$$

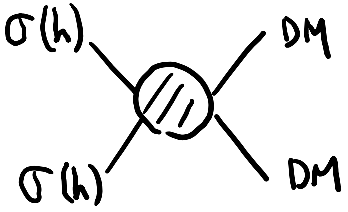
Reaction rate $R(T)$

✓ Two kinds of production mechanism

Thermal production = from radiation
Efficient both during and after RH



Non-thermal production = from inflaton
Efficient during RH



✓ In both cases, graviton exchange diagram is important

S. Clery, Y. Mambrini, K. A. Olive, S. Verner' 21

Summary $Y \equiv n_X T^{-3}$

- Thermal production after RH

$$Y_{\rm After\ RH} \simeq \frac{\sqrt{10}}{20480\pi^4 g_{\rm reh}^{1/2}} \frac{4m_X^4 + 45M_{\rm Pl}^4\left(\lambda_{hX} + 18\kappa\tilde{\eta}\tilde{\xi}\right)^2}{m_X M_{\rm Pl}^3} + \frac{209\sqrt{10}}{240\pi^6 g_{\rm reh}^{1/2}} \frac{T_{\rm reh}^3}{M_{\rm Pl}^3}$$

IR Freeze-in ≪ UV Freeze-in (for $\tilde{\eta} \sim \lambda_{hX} \sim 0$)

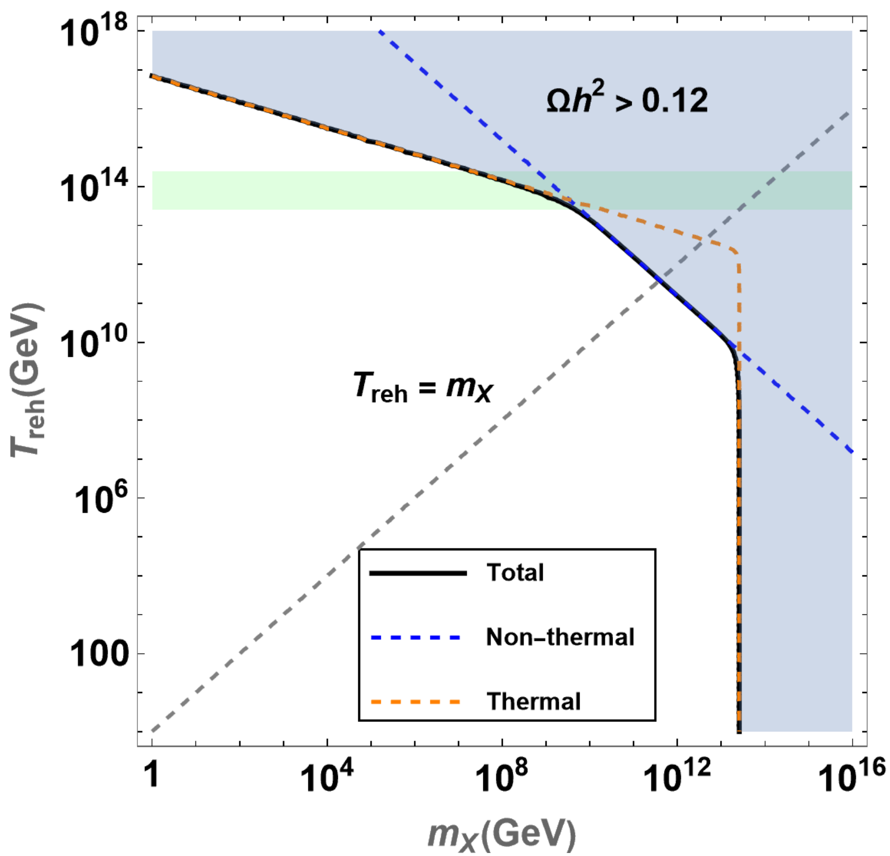
- Thermal production during RH

$$Y_{\rm During\ RH,\ thermal} \simeq \frac{69\sqrt{10}}{40\pi^6 g_{\rm reh}^{1/2}} \frac{T_{\rm reh}^3}{M_{\rm Pl}^3}$$

- Non-thermal production during RH

$$Y_{\rm During\ RH,\ non-thermal} \simeq \frac{\sqrt{3}\pi g_{\rm reh}}{2239488} \frac{T_{\rm reh}}{\kappa^2 M_{\rm Pl}^{11}} \frac{\rho_{\sigma,\rm end}^4}{\rho_{\rm end}^{3/2}}$$

Parameter space ($\tilde{\eta} = \lambda_{hX} = 0$)

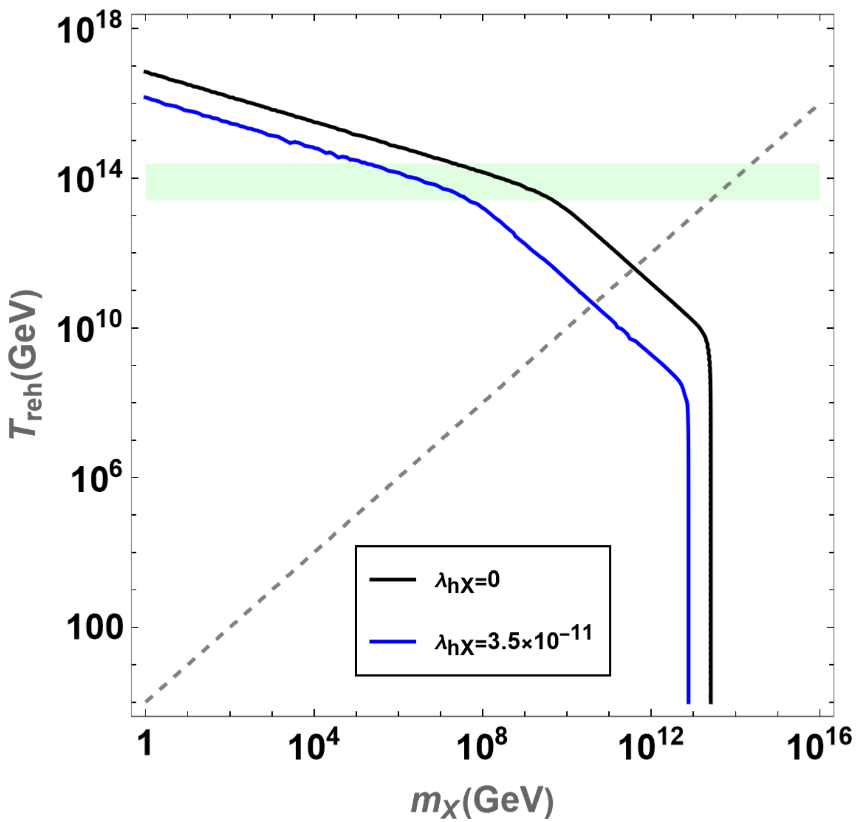


Green band
(predicted RH temp. in this model)

- Non-thermal << Thermal
- $10^7 < m_X < 10^{10}$(GeV) can explain DM abundance

Effects of $\tilde{\eta}$ and λ_{hX}

To avoid overproduction ⇒ $|\lambda_{hX}| \lesssim 10^{-10}$ & $|\tilde{\eta}| \lesssim 10^{-6}$



Smaller DM mass
for correct relic abundance

Summary

- Higgs-R^2 = UV completion of Higgs inflation
- Study post inflationary dynamics (Reheating, DM genesis)
- temperature evolution during/after reheating
- Scalar FIMP DM ($\tilde{\eta}, \lambda_{hX} \sim 0$)
- Evaluate relic abundance including thermal/non-thermal production, graviton exchange, temperature evolution, etc.
- Non-thermal << Thermal production
- $T_{\mathrm{reh}} \sim 10^{14}$(GeV) $\Rightarrow 10^7 < m_X < 10^{10}$ can explain a correct relic density

Future direction

- spinning DM ?
- Baryogenesis ?
- Preheating effects ?

Thank you !!

Backup

Dual picture

Y. Ema, K. Mukaida, J. van de Vis, 2002.11739
SA, H. M. Lee, A. G. Menkara, 2104.10390,2108.00222

$$\mathcal{L}/\sqrt{-g_J} = \frac{1}{2}(M_{\rm Pl}^2 + \xi \hat{h}^2) R_J - \frac{1}{2}(\partial_\mu \hat{h})^2 - \frac{\lambda}{4}\hat{h}^4 + \underline{\alpha R_J^2}$$

$$\mathcal{L}/\sqrt{-g_J} = \frac{1}{2}(M_{\rm Pl}^2 + \xi \hat{h}^2 \underline{+ 4\alpha\hat{\chi}}) R_J - \frac{1}{2}(\partial_\mu \hat{h})^2 - \frac{\lambda}{4}\hat{h}^4 \underline{- \alpha\hat{\chi}^2}.$$

Conformal transformation :

$$g_{J\mu\nu} = \Delta^{-2} g_{L\mu\nu}, \quad \hat{h} = \Delta h, \quad \hat{\chi} = \Delta^2 \chi,$$

$$\Delta^{-2} = \left(1 + \frac{\sigma}{\sqrt{6}M_{\rm Pl}}\right)^2$$

$$\left(1 + \frac{\sigma}{\sqrt{6}M_{\rm Pl}}\right)^2 + \xi\frac{h^2}{M_{\rm Pl}^2} + 4\alpha\frac{\chi}{M_{\rm Pl}^2} = 1 - \frac{h^2}{6M_{\rm Pl}^2} - \frac{\sigma^2}{6M_{\rm Pl}^2}.$$

Dual picture

Higgs-σ system (linear-sigma frame)

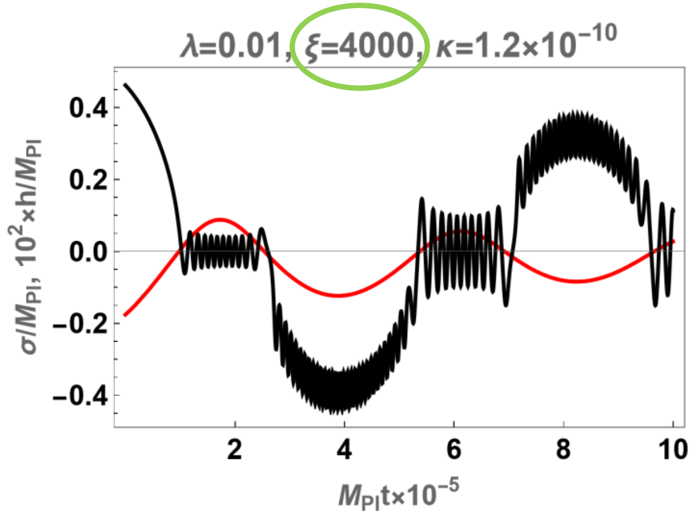
$$\mathcal{L}/\sqrt{-g_L} = \frac{M_{\rm Pl}^2}{2}\left(1-\frac{h^2}{6M_{\rm Pl}^2}-\frac{\sigma^2}{6M_{\rm Pl}^2}\right)R_L-\frac{1}{2}\left(\partial_\mu\sigma\right)^2-\frac{1}{2}\left(\partial_\mu h\right)^2-\frac{\lambda}{4}h^4$$
$$-\frac{\kappa}{4}\left[\sigma(\sigma+\sqrt{6}M_{\rm Pl})+3\left(\xi+\frac{1}{6}\right)h^2\right]^2, \qquad \kappa \equiv 1/(36\alpha)$$

- $R^2 \Rightarrow \sigma$-field
- manifest unitarity and perturbativity if

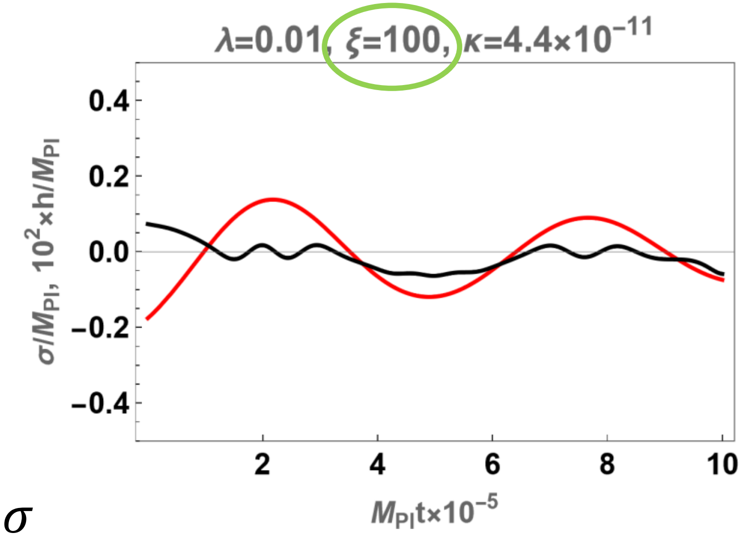
$$\kappa \lesssim 1, \quad \lambda_{\rm eff} \equiv \lambda + 9\kappa\left(\xi+\frac{1}{6}\right)^2 \lesssim 1, \quad 6\kappa\left(\xi+\frac{1}{6}\right) \lesssim 1.$$

Numerical solutions after inflation ($\Gamma = 0$)

Higgs-like



R^2-like



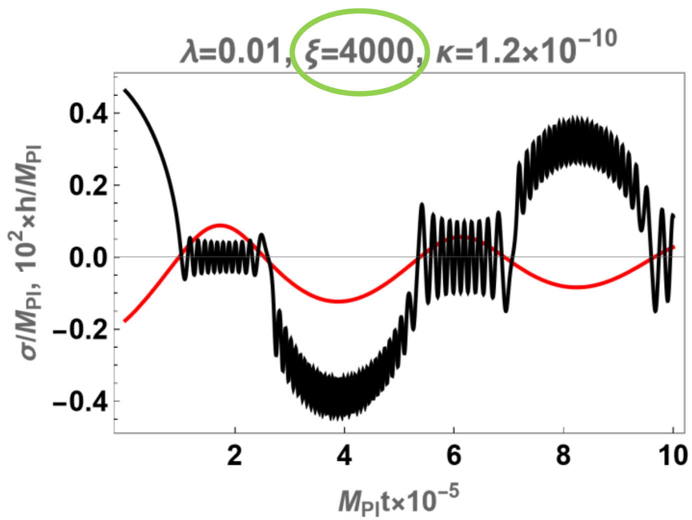
Red : σ

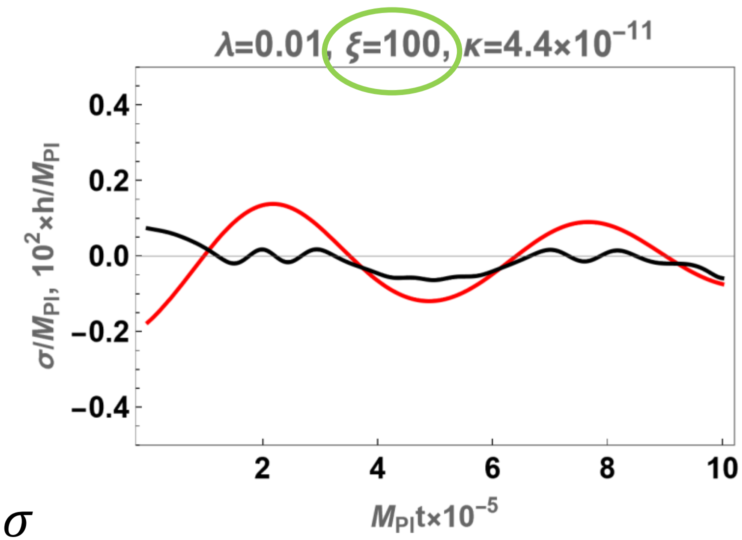
Black : h

$$V = \frac{1}{\Omega^4}\left[\frac{1}{4}\kappa\left(\sigma(\sigma + \sqrt{6}M_{\rm Pl}) + 3\left(\xi + \frac{1}{6}\right)h^2\right)^2 + \frac{1}{4}\lambda h^4\right]$$

Tachyonic mass of h for $\sigma < 0$

Numerical solutions after inflation ($\Gamma = 0$)





Red : σ

Black : h

$$m_\sigma^2 = \begin{cases} 3\kappa M_{\rm Pl}^2 \equiv m_{\sigma,+}^2 & , \quad \sigma_0 > 0, \\ \frac{3\kappa\lambda M_{\rm Pl}^2}{\lambda + 9\kappa\tilde{\xi}^2} \equiv m_{\sigma,-}^2 & , \quad \sigma_0 < 0 \end{cases} \qquad m_h^2 = \begin{cases} 3\sqrt{6}\kappa\tilde{\xi} M_{\rm Pl}\sigma_0 \equiv m_{h,+}^2 & , \quad \sigma_0 > 0, \\ 6\sqrt{6}\kappa\tilde{\xi}(-M_{\rm Pl}\sigma_0) \equiv m_{h,-}^2 & , \quad \sigma_0 < 0, \end{cases}$$

Boltzmann equations

EOM of σ : $$\ddot{\sigma} + \frac{\sigma}{3\Omega^2 M_{\rm Pl}^2}\dot{\sigma}^2 + \frac{h}{3\Omega^2 M_{\rm Pl}^2}\dot{\sigma}\dot{h} + (3H + \Gamma_{\sigma_0})\dot{\sigma} + \frac{2\sigma}{3\Omega^2 M_{\rm Pl}^2}U + \frac{1}{\Omega^2}\left(1 - \frac{\sigma^2}{6M_{\rm Pl}^2}\right)U_\sigma - \frac{h\sigma}{6\Omega^2 M_{\rm Pl}^2}U_h = 0,$$

$$U \equiv V\Omega^4$$

EOM of h : $$\ddot{h} + \frac{h}{3\Omega^2 M_{\rm Pl}^2}\dot{h}^2 + \frac{\sigma}{3\Omega^2 M_{\rm Pl}^2}\dot{\sigma}\dot{h} + (3H + \Gamma_{h_{\rm osc}})\dot{h} + \frac{2h}{3\Omega^2 M_{\rm Pl}^2}U + \frac{1}{\Omega^2}\left(1 - \frac{h^2}{6M_{\rm Pl}^2}\right)U_h - \frac{h\sigma}{6\Omega^2 M_{\rm Pl}^2}U_\sigma = 0,$$

Radiation ρ_r : $$\dot{\rho}_r + 4H\rho_r - \frac{\Gamma_{\sigma_0}}{\Omega^4}\left[\left(1 - \frac{h^2}{6M_{\rm Pl}^2}\right)\dot{\sigma}^2 + \frac{h\sigma}{6M_{\rm Pl}^2}\dot{\sigma}\dot{h}\right] - \frac{\Gamma_{h_{\rm osc}}}{\Omega^4}\left[\left(1 - \frac{\sigma^2}{6M_{\rm Pl}^2}\right)\dot{h}^2 + \frac{h\sigma}{6M_{\rm Pl}^2}\dot{\sigma}\dot{h}\right] = 0,$$

Friedmann : $3H^2 M_{\rm Pl}^2 = \rho_{\sigma+h} + \rho_r$

$\rho_{\sigma+h}$: total energy density of inflatons

Energy density and Pressure

$$\rho_\sigma = \frac{1}{2}\dot{\sigma}^2 + \frac{1}{2}m_{\sigma,+}^2\sigma^2, \quad \rho_h = \frac{1}{2}\dot{h}^2 + \frac{1}{2}m_{h,+}^2 h^2 + \frac{\lambda_{\rm eff}}{4}h^4,$$
$$p_\sigma = \frac{1}{2}\dot{\sigma}^2 - \frac{1}{2}m_{\sigma,+}^2\sigma^2, \quad p_h = \frac{1}{2}\dot{h}^2 - \frac{1}{2}m_{h,+}^2 h^2 - \frac{\lambda_{\rm eff}}{4}h^4,$$

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$$m_h^2 = \begin{cases} 3\sqrt{6}\kappa\tilde{\xi} M_{\rm Pl}\sigma_0 \equiv m_{h,+}^2 & , \quad \sigma_0 > 0, \\ 6\sqrt{6}\kappa\tilde{\xi}(-M_{\rm Pl}\sigma_0) \equiv m_{h,-}^2 & , \quad \sigma_0 < 0, \end{cases}$$

Inflaton potential

Higgs can be integrated out during inflation → only σ-field remains

Effective potential:

canonical field ϕ

$$V_{\rm eff}(\phi) \simeq V_I \left(1 - \frac{2\lambda + \kappa(3\xi+1)(6\xi+1)}{\lambda + 9\kappa\left(\xi+\frac{1}{6}\right)^2} \cdot e^{-\frac{2\phi}{\sqrt{6}M_{\rm Pl}}} + \cdots\right) \quad V_I \equiv \frac{9\kappa\lambda M_{\rm Pl}^4}{4\left(\lambda + 9\kappa\left(\xi+\frac{1}{6}\right)^2\right)}.$$

↓

CMB normalization: $$\frac{\lambda + 9\kappa\left(\xi+\frac{1}{6}\right)^2}{\kappa\lambda} = 2.25 \times 10^{10}.$$

Derivation of Boltzmann equation

L. J. Hall, K. Jedamzik, J. March-Russell and S. M. West, 0911.1120

$$B_1 \to B_2 X.$$

$$\dot{n}_X + 3Hn_X = \int d\Pi_X d\Pi_{B_1} d\Pi_{B_2} (2\pi)^4 \delta^4(p_X + p_{B_2} - p_{B_1})$$
$$\times \left[|M|^2_{B_1 \to B_2 + X} f_{B_1} (1 \pm f_{B_2})(1 \pm \cancel{f_X}) - |M|^2_{B_2 + X \to B_1} f_{B_2} \cancel{f_X} (1 \pm f_{B_1}) \right]$$

Previous works

Reheating

- ✓ Perturbative reheating

  M. He, 2010.11717
  D.Y. Cheong, talk at APPPC 2021

- ✓ Preheating

  M. He, R. Jinno, K. Kamada, S. C. Park, A. A. Starobinsky, J. Yokoyama ’18
  F. Bezrukov and C. Shepherd, ’20

DM production

- ✓ Primordial black hole

  D. Y. Cheong, S. M. Lee, and S. C. Park , ’19

After reheating

radiation dominated, only thermal production

$R(T)$ for thermal production :

L. J. Hall, K. Jedamzik, J. March-Russell and S. M. West, 0911.1120

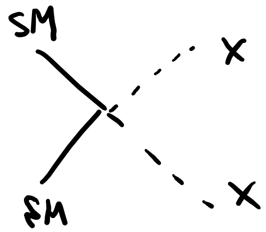
$$R = \frac{T}{2^{11}\pi^6} \int_{4m_X^2}^{\infty} ds\, d\Omega\, K_1\left(\frac{\sqrt{s}}{T}\right) \sqrt{s - 4m_X^2}\, \overline{|\mathcal{M}_{i_1+i_2\to X+X}|^2},$$

amplitude

After reheating

- contact terms

$$-\frac{X^2}{12M_{\rm Pl}^2}(\partial_\mu h)^2 - \frac{h^2}{12M_{\rm Pl}^2}(\partial_\mu X)^2 - \frac{hX}{6M_{\rm Pl}^2}\partial_\mu h \partial^\mu X + c_{hhXX}h^2X^2$$



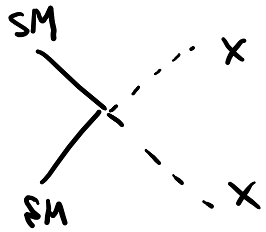
Higgs-DM interactions

$$c_{hhXX} = -\frac{m_X^2}{6M_{\rm Pl}^2} - \frac{9}{2}\kappa\tilde{\xi}\tilde{\eta} - \frac{\lambda_{hX}}{4},$$

After reheating

- contact terms

$$-\frac{X^2}{12M_{\rm Pl}^2}(\partial_\mu h)^2 - \frac{h^2}{12M_{\rm Pl}^2}(\partial_\mu X)^2 - \frac{hX}{6M_{\rm Pl}^2}\partial_\mu h \partial^\mu X + c_{hhXX} h^2 X^2$$



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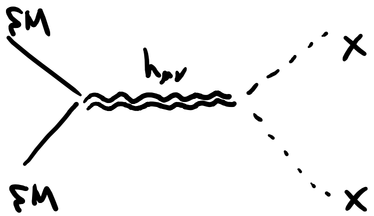
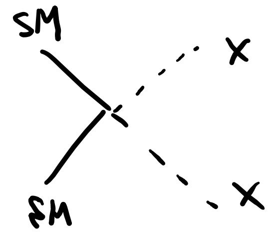
$$c_{hhXX} = -\frac{m_X^2}{6M_{\rm Pl}^2} - \frac{9}{2}\kappa\tilde{\xi}\tilde{\eta} - \frac{\lambda_{hX}}{4},$$

- graviton exchange S. Clery, Y. Mambrini, K. A. Olive, S. Verner' 21



After reheating

- contact terms

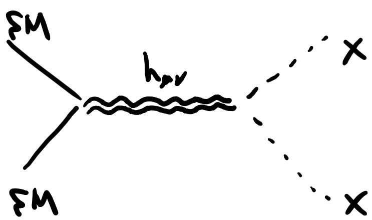


$$-\frac{X^2}{12M_{\rm Pl}^2}(\partial_\mu h)^2 - \frac{h^2}{12M_{\rm Pl}^2}(\partial_\mu X)^2 - \frac{hX}{6M_{\rm Pl}^2}\partial_\mu h \partial^\mu X + c_{hhXX} h^2 X^2$$

Higgs-DM interactions

$$c_{hhXX} = -\frac{m_X^2}{6M_{\rm Pl}^2} - \frac{9}{2}\kappa\tilde{\xi}\tilde{\eta} - \frac{\lambda_{hX}}{4},$$

- graviton exchange S. Clery, Y. Mambrini, K. A. Olive, S. Verner' 21



$$|\mathcal{M}^{\rm total}_{h+h\to X+X}|^2 = \left(\frac{s+2m_X^2}{6M_{\rm Pl}^2} + 18\kappa\tilde{\eta}\tilde{\xi} + \lambda_{hX} + \frac{(t-m_X^2)(s+t-m_X^2)}{sM_{\rm Pl}^2}\right)^2$$

$$|\mathcal{M}^{G}_{f+f\to X+X}|^2 = \frac{-1}{2M_{\rm Pl}^4 s^2}\left(s+2t-2m_X^2\right)^2\left(\left(t-m_X^2\right)^2+st\right),$$

$$|\mathcal{M}^{G}_{V+V\to X+X}|^2 = \frac{2}{M_{\rm Pl}^4 s^2}\left(m_X^4 - 2m_X^2 t + t(s+t)\right)^2.$$

After reheating

Integrate Boltzmann eq. from $T_{\rm reh}$ to T_* (Note : depends only on $T_{\rm reh}$)

$$Y \equiv n_X T^{-3}$$

$$Y_{\rm After\ RH} \simeq \frac{\sqrt{10}}{20480\pi^4 g_{\rm reh}^{1/2}} \frac{4m_X^4 + 45 M_{\rm Pl}^4 \left(\lambda_{hX} + 18\kappa\tilde{\eta}\tilde{\xi}\right)^2}{m_X M_{\rm Pl}^3} + \frac{209\sqrt{10}}{240\pi^6 g_{\rm reh}^{1/2}} \frac{T_{\rm reh}^3}{M_{\rm Pl}^3}$$

for $\tilde{\eta} \sim \lambda_{hX} \sim 0$ IR Freeze-in $\ll$ UV Freeze-in

During reheating

Inflaton dominated, both thermal and non-thermal production

$$Y_{\rm thermal}(T_{\rm reh}) + Y_{\rm non-thermal}(T_{\rm reh})$$

- Thermal production → similar but
  - Higgs is absent in radiation
  - $H = \sqrt{\frac{\pi^2 g_*}{90}} \frac{T^4}{T_{\rm reh}^2}$

$$Y_{\rm thermal}(T_{\rm reh}) \simeq \frac{69\sqrt{10}}{40\pi^6 g_{\rm reh}^{1/2}} \frac{T_{\rm reh}^3}{M_{\rm Pl}^3}$$

During reheating

- Non-thermal production

M. A. G. Garcia, K. Kaneta, Y. Mambrini and K. A. Olive, 2012.10756

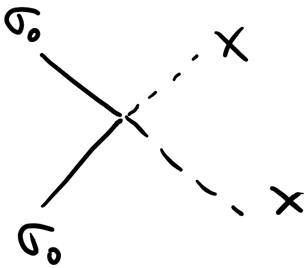
$$\frac{d}{da}(n_X a^3) \simeq \sqrt{\frac{3}{\rho_{\rm end}}} M_{\rm Pl} a^2 \left(\frac{a}{a_{\rm end}}\right)^{3/2} R(a)$$

$$R = \frac{1}{8\pi} \sum_{n=1}^{\infty} |\mathcal{M}_n|^2 \sqrt{1 - \frac{4m_{X,\rm eff}^2}{n^2\omega^2}},$$

only $n = 1$ for harmonic oscillation
ω : oscillation frequency
M_n: transition amplitude of inflaton with Fourier mode n

During reheating

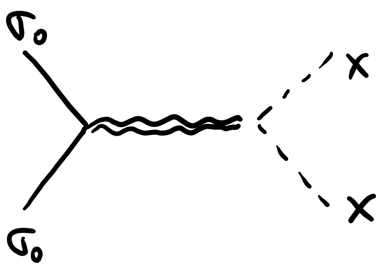
- contact terms



$$\mathcal{L} \supset \begin{cases} -\frac{\kappa}{2}\sigma_0^2 X^2 & , \quad \sigma_0 > 0, \\ -\frac{\kappa}{2}\frac{\lambda}{\lambda+9\kappa\tilde{\xi}^2}\sigma_0^2 X^2 & , \quad \sigma_0 < 0, \end{cases}$$

$$\mathcal{L} \supset -\frac{1}{12M_{\rm Pl}^2}X^2(\partial_\mu\sigma_0) - \frac{1}{6M_{\rm Pl}^2}X\sigma\partial_\mu X\partial^\mu\sigma_0 - \frac{1}{12M_{\rm Pl}^2}\sigma_0^2(\partial_\mu X)$$

- graviton exchange



Note : Higgs condensate effects are negligible

During reheating

- contact terms

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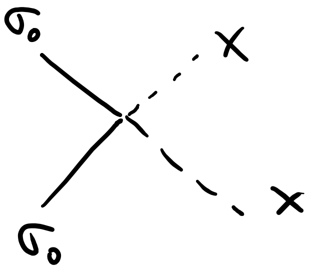
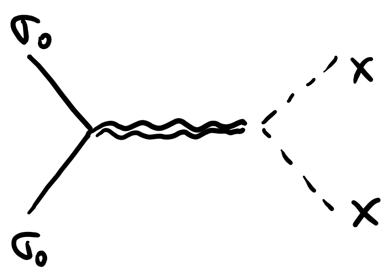


$$\mathcal{L} \supset -\frac{1}{12M_{\rm Pl}^2}X^2(\partial_\mu\sigma_0) - \frac{1}{6M_{\rm Pl}^2}X\sigma\partial_\mu X\partial^\mu\sigma_0 - \frac{1}{12M_{\rm Pl}^2}\sigma_0^2(\partial_\mu X)$$

- graviton exchange



Note : Higgs condensate effects are negligible

$$Y_{\rm non-thermal}(T_{\rm reh}) \simeq \frac{\sqrt{3}\pi g_{\rm reh}}{2239488}\frac{T_{\rm reh}}{\kappa^2 M_{\rm Pl}^{11}}\frac{\rho_{\sigma,\rm end}^4}{\rho_{\rm end}^{3/2}}$$