

Primordial Black Holes and Gravitational Waves from the Tachyonic Instability in Higgs- R^2 Inflation

“The Inflaton that Could”



Dhong Yeon Cheong (Yonsei University)

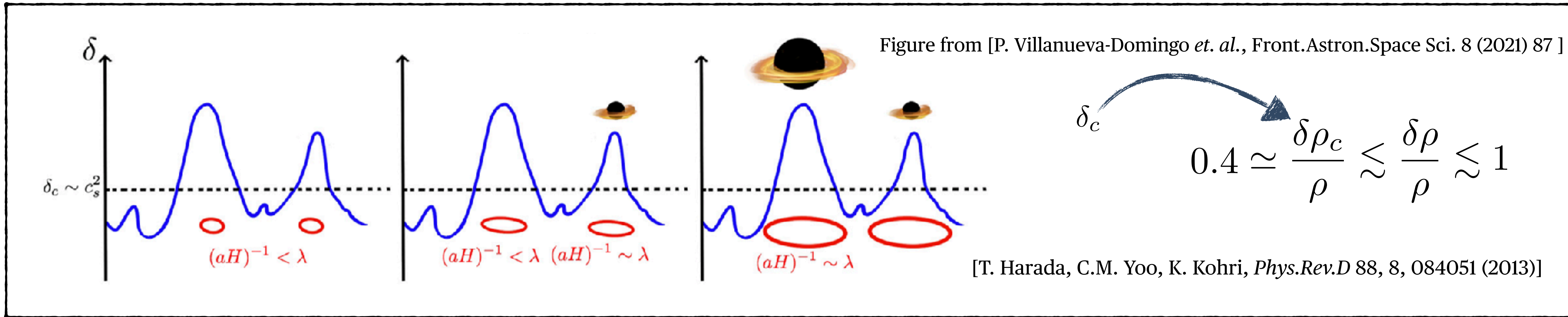
with Kazunori Kohri (KEK, IPMU), Seong Chan Park (Yonsei University)

DYC, K.Kohri, S.C.Park, *JCAP* 10 (2022) 015 (arXiv : 2205.14813)

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Introduction - Primordial Black Holes



PBHs form through the collapse of large over-densities in the density perturbation

$$k \leftrightarrow M_{\text{PBH}} \rightarrow \mathcal{P}_{\mathcal{R}}(k) \rightarrow \mathcal{P}_{\delta}(k) \rightarrow \beta(M) = \frac{\rho_{\text{PBH}}}{\rho_{\text{tot}}} \quad \frac{M_{\text{PBH}}}{M_{\odot}} \sim \left(\frac{k_*}{3.3 \times 10^6 \text{Mpc}^{-1}} \right)^{-2} \sim \left(\frac{f_*}{5.0 \times 10^{-9} \text{Hz}} \right)^{-2}$$

Many mechanisms to produce PBHs —> Inflation is appealing!



Inflation, phase transition, reheating, etc etc..

Introduction - Primordial Black Holes

Then, how large should the density perturbations / curvature perturbations be?

$$\beta \sim \frac{\sigma}{\sqrt{2\pi}\delta_c} e^{-\delta_c^2/(2\sigma^2)} \sim e^{-\delta_c^2/\mathcal{P}_{\mathcal{R}}} \leftarrow \frac{\rho_{\text{PBH}}}{\rho_{\text{tot}}}\Big|_{\text{form}}$$

$$\sigma^2(R) = \int_0^\infty W^2(kR) \mathcal{P}_\delta(k) d\ln k$$

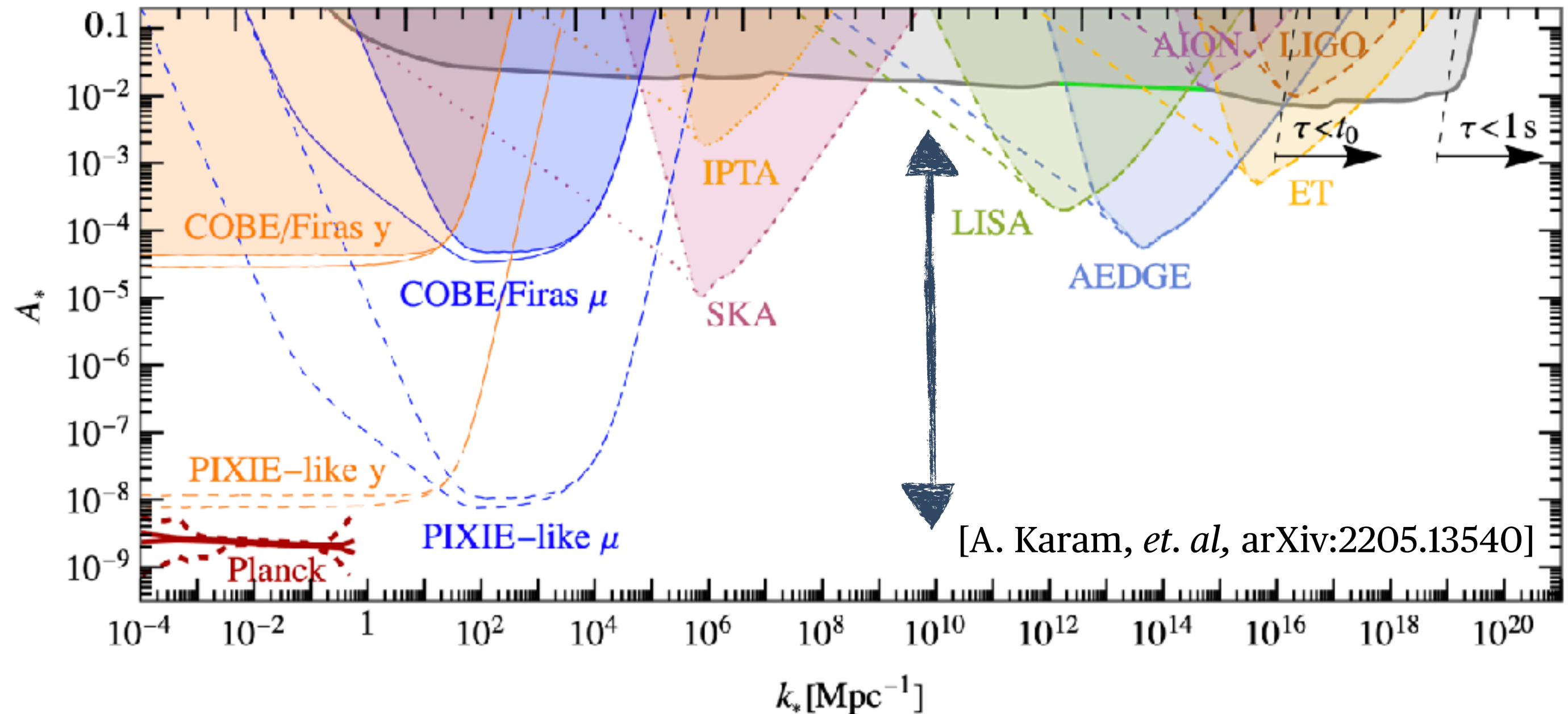
Variance, Naive! $\sigma^2 \sim \mathcal{P}_{\mathcal{R}}$ Exponential dependence

$$f_{\text{PBH}} \equiv \frac{\rho_{\text{PBH}}}{\rho_{\text{DM}}} = \left(\frac{a_{\text{eq}}}{a_{\text{form}}} \right) \beta(M)$$

Take $f_{\text{PBH}} \sim \mathcal{O}(1) \rightarrow \beta(M) \sim 10^{-8}$ then

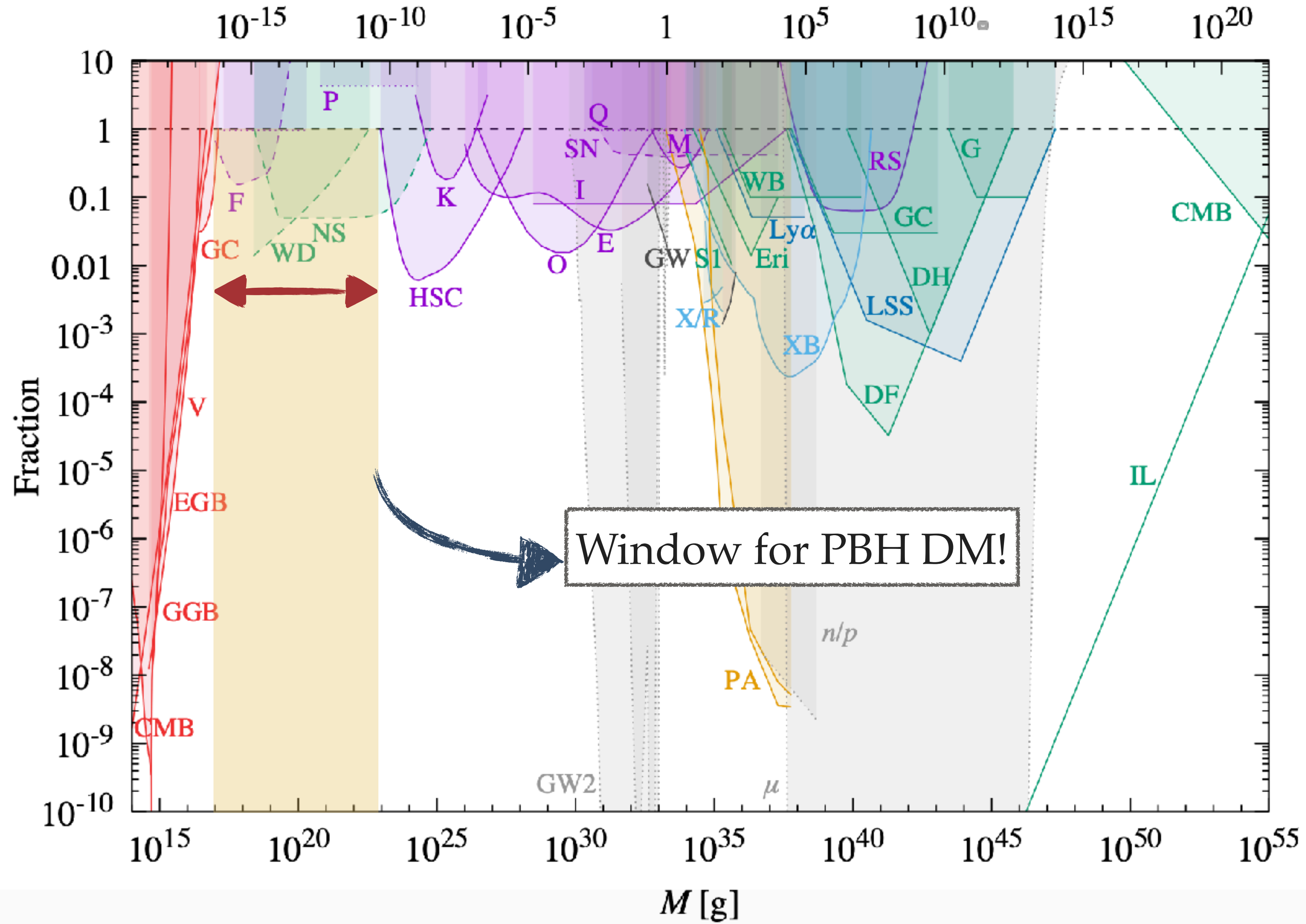
$$\mathcal{P}_{\mathcal{R}} \sim \mathcal{O}(10^{-2})$$

For solar-mass black holes, $a_{\text{eq}}/a_{\text{form}} \sim 10^8$ in RD



Introduction - Primordial Black Holes

M/M_{\odot} [B. Carr, K. Kohri, Y. Sendouda, J. Yokoyama, *Rept.Prog.Phys.* 84, 11, 116902 (2021)]



Introduction - Second Order GWs

Scalar and tensor perturbations couple at “second order metric perturbations”.

$$\Omega_{\text{GW}}(\eta_0, k) = c_g \frac{\Omega_{r,0}}{6} \int_0^\infty dv \int_{|1-v|}^{1+v} du \left(\frac{4v^2 - (1 + v^2 - u^2)^2}{4uv} \right)^2 \overline{\mathcal{I}^2(v, u)} \mathcal{P}_{\mathcal{R}}(kv) \mathcal{P}_{\mathcal{R}}(ku)$$

[K. Kohri, T. Terada, Phys. Rev. D **97**, 123532 (2018)]

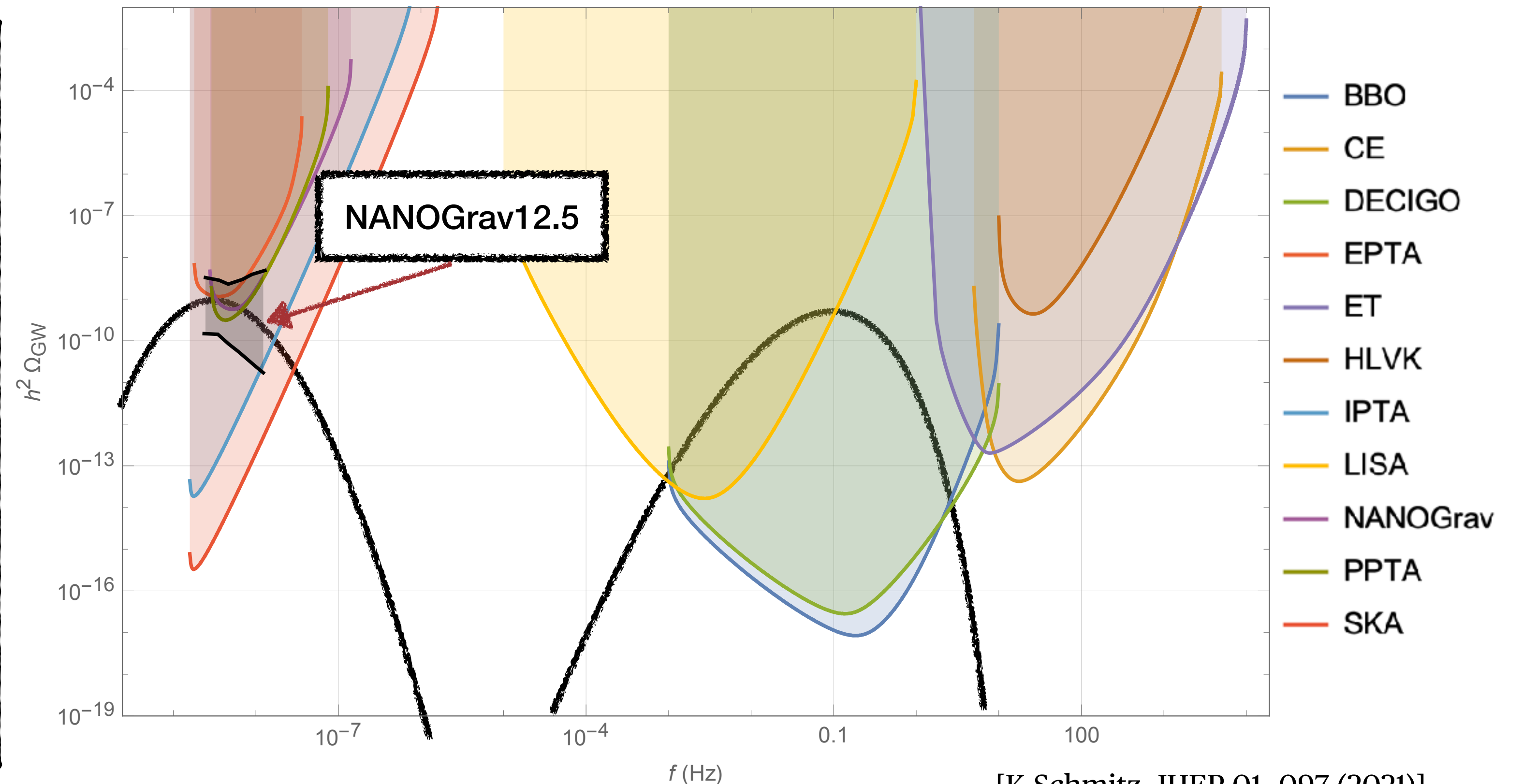
$$\Omega_{\text{GW}} h^2 \sim \frac{1}{12} \Omega_{r,0} h^2 \times \mathcal{P}_{\mathcal{R}}^2 \sim 10^{-6} \mathcal{P}_{\mathcal{R}}^2$$

DECIGO, SKA: $\mathcal{P}_{\mathcal{R}} \sim 10^{-5}$

LISA, CE, ET: $\mathcal{P}_{\mathcal{R}} \sim 10^{-4}$

NANOGrav12.5: $\mathcal{P}_{\mathcal{R}} \sim 10^{-2}$

Future GW observatories well involved
in probing the small scale.



[K.Schmitz, JHEP 01, 097 (2021)]

[Z. Arzoumanian et al, Astrophys.J.Lett. 905, 2, L34 (2020)]

Higgs- R^2 Inflation

Q1) What can be our inflaton?

Considering dim-4 operators, R^2 present as well!

[Y. Ema, *Phys.Lett.B* 770 (2017) 403-411]

[Y-C. Wang, T. Wang, *Phys.Rev.D* 96 (2017) 12, 123506]

[M.He, A. A. Starobinsky, J. Yokoyama, *JCAP* 05 (2018) 064]

$$S_J = \int d^4x \sqrt{-g_J} \left[\frac{M_P^2}{2} \left(R_J + \frac{\xi h^2}{M_P^2} R_J + \frac{R_J^2}{6M^2} \right) - \frac{1}{2} g^{\mu\nu} \nabla_\mu h \nabla_\nu h - \frac{\lambda}{4} h^4 \right],$$

non minimal coupling
 R^2 term
Higgs

Conformal transformation into Einstein frame

$$S_E = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R - \frac{1}{2} g^{\mu\nu} \nabla_\mu s \nabla_\nu s - \frac{1}{2} e^{-\sqrt{\frac{2}{3}} \frac{s}{M_P}} g^{\mu\nu} \nabla_\mu h \nabla_\nu h - U(s, h) \right]$$

$$U(s, h) \equiv e^{-2\sqrt{\frac{2}{3}} \frac{s}{M_P}} \left\{ \frac{3}{4} M_P^2 M^2 \left(e^{\sqrt{\frac{2}{3}} \frac{s}{M_P}} - 1 - \frac{\xi h^2}{M_P^2} \right)^2 + \frac{\lambda_{\text{eff}}(\mu)}{4} h^4 \right\}$$

Higgs- R^2 Inflation

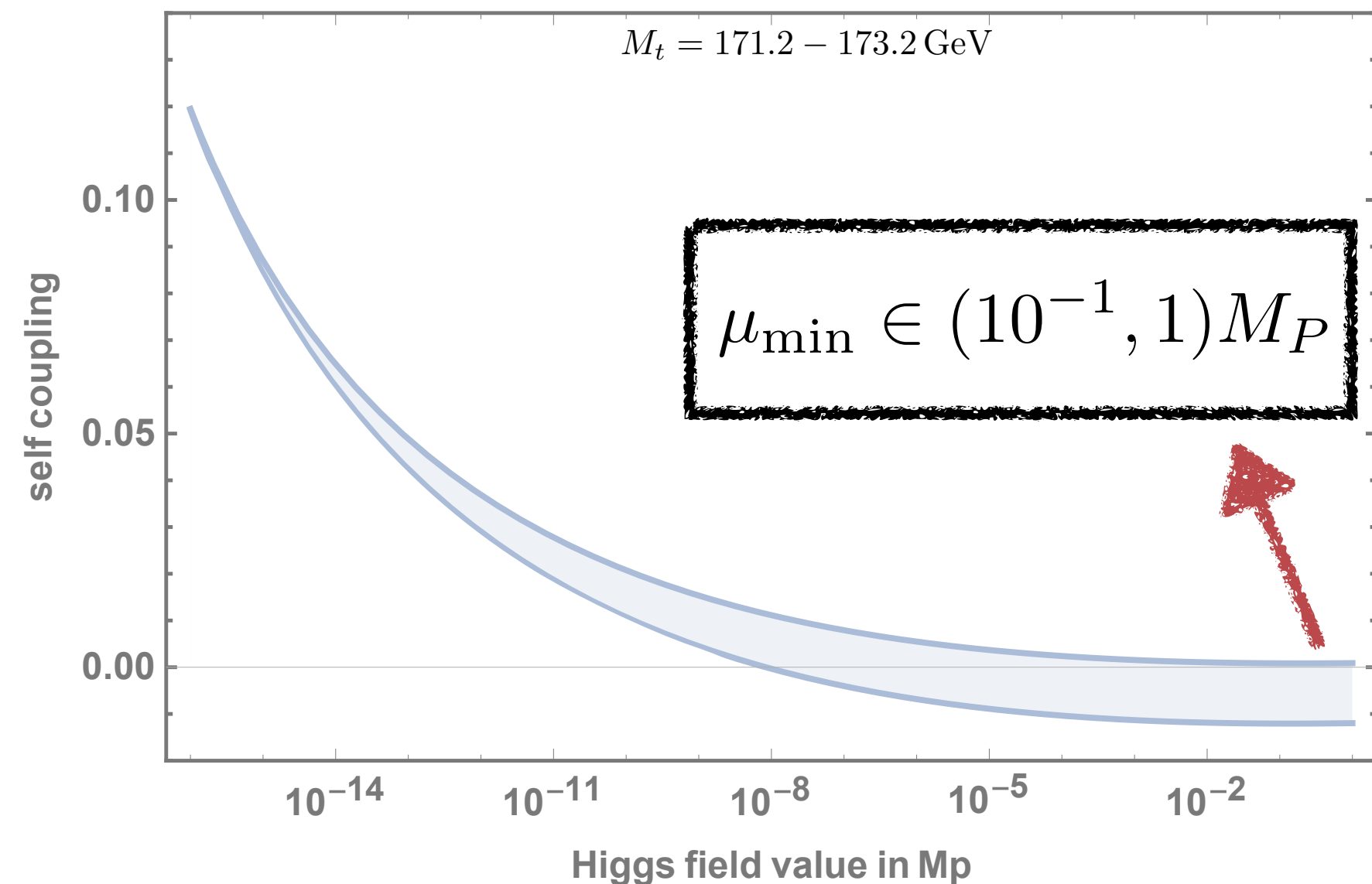
Q1) What can be our inflaton? \rightarrow Higgs + R^2

$$U(s, h) \equiv e^{-2\sqrt{\frac{2}{3}}\frac{s}{M_P}} \left\{ \frac{3}{4}M_P^2 M^2 \left(e^{\sqrt{\frac{2}{3}}\frac{s}{M_P}} - 1 - \frac{\xi h^2}{M_P^2} \right)^2 + \frac{\lambda}{4}h^4 \right\}$$

$(s, h) \simeq (0, 0)$

$$\simeq \frac{\lambda}{4}h^4 + \frac{3\xi^2 M^2}{4M_P^2}h^4 + \frac{1}{2}M^2 s^2 + \dots - \frac{\lambda}{\sqrt{6}M_P}sh^4 - \frac{M^2}{6\sqrt{6}M_P^3}s^5 + \left(\frac{\lambda}{3M_P^2} + \frac{\xi^2 M^2}{M_P^4} \right) h^4 s^2 + \dots$$

$$\Lambda \sim \mathcal{O}\left(\frac{M_P^2}{\xi^2 M^2}\right) M_P > M_P \quad \text{Theory unitarized through the scalaron!}$$



Choosing $\mu = h$

$$\lambda(\mu)|_{\mu=h} = \lambda_m + b \ln^2(|h|/h_m)$$

$$\beta_2^{SM} \simeq 0.5$$

$$b = \frac{\beta_2}{(16\pi^2)^2}$$

Critical values where $\lambda_m \sim \mathcal{O}(10^{-6})$ possible

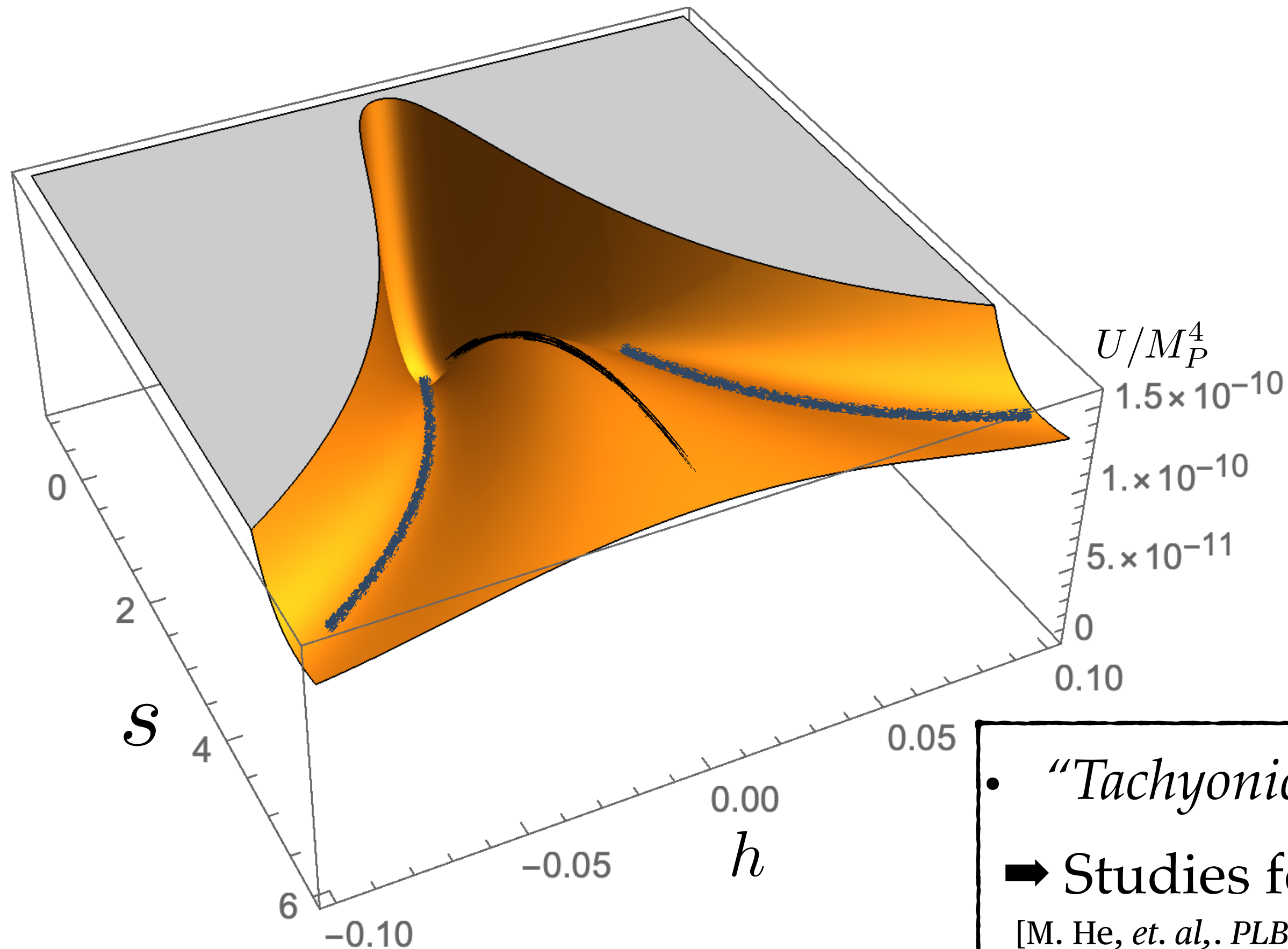
Single-field case : “Critical Higgs inflation”

[Y. Hamada, H. Kawai, K-y. Oda, S.C. Park *Phys.Rev.Lett.* 112 (2014) 24]

[F. Bezrukov, M. Shaposhnikov *Phys.Lett.B* 734 (2014) 249-254]

Higgs- R^2 Inflation

- Shape of the potential? Focus on constant λ for simplicity



- “Valley” structure, trajectory (initially) follows,

$$\frac{\partial U(s, h)}{\partial h} = 0 \quad h_v^2 = \frac{e^{\sqrt{\frac{2}{3}} \frac{s}{M_P}} - 1}{\frac{\xi}{M_P^2} + \frac{\lambda}{3\xi M^2}} \quad \text{for } \phi > 0$$

- ➔ Isocurvature mass heavier than the Hubble scale, suppression in perturbations.
- ➔ Flat plateau induced in the large s limit.
- ➔ Running of λ can induce a USR phase.

[DYC, S.M. Lee, S.C. Park, *JCAP* 01 (2021), 032]

- “Tachyonic hill” at $h = 0$ for $s > 0$

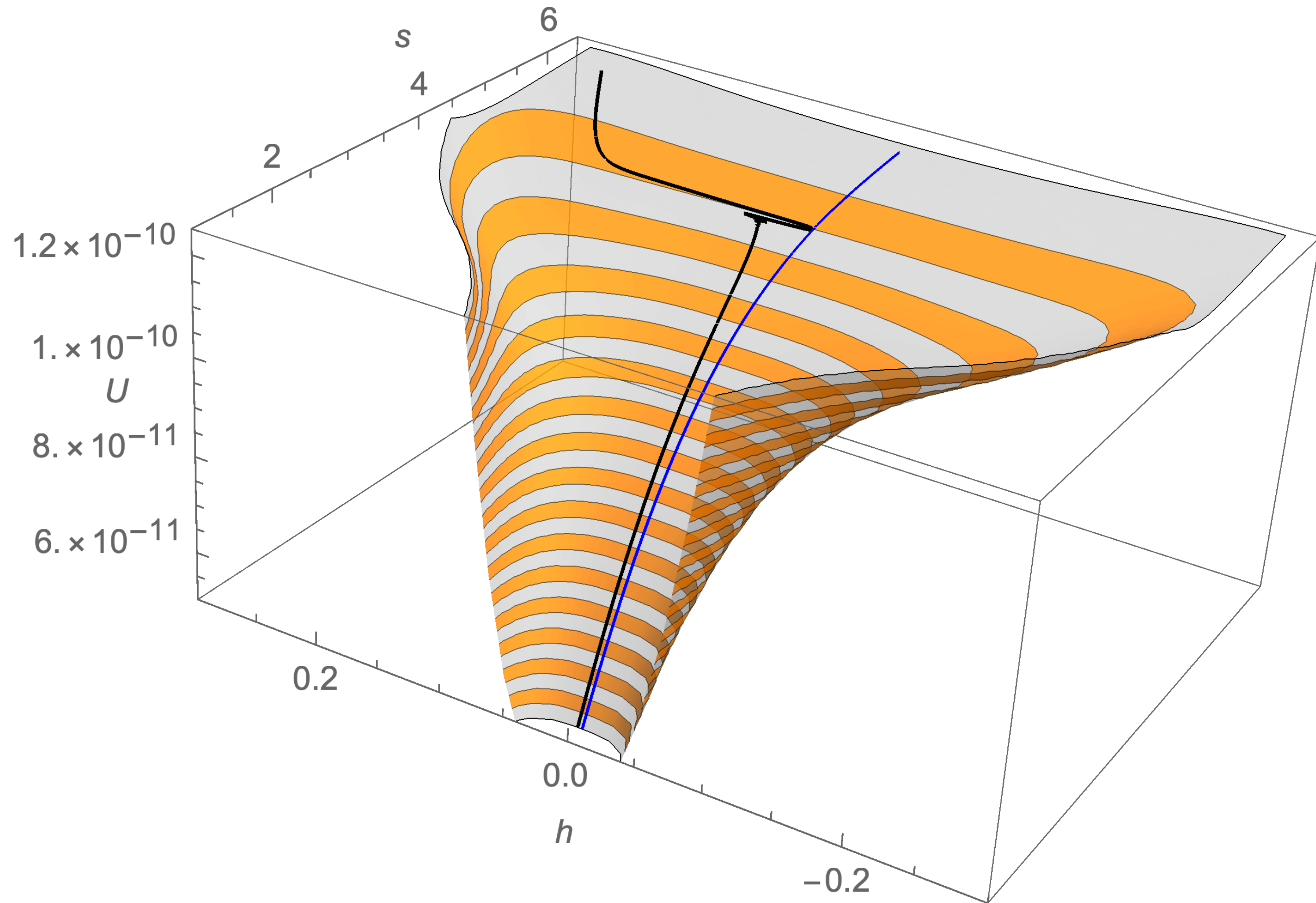
- ➔ Studies focused on a “tachyonic instability” during (p)reheating

[M. He, *et. al.*, *PLB*, 791, 36-42 (2019).] [F. Bezrukov, *et. al.*, *PLB*, 795, 657-665 (2019).]

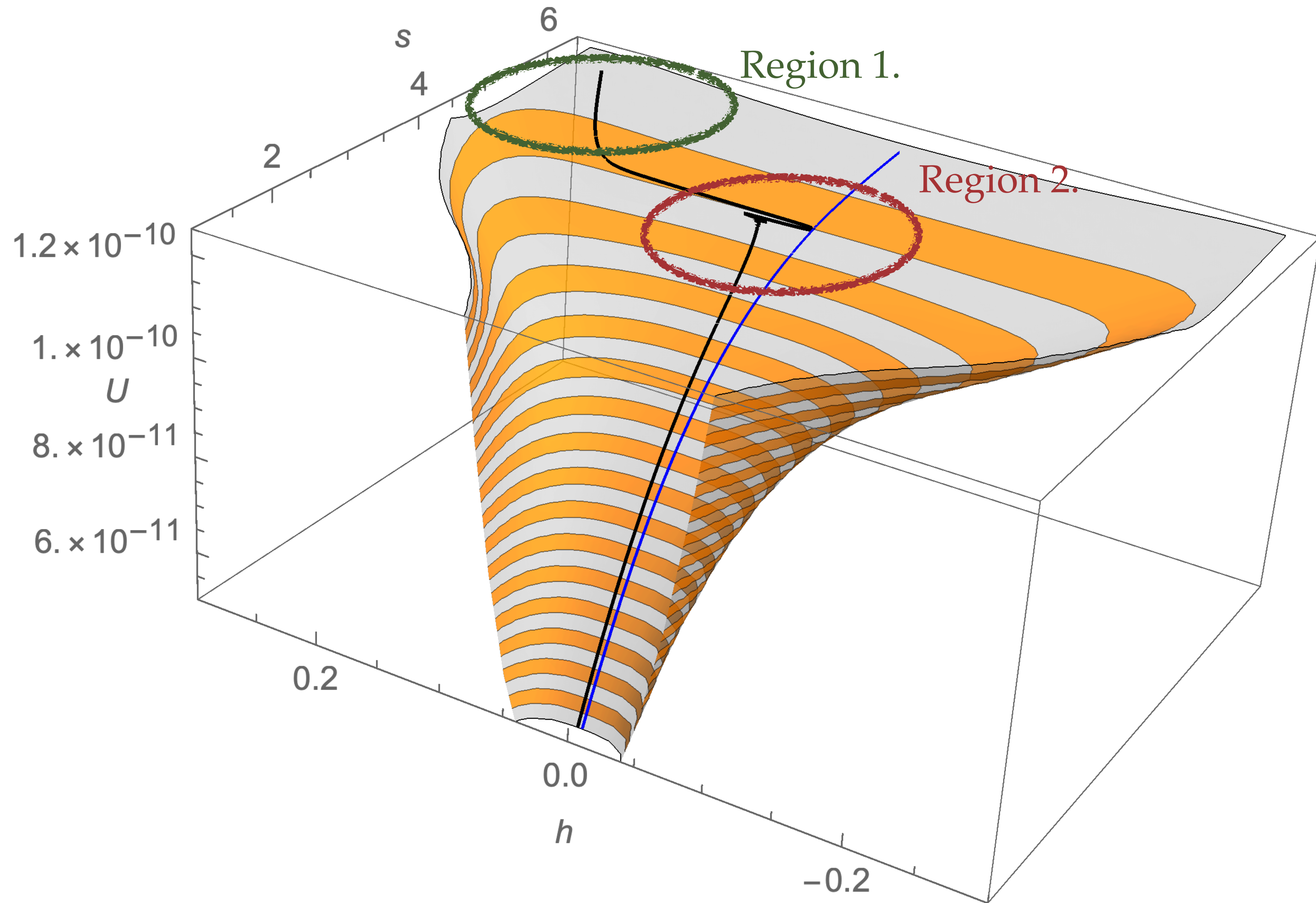
[M. He, *et. al.*, *JCAP* 01 (2021) 066.] [F. Bezrukov, *et. al.*, *JCAP* 12 (2020) 028.] [S. Aoki *et. al.*, *JHEP* 05 (2022) 121]

- ➔ Instability “during inflation” induced by λ -running
—> exponential growth of perturb.

Higgs- R^2 Inflation, Trajectory



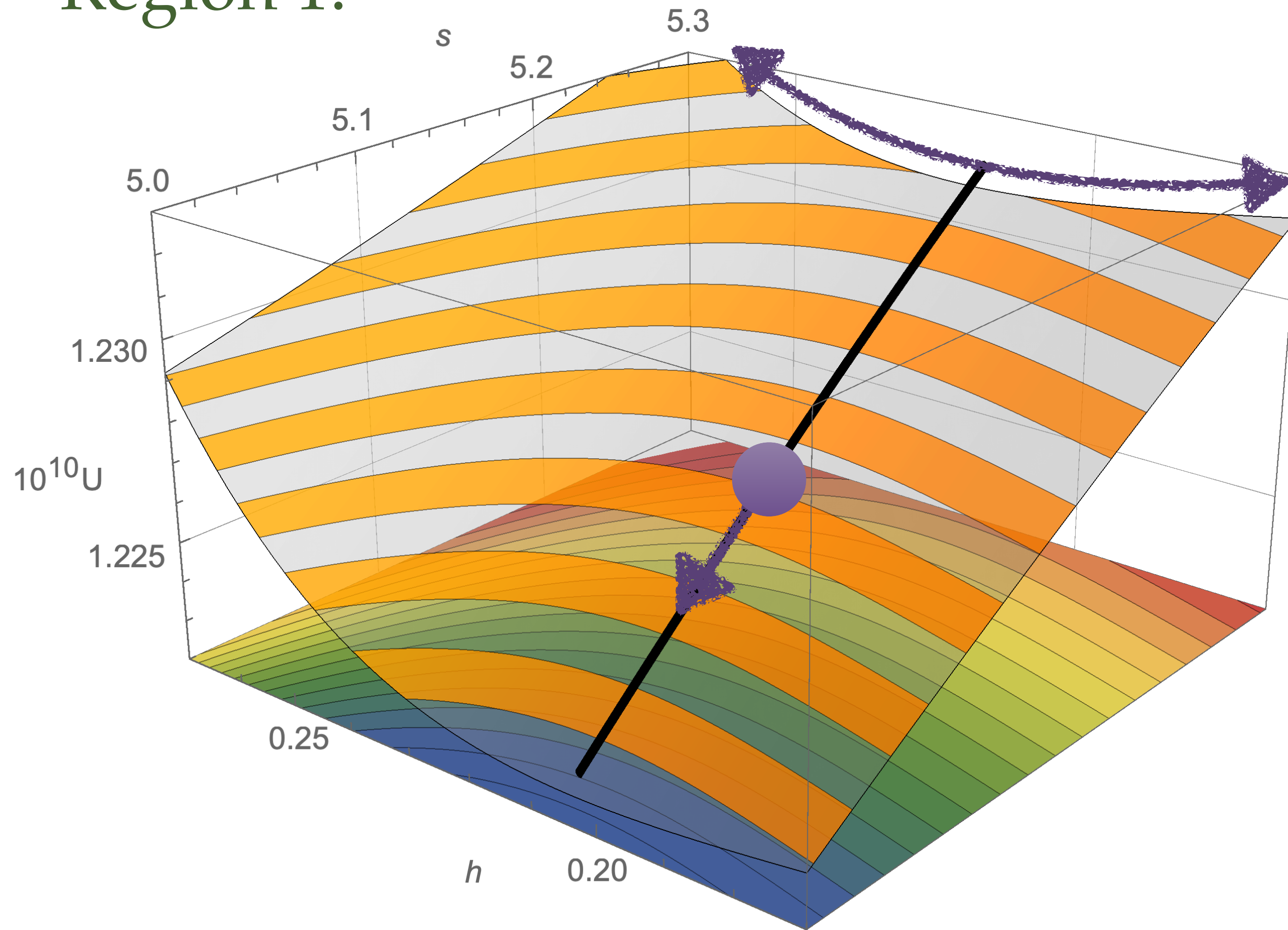
Higgs- R^2 Inflation, Trajectory



Higgs- R^2 Inflation, Trajectory

[DYC, K. Kohri, S.C. Park, *JCAP* 10 (2022) 015]

Region 1.



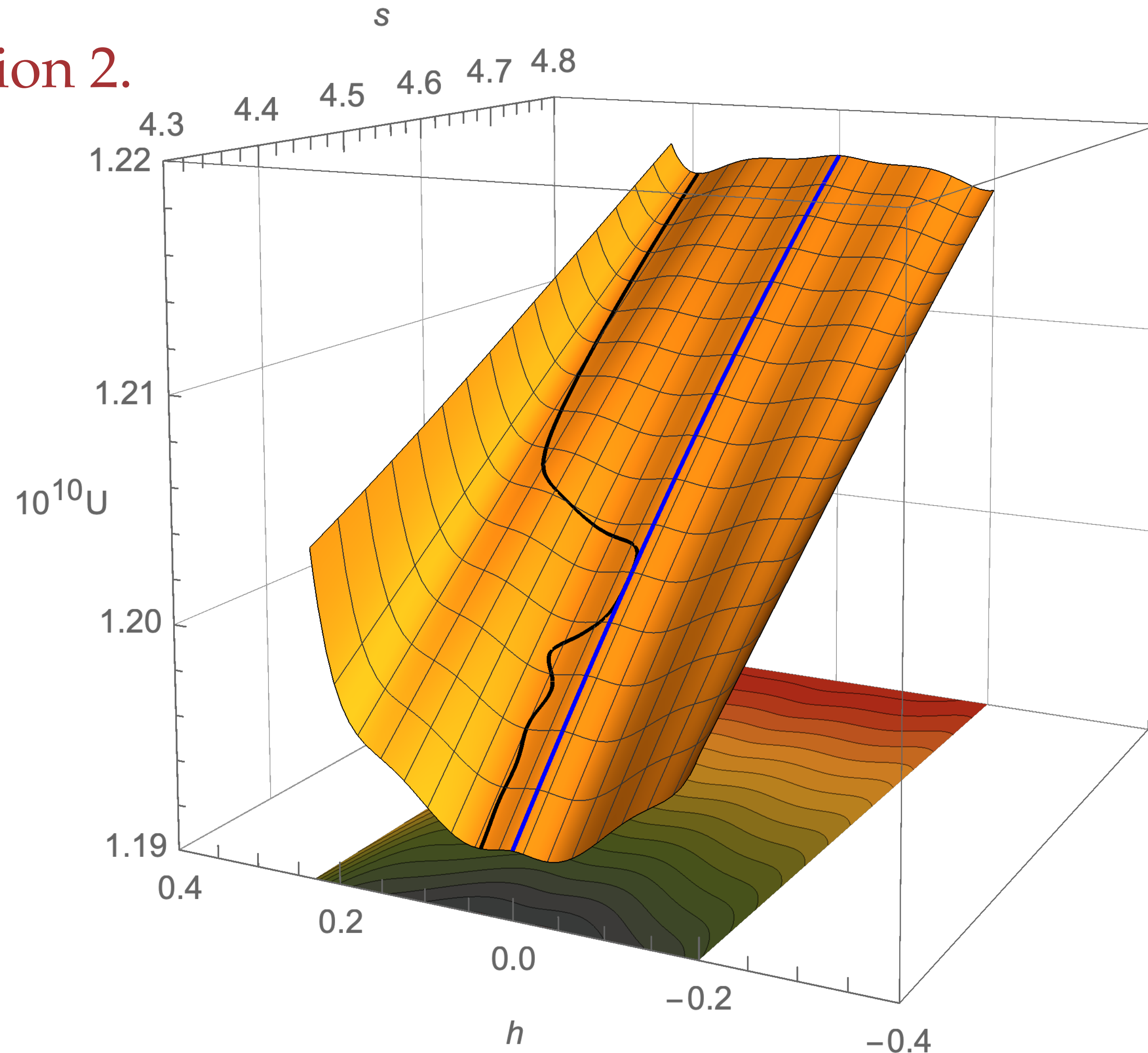
- “Valley” structure persists, exhibits an “attractor” behavior.
- *Isocurvature perturbations suppressed*
- CMB observables resemble the predictions of an “effective single-field” setup.
- “Slightly larger” n_s, r compared to constant λ .

$$n_s \approx 1 - \frac{2}{N_{\text{inf}}} - \frac{9}{2N_{\text{inf}}^2} + \frac{2M^2\xi^2 b}{\lambda_m(\lambda_m + 3M^2\xi^2)} + \dots \quad r \approx \frac{12}{N_{\text{inf}}^2} + \frac{24M^2\xi^2 b}{\lambda_m(\lambda_m + 3M^2\xi^2)N_{\text{inf}}} \ln \left(\frac{4M^2\xi N_{\text{inf}}}{(\lambda_m + 3M^2\xi^2)h_m^2} \right) + \dots$$

Higgs- R^2 Inflation, Trajectory

[**DYC**, K. Kohri, S.C. Park, *JCAP* 10 (2022) 015]

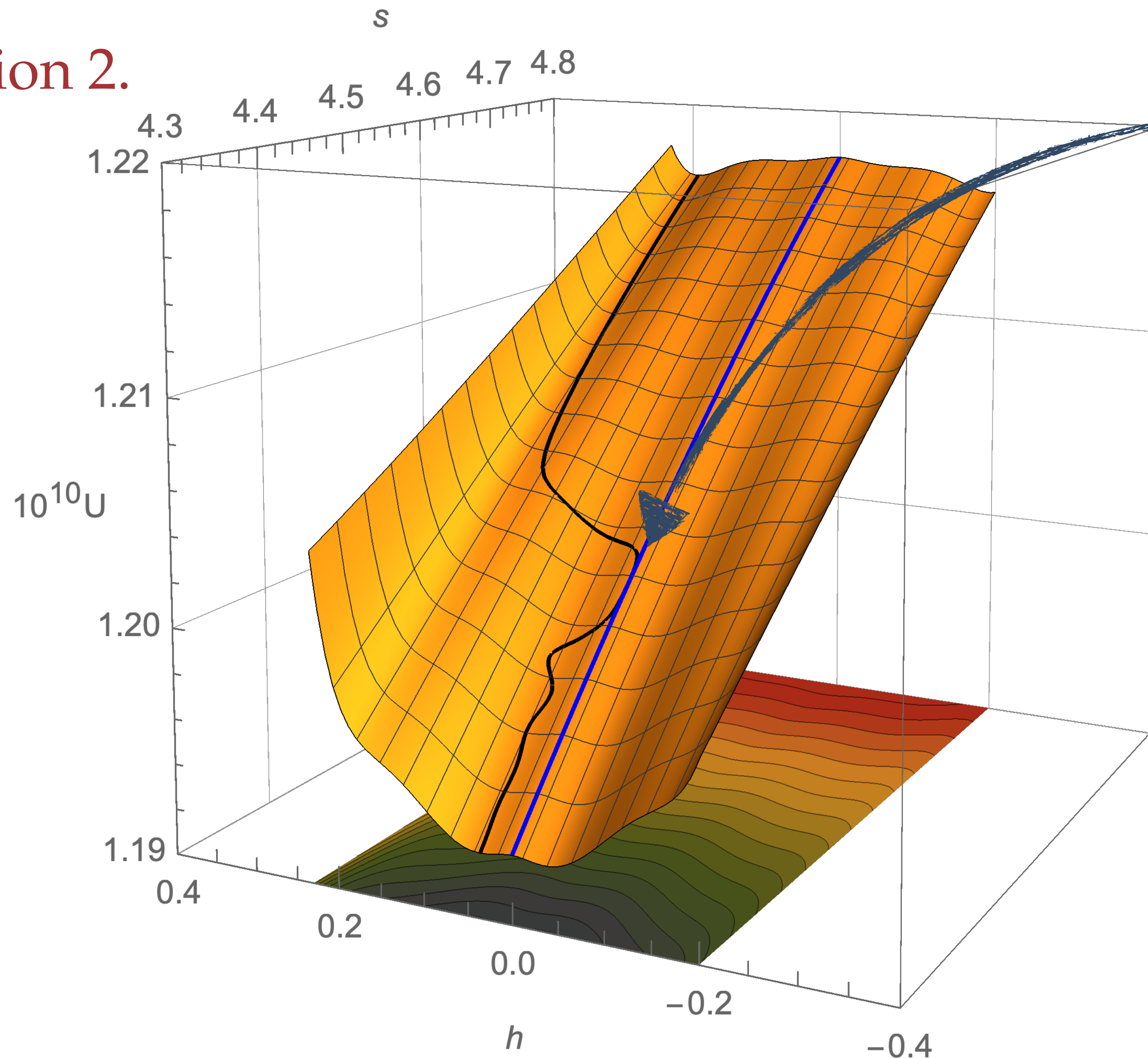
Region 2.



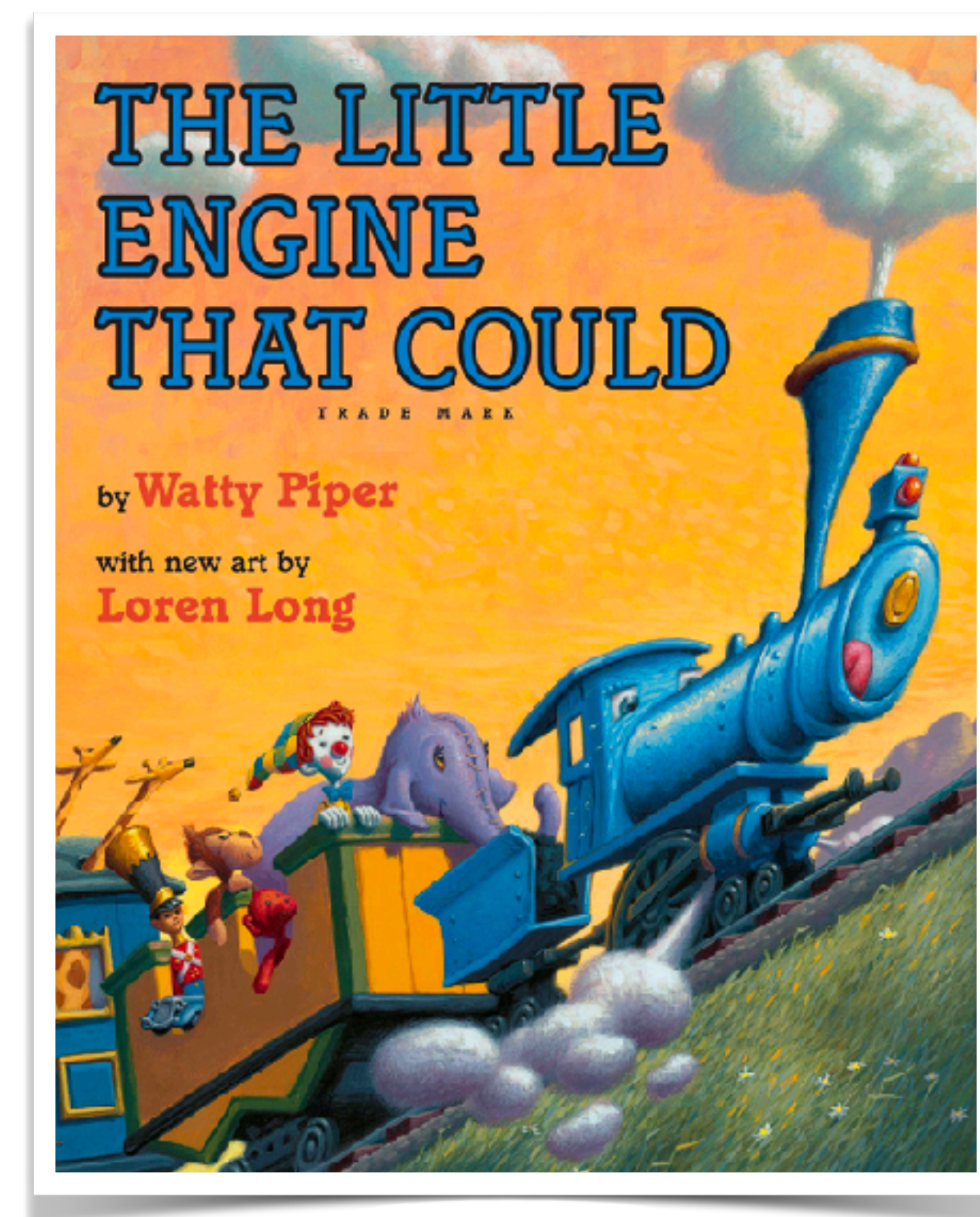
Higgs- R^2 Inflation, Trajectory

[DYC, K. Kohri, S.C. Park, *JCAP* 10 (2022) 015]

Region 2.



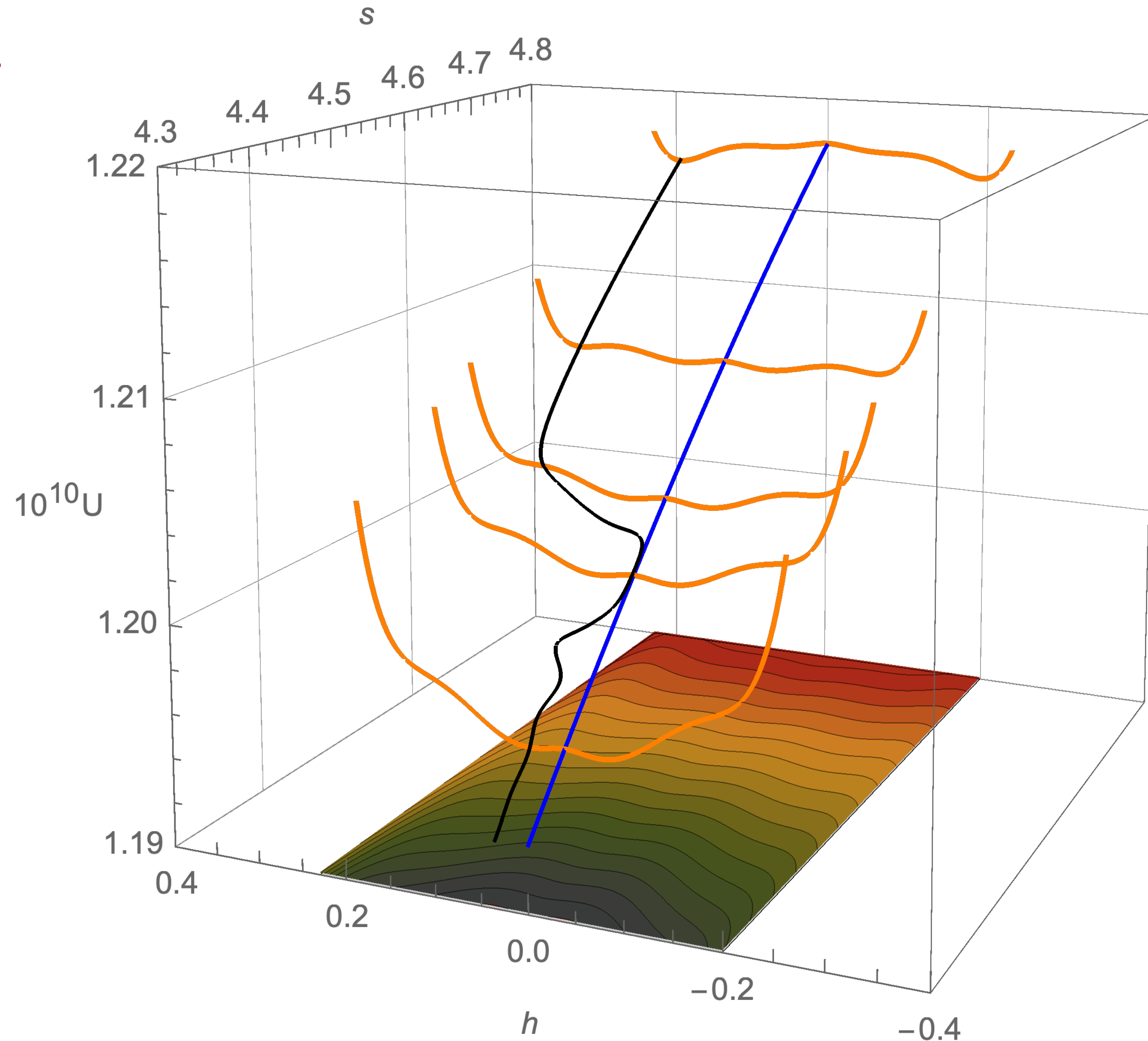
Inflaton “approaching” the $h = 0$ hill.



Higgs- R^2 Inflation, Trajectory

[DYC, K. Kohri, S.C. Park, *JCAP* 10 (2022) 015]

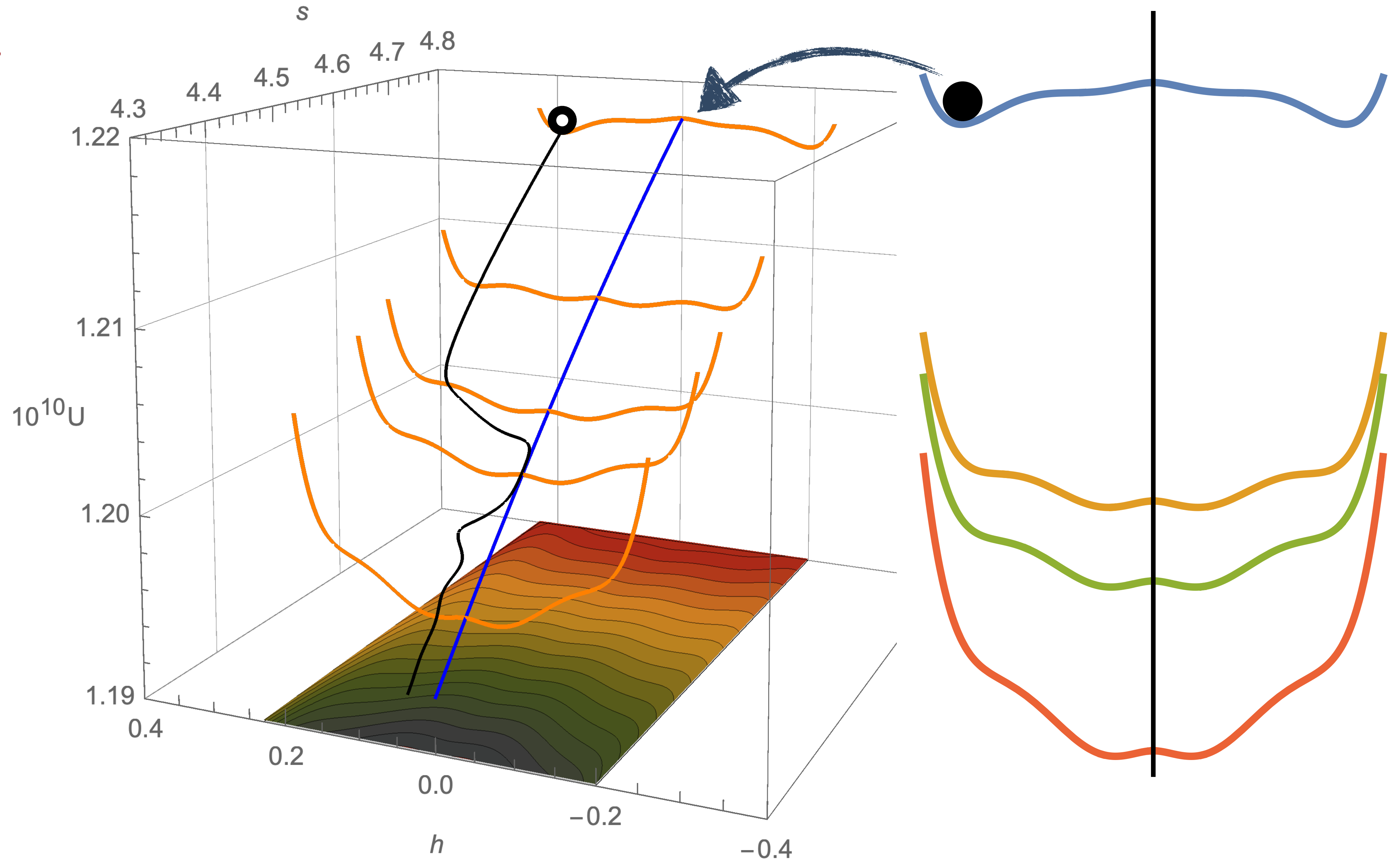
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[DYC, K. Kohri, S.C. Park, *JCAP* 10 (2022) 015]

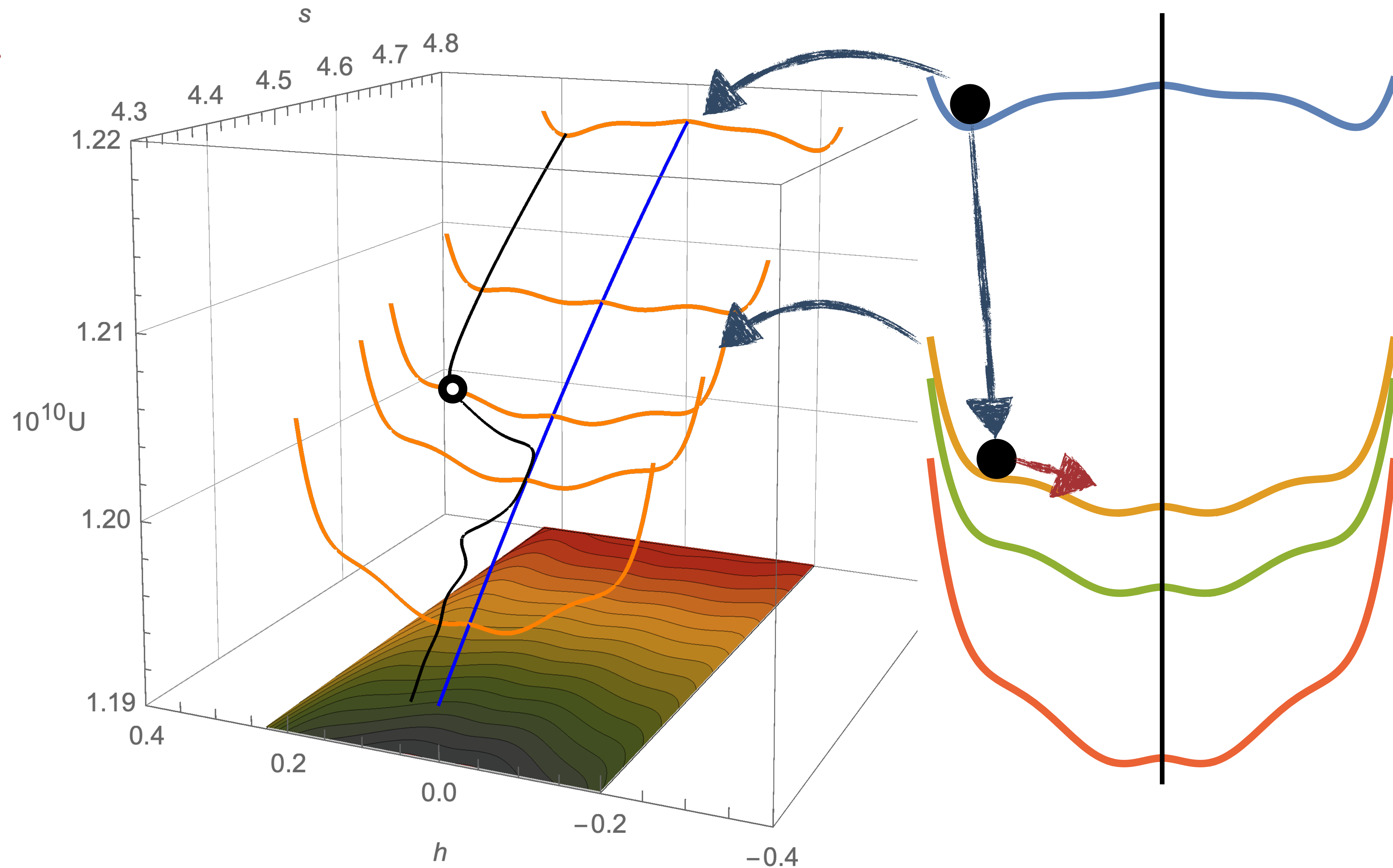
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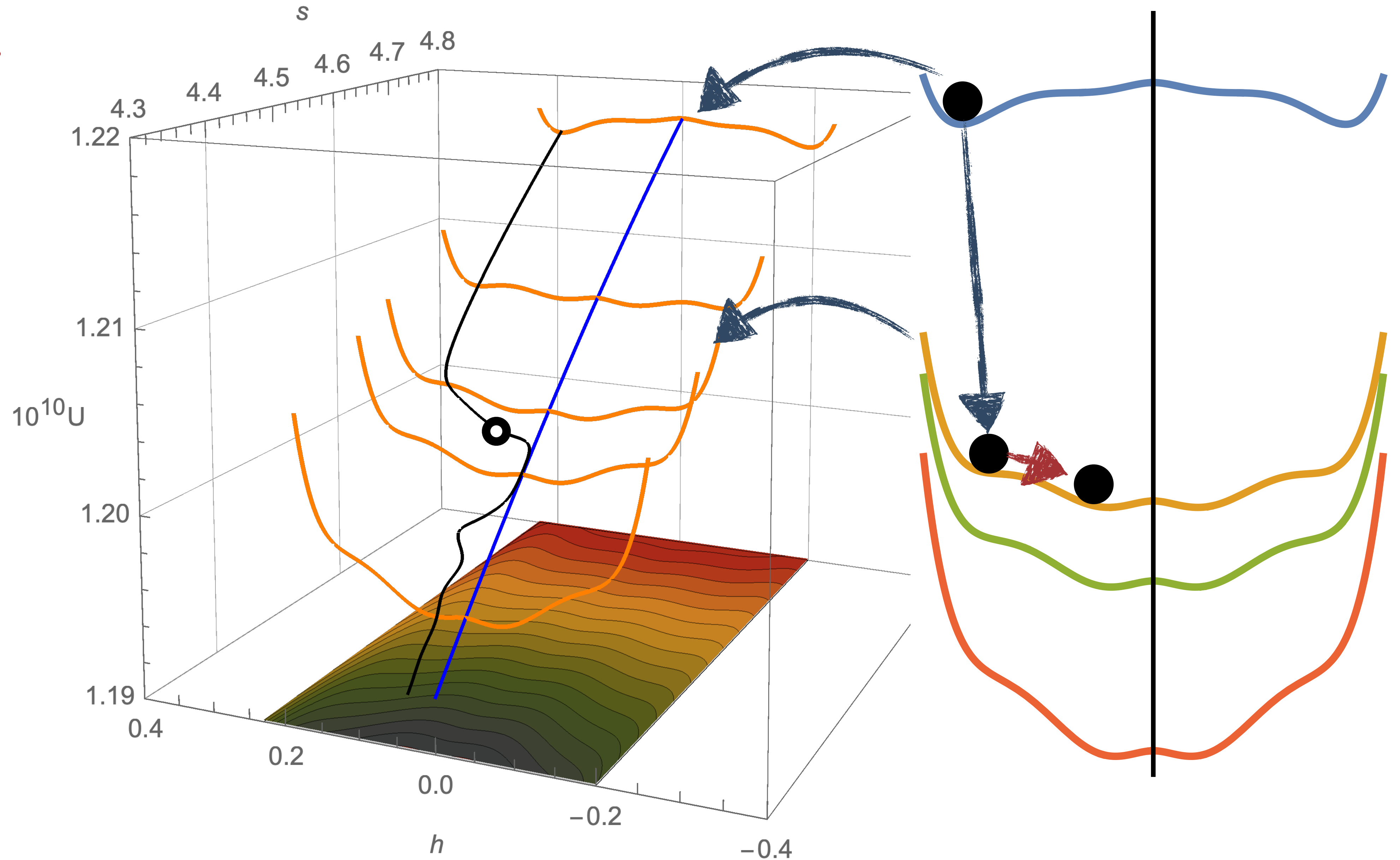
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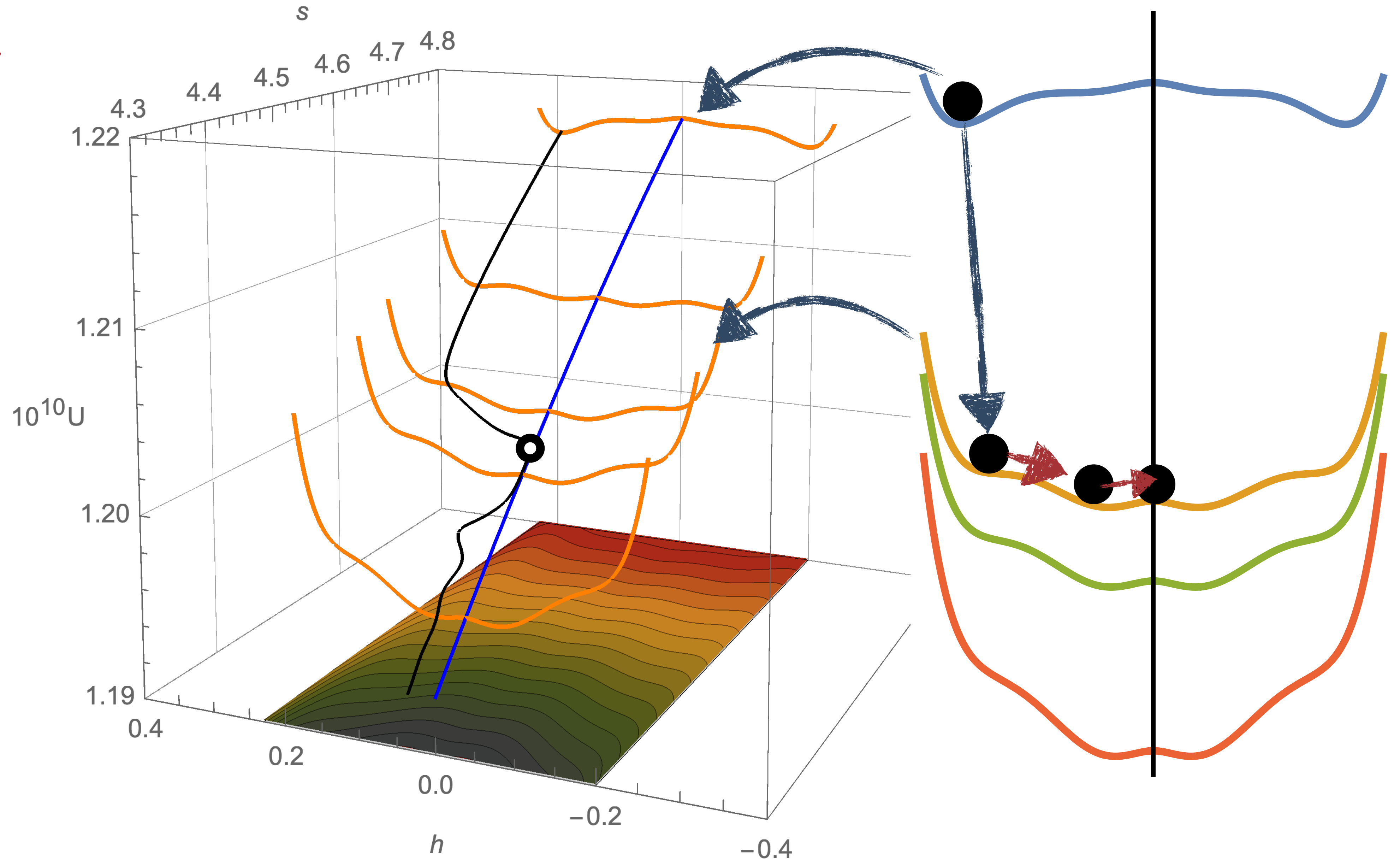
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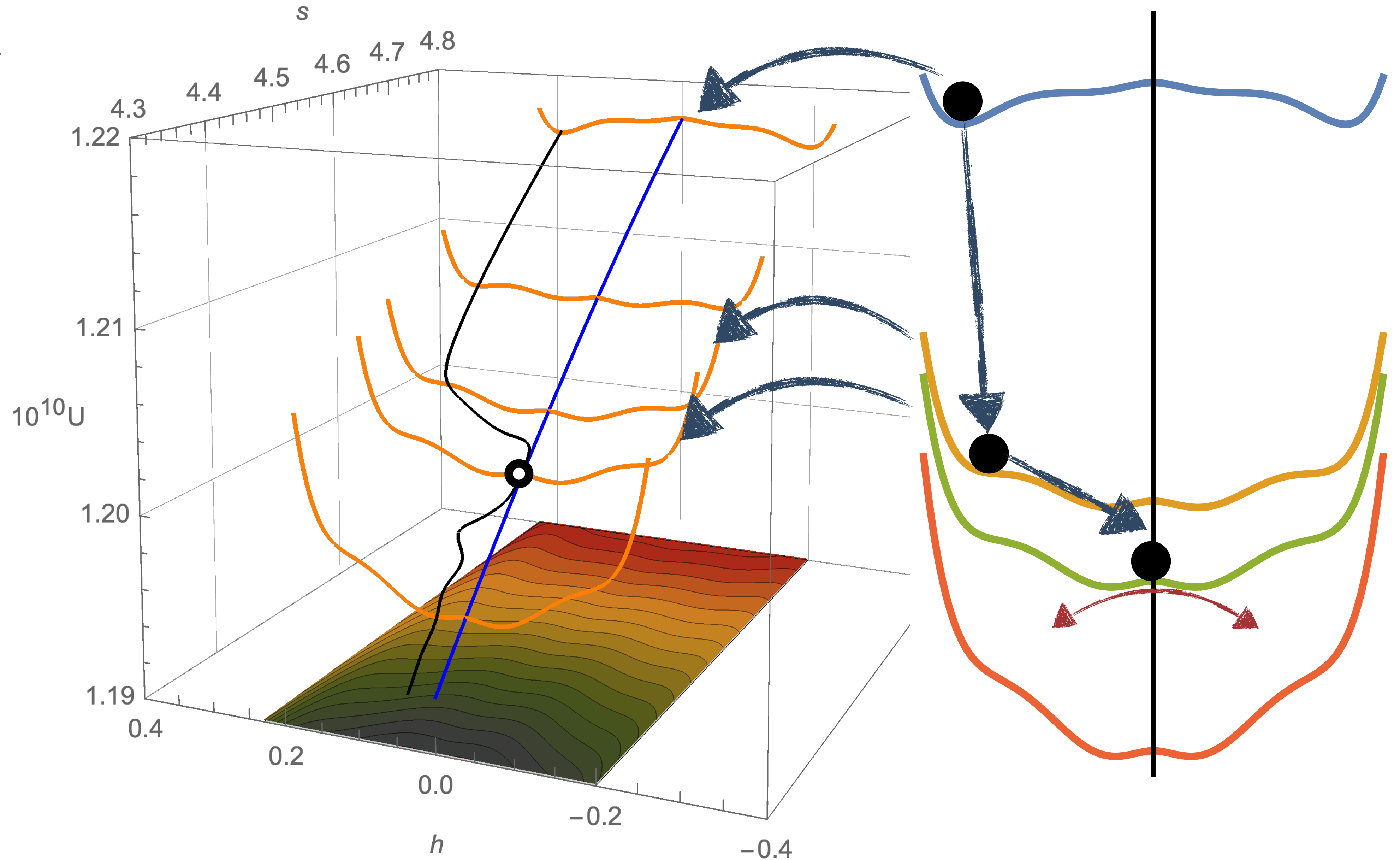
Region 2.



Higgs- R^2 Inflation, Trajectory

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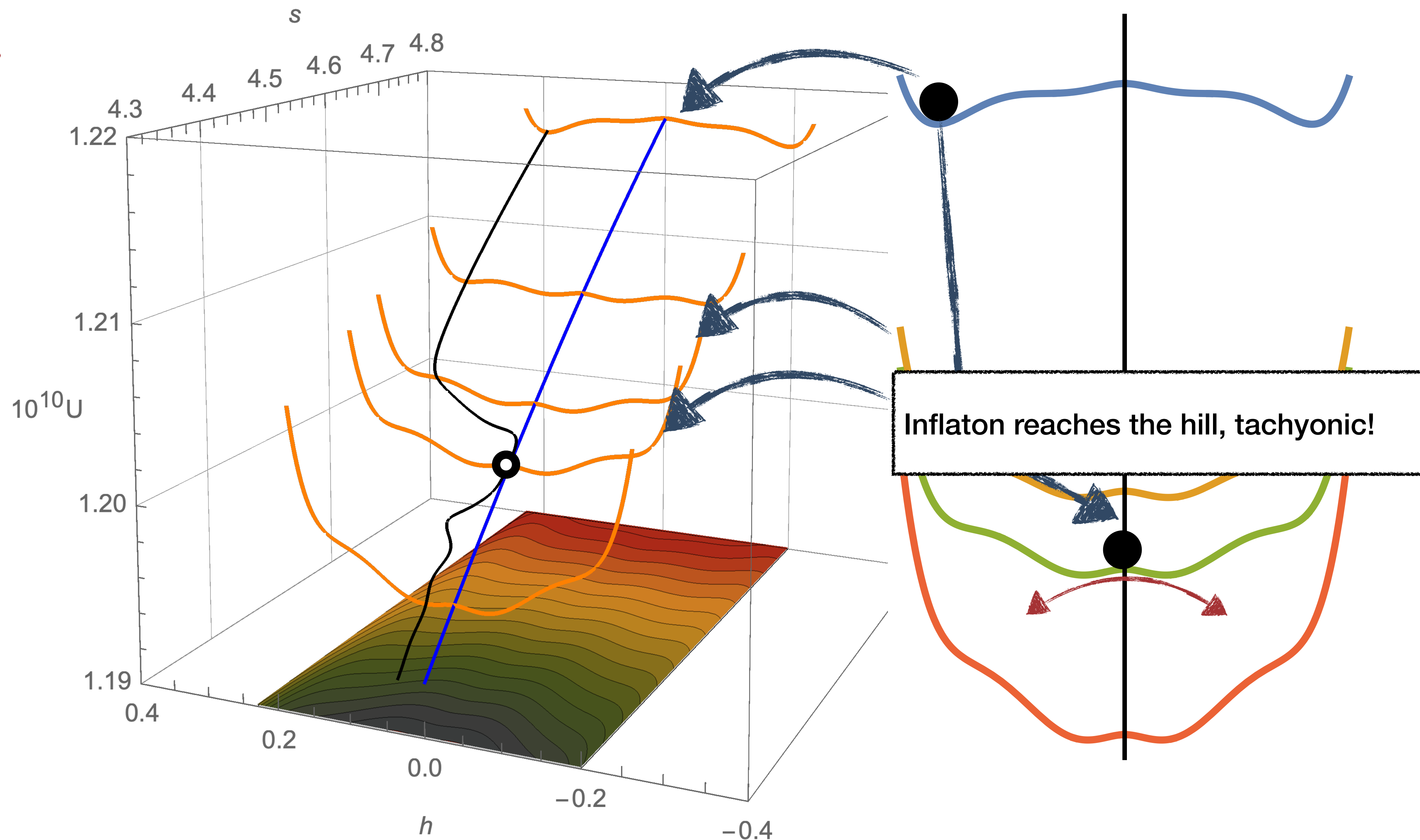
Region 2.



Higgs- R^2 Inflation, Trajectory

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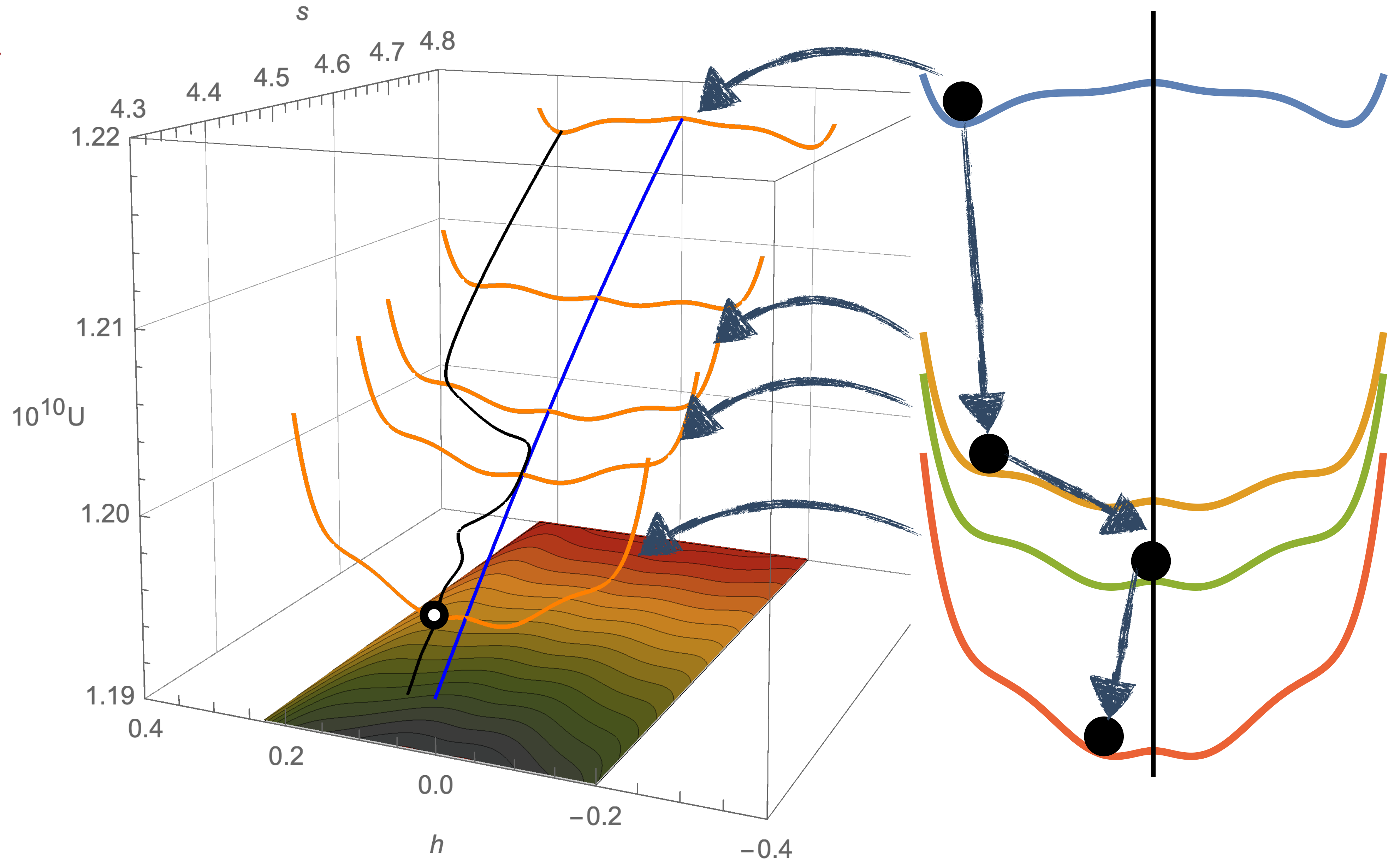
Region 2.



Higgs- R^2 Inflation, Trajectory

[DYC, K. Kohri, S.C. Park, *JCAP* 10 (2022) 015]

Region 2.



Higgs- R^2 Inflation, Perturbations

Second order perturbation with $\phi^a(t, \vec{x}) = \phi_0^a(t) + \delta\phi^a(t, \vec{x})$, $ds^2 = -(1 + 2\psi)dt^2 + a(t)^2(1 - 2\psi)\delta_{ij}dx^i dx^j$.

$$\ddot{\mathcal{R}} + (3 + 2\epsilon - 2\eta_{\parallel}) H \dot{\mathcal{R}} + \frac{k^2}{a^2} \mathcal{R} = -2 \frac{H^2}{\dot{\phi}_0} \eta_{\perp} \left[\dot{Q}_N + \left(3 - \eta_{\parallel} + \frac{\dot{\eta}_{\perp}}{H \eta_{\perp}} \right) H Q_N \right]$$

$$\ddot{Q}_N + 3H \dot{Q}_N + \left(\frac{k^2}{a^2} + M_{\text{eff}}^2 \right) Q_N = 2\dot{\phi}_0 \eta_{\perp} \dot{\mathcal{R}}.$$

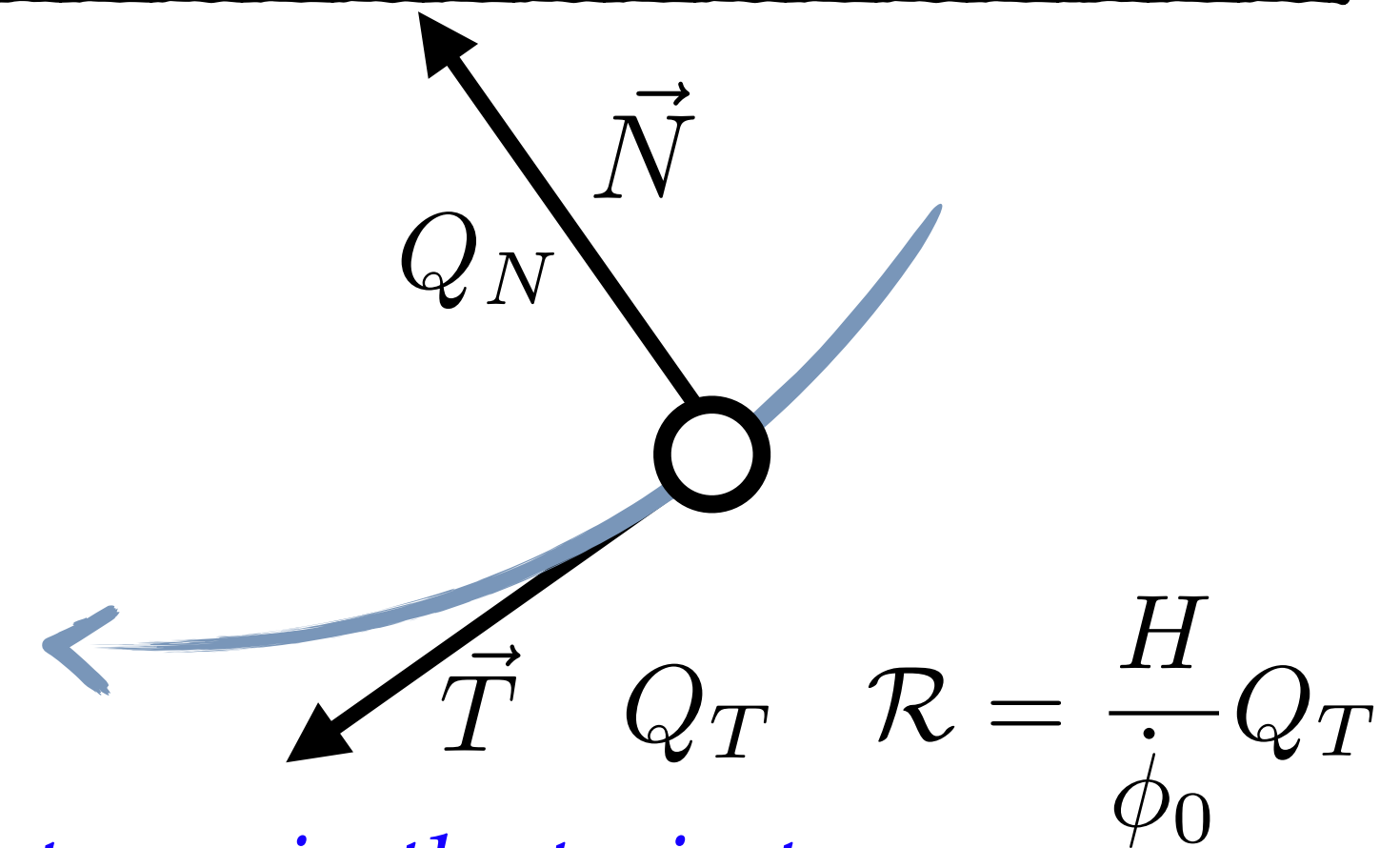
[S. Cespedes et. al, JCAP 05 (2012) 008]

[A. Achucarro et. al, Phys. Rev. D86 (2012) 121301]

[S. Groot Nibbelink and B.J.W. van Tent, Class. Quant. Grav. 19 (2002) 613] and many more...

$$\eta_{\parallel} \equiv -\frac{\ddot{\phi}_0}{\dot{\phi}_0 H} \quad \eta_{\perp} \equiv \frac{U_N}{\dot{\phi}_0 H} \quad Q^a \equiv \delta\phi^a + \frac{\dot{\phi}^a}{H} \psi \quad \dot{\theta} \equiv H \eta_{\perp}$$

$$M_{\text{eff}}^2 = U_{NN} + H^2 \epsilon_{\mathcal{R}} - \dot{\theta}^2.$$

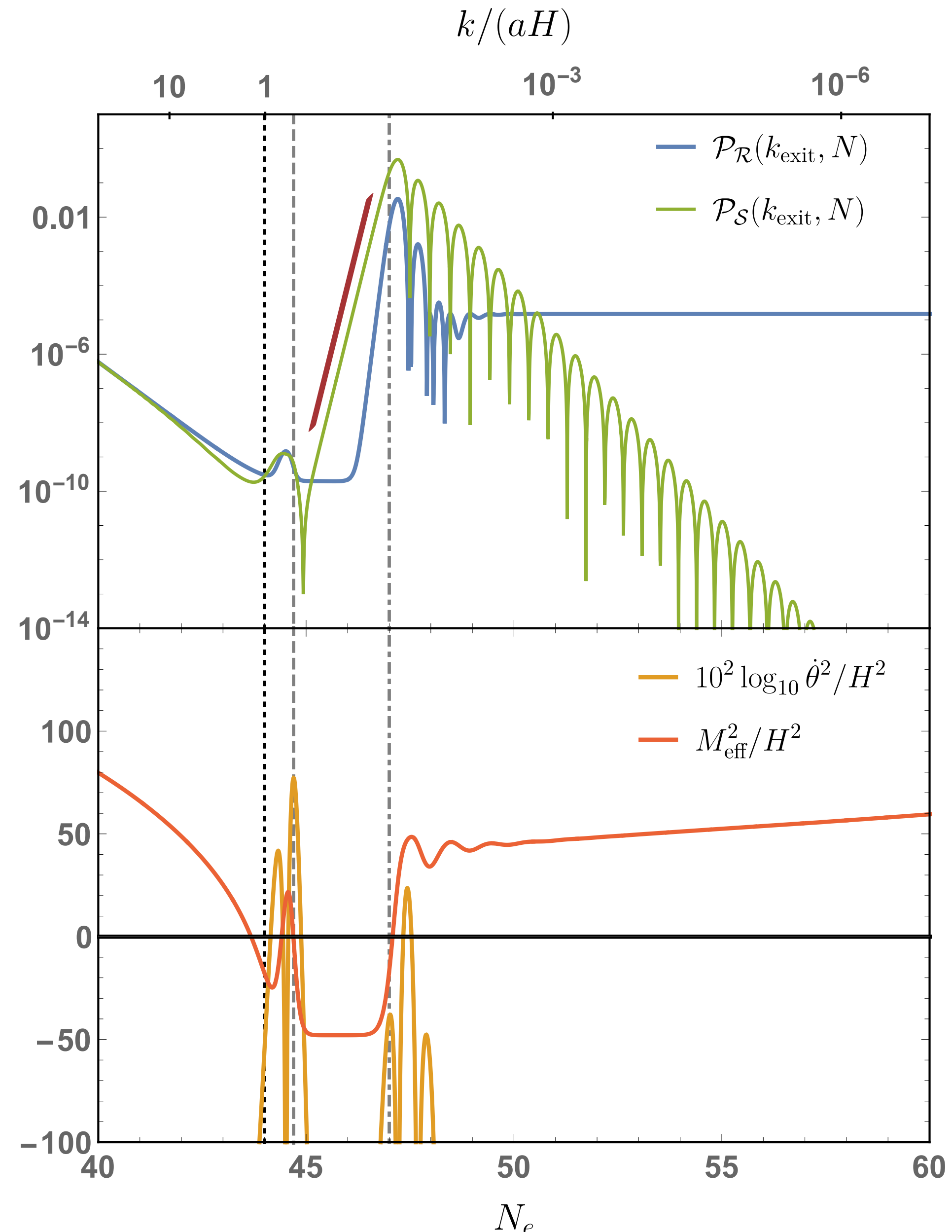


- $M_{\text{eff}}^2 < 0$ leads to *tachyonic growth of Q_N* , then gets *sourced to \mathcal{R}* through *turns in the trajectory*.

- Tachyonic mass at the “hill” of the potential at $h = 0$, $M_{\text{eff}}^2 \simeq -3M^2 \xi \left(1 - e^{-\sqrt{\frac{2}{3}} s} \right)$

Higgs- R^2 Inflation, Perturbations

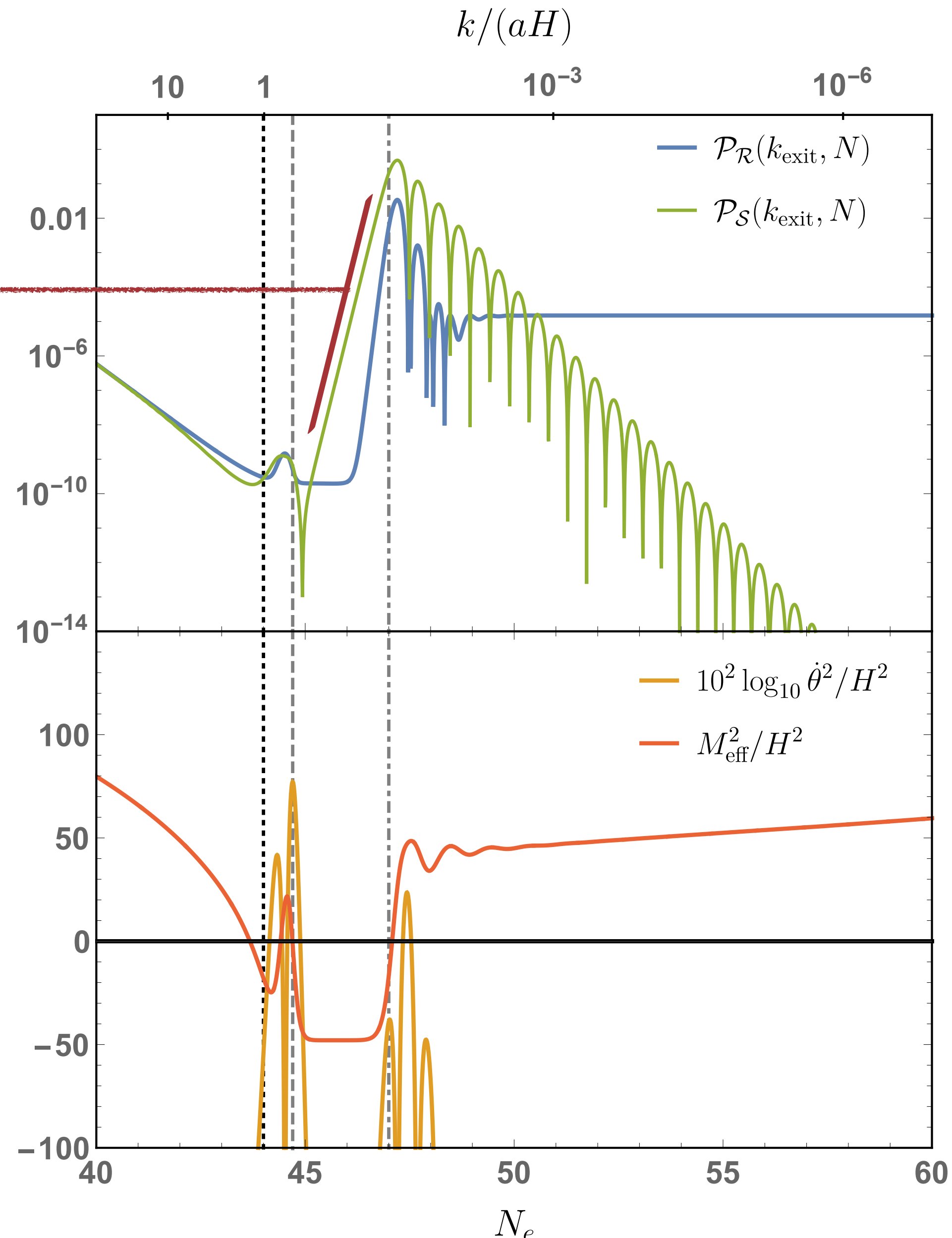
[DYC, K. Kohri, S.C. Park, *JCAP* 10 (2022) 015]



Higgs- R^2 Inflation, Perturbations

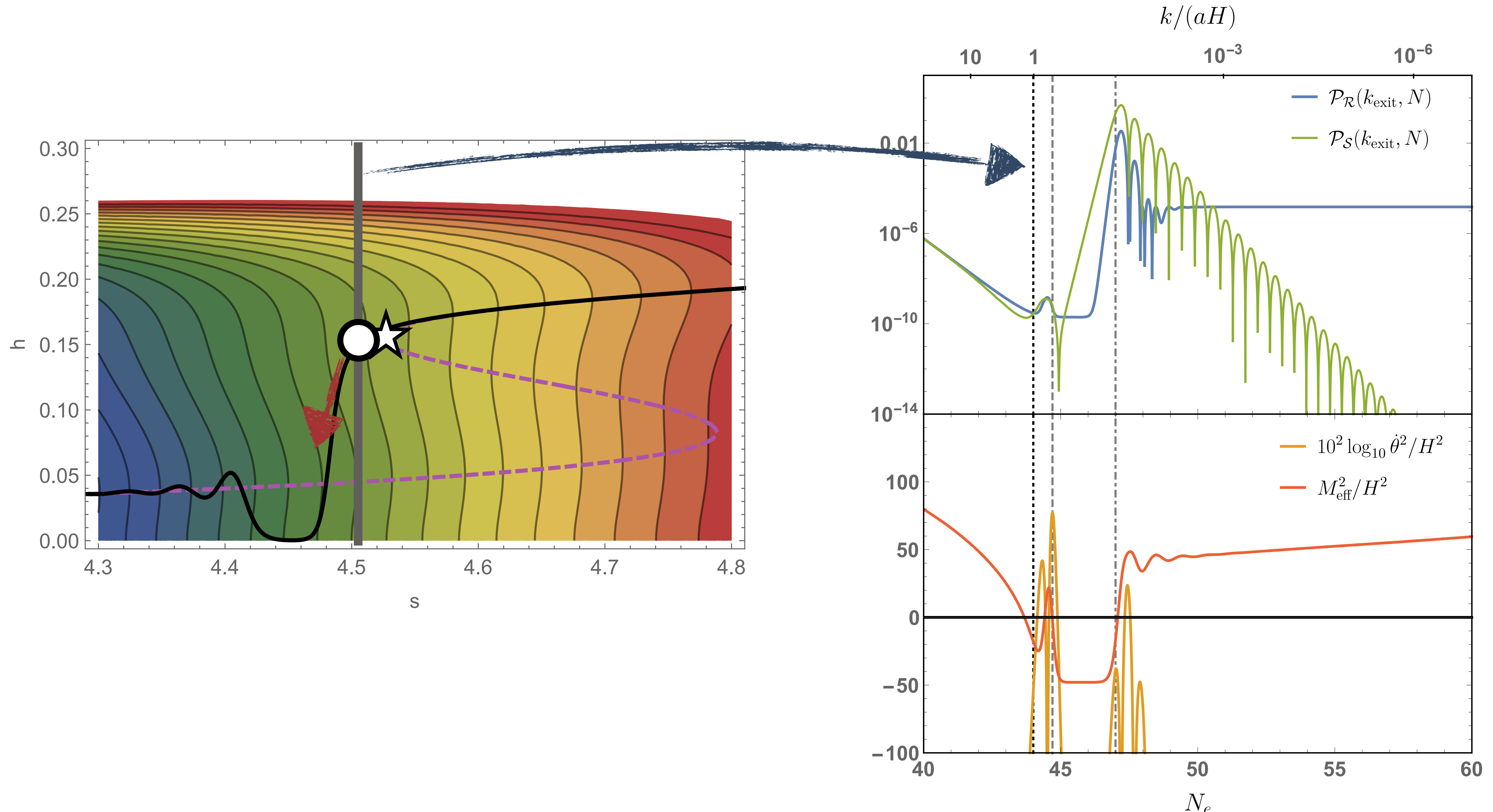
[DYC, K. Kohri, S.C. Park, *JCAP* 10 (2022) 015]

$$\mathcal{P}_S \propto \exp \left[\left(\frac{2|M_{\text{eff}}|}{H} - 3 \right) N_e \right]$$



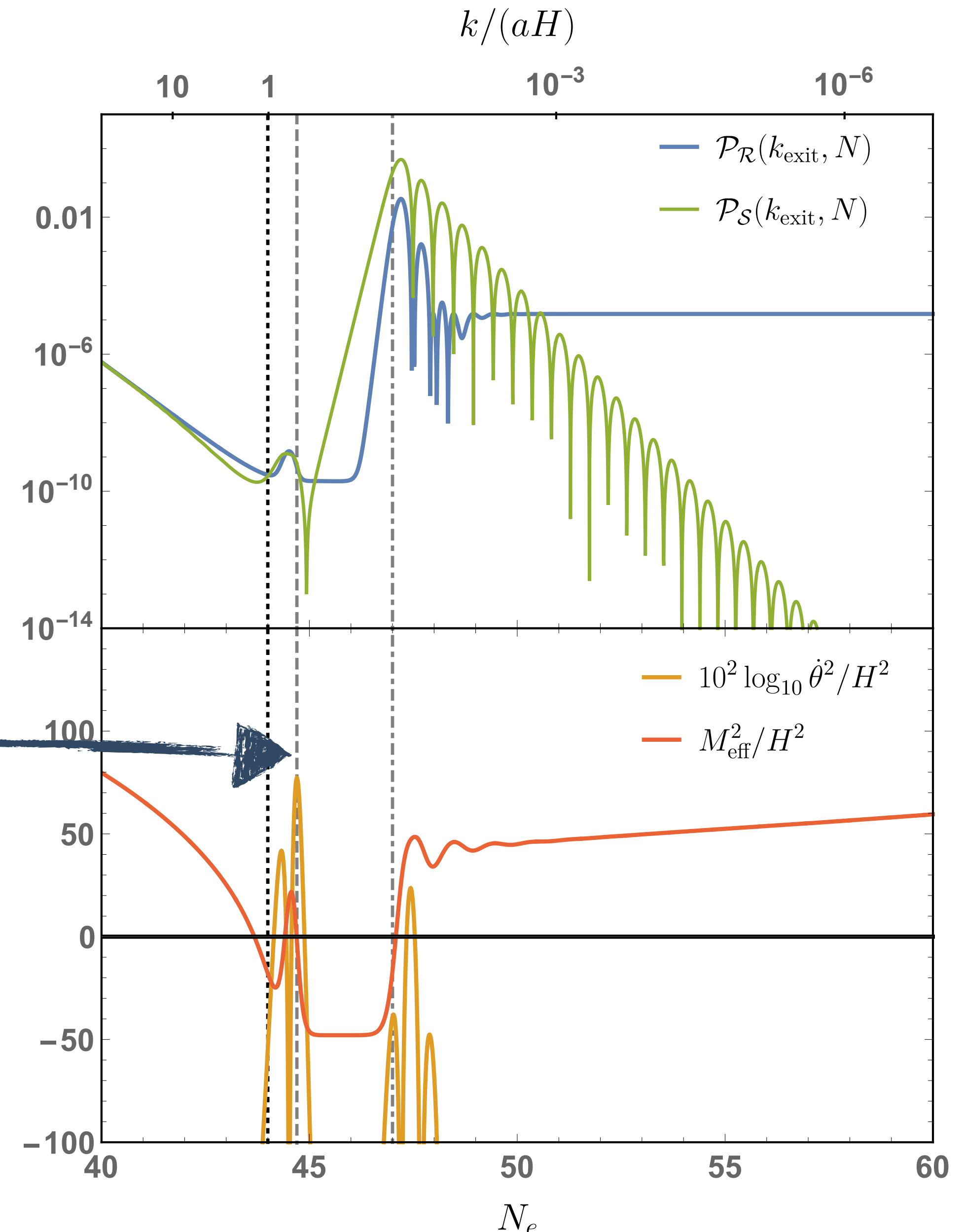
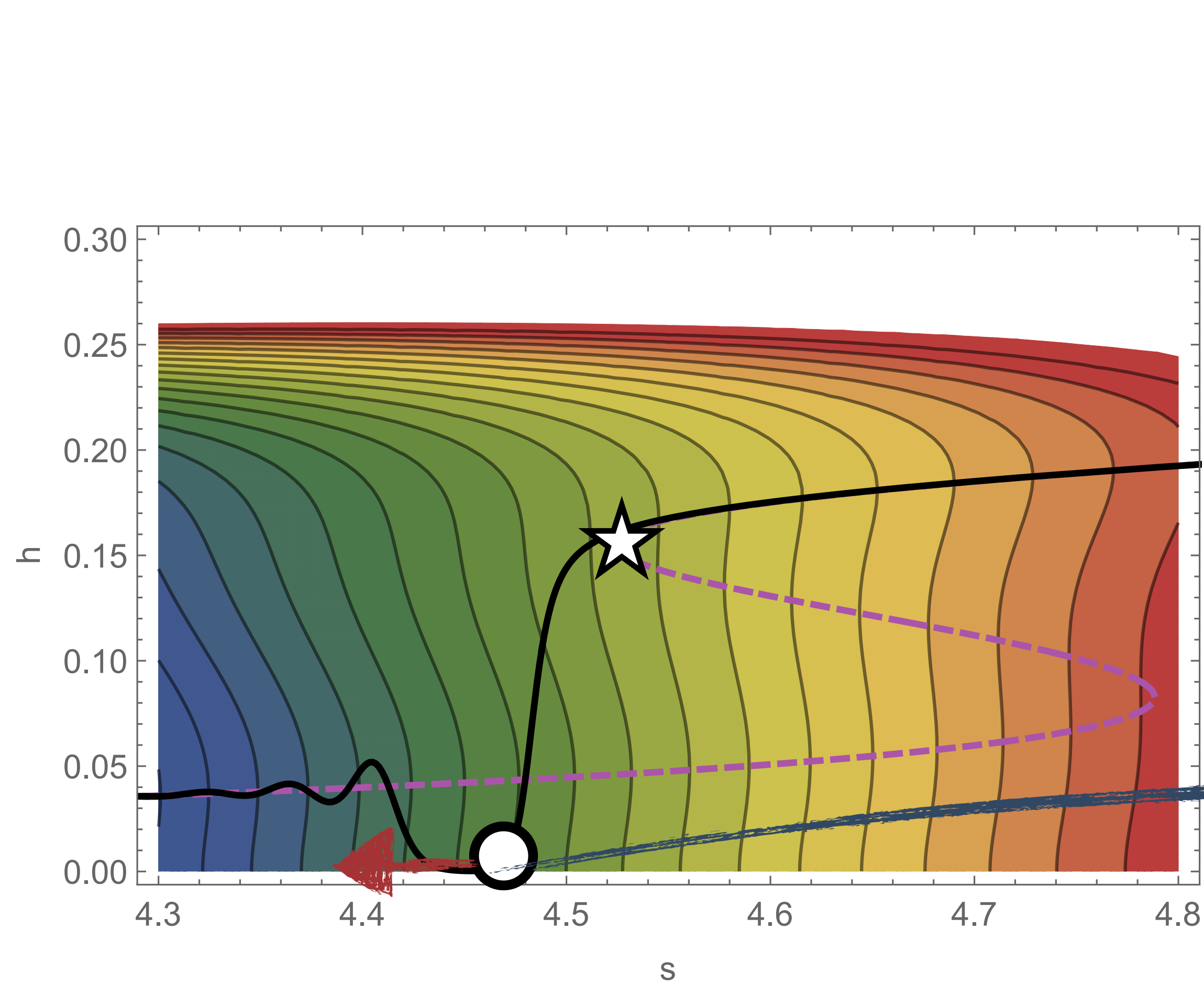
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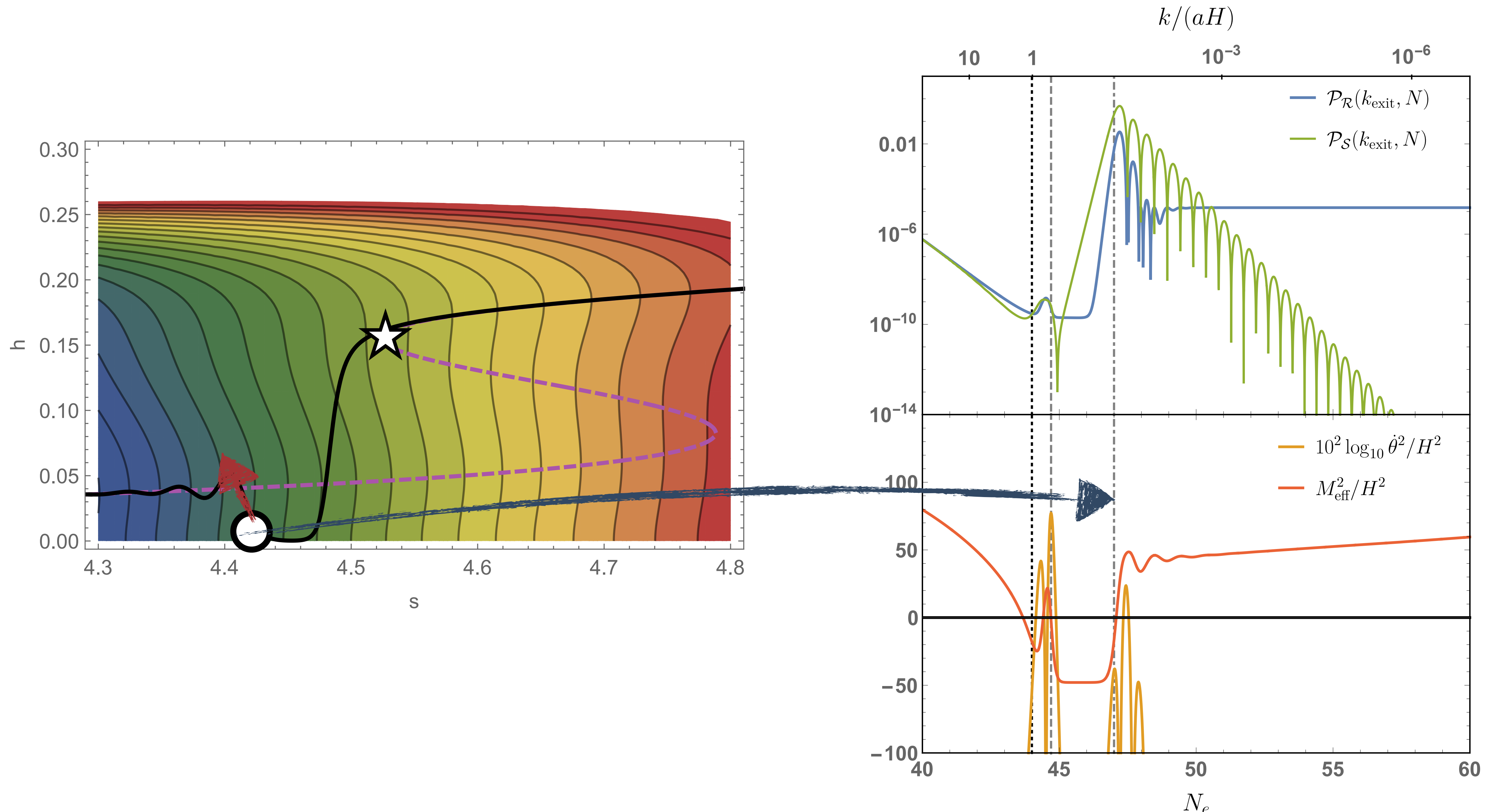
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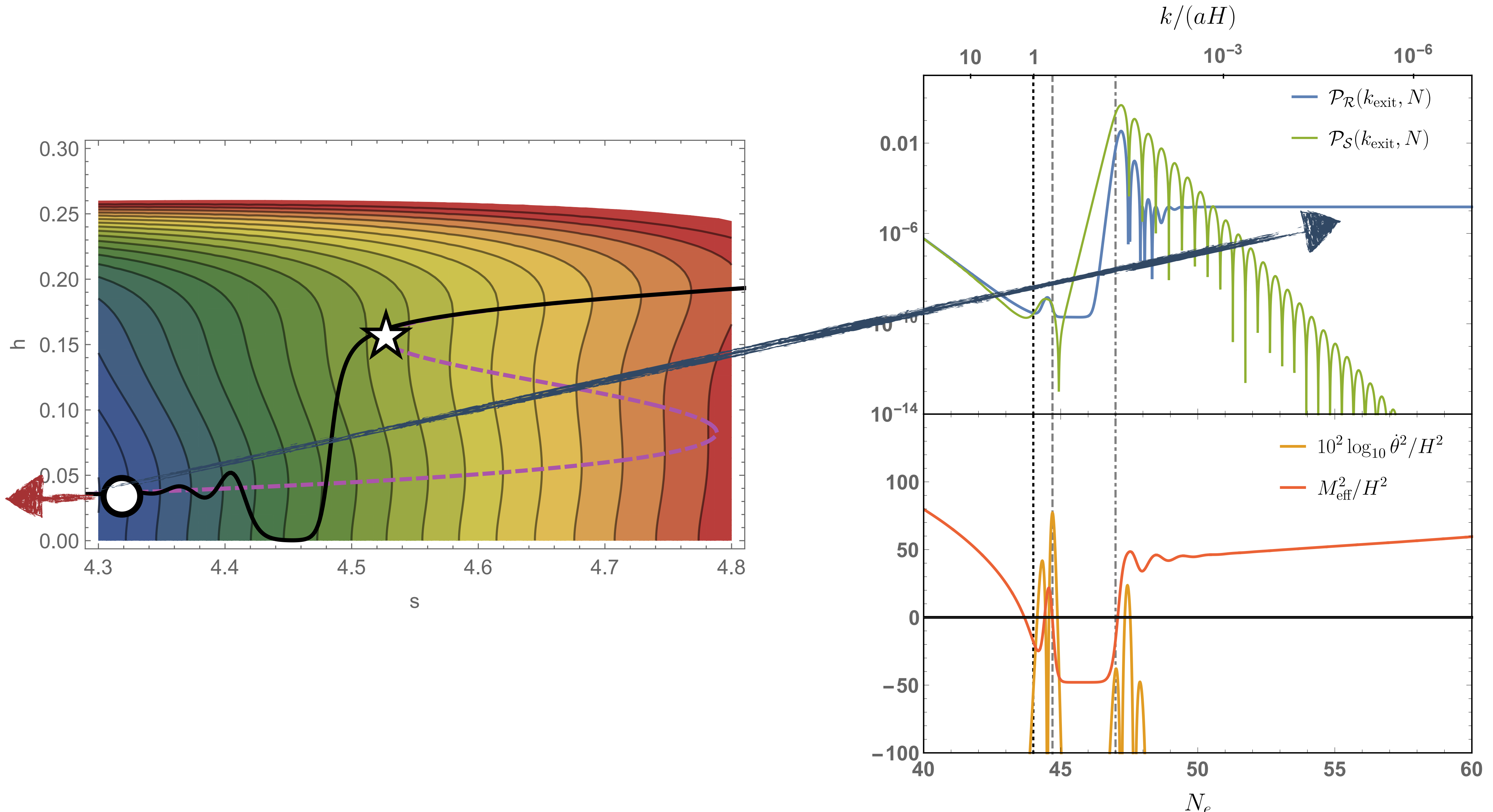
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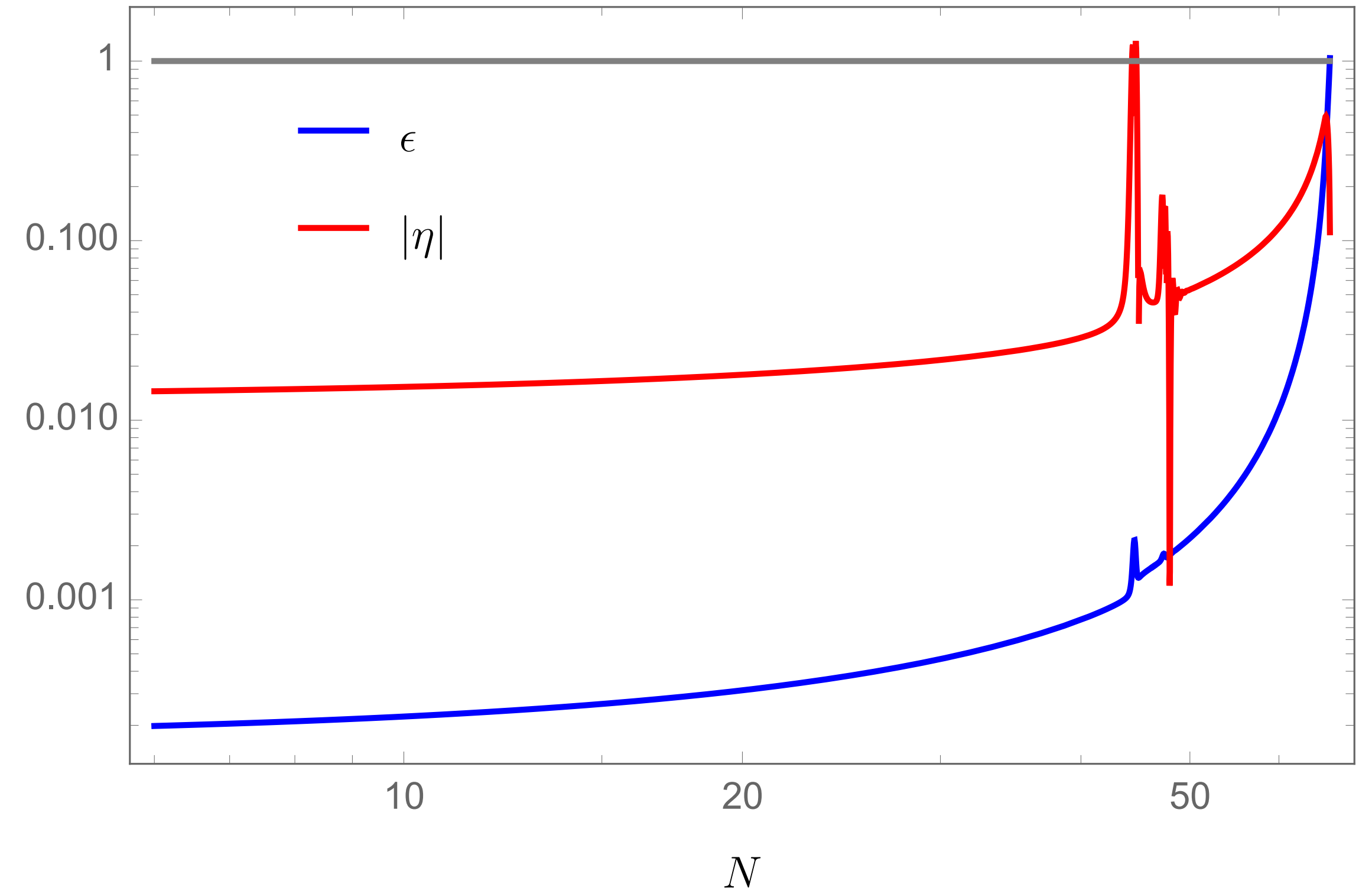
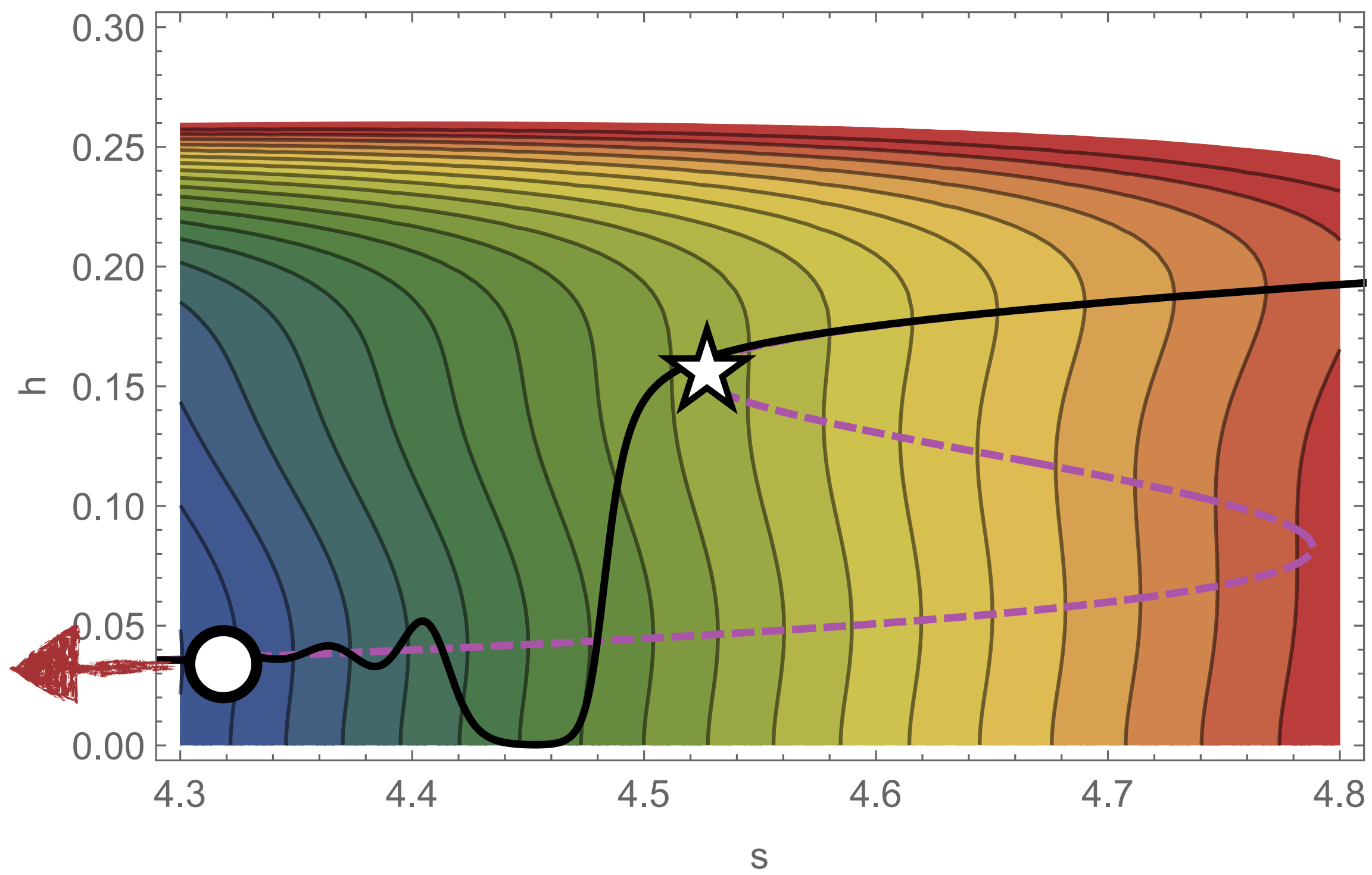
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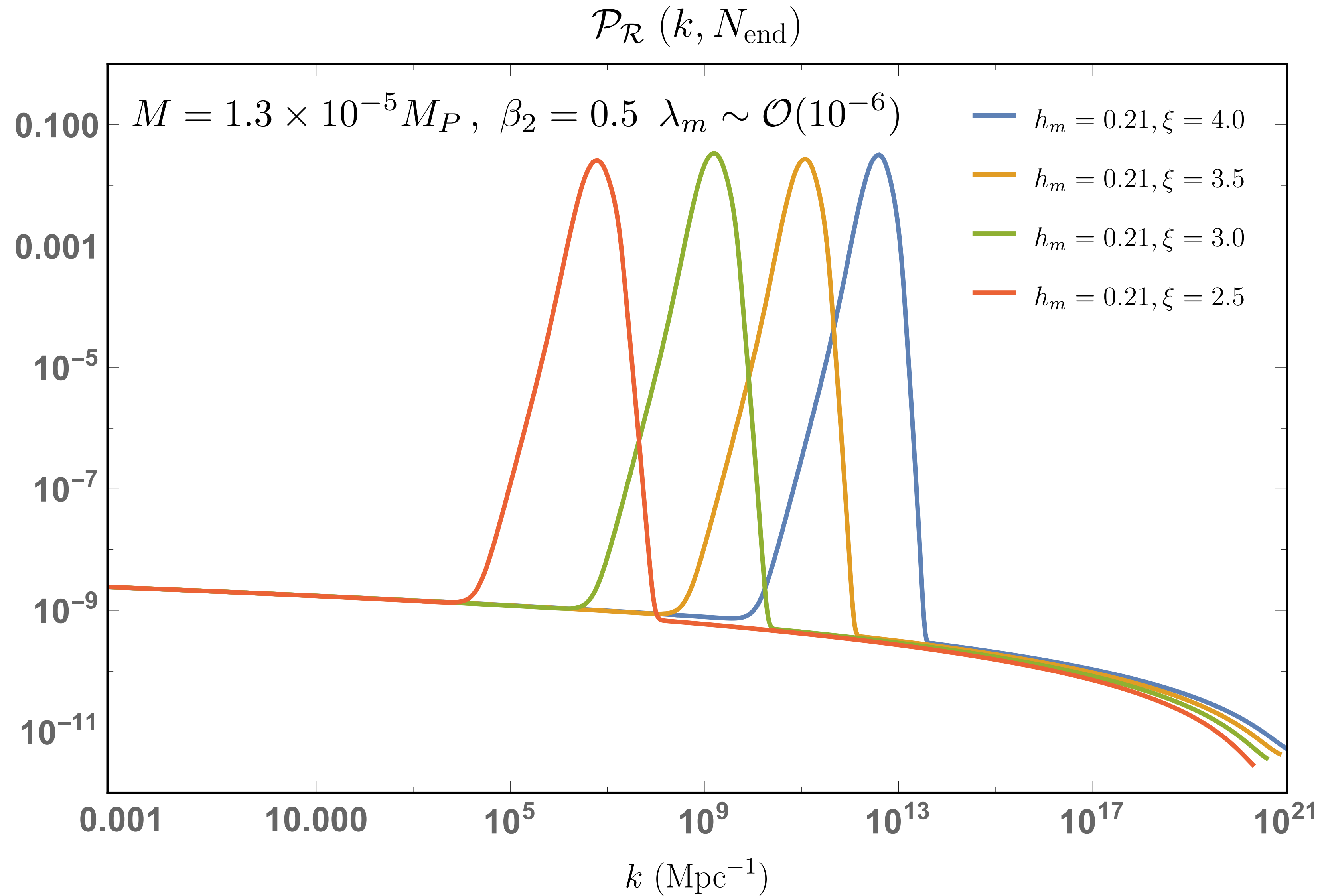
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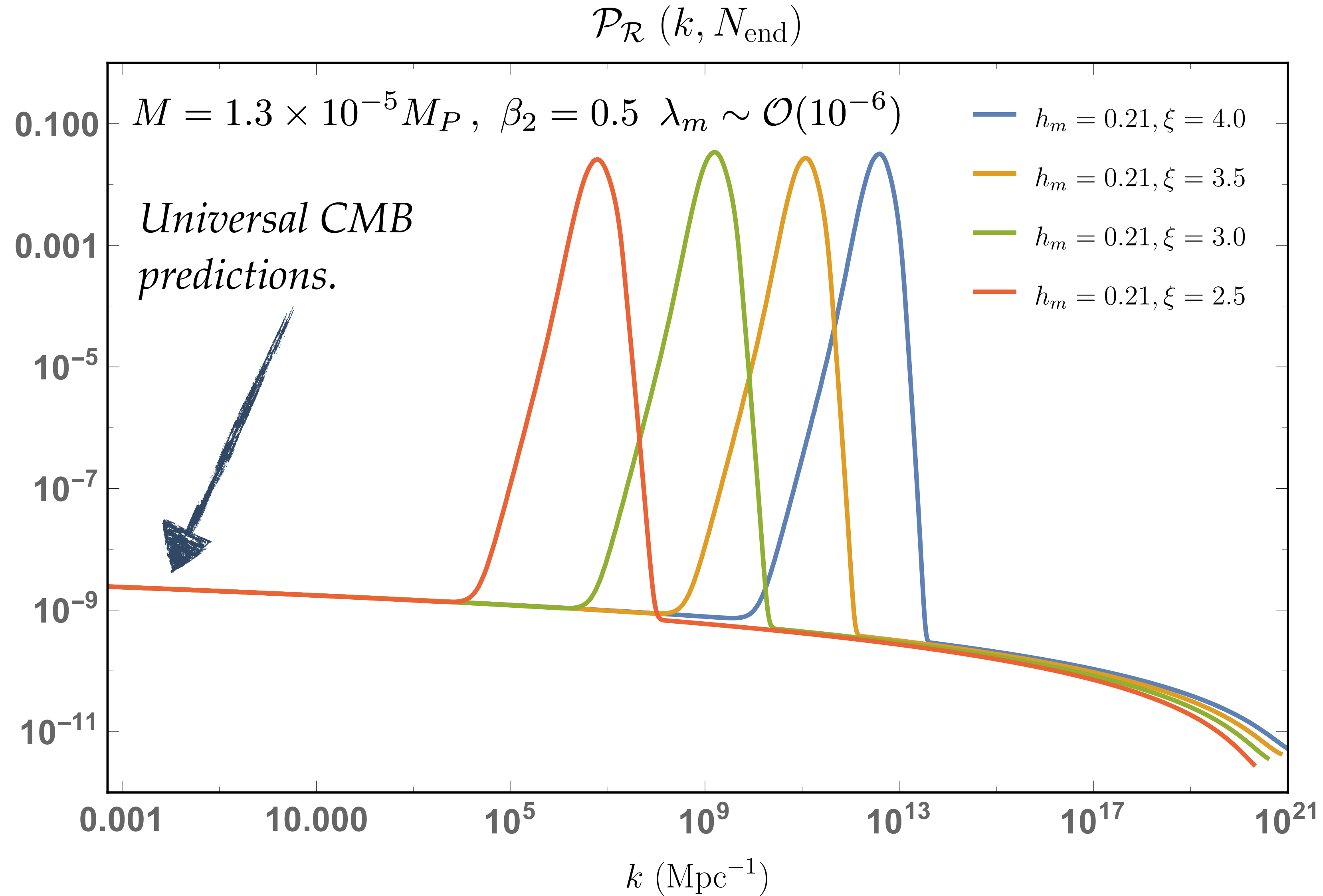
Higgs- R^2 Inflation, Power Spectrum

[DYC, K. Kohri, S.C. Park, *JCAP* 10 (2022) 015]



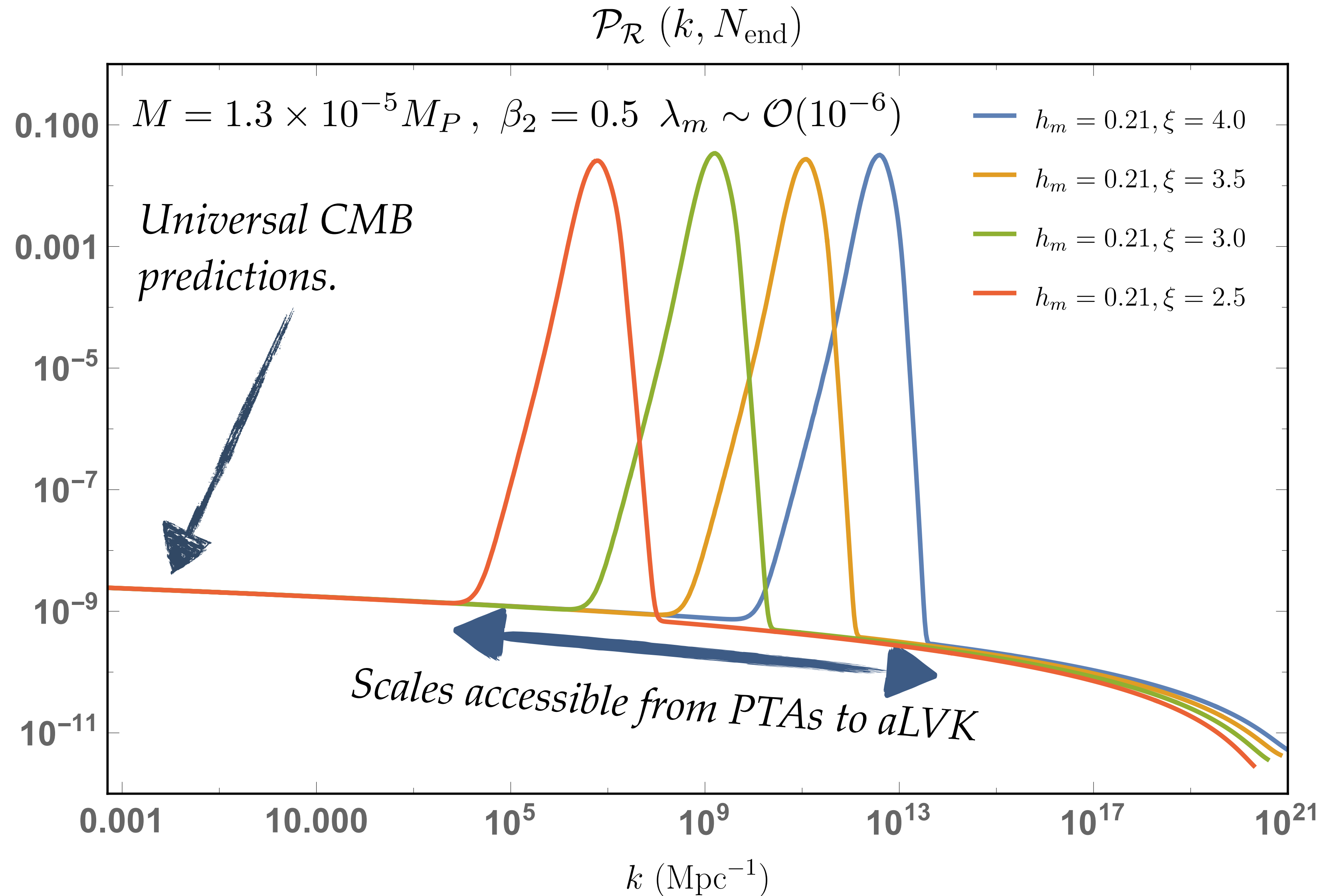
Higgs- R^2 Inflation, Power Spectrum

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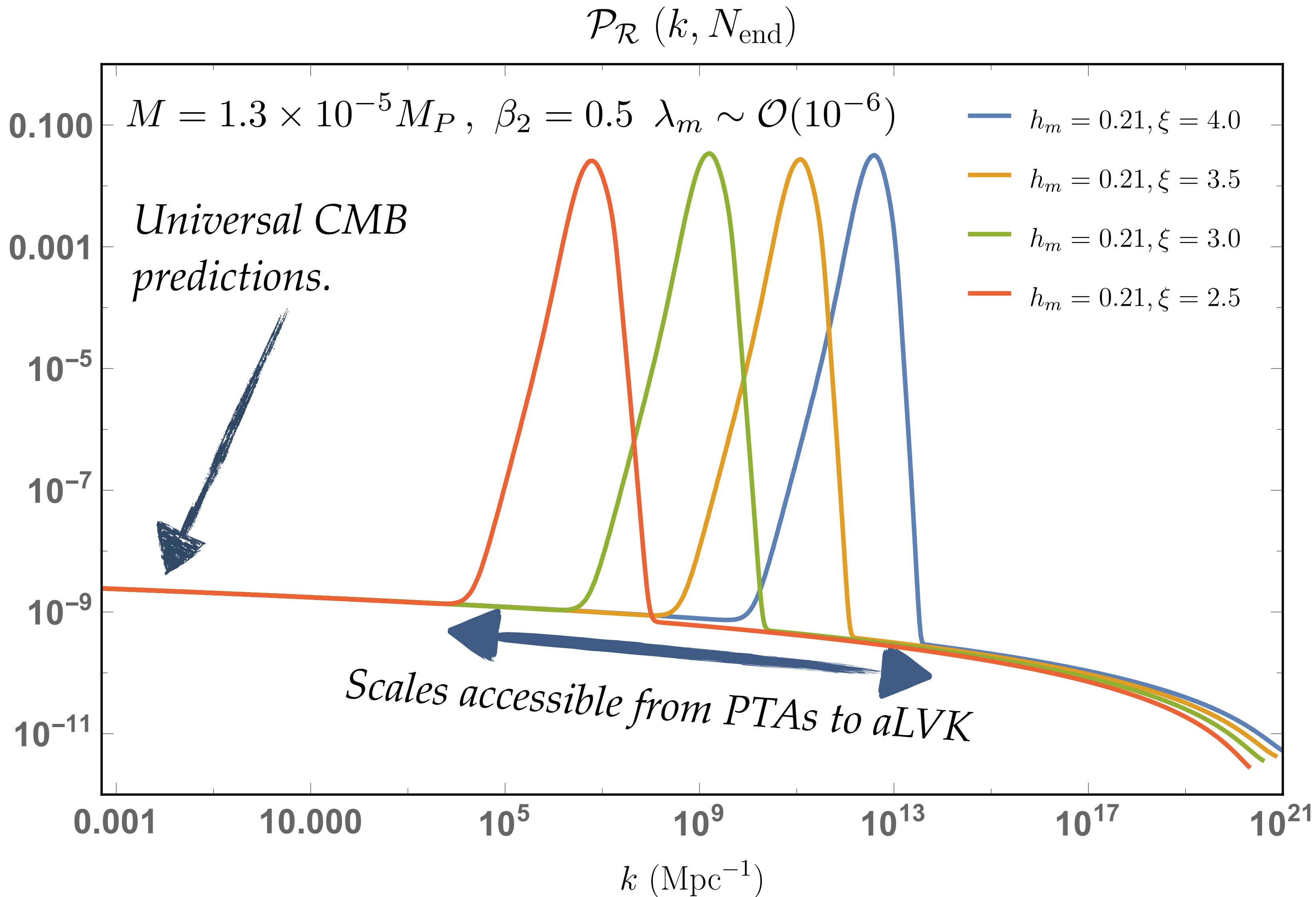


Higgs- R^2 Inflation, Power Spectrum

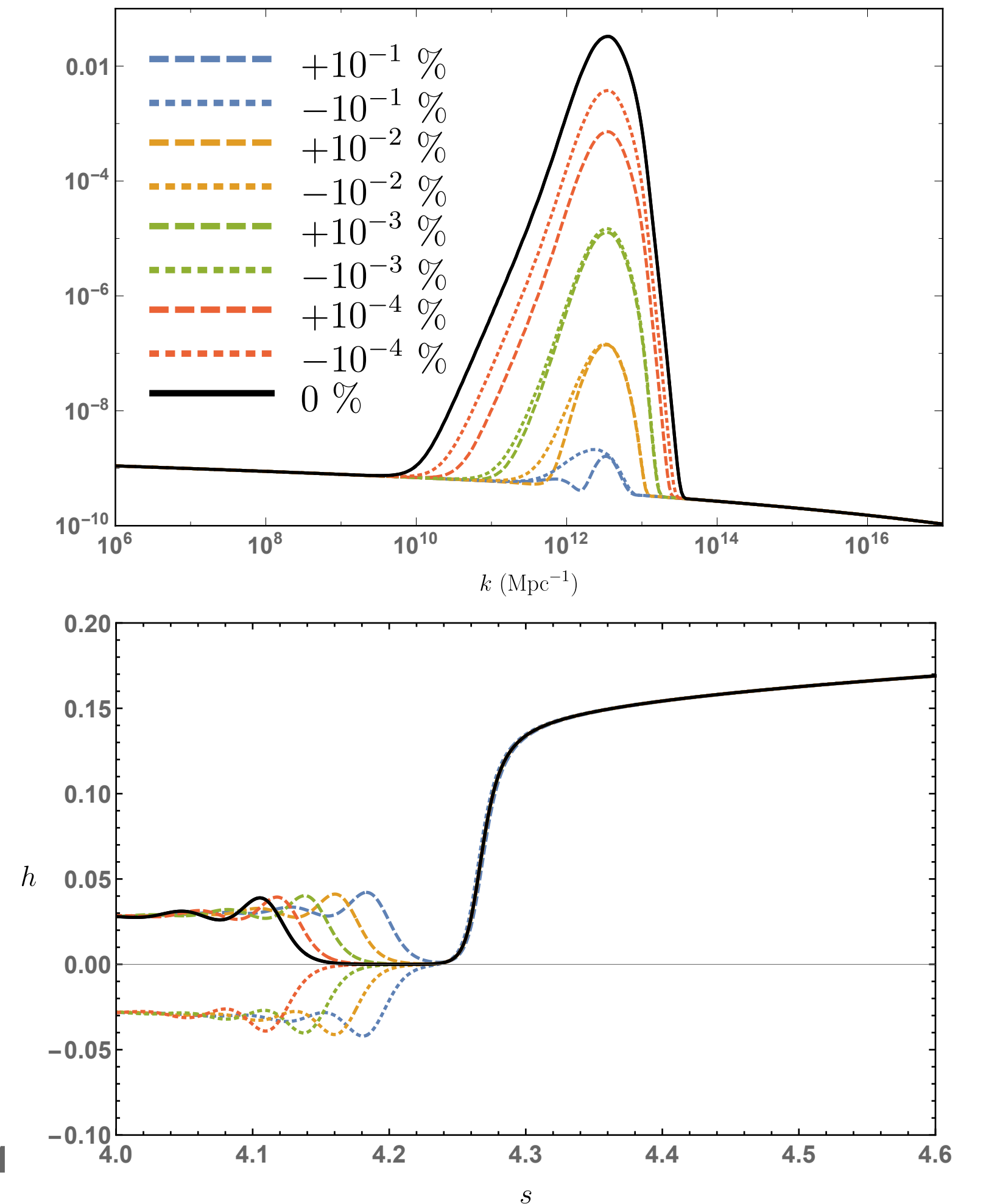
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Higgs- R^2 Inflation, Power Spectrum



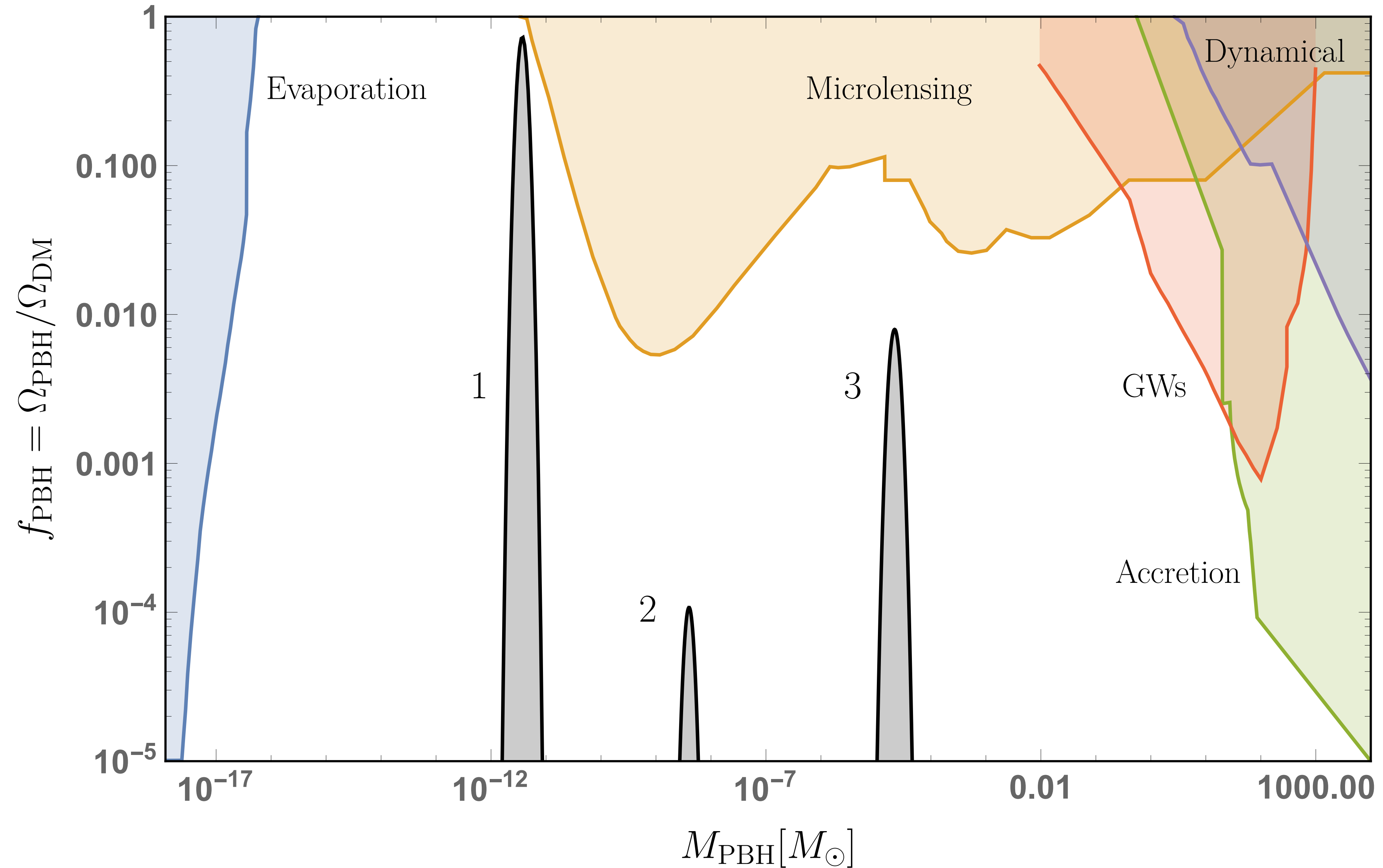
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$$\frac{\delta \lambda_m}{\lambda_m} \equiv \frac{\lambda_m^{\text{dev}} - \lambda_m}{\lambda_m} \sim 10^{-4} \%$$

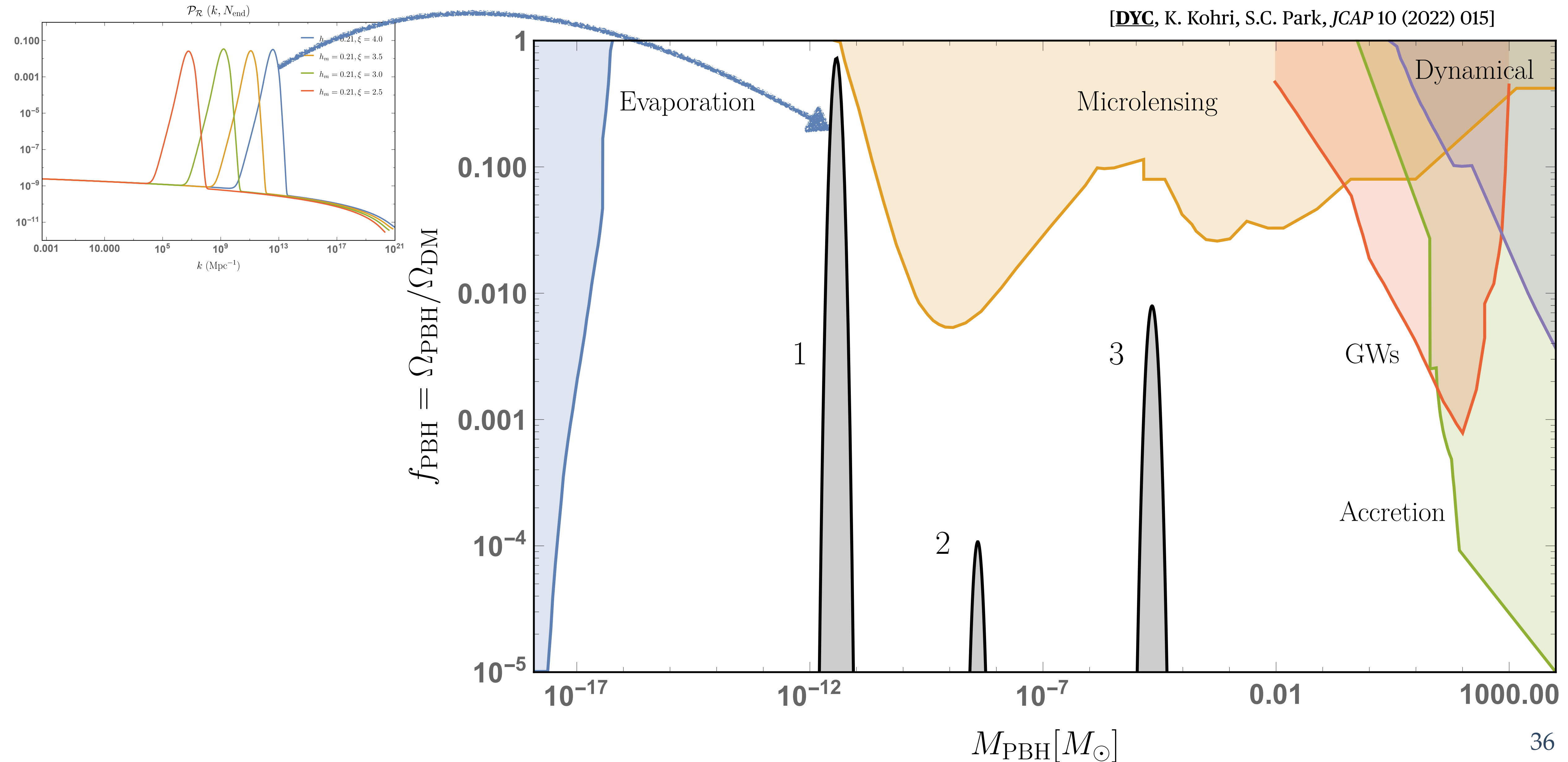
Phenomena — Primordial Black Holes

[DYC, K. Kohri, S.C. Park, *JCAP* 10 (2022) 015]



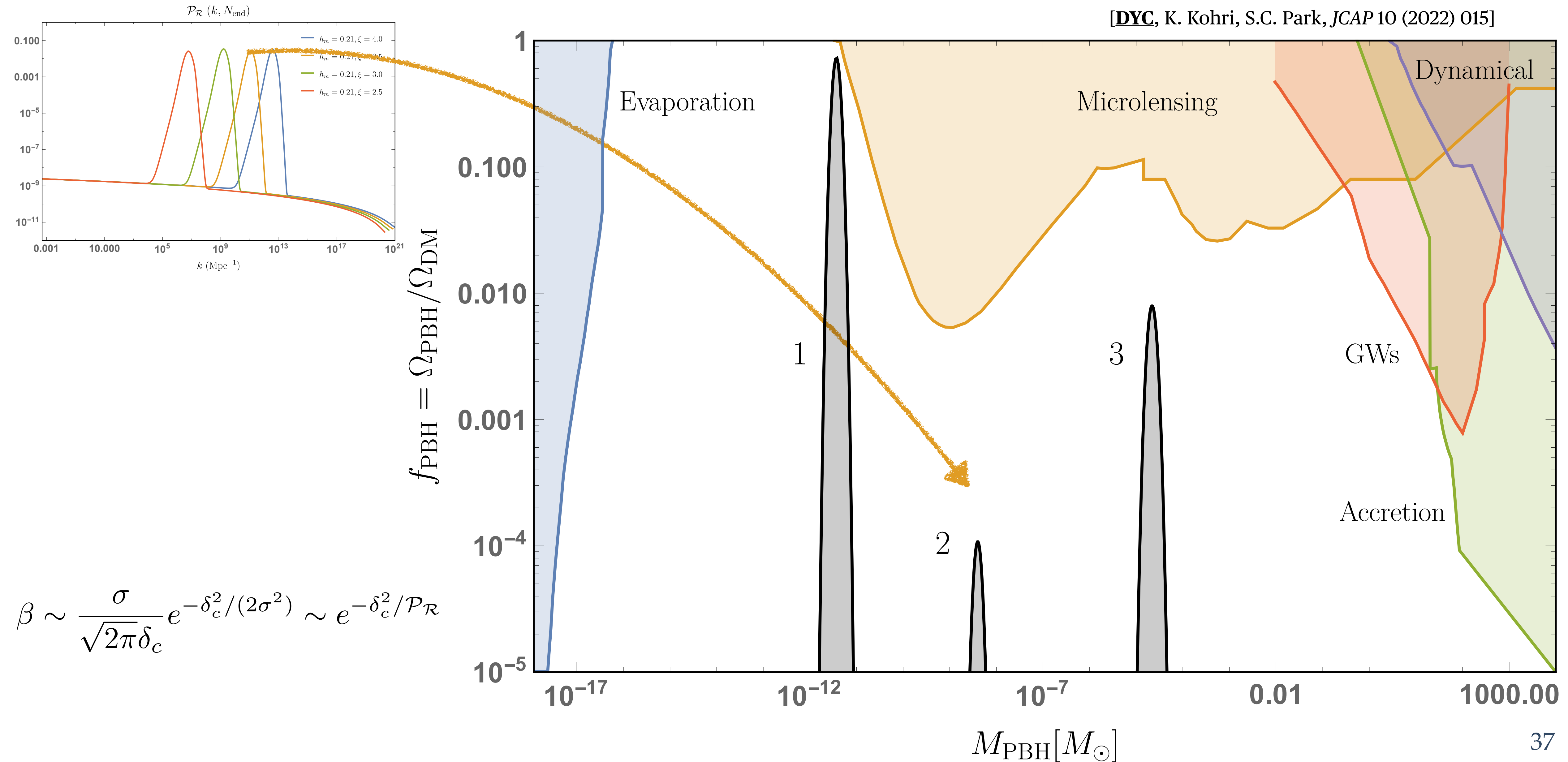
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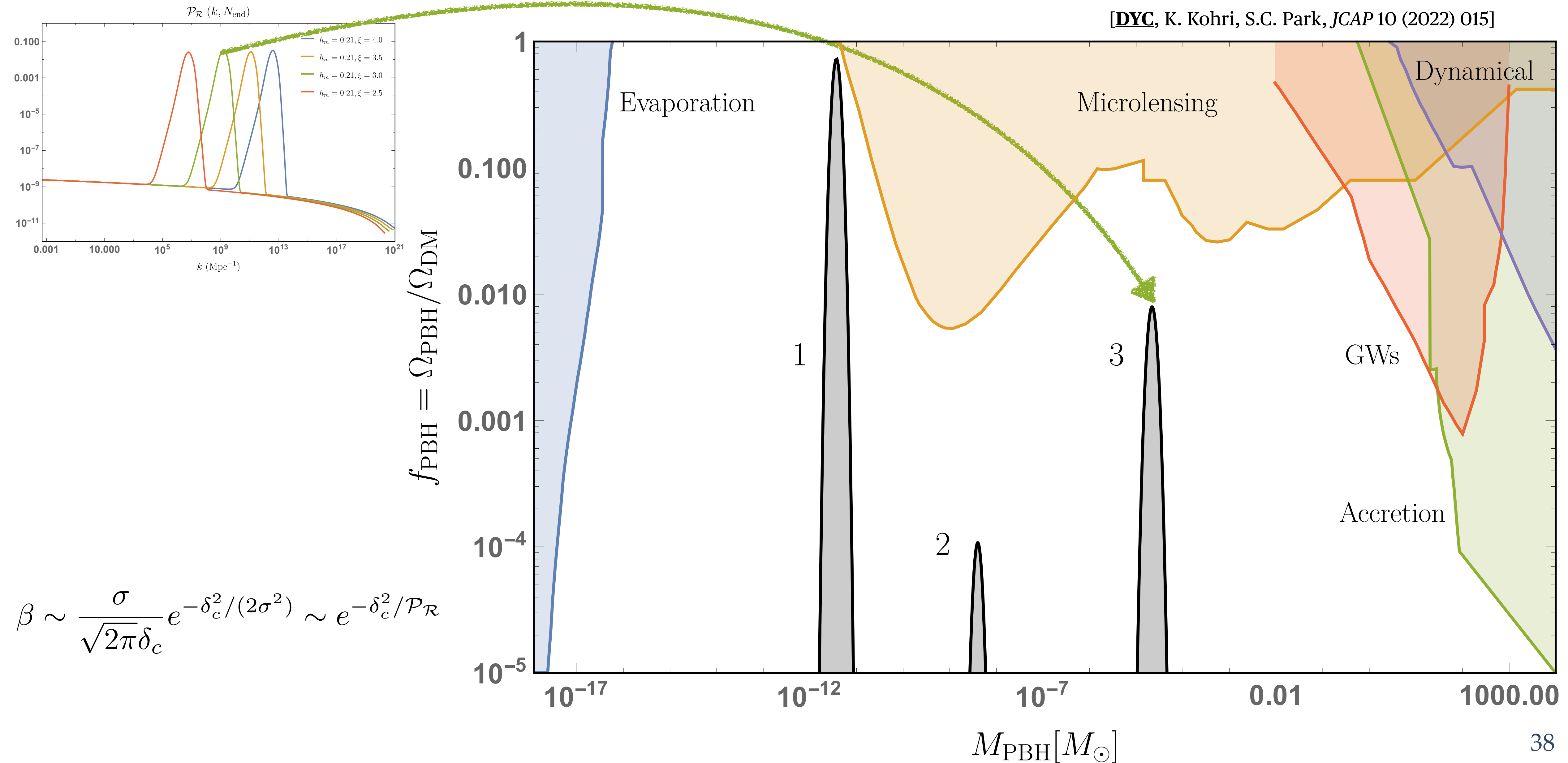
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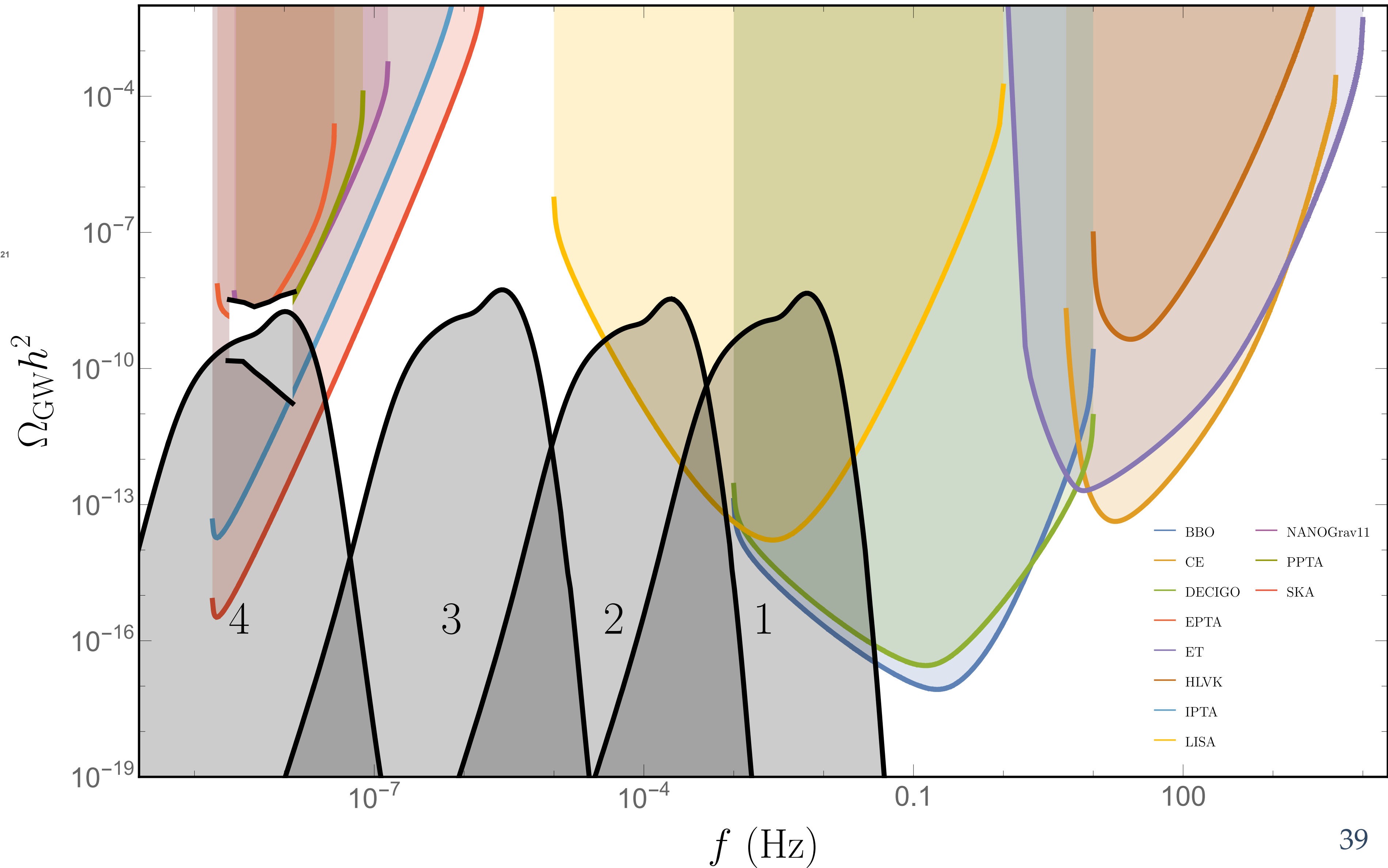
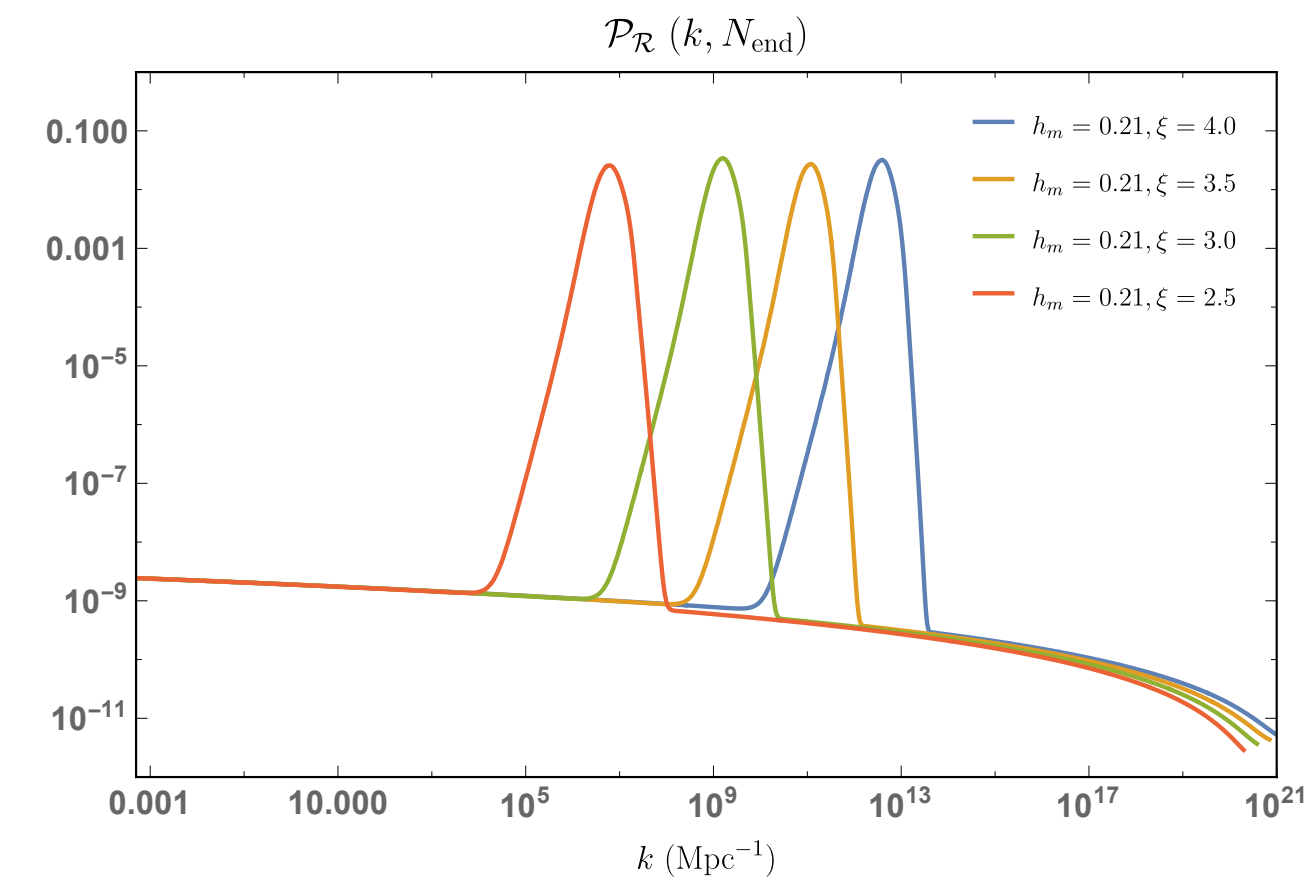
Phenomena — Primordial Black Holes

[DYC, K. Kohri, S.C. Park, *JCAP* 10 (2022) 015]



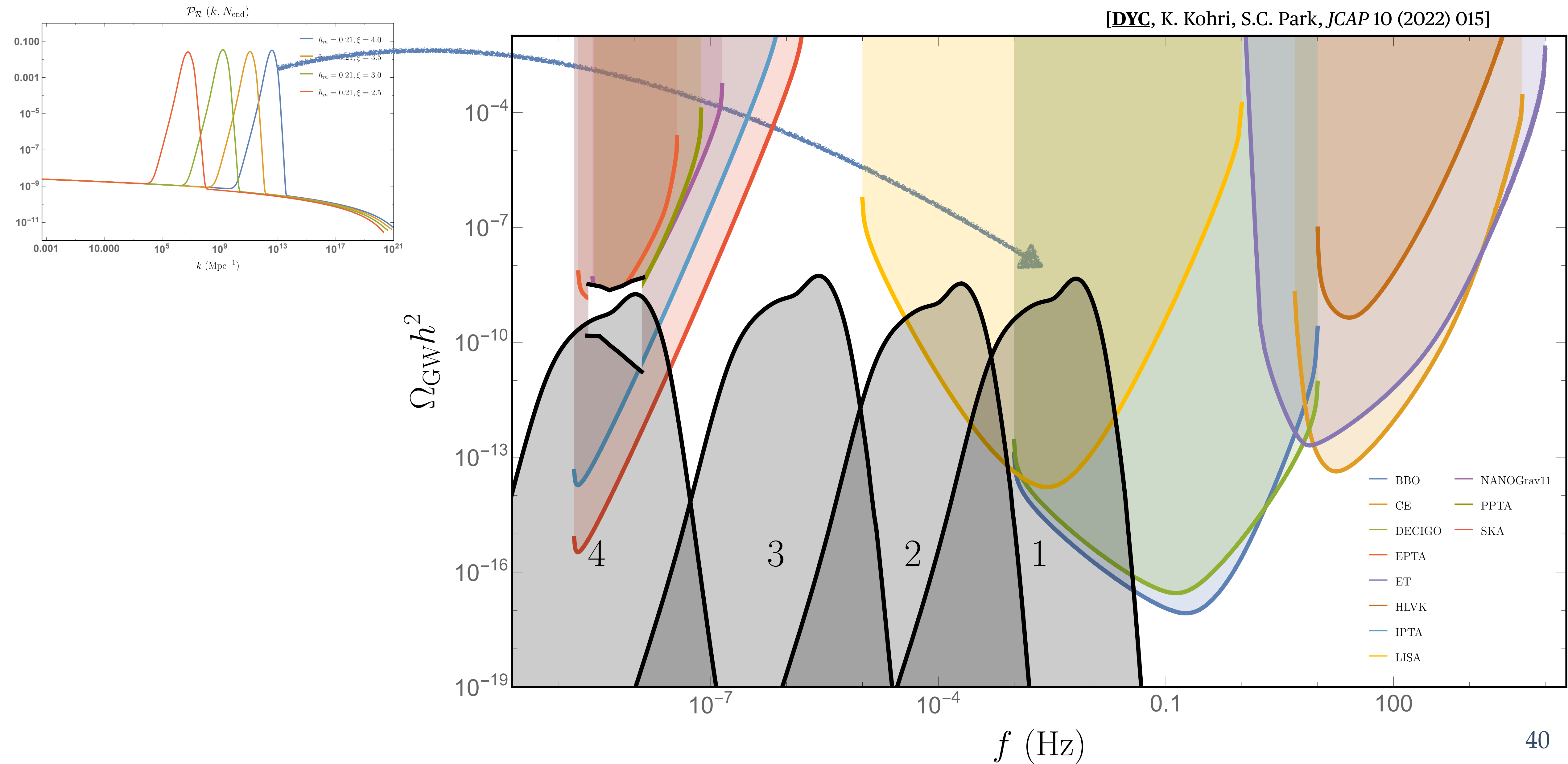
Phenomena — Second order GWs

[DYC, K. Kohri, S.C. Park, *JCAP* 10 (2022) 015]



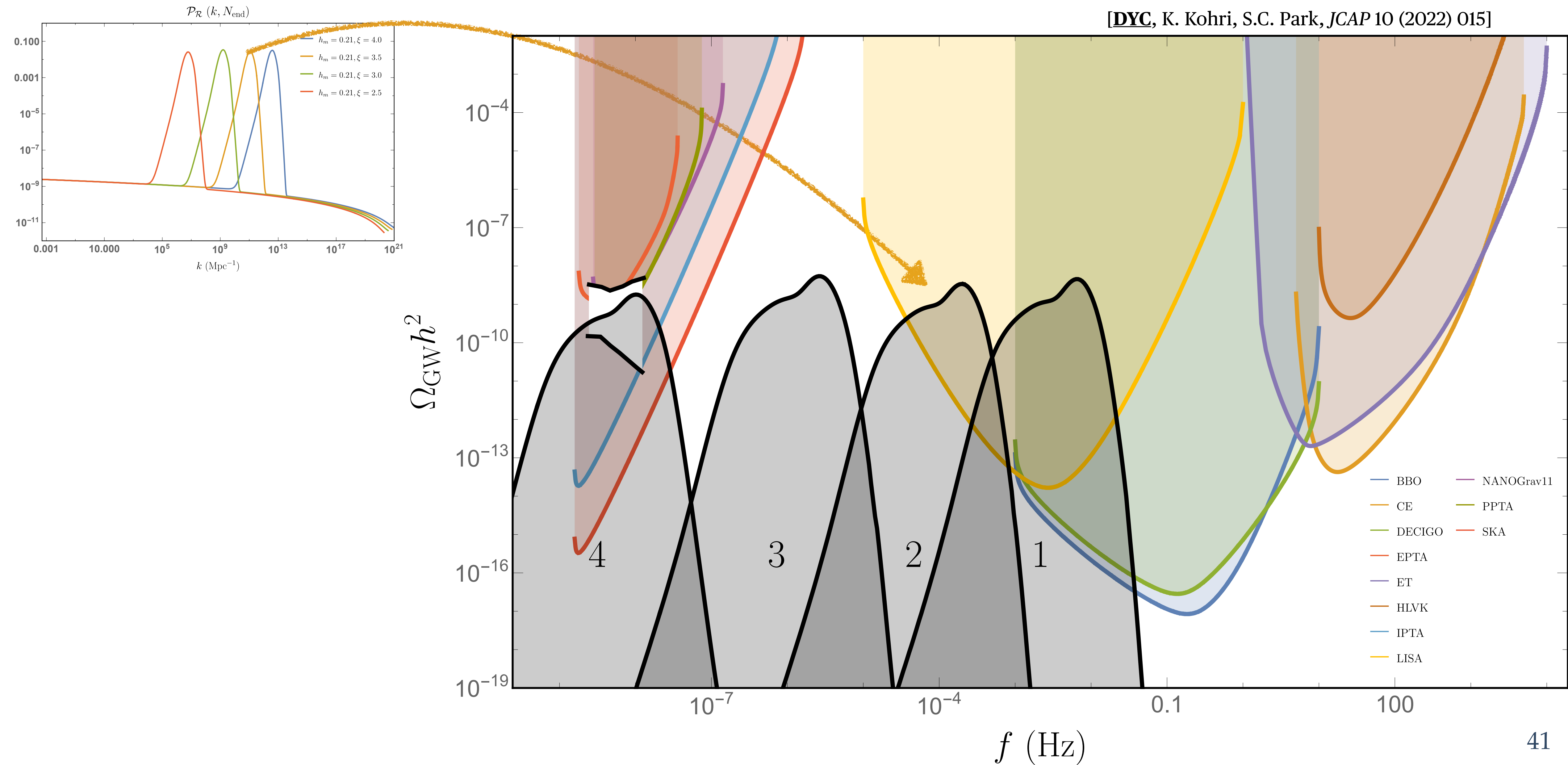
Phenomena — Second order GWs

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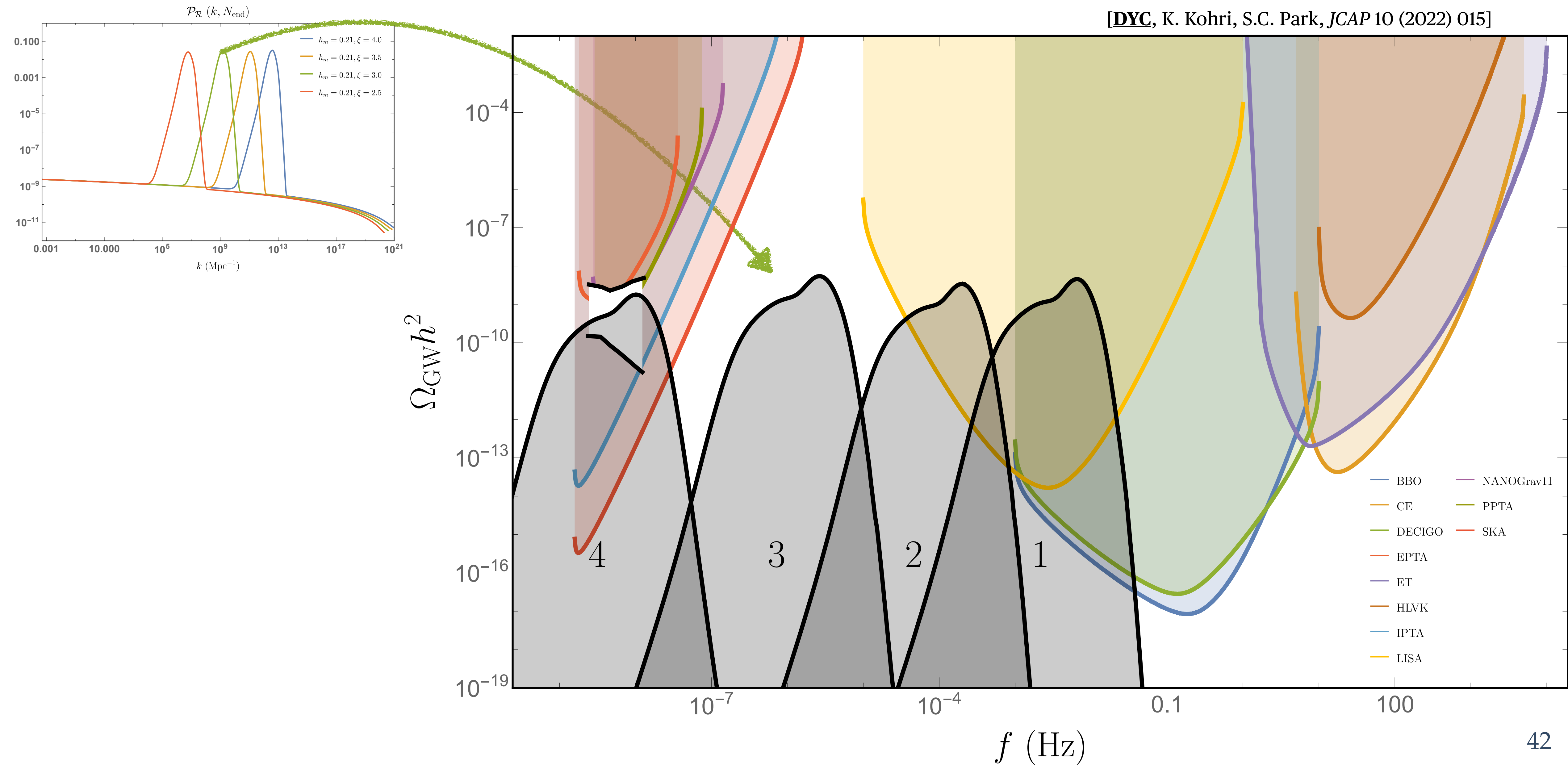
Phenomena — Second order GWs

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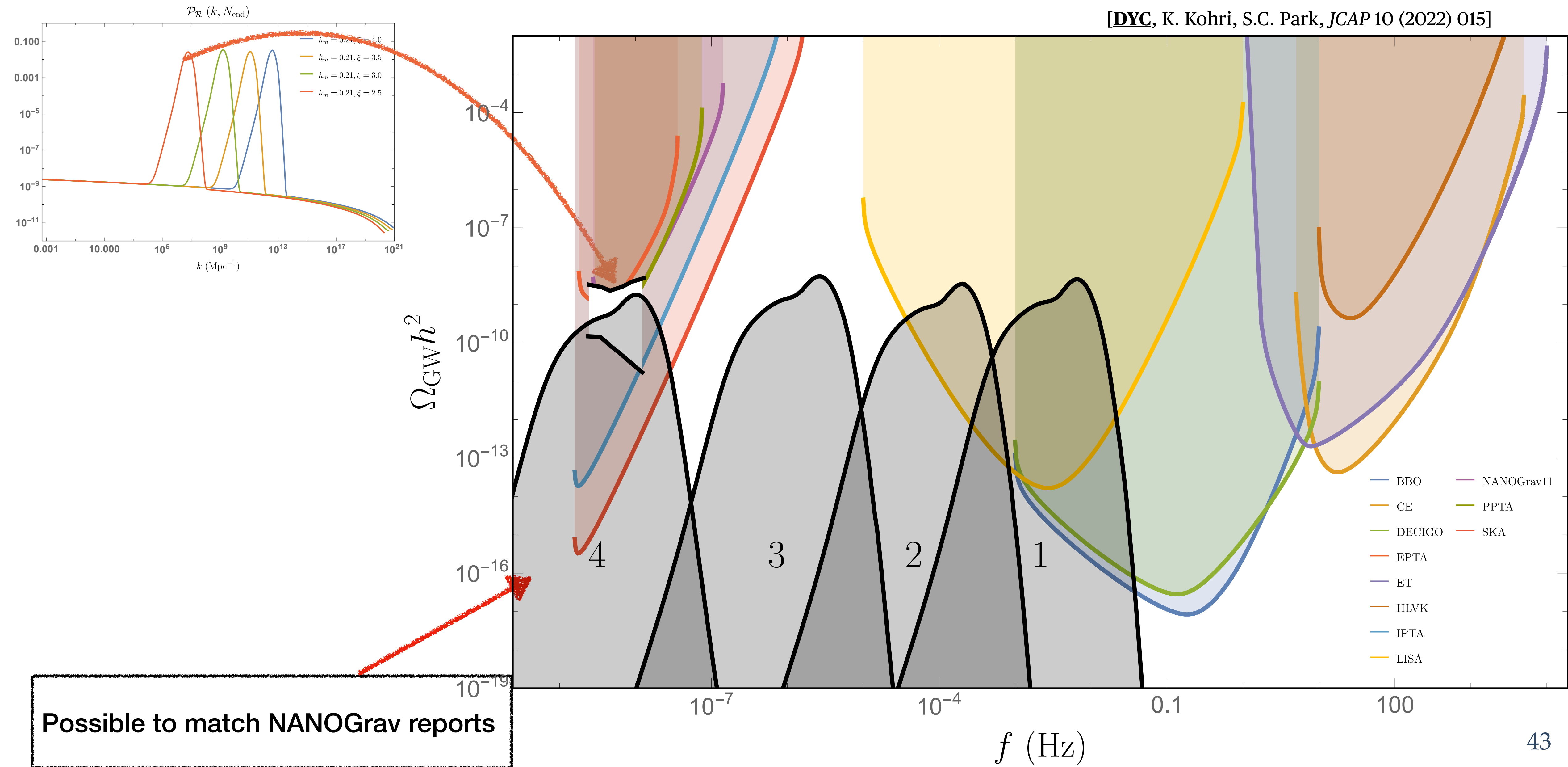
Phenomena — Second order GWs

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Phenomena — Second order GWs

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Possible to match NANOGrav reports

Summary & Outlooks

- ❖ Primordial Black Holes, an appealing DM candidate, can be produced through large curvature perturbations, that can be associated with second order gravitational waves.
- ❖ The Higgs- R^2 model, featuring both a *distinctive valley* and a *noticeable hill*, can exhibit a *tachyonic instability* induced by the *running λ* .
- ❖ *Tachyonic instability* can produce a wide range of PBHs / GWs from *LIGO/Virgo to PTA frequencies*.
- ❖ Backreaction considerations, non-Gaussianity?
- ❖ Further distinguishable features? Direct correlation to collider observables? (m_{top})

“Thank you!”