Revisit the gravitational lensing effect using radio wave polarization

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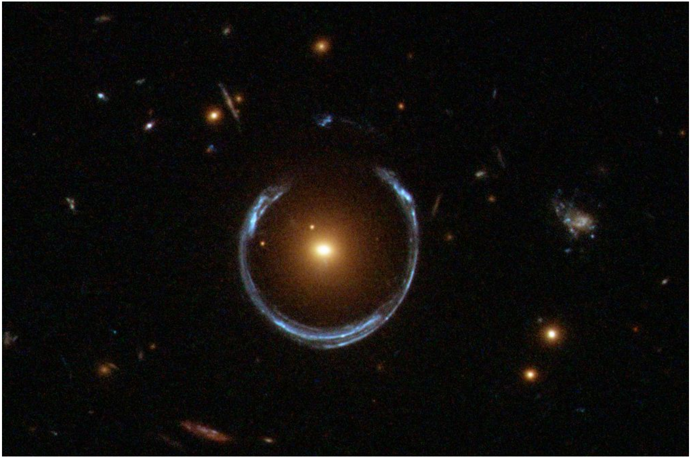
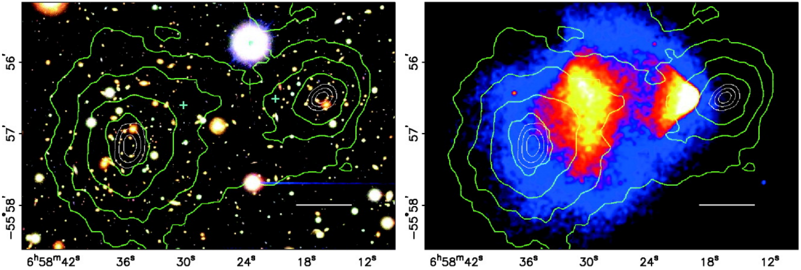
Table of contents

Gravitational lensing



A Horseshoe Einstein Ring from Hubble Space Telescope.
https://commons.wikimedia.org/wiki/File:A_Horseshoe_Einstein_Ring_from_Hubble.JPG

Bullet Cluster, a direct empirical proof?



D. Clowe, M. Bradac, A. H. Gonzalez, M. Markevitch, S. W. Randall, C. Jones and D. Zaritsky, “A direct empirical proof of the existence of dark matter,” Astrophys. J. Lett. **648** (2006), L109-L113 doi:10.1086/508162 [arXiv:astro-ph/0608407 [astro-ph]].

Convergence and shear

θ_1, θ_2: the observed positions in terms of the two orthogonal coordinates on the sky. β_1, β_2: the actual positions

$$A_{ij} \equiv \frac{\partial \beta_i}{\partial \theta_j}$$

$$A = \begin{pmatrix} 1-\kappa+\gamma_1 & \gamma_2 \\ \gamma_2 & 1-\kappa-\gamma_1 \end{pmatrix}$$

κ:convergence (size change) γ: shear (shape change)
Eigenvalues ($\gamma = \sqrt{\gamma_1^2+\gamma_2^2}$)

$$a_+ = 1-\kappa+\gamma, \quad a_- = 1-\kappa-\gamma$$

size change: not directly observable
shape change: directly observable

Reduced shear

Reduced shear

$$g_\alpha \equiv \frac{\gamma_\alpha}{1-\kappa}$$

$$A = \begin{pmatrix} 1-\kappa+\gamma_1 & \gamma_2 \\ \gamma_2 & 1-\kappa-\gamma_1 \end{pmatrix} = (1-\kappa)\begin{pmatrix} 1+g_1 & g_2 \\ g_2 & 1-g_1 \end{pmatrix}$$

$$\nabla \ln(1-\kappa) = \frac{1}{1-g_1^2-g_2^2}\begin{pmatrix} 1+g_1 & g_2 \\ g_2 & 1-g_1 \end{pmatrix}\begin{pmatrix} g_{1,1}+g_{2,2} \\ g_{2,1}-g_{1,2} \end{pmatrix}$$

Current method to estimate reduced shear

$I(\theta)$: the brightness distribution.
$\bar{\theta}_i$: is the center of light of the galaxy image

$$\int d^2\theta I(\theta)(\theta_i - \bar{\theta}_i) = 0$$

The quadrupole moment:

$$Q_{ij} = \frac{\int d^2\theta I(\theta)(\theta_i - \bar{\theta}_i)(\theta_j - \bar{\theta}_j)}{\int d^2\theta I(\theta)}$$

The ellipticity:

$$e_\alpha \equiv Q_\alpha / T$$

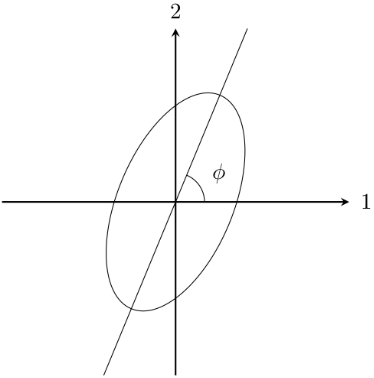
where

$$Q_1 \equiv Q_{11} - Q_{22}, \quad Q_2 \equiv 2Q_{12}, \quad T = Q_{11} + Q_{22}$$

This ellipticity has two components, which can be positive or negative.

Current method to estimate reduced shear

ϕ: the angle between the axis 1 and the major axis of the observed elliptical image of galaxy



$$\tan(2\phi) = \frac{2Q_{12}}{Q_{11} - Q_{22}} = \frac{e_2}{e_1}$$

Current method to estimate reduced shear

Gravitational lensing changes e_α

$$\delta e_\alpha = P^\gamma_{\alpha\beta} g_\beta$$

$P^\gamma_{\alpha\beta}$: the shear susceptibility tensor
g_β (g_1 and g_2): the reduced shear.

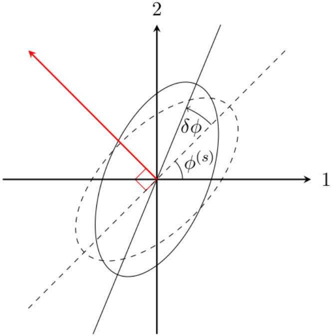
$$g_\beta = P^{\gamma-1}_{\alpha\beta}(e_\alpha - e^{(s)}_\alpha)$$

(s) denotes the "source." If the orientation of galaxies are random

$$\langle e^{(s)}_\alpha \rangle = 0$$

Average of $P^{\gamma-1}_{\alpha\beta} e_\alpha$ is g_β.

Image rotation from lensing [arXiv:2106.08631] (Francfort et al., *Class.Quant.Grav*, 38 (2021) 24, 245008)



$$\theta_{\rm pol} = \phi^{(s)} + 90^\circ$$

$$\gamma_2 \cos 2\phi^{(s)} - \gamma_1 \sin 2\phi^{(s)} = \frac{\varepsilon^2}{2-\varepsilon^2}\delta\phi$$

ε: the usual ellipticity

$$\varepsilon = \sqrt{1 - \frac{b^2}{a^2}}$$

Not the shear, but the reduced shear

Francfort et al. deliberately ignored the size change (shear).

$$D = D_s \begin{pmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{pmatrix} \exp\begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & -\gamma_1 \end{pmatrix} \tag{1}$$

The exponential is given by

$$\exp\begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & -\gamma_1 \end{pmatrix} = \begin{pmatrix} 1+\gamma_1 & \gamma_2 \\ \gamma_2 & 1-\gamma_1 \end{pmatrix} \tag{2}$$

$$A = \begin{pmatrix} 1-\kappa+\gamma_1 & \gamma_2 \\ \gamma_2 & 1-\kappa-\gamma_1 \end{pmatrix} = (1-\kappa)\begin{pmatrix} 1+g_1 & g_2 \\ g_2 & 1-g_1 \end{pmatrix}$$

$$\log(a_+/a_-)/2 = \log((1+\gamma)/(1-\gamma))/2 = \gamma + \mathcal{O}(\gamma^3)$$

$$\frac{1}{2}\log\left(\frac{1-\kappa+\gamma}{1-\kappa-\gamma}\right) = \frac{1}{2}\log\left(\frac{1+g}{1-g}\right) = g + \mathcal{O}(g^3)$$

Not the shear, but the reduced shear

If they considered the size change, the shear in their formula must be replaced by the reduced shear.

$$g_2 \cos 2\phi^{(s)} - g_1 \sin 2\phi^{(s)} = \frac{\varepsilon^2}{2 - \varepsilon^2} \delta\phi$$

"Linear regression"

Discussions and Conclusions

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Discussions and Conclusions

- In the previous method, (theoretically) infinite data points are needed for the assumption that the average ellipticity is zero.
- In the new method, only two data points are needed (theoretically) to determine g_1 and g_2 by linear regression.
- If the gravitational lensing effect at the Bullet Cluster is reanalyzed, using the polarization data of radio wave, we will be more sure whether dark matter exists or not.