POSITIVITY BOUNDS FOR SCALAR DARK MATTER

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Workshop on Physics of Dark Cosmos:
dark matter, dark energy, and all
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Effective Field Theory (EFT)

- EFT

  heavy degrees of freedom decouple

  for large-distance phenomena
  or small momentum scale

- EFT interaction terms:

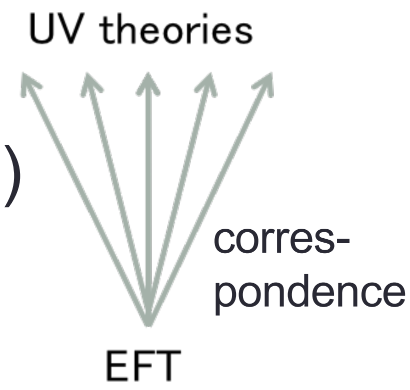
$$\mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \mathcal{L}^{(7)} + \mathcal{L}^{(8)} + \cdots$$

Wilson coefficients

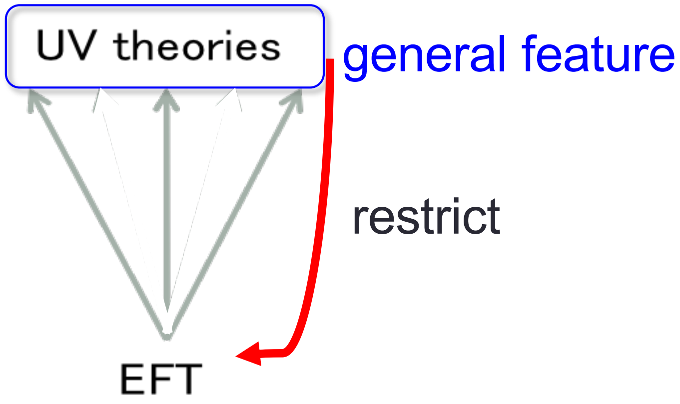
$$\mathcal{L} = \sum_{i=1}^{n_d} \frac{c_i^5}{\Lambda}\mathcal{O}_i^{(5)} + \frac{c_i^6}{\Lambda^2}\mathcal{O}_i^{(6)} + \frac{c_i^7}{\Lambda^3}\mathcal{O}_i^{(7)} + \frac{c_i^8}{\Lambda^4}\mathcal{O}_i^{(8)} + \cdots$$

Positivity Bounds

- EFT is for the energy scale of
  E << Λ (typical energy scale of the UV physics)
- Many UV models correspond with EFT



- From the general feature of UV theory,
  can we bound on Wilson coefficients of EFT?



If we base on the local Quantum Field Theory(QFT) for the general feature of UV theory,

1. Special relativity ⟶ Lorentz invariance
2. Conservation of probability ⟶ Unitarity
3. Causality - - - → Analyticity

Positivity Bounds

A. Adams, N. Arkani-Hamed, S. Dubovsky, A. Nicolis, R.Rattazzi, JHEP **0610**, 014(2006)

- One of the way to do this is Positivity bounds
- Positivity bounds: the signs of certain combinations of Wilson coefficients in EFT have to be positive, e.g. W^4 operators:

$$\frac{F_{T,0}}{\Lambda^4}\mathrm{Tr}[\hat{\mathrm{W}}_{\mu\nu}\hat{\mathrm{W}}^{\mu\nu}]\mathrm{Tr}[\hat{\mathrm{W}}_{\alpha\beta}\hat{\mathrm{W}}^{\alpha\beta}] \qquad \frac{F_{T,1}}{\Lambda^4}\mathrm{Tr}[\hat{\mathrm{W}}_{\alpha\nu}\hat{\mathrm{W}}^{\mu\beta}]\mathrm{Tr}[\hat{\mathrm{W}}_{\mu\beta}\hat{\mathrm{W}}^{\alpha\nu}]$$

$$\frac{F_{T,2}}{\Lambda^4}\mathrm{Tr}[\hat{\mathrm{W}}_{\alpha\mu}\hat{\mathrm{W}}^{\mu\beta}]\mathrm{Tr}[\hat{\mathrm{W}}_{\beta\nu}\hat{\mathrm{W}}^{\nu\alpha}] \qquad \frac{F_{T,10}}{\Lambda^4}\mathrm{Tr}[\hat{\mathrm{W}}_{\mu\nu}\tilde{\mathrm{W}}^{\mu\nu}]\mathrm{Tr}[\hat{\mathrm{W}}_{\alpha\beta}\tilde{\mathrm{W}}^{\alpha\beta}]$$

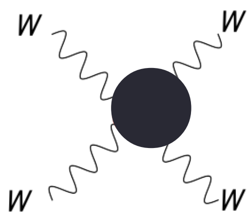
$$\hat{W}^{\mu\nu} \equiv ig\frac{\sigma^I}{2}W^{I,\mu\nu} \qquad \tilde{W}^{\mu\nu} \equiv ig\frac{\sigma^I}{2}\left(\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}W^{I,\rho\sigma}\right)$$

One of the positivity bounds:

$$2F_{T,0} + 2F_{T,1} + F_{T,2} \geq 0$$

KY, C. Zhang, S. Y. Zhou, JHEP **01**, 095 (2021)

Positivity Bounds

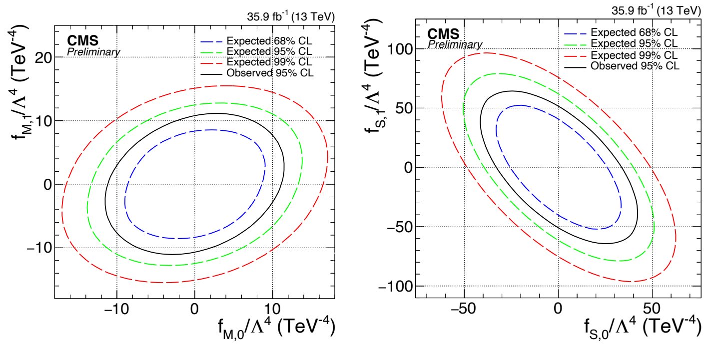


[FFFF]=Dim 8

- Positivity bounds can apply for dim-8 operators in tree-level ← Froissart Bound (⇦Analyticity)

- Dim-8 operators are more suppressed by Λ than lower dimensional ones, however, for dim-8 aQGC operators, LHC experimentalists have been and currently working on constraining them

CMS-PAS-SMP-18-001



- In the future, more dim-8 effects may become accessible

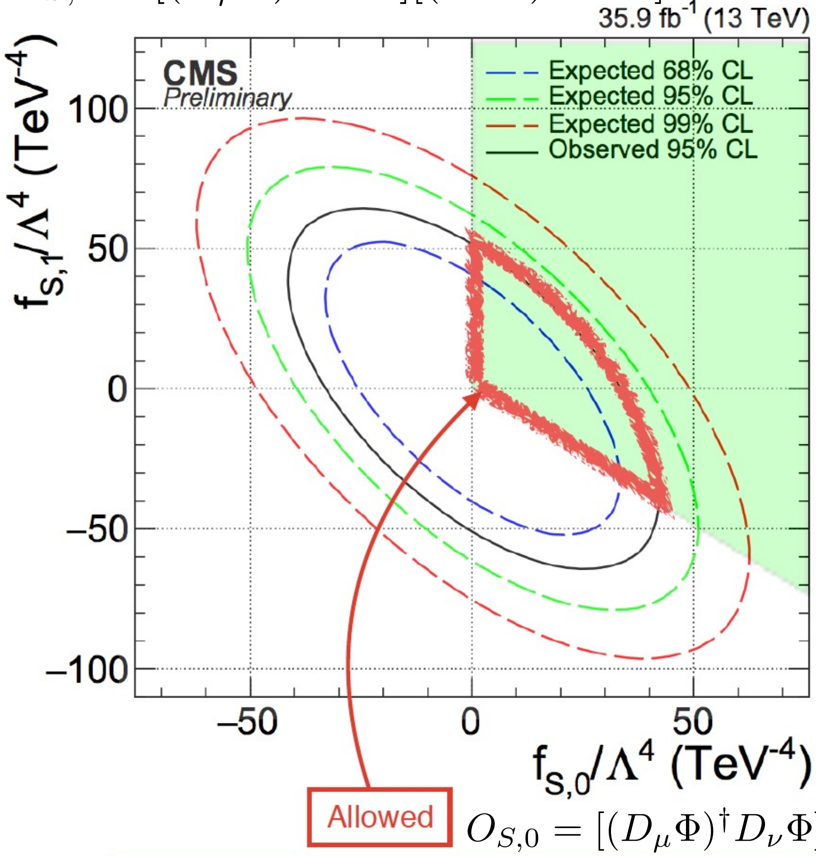
(e.g. new observable proposed for DY process:
Alioli, Boughezal, Mereghetti, Petriello, Phys. Lett. B **809**, 135703 (2020),
X. Li, K. Mimasu, KY, C. Yang, C. Zhang, S. Y. Zhou, JHEP**10**(2022)107)

Positivity Bounds

Positivity bounds are important as they offer complementary bounds to the experiments Q. Bi, C. Zhang, S.-Y. Zhou JHEP **1906** (2019) 137

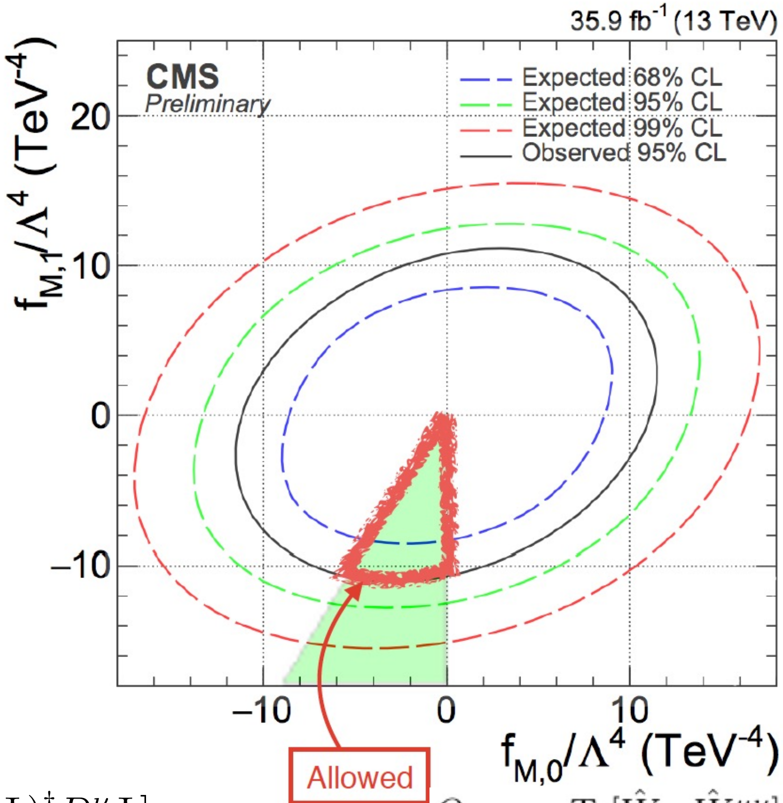
E.g. WZjj (CMS-PAS-SMP-18-001)

$O_{S,1} = [(D_\mu\Phi)^\dagger D^\mu\Phi][(D_\nu\Phi)^\dagger D^\nu\Phi]$



$O_{S,0} = [(D_\mu\Phi)^\dagger D_\nu\Phi][(D^\mu\Phi)^\dagger D^\nu\Phi]$

$O_{M,1} = \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\nu\beta}][(D_\beta\Phi)^\dagger D^\mu\Phi]$



$O_{M,0} = \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}][(D_\beta\Phi)^\dagger D^\beta\Phi]$

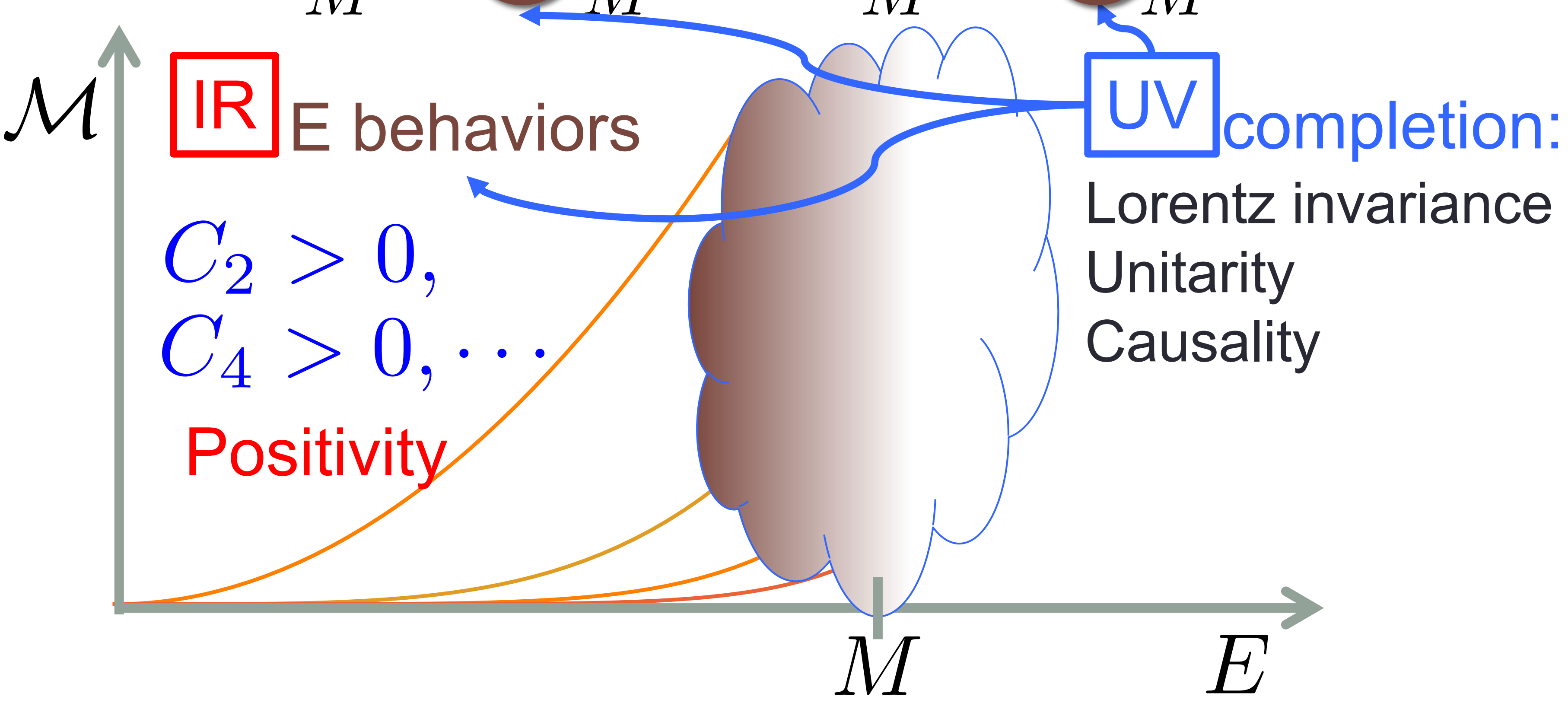
Positivity restricts the directions in which SM deviation is possible

Positivity Bounds

Ref: Slides by Francesco Riva
https://indico.ph.tum.de/event/4408/contributions/3825/attachments/3292/3974/Berlin-2.pdf

- Effective Theory Forward Amplitude (IR):

$$\mathcal{M} = C_0 + C_1 \frac{s}{M^2} + C_2 \frac{s^2}{M^4} + C_3 \frac{s^3}{M^6} + C_4 \frac{s^4}{M^8} + \cdots$$



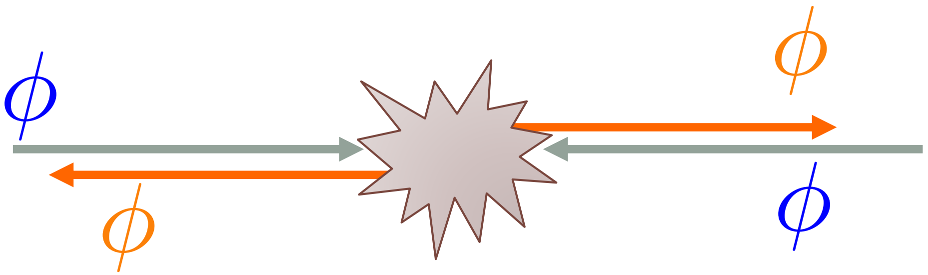
Positivity Bounds

$$\mathcal{M} = C_0 + C_1 \frac{s}{M^2} + C_2 \frac{s^2}{M^4} + C_3 \frac{s^3}{M^6} + C_4 \frac{s^4}{M^8} + \cdots$$

>0

massless scalar 2-2 forward elastic scattering:

forward: *t*=0



I+II → I+II

elastic

Let us consider the amplitude of this: $\frac{\mathcal{M}(s,0)}{s^3}$

Positivity Bounds

Forward limit positivity bounds are from:

1. Lorentz Invariance

2. Unitarity ⇨ Optical theorem:
   e.g., elastic case,

$$\mathrm{Im}\mathcal{M}(k_1,k_2\to k_1,k_2)=\underline{\underline{s\sigma_{\mathrm{tot}}(k_1,k_2\to\text{anything})}}$$

Positive

1. Analyticity* ⇨ Froissart Bound:

$$|\mathcal{M}(s,\underline{\underline{\cos\theta=1}})|<\mathrm{Const.}\ s(\ln s)^2$$

forward

Froissart, Martin 1960's

(for real s→∞)

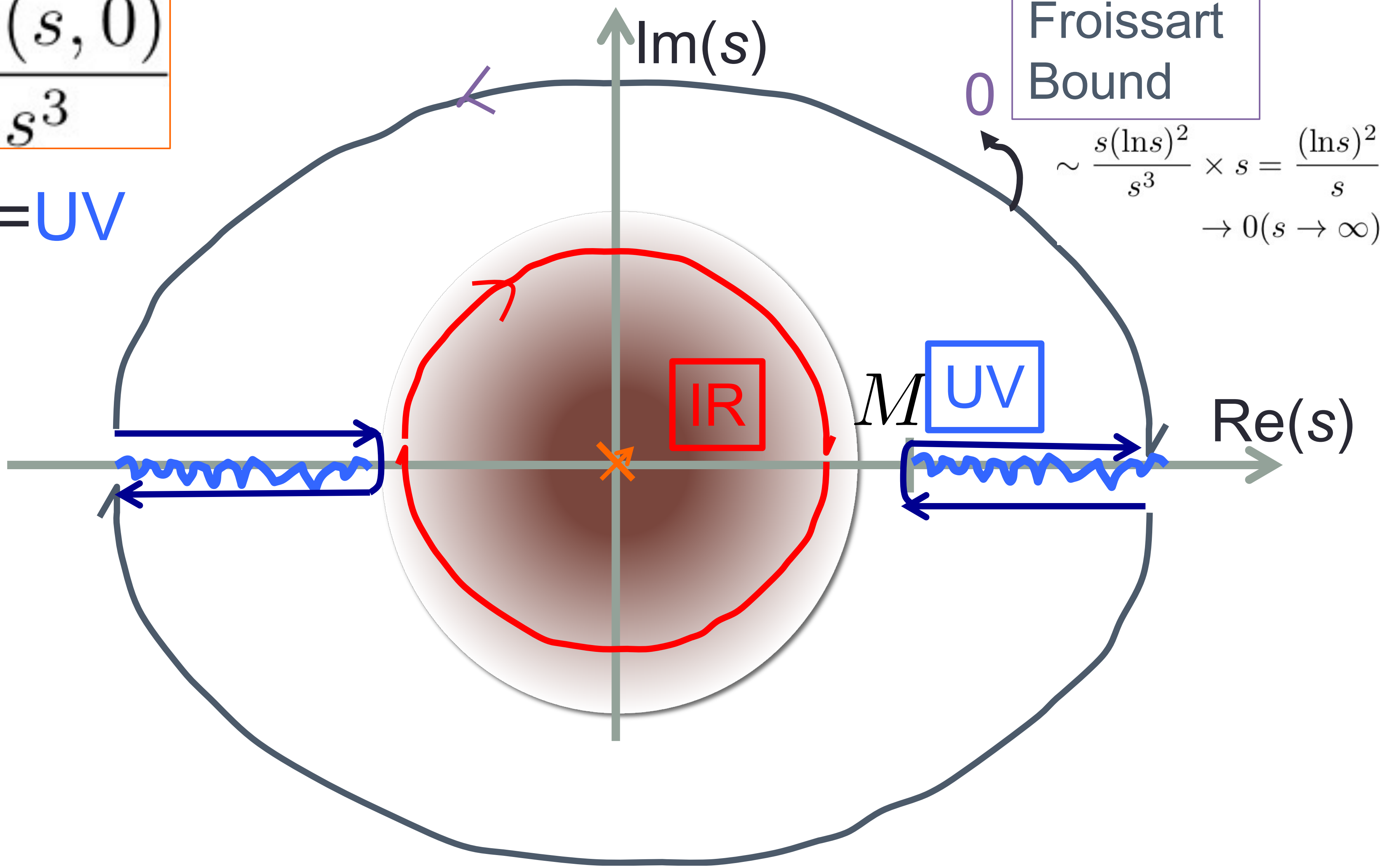
*Analyticity of the amplitude besides poles and branch cuts on real axis

Positivity Bounds

massless scalar 2-2 forward elastic scattering amplitude:

$$\frac{\mathcal{M}(s,0)}{s^3}$$

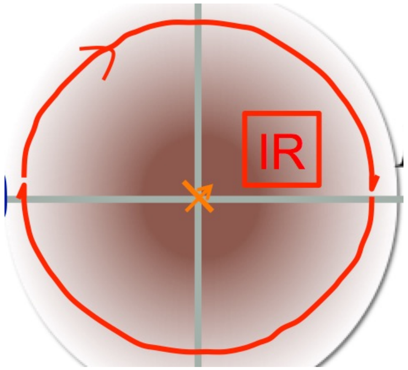
IR=UV



Positivity Bounds

IR

$$\frac{1}{2\pi i}\oint ds \frac{\mathcal{M}(s,0)}{s^3} = \frac{C_2}{M^4}$$



$$\mathcal{M} = C_0 + C_1\frac{s}{M^2} + C_2\frac{s^2}{M^4} + C_3\frac{s^3}{M^6} + C_4\frac{s^4}{M^8} + \cdots$$

Positivity Bounds

UV

$$\frac{1}{2\pi i}\int_M^\infty ds\,\left(M(s+i\epsilon,0)-M(s-i\epsilon,0)\right)/s^3$$

2)&3)
= (2i)Im M(s,0)
= (2i)s $\sigma_{tot}(s)$

|| 1)

$$\frac{1}{2\pi i}\int_{-\infty}^{-M} ds\,\left(M(-s-i\epsilon,0)-M(-s+i\epsilon,0)\right)/s^3$$

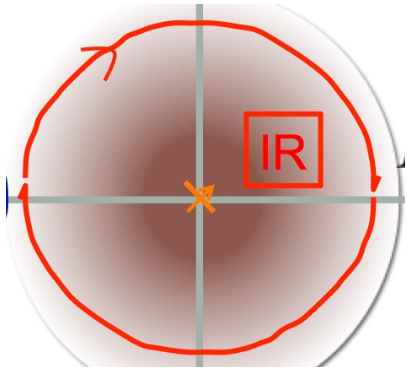
1. Crossing Symmetry: M(s,0)=M(-u,0),
2. Schwarz reflection principle: M(s*,0)=M(s,0)*
3. Optical theorem: Im M(s,0) = s σ_{tot}(s)

$$= \frac{2}{\pi}\int_M^\infty ds\frac{s\sigma_{tot}(s)}{s^3} > 0$$

Positivity Bounds

IR

$$\frac{1}{2\pi i}\oint ds \frac{\mathcal{M}(s,0)}{s^3} = C_2$$

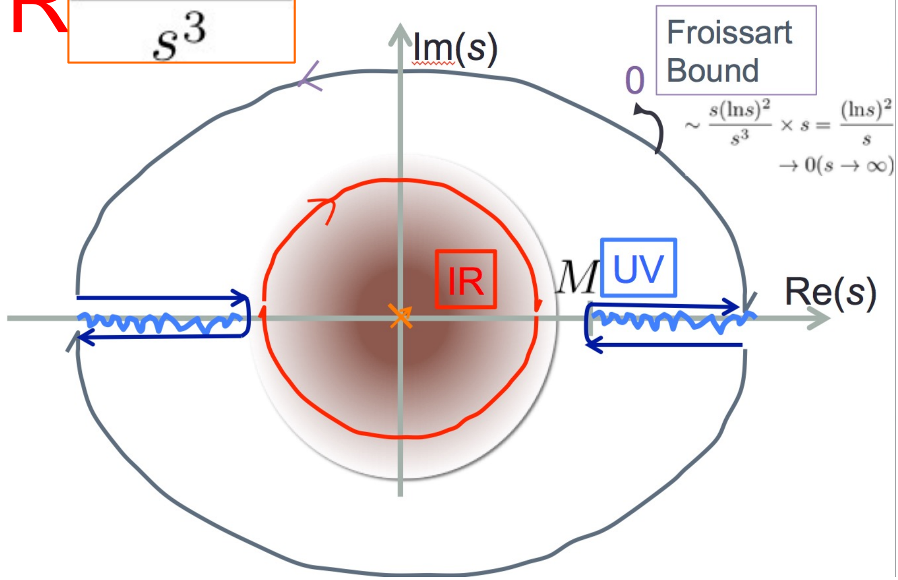


$$\mathcal{M} = C_0 + C_1\frac{s}{M^2} + C_2\frac{s^2}{M^4} + C_3\frac{s^3}{M^6} + C_4\frac{s^4}{M^8} + \cdots$$

$1/(2\pi i)\int M_{IR}/s^3\ (=C_2/M^4)$…IR $\frac{\mathcal{M}(s,0)}{s^3}$

$=1/(2\pi i)\int M_{UV}/s^3 > 0$ …UV

→ $C_2>0$ …IR



Positivity Bounds

Example of Positivity

W. Heisenberg, H. Euler, Z. Phys. **98**, 714 (1936)

Heisenberg-Euler Lagrangian:

$$X = \sqrt{2(\mathcal{F} + i\mathcal{G})},$$

$$\mathcal{F} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} = \frac{1}{2}\left(\vec{H}^2 - \vec{E}^2\right)$$

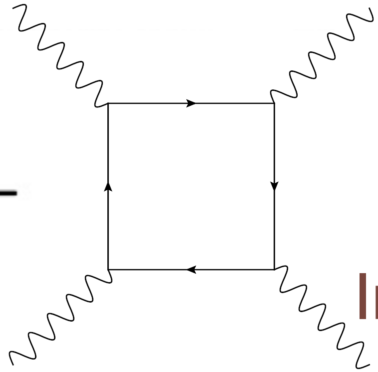
$$\mathcal{G} = \frac{1}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} = \vec{E} \cdot \vec{H}$$

$$\mathfrak{L} = -\mathfrak{F} - \frac{1}{8\pi^2} \int_0^\infty ds s^{-3} \exp(-m^2 s) \times \left[(es)^2 \mathfrak{G} \frac{\mathrm{Re}\cosh esX}{\mathrm{Im}\cosh esX} - 1 - \tfrac{2}{3}(es)^2 \mathfrak{F} \right]$$

$$= \tfrac{1}{2}(\mathbf{E}^2 - \mathbf{H}^2) + \frac{2\alpha^2}{45} \frac{(\hbar/mc)^3}{mc^2} \times [(\mathbf{E}^2 - \mathbf{H}^2)^2 + 7(\mathbf{E}\cdot\mathbf{H})^2] + \cdots$$

>0

from J. Schwinger, Phys. Rev. **82**, 664 (1951)

$$\mathcal{L}_{\rm eff} = -\frac{1}{4} F_{\mu\nu}^2 +$$  $$+ \quad \cdots$$

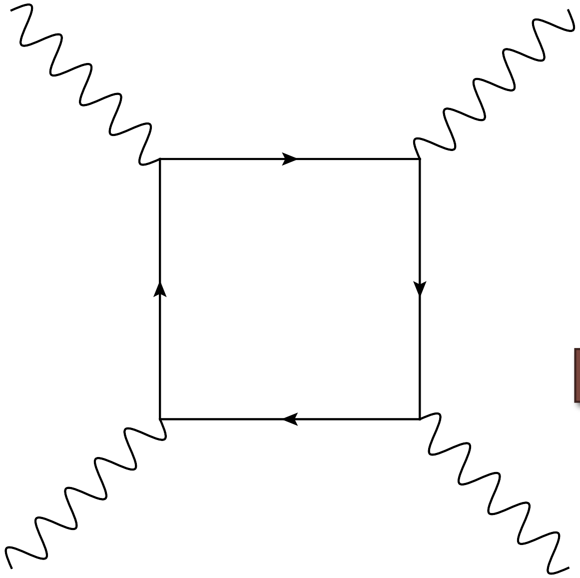
Including this

Positivity Bounds

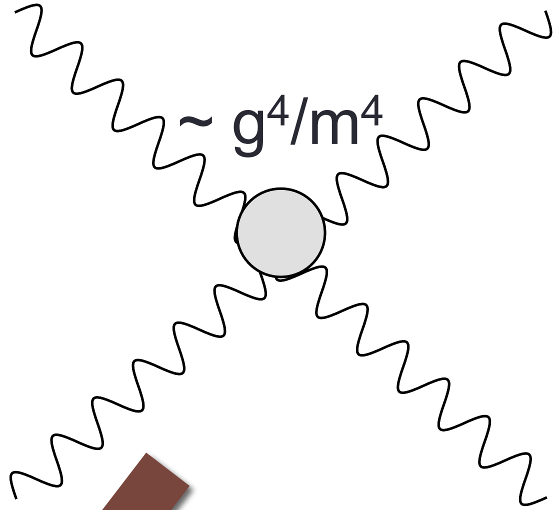
Example of Positivity

$$\mathcal{F} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}\left(\vec{B}^2 - \vec{E}^2\right) \qquad \tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\rho}F^{\lambda\rho}$$

$$\mathcal{G} = \frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} = \vec{E}\cdot\vec{B}$$



Photon Energy >> Fermion Mass



$$\mathcal{L}_{\rm eff} = -\mathcal{F} + a\mathcal{F}^2 + b\mathcal{G}^2$$

CP even case

Consistent with QED

Positivity bounds: a>0, b>0

Dispersion Relation (for Positivity Bounds)

Forward scattering amp, at low energy (EFT)

(Amp by Dim.8) ∝ $(F/\Lambda^4)\, s^2$

$$M^2 = m_i^2 + m_j^2 + m_k^2 + m_l^2$$

$$M_{ijkl} = \frac{1}{2}\frac{d^2}{ds^2} M_{ij\to kl}\left(s = \frac{1}{2}M^2, t = 0\right) + c.c.$$

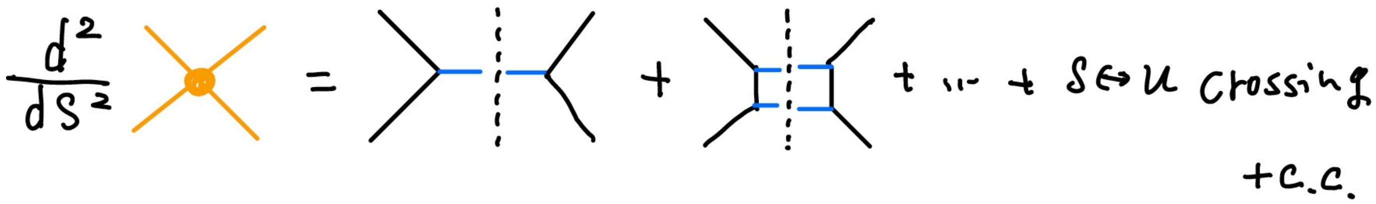
$$= \sum_X \int_{(\epsilon\Lambda)^2}^{\infty} \frac{ds\, M_{ij\to X} M^*_{kl\to X}}{2\pi s^3} + (j \leftrightarrow l) + c.c$$

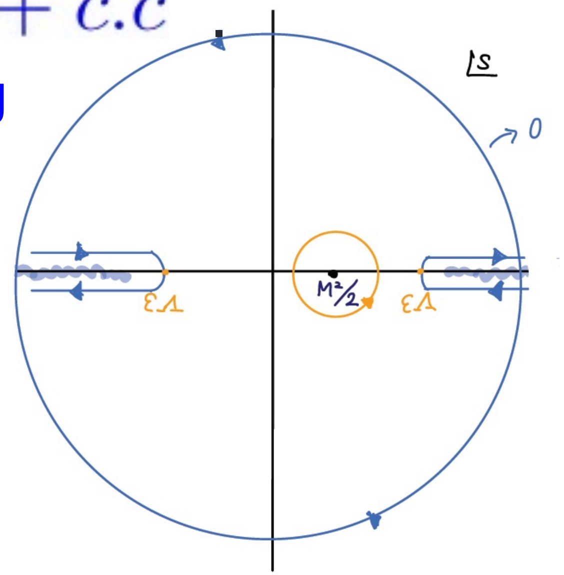
$\epsilon \lesssim 1$

Amplitude of SM →X

Σ_X: BSM states, X summation& LIPS integration

s⇆u crossing





Dispersion Relation (for Positivity Bounds)

- Useful to rewrite Dispersion Relation for Positivity Bounds

(Amp by Dim.8)
∝ $(F/\Lambda^4)\, s^2$

$$M_{ijkl} = \frac{F_\alpha M_\alpha^{ijkl}}{\Lambda^4}$$

$$M^{ijkl} = \int_{(\epsilon\Lambda)^2}^{\infty} \sum_X{}' \sum_{K=R,I} \frac{d\mu\, m_{K\,X}^{\,ij} m_{K\,X}^{\,kl}}{\pi\mu^3} + (j \leftrightarrow l)$$

where $M(ij \to X) \equiv m_{R_X}^{ij} + i m_{I_X}^{ij}$

- When i=k, j=l, RHS complete squares >=0

$$M^{ijij} \geq 0 \quad \text{because} \quad m_{K\,X}^{\,ij} m_{K\,X}^{\,ij} \geq 0$$

- More generally,
  Elastic Forward Scattering between Superposed States :

$M(ab \to ab)$ with $|a\rangle = u^i\,|i\rangle\,, \quad |b\rangle = v^i\,|i\rangle$

||

$$u^i v^j u^{*k} v^{*l} M^{ijkl} = \int_{(\epsilon\Lambda)^2}^{\infty} \sum_X{}' \sum_{K=R,I} \frac{d\mu}{\pi\mu^3} \left[|u \cdot m_{K_X} \cdot v|^2 + |u \cdot m_{K_X} \cdot v^*|^2 \right] \geq 0$$

(generalized) Elastic Positivity Bounds

Summary and Outlook

- We consider Higgs portal dark matter derivative coupled interactions and apply the positivity conditions to them
- We see relic density and relation to the massive graviton case as the partial UV completion
- We obtained constraints from the current LHC
- For HL-LHC search, utilizing the kinematical distributions may be useful
- Constraints from direct and indirect detections will be checked