Non-minimally assisted chaotic inflation

Sang Chul Hyun (Yonsei University)

in collaboration with

Jinsu Kim (CERN),

Seong Chan Park (Yonsei University / KIAS),

Tomo Takahashi (Saga University)

Based on JCAP 05 (2022) 045 **(arXiv : 2203.09201)**

Workshop on Physics of Dark Cosmos (in Busan, October 21~23 2022)

2022.10.22.

Chaotic Inflation

Chaotic Inflationary model is the most simplest single-field inflationary model, where is dictated by :

Two free parameters

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{Pl}^2 R - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)\right] \qquad V(\varphi) = \lambda_\varphi M_{Pl}^4\left(\frac{\varphi}{M_{Pl}}\right)^n$$

A. D. Linde (1983)

In this model, theoretical calculations of CMB observables are given by

Scalar Spectral Index Tensor-to-Scalar Ratio

$$n_s \simeq 1 - \frac{2(n+2)}{n+4N}, r \simeq \frac{16n}{n+4N} \quad \text{with} \quad \lambda_\varphi \simeq \begin{cases} 10^{-11}, & n=2 \\ 10^{-13}, & n=4 \end{cases}$$

where $N = \int_\star^e H dt$ expresses the number of e-folds during evolution of inflaton field from horizon crossing point(denoted by $t_\star$) to the end-of-inflation point(denoted by t_e).

Issues of Chaotic Inflation

Planck Normalization condition(required to match amplitude for scalar power spectrum with CMB data) determines extremely small coefficients of inflaton potential. For example,

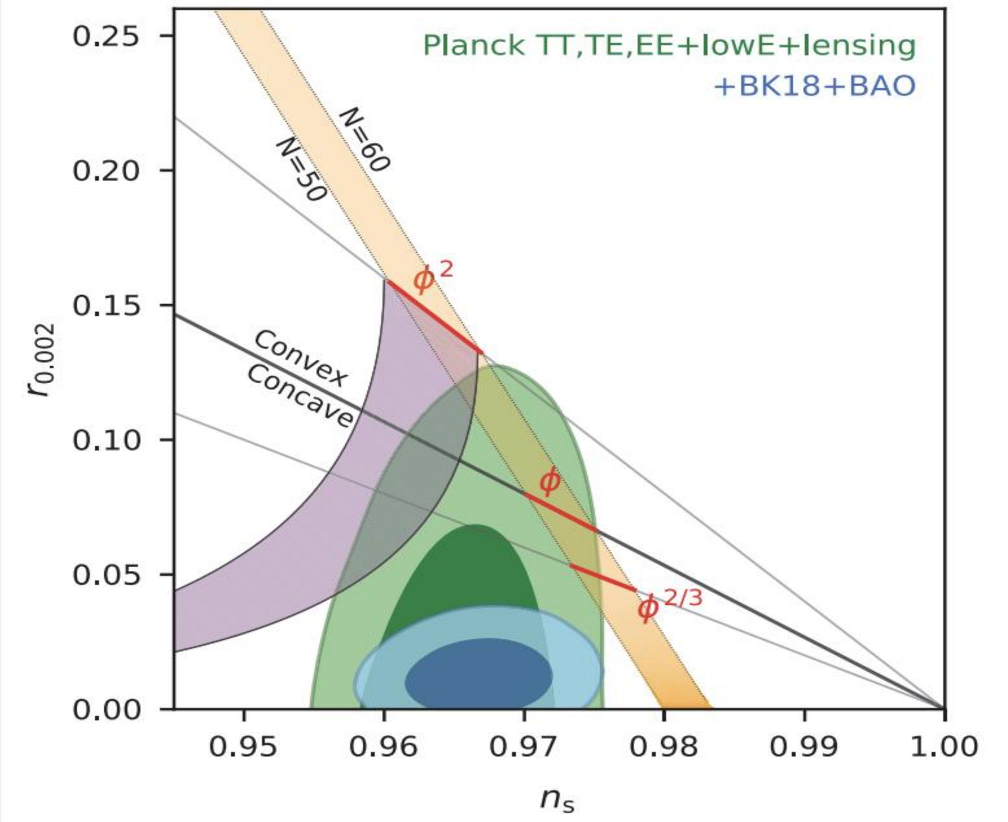
$$\lambda_\varphi \simeq \begin{cases} 10^{-11}, & n=2 \\ 10^{-13}, & n=4 \end{cases}$$

Chaotic inflationary model with any value of exponent n is ruled out by experimental constraints given by Planck + BICEP/Keck Array.

Scalar Spectral Index

$$n_s \simeq 1 - \frac{2(n+2)}{n+4N}, r \simeq \frac{16n}{n+4N}$$

Tensor-to-Scalar Ratio



· Y. Akrami et al. (2020), P. A. R. Ade et al. (2021)

Our idea

To solve aforementioned problems that original chaotic inflation has, we aim to modify that scenario. There are lots of methods by which we can change chaotic inflationary model.

In this work, we planned to add another scalar field named s. Then, action would be

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{Pl}^2 R - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)\right]$$

$$\rightarrow S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{Pl}^2\Omega^2(s,\varphi)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g^{\mu\nu}\partial_\mu s\partial_\nu s - V(s,\varphi)\right]$$

Generally, we can both consider effects between non-minimal coupling of two fields between gravitational sector.

$$\Omega^2(s,\varphi) = 1 + g\left(\frac{\varphi}{M_{Pl}}\right) + f\left(\frac{s}{M_{Pl}}\right)$$

Our idea

We think newly introduced field s as assistant field.

$$V(s,\varphi) \rightarrow V(\varphi)$$
$$\Omega^2(s,\varphi) = 1 + g\left(\frac{\varphi}{M_{Pl}}\right) + f\left(\frac{s}{M_{Pl}}\right) \rightarrow 1 + f\left(\frac{s}{M_{Pl}}\right)$$

Our action then becomes

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{Pl}^2\Omega^2(s,\varphi)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g^{\mu\nu}\partial_\mu s\partial_\nu s - V(s,\varphi)\right]$$
$$\rightarrow S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{Pl}^2\left(1 + f\left(\frac{s}{M_{Pl}}\right)\right)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g^{\mu\nu}\partial_\mu s\partial_\nu s - V(\varphi)\right]$$

Small field approximation

Non-minimal coupling

$$\Omega^2(s,\varphi) = 1 + f\left(\frac{s}{M_{Pl}}\right)$$

In this work, we are interested at the situation where the value of assistant field during inflation is extremely small, unlike original inflaton φ.

$$s \ll M_{Pl}$$

Thanks to above assumption, we can do Taylor expansion of non-minimal coupling function.

$$f\left(\frac{s}{M_{Pl}}\right) = f(0) + f'(0)\frac{s}{M_{Pl}} + \frac{1}{2!}f''(0)\frac{s^2}{M_{Pl}^2} + \cdots$$
$$= \xi_2\frac{s^2}{M_{Pl}^2} + \xi_4\frac{s^4}{M_{Pl}^4} + \cdots = \sum \xi_m \frac{s^m}{M_{Pl}^m}$$

Z_2 symmetry ($s \rightarrow -s$)

In this work, we have considered two cases.

$$\Omega^2(s) \approx 1 + \xi_2\frac{s^2}{M_{Pl}^2}$$ Quadratic Case $(m=2)$

$$\Omega^2(s) \approx 1 + \xi_4\frac{s^4}{M_{Pl}^4}$$ Quartic Case $(m=4)$

Weyl transformation

Converting Jordan-frame action into Einstein-frame one,

$$S_J = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{Pl}^2\Omega^2(s)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g^{\mu\nu}\partial_\mu s\partial_\nu s - V(\varphi)\right]$$

$$g_{E,\mu\nu} = \Omega^2 g_{J,\mu\nu},\, \Omega^2 \equiv 1 + \xi_m(s/M_{Pl})^m$$

$$R = \Omega^2[R_E - 6g^{E,\mu\nu}\nabla_\mu\nabla_\nu \ln\Omega - 6g^{E,\mu\nu}\nabla_\mu \ln\Omega\nabla_\nu \ln\Omega]$$

$$S_E = \int d^4x\sqrt{-g_E}\left[\frac{M_{Pl}^2}{2}R_E - \frac{1}{2}\mathcal{K}_1 g_E^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}\mathcal{K}_2 g_E^{\mu\nu}\partial_\mu s\partial_\nu s - V_E(\varphi, s)\right]$$

Einstein-frame potential

$$V_E(\varphi, s) = \frac{V(\varphi)}{\Omega^4} = \frac{V(\varphi)}{(1+\xi_m s^m/M_{Pl}^m)^2} \qquad \mathcal{K}_1 \equiv \frac{1}{\Omega^2}, \quad \mathcal{K}_2 \equiv \frac{\Omega^2 + (3/2)m^2\xi_m^2(s/M_{Pl})^{2m-2}}{\Omega^4}$$

Canonical field

Introducing another scalar field σ to canonically normalize kinetic term of assistant field,

$$S_E = \int d^4x\sqrt{-g_E}\left[\frac{M_{Pl}^2}{2}R_E - \frac{1}{2}\mathcal{K}_1 g_E^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}\mathcal{K}_2 g_E^{\mu\nu}\partial_\mu s\partial_\nu s - V_E(\varphi, s)\right]$$

$$= \int d^4x\sqrt{-g_E}\left[\frac{M_{Pl}^2}{2}R_E - \frac{1}{2\Omega^2}(\partial\varphi)^2 - \frac{1}{2}(\partial\sigma)^2 - V_E(\varphi, \sigma(s))\right]$$

Einstein-frame potential

$$V_E(\varphi, s) = \frac{V(\varphi)}{\Omega^4} = \frac{V(\varphi)}{(1+\xi_m s^m/M_{Pl}^m)^2}$$

Normalized field σ

$$\left(\frac{\partial\sigma}{\partial s}\right)^2 = \frac{1+\xi_m s^m/M_{Pl}^m + (3/2)m^2\xi_m^2(s/M_{Pl})^{2m-2}}{(1+\xi_m s^m/M_{Pl}^m)^2}$$

Connecting to Natural Inflation

$$S_E = \int d^4x \sqrt{-g_E} \left[\frac{M_{Pl}^2}{2} R_E - \frac{1}{2\Omega^2} (\partial\varphi)^2 - \frac{1}{2} (\partial\sigma)^2 - V_E(\varphi, \sigma(s)) \right]$$

When $m = 2, \xi_m s^m / M_{Pl}^m \ll 1$,

$$V_E(\varphi, \sigma(s)) = \frac{\lambda_\varphi M_{Pl}^4 (\varphi / M_{Pl})^n}{(1 + \xi_m s^m / M_{Pl}^m)^2} \simeq \lambda_\varphi M_{Pl}^4 \left(\frac{\varphi}{M_{Pl}} \right)^n \left(1 - 2\xi_2 \frac{s^2}{M_{Pl}^2} \right)$$

$$\simeq \lambda_\varphi M_{Pl}^4 \left(\frac{\varphi}{M_{Pl}} \right)^n \left[1 + \cos\left(\frac{s}{f} \right) \right]$$

Chaotic Inflation Natural Inflation

$$f \equiv (2\sqrt{2\xi_2})^{-1} M_{Pl}$$

Einstein-frame potential

$$V_E(\varphi, s) = \frac{V(\varphi)}{\Omega^4} = \frac{V(\varphi)}{(1 + \xi_m s^m / M_{Pl}^m)^2}$$

Normalized field σ

$$\left(\frac{\partial\sigma}{\partial s} \right)^2 = \frac{1 + \xi_m s^m / M_{Pl}^m + (3/2) m^2 \xi_m^2 (s / M_{Pl})^{2m-2}}{(1 + \xi_m s^m / M_{Pl}^m)^2}$$

Slow-roll analysis & δN formalism

- We transformed action in Jordan frame to Einstein frame and earned approximated equation of motions by using slow-roll assumption.

Slow-roll assumption : $\{\epsilon^i, |\eta^{ij}|, \epsilon^b\} \ll 1$

$$\epsilon^\sigma \equiv \frac{M_{Pl}^2}{2}\left(\frac{V_{,\sigma}}{V}\right)^2, \epsilon^\varphi \equiv \frac{M_{Pl}^2}{2}\left(\frac{V_{,\varphi}}{V}e^{-b}\right)^2, \eta^{\sigma\sigma} \equiv M_{Pl}^2\frac{V_{,\sigma\sigma}}{V},$$

$$\eta^{\varphi\varphi} \equiv M_{Pl}^2\frac{V_{,\varphi\varphi}}{V}e^{-2b}, \eta^{\varphi\sigma} \equiv M_{Pl}^2\frac{V_{,\varphi\sigma}}{V}e^{-b}, \epsilon^b \equiv 8M_{Pl}^2 b_{,\sigma}^2$$

- We calculated three CMB observables : spectral index n_s, tensor-to-scalar ratio r and local-type nonlinearity parameter $f_{NL}^{(local)}$ to match with latest constraints. We used δN formalism to calculate these and plotted them numerically.

· *A. A. Starobinsky, PLB117, 175. (1982),*
· *D. S. Salopek, J. R. Bond, PRD42, 3936 (1990),*
· *M. Sasaki and E. D. Stewart, PTP 95, 71. (1996)*

Cosmological observables

Expressions for the cosmological observables in the δN formalism ($N_{,i} \equiv \frac{\partial N}{\partial \varphi^i}$ $(\varphi^i = \{\sigma, \varphi\})$)

❖Curvature power spectrum : $\mathcal{P}_\zeta = \left(\frac{H}{2\pi}\right)^2 G^{ij} N_{,i} N_{,j}$

❖Scalar Spectral Index : $n_s = 1 + 2\frac{\dot{H}}{H^2} - 2\frac{1 + N_{,k}\left(\frac{M_{Pl}^6}{3} R^{kmnl} V_{,m} V_{,n}/V^2 - M_{Pl}^4 V^{;kl}/V\right) N_{,l}}{G^{ij} N_{,i} N_{,j} M_{Pl}^2}$

❖Tensor-to-Scalar Ratio : $r = \frac{8}{M_{Pl}^2 G^{ij} N_{,i} N_{,j}}$

❖Local-type nonlinearity parameter : $f_{NL}^{local} = -\frac{5}{6}\frac{G^{ij} G^{mn} N_{,i} N_{,m} N_{,jn}}{(G^{kl} N_{,k} N_{,l})^2}$

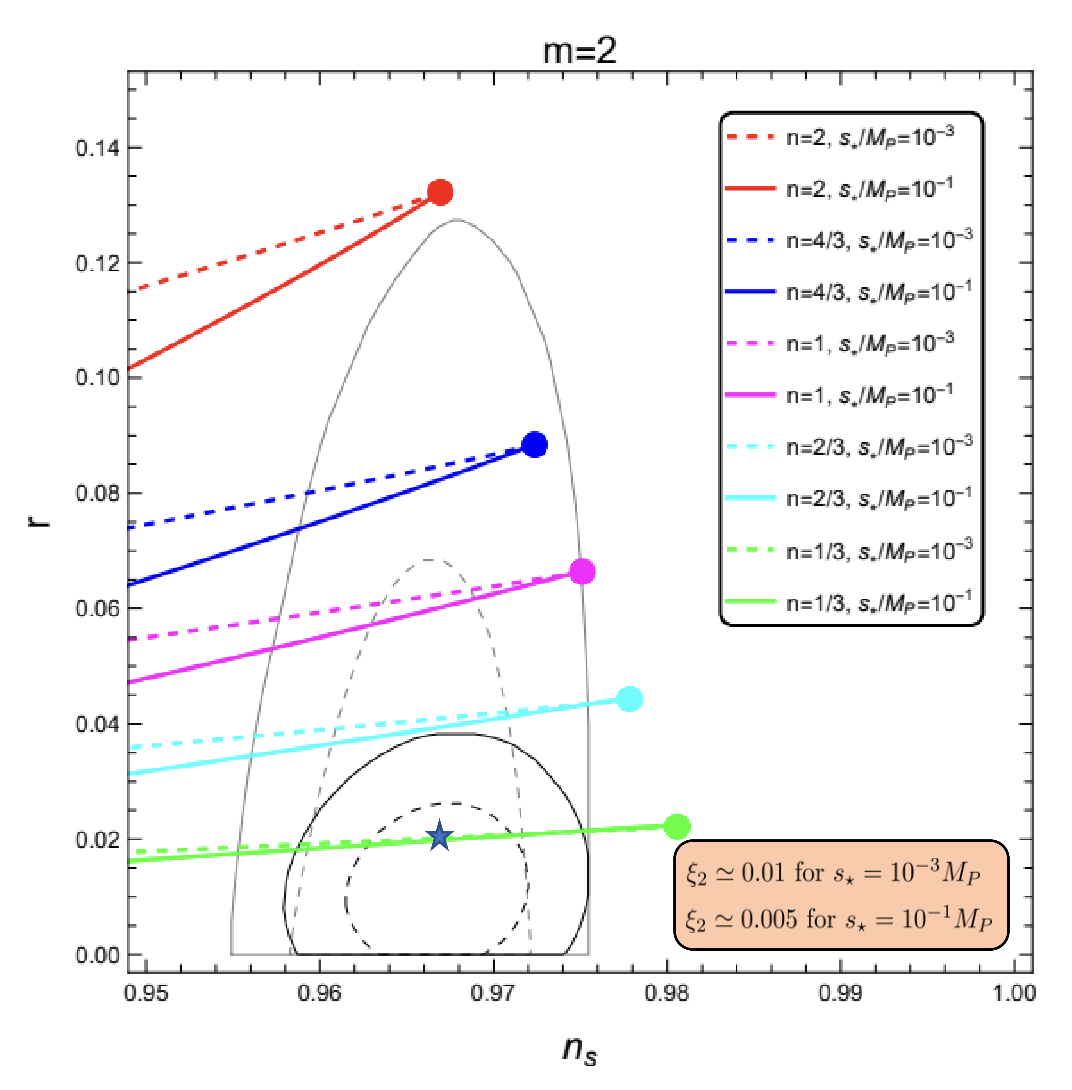
- There exist three independent parameters : $n, s_\star, \xi_m$.
- We set the number of e-folds from horizon crossing point($\star$) to end-of-inflation point(e) to be equal to 60. ($N = \int_\star^e H dt = 60$)

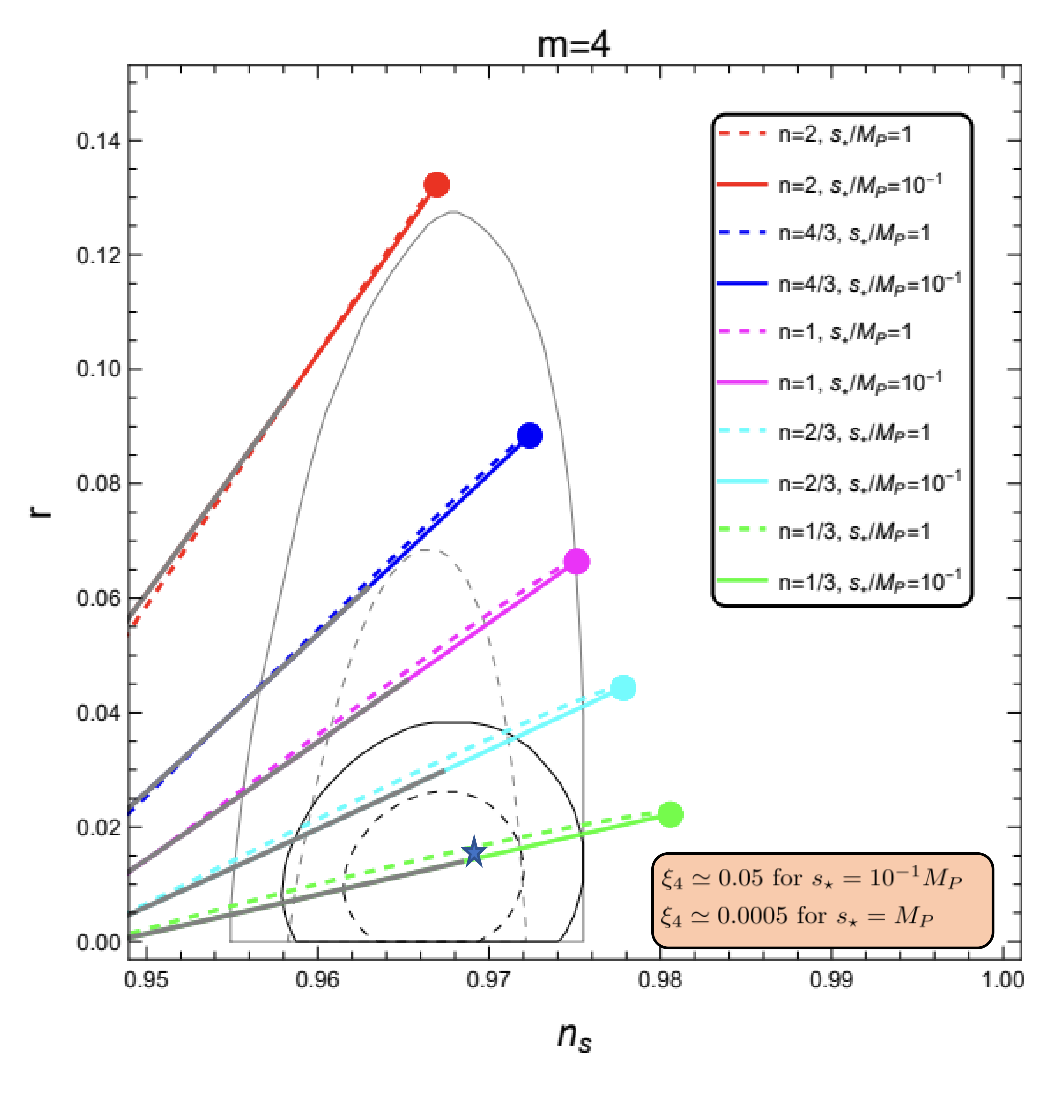
Field Space metric

$$G_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & e^{2b(s)} \end{pmatrix}, b(s) \equiv \frac{1}{2}\ln \mathcal{K}_1 = \frac{1}{2}\ln\left(\frac{1}{1 + \xi_m s^m / M_{Pl}^m}\right)$$

J. Kim, Y. Kim and S. C. Park, CQG31, 135004 (2014)

Results





· *Planck (2020), Planck + BICEP/Keck (2021)*

Summary

- Chaotic inflation with any power $V \sim \varphi^n$ is ruled out by the recent Planck-BICEP/Keck constraints.
- In order to rescue this problem, we introduced assistant field with original inflaton. Only assistant field which couples to gravity non-minimally, unlike original inflaton field.
- Both $m = 2$ and $m = 4$ cases, it is possible to rescue chaotic inflation with certain value of n and $s_\star$, just by effect coming from non-minimal coupling, without changing potential.

Thank you for your attention!