APCTP-LDU-CCCP

WORKSHOP ON PHYSICS OF DARK COSMOS : DARK MATTER, DARK ENERGY, AND ALL

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Non-Standard neutrino Interaction (NSI) and future atmospheric neutrino oscillation experiments

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P. Bakhti and M. Rajaee, S. Shin, "Non-Standard Interaction of atmospheric neutrino in future experiments", arXiv:2206.02594. Will be published in PRD.

1 Introduction

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Neutrino Oscillation (NO) provides a very strong evidence for physics BSM.



Figure: https://www.symmetrymagazine.org/

• Neutrinos oscillate: after being produced, they change their flavours.

Introduction

Standard three-neutrino mixing paradigm

There is a mixing between mass and flavor states

• Neutrino oscillation occurs since Flavor eigenstates (of weak interactions) and mass eigenstates (of free particle Hamiltonian) are not aligned for neutrinos.

$$|\nu_{\alpha}\rangle = \sum_{i} U_{\alpha i}^{*} |\nu_{i}\rangle, \ \alpha = e, \mu, \tau, \quad i = 1, 2, 3$$
(1)

PMNS mixing matrix

• Flavor states and mass states are related by mixing matrix called PMNS matrix. (Pontecorvo-Maki-Nakagawa-Sakata matrix) This matrix can be parametrized by 4 parameters (3 angles and at least one phase).

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix} = U \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

Neutrino oscillation introduces more parameters to SM.

In the standard parametrization, we can write the PMNS matrix as a product of three mixing matrices.



The goal of current and future oscillation experiments is to measure these parameters.

Probability to detect neutrino with flavor β at distance L:

$$\mathcal{P}(
u_{lpha}
ightarrow
u_{eta}) = \sin^2 2 heta \sin^2(\Delta m_{ij}^2 L/4E), \ \Delta m_{ij}^2 = m_i^2 - m_j^2$$



Current status of neutrino parameters:

		Normal Ore	lering (best fit)	Inverted Ordering $(\Delta \chi^2 = 7.0)$		
		$bfp \pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range	
with SK atmospheric data	$\sin^2 \theta_{12}$	$0.304^{+0.012}_{-0.012}$	$0.269 \rightarrow 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \rightarrow 0.343$	www.
	$\theta_{12}/^{\circ}$	$33.45_{-0.75}^{+0.77}$	$31.27 \rightarrow 35.87$	$33.45_{-0.75}^{+0.78}$	$31.27 \rightarrow 35.87$	
	$\sin^2 \theta_{23}$	$0.450^{+0.019}_{-0.016}$	$0.408 \rightarrow 0.603$	$0.570^{+0.016}_{-0.022}$	$0.410 \to 0.613$	
	$\theta_{23}/^{\circ}$	$42.1^{+1.1}_{-0.9}$	$39.7 \rightarrow 50.9$	$49.0^{+0.9}_{-1.3}$	$39.8 \rightarrow 51.6$	
	$\sin^2 \theta_{12}$	$0.02246^{+0.00062}$	$0.02060 \rightarrow 0.02435$	$0.02241^{+0.00074}$	$0.02055 \rightarrow 0.02457$	
	$\theta_{13}/^{\circ}$	$8.62^{+0.12}_{-0.12}$	$8.25 \rightarrow 8.98$	$8.61^{+0.14}_{-0.12}$	$8.24 \rightarrow 9.02$	
	δcp/°	230^{+36}	$144 \rightarrow 350$	278^{+22}	$194 \rightarrow 345$	
	•CP/	200-25	111 / 555	210-30	151 / 015	
	$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \rightarrow 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \rightarrow 8.04$	
	$\Delta m_{3\ell}^2$	$\pm 2.510^{\pm0.027}$	$\pm 2.430 \rightarrow \pm 2.503$	$-2.400^{+0.026}$	$-2574 \rightarrow -2410$	
	10^{-3} eV^2	+2.010-0.027	T2.450 -7 T2.595	-2.430-0.028	-2.514 -7 -2.410	

Figure: JHEP 09 (2020) 178 [arXiv:2007.14792], NuFIT 5.1 (2021), www.nu-fit.org

Neutrino Oscillation parameters

There is very good agreement between different experiments in the measurement of mixing angle and the mass splitting, although different in baseline and sources.



Neutrino mass ordering is unkown



mass ordering unknown



Neutrino mass ordering could be normal which means that the lightest neutrino has the largest amount of electron neutrino or inverted which means the lightes neutrino has the least amount of electron neutrino.

Introduction

Delta or the CP phase is the least known parameter among oscillation parameters. One of the major plans of the oscillation experiments is to measure this parameter.



A. Himmel 10.5281/zenodo.3959581

- T2K and NOvA both prefer Normal Ordering.
- Disagreement at about 2σ .
- T2K prefers $\delta = 3\pi/2$
- NOvA prefers $\delta = \pi$

Introduction

Neutrino Oscillation (Conversion) in Matter

- Neutrinos propagating in matter interact with particles in the medium.
- These interactions are encoded in an effective potential which depends on the density of fermions in the medium.
- Presence of matter potential modifies the mixing of neutrinos and lead to an effective mixing angle in matter which can be large even for small values of the vacuum mixing angle.
- Matter can enhance the flavor conversion of neutrinos and give rise to a resonant effect under certain conditions of medium density and neutrino energy. [Mikheyev-Smirnov-Wolfenstein (MSW) effect]

$$\tan 2\theta_m = \frac{\frac{\Delta m^2}{2E}\sin 2\theta}{\frac{\Delta m^2}{2E}\cos 2\theta - \sqrt{2}G_F N_e}$$

NSI

- NSIs provide a general effective field theory (EFT) style framework to quantify new physics in the neutrino sector. NSIs represent new physics beyond mass generation.
- NC NSI was first proposed by Wolfenstein in 1977 to explain the neutrino oscillation in matter [L. Wolfenstein, Phys. Rev. D 17 (1978) 2369].
- In general, NSI could be devided into Charged current NSI which mostly affects production and detection processes and Neutral current NSI which is also relevant for detection and can affect propagation through a medium.



- NC NSI can be parametrised as new contributions to the MSW matrix in the neutrino-propagation Hamiltonian
- The modified effective Hamiltonian for neutrino propagation through matter is

$$H_{\rm eff} = \frac{1}{2E} U \begin{pmatrix} 0 & 0 & 0\\ 0 & \Delta m_{21}^2 & 0\\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} U^{\dagger} + V_{\rm CC} \begin{pmatrix} 1 + \varepsilon_{ee} & \varepsilon_{e\mu} & \varepsilon_{e\tau}\\ \varepsilon_{e\mu}^* & \varepsilon_{\mu\mu} & \varepsilon_{\mu\tau}\\ \varepsilon_{e\tau}^* & \varepsilon_{\mu\tau}^* & \varepsilon_{\tau\tau} \end{pmatrix},$$
(2)

- $V_{CC} \equiv \pm \sqrt{2} G_F N_e$ is the effective matter potential
- $\bullet~\pm$ denotes the cases for neutrinos and anti-neutrinos, respectively.

Interaction (NSI)

Best probes for NSI paraeters:

- Atmospheric: $\epsilon_{\mu\tau}$ and $|\epsilon_{\mu\mu} \epsilon_{\tau\tau}|$
- LBL, atmospheric: $\epsilon_{e\mu}$ and $\epsilon_{e\tau}$
- Current constraints on NC NSI parameters [I. Esteban et al, arXiv:1905.05203]

 $|\epsilon|$

$$|e_{e\mu}| < 0.12$$

 $|e_{e\tau}| < 0.26$ (3
 $|\mu_{\mu\tau}| < 0.02$

$$\begin{array}{rclcrcrcrc} -0.018 & < & \epsilon_{\tau\tau} - \epsilon_{\mu\mu} & < & 0.054 \\ 0.35 & < & \epsilon_{ee} - \epsilon_{\mu\mu} & < & 0.93. \end{array}$$
(4)

Experiment	90% C.L. bounds
IceCube	$-0.018 < \varepsilon_{\mu\tau} < 0.0162$
DeepCore	$-0.0201 < \varepsilon_{\mu\tau} < 0.0243$
Super-K	$ \varepsilon_{\mu\tau} < 0.033$

M. Aartsen et al, 2018, J. Salvado et al, 2017.

Interaction (NSI)

NSI in Atmospheric neutrino experiments

- NSI in the context of atmospheric neutrinos has been studied in the literatue [Friedland et al, 2004, GonzalezGarcia et al, 2011, Esmaili et al, 2013 and Choubey et al, 2014, etc...]
- This work: The presence of NSI via its impact on the propagation of atmospheric neutrinos through the Earth using future atmospheric experiments.

	$E < 10 { m GeV}$	10 GeV < E < 200 GeV	$E > 200 { m GeV}$
depends on δ_{CP}	Yes	No	No
ν_{atm} Statistics	High	High	Low
ν_{LBL} Statistics	High	Low	Non

Atmospheric neutrinos are typically produced around 15 kilometers above Earth's surface.



Atmospheric neutrinos have the maximum matter effect while propagating through earth matter

Interaction (NSI)

NSI in Atmospheric neutrino experiments

- For energies E > 10 GeV, energies much above the θ_{13} resonance: 2ν
- The only relevant NSI parameters are $\varepsilon_{\mu\tau}$ and $|\varepsilon_{\mu\mu} \varepsilon_{\tau\tau}|$.
- In this energy limit, we will consider ν_{μ} and ν_{τ} detection to constrain $\varepsilon_{\mu\tau}$ and $| \varepsilon_{\mu\mu} - \varepsilon_{\tau\tau} |$.



Interaction (NSI)

Atmospheric neutrino oscillation probability



• Oscillation probability is not sensitive to the value of δ_{CP} for energies higher than 10 GeV.



$$\Delta m^2_{31,m} \approx \Delta m^2_{31} \left(1 + 2\varepsilon_{\mu\tau} A_{CC} + \frac{1}{2} \frac{|\varepsilon_{\mu\mu} - \varepsilon_{\tau\tau}|^2 A^2_{CC}}{1 + \varepsilon_{\mu\tau} A_{CC}} \right) \,,$$

- The relevant NSI parameters are $\epsilon_{\mu\tau}$ and $\epsilon_{\mu\mu} \epsilon_{\tau\tau}$ with $\epsilon_{\mu\tau}$ being the most important NSI parameter.
- $\varepsilon_{\mu\tau} \equiv |\varepsilon_{\mu\tau}| e^{i\phi_{\mu\tau}}$
- Domainant disappearance channel: $\nu_{\mu} \rightarrow \nu_{\mu}$ and $\bar{\nu_{\mu}} \rightarrow \bar{\nu_{\mu}}$,
- In the leading order $\epsilon_{\mu\tau}$ appears as $|\epsilon_{\mu\tau}| \cos(\phi_{\mu\tau})$. Considering the complex phase can vary the effective value of $\epsilon_{\mu\tau}$ between $|\epsilon_{\mu\tau}|$ and $+ |\epsilon_{\mu\tau}|$.

Details of the experiments

Experiments	$E_{\rm th}$ for ν_{μ}	$E_{\rm th}$ for ν_e	fiducial volume
DUNE	$135 { m MeV}$	$10 { m MeV}$	$40 \mathrm{~kt}$
HK	$110 { m MeV}$	$6.5 { m MeV}$	260 kt
KNO	$110 { m MeV}$	$6.5 { m MeV}$	260 kt
ORCA	$3 {\rm GeV}$	$3 { m GeV}$	6 Mton

Table: The detailed information of the reference experiments for this analysis [DUNE 2020, Kelly, et al. 2021, KM3Net 2016, Hyper-Kamiokande 2018].

Experiments	ν_e and ν_μ detection efficiency
DUNE	85%
HK	80%
KNO	80%
ORCA	50% (E < 10 GeV) , 90% (E > 10 GeV)



Atmospheric neutrino oscillation probability



Figure: Oscillograms for the probability $P_{\mu\mu}$ assuming $\delta_{CP} = 0^{\circ}$. We have considered known normal mass ordering, and the oscillation parameters are set as [nufit 2020].

- Left: $P_{\mu\mu}$ in the absence of NSI parameters.
- Right panel: we have assumed non rezo $| \varepsilon_{\mu\mu} \varepsilon_{\tau\tau} |$ while setting other NSI parameters to zero.

Atmospheric neutrino oscillation probability



Figure: Oscillograms for the probability $P_{\mu\mu}$ assuming $\delta_{CP} = 0^{\circ}$.

- The huge difference in oscillation probability in the high energies (E > 10 GeV) as well as in larger zenith angles.
- More significant in the case of $\varepsilon_{\mu\tau} = 0.01$ compared to $|\varepsilon_{\mu\mu} \varepsilon_{\tau\tau}| = 0.1$ case.







Constraints on $\epsilon_{\mu\tau}$ with DUNE

DUNE can detect $\tau\text{-neutrinos}$

Does including τ -neutrinos detection channel can improve the sensitivity to constrain NSI parameters significantly?



Figure: χ^2 after 10 years of running of the DUNE experiment as a function of the NSI parameters $\varepsilon_{\mu\tau}$. We assume the τ neutrino detection efficiency of 30% in the left panels and 100% in the right panels.

Constraints on $| \varepsilon_{\mu\mu} - \varepsilon_{\tau\tau} |$ with DUNE

Does including τ -neutrinos detection channel can improve the sensitivity to constrain NSI parameters significantly?



In case of $|\varepsilon_{\mu\mu} - \varepsilon_{\tau\tau}|$, including ν_{τ} events does not improve the sensitivity.

Constraints on $\epsilon_{\mu\tau}$



Figure: The expected Chi-squared for NSI as a function of $|\varepsilon_{\mu\tau}|$ assuming 10 years of data taking. The neutrino mass hierarchy is assumed to be known and normal.

The experimental sensitivities can be significantly improved by around two orders of magnitude from the current bounds [E. Esteban, et al. 2019].

Constraints on $|\epsilon_{\mu\mu} - \epsilon_{\tau\tau}|$



Figure: The expected Chi-squared for NSI as a function of $|\varepsilon_{\mu\mu} - \varepsilon_{\tau\tau}|$.

- ORCA has the best sensitivity to constrain NSI parameters $\varepsilon_{\mu\tau}$ and $| \varepsilon_{\mu\mu} \varepsilon_{\tau\tau} |$.
- KNO+ HK with a larger volume and in consequence higher statistics, will be more sensitive to constrain NSI parameters.

Take aways

- For energies more than 10 GeV, the only relevant NSI parameters are $\varepsilon_{\mu\tau}$ (more sensitive) and $|\varepsilon_{\mu\mu} \varepsilon_{\tau\tau}|$.
- Effect of τ neutrino detection is important for constraining $\varepsilon_{\mu\tau}$, assuming 100% detection efficiency for τ neutrinos at DUNE.
- ORCA has the best sensitivity to constrain relevant NSI parameters $\varepsilon_{\mu\tau}$ and $|\varepsilon_{\mu\mu} - \varepsilon_{\tau\tau}|$.
- HK+KNO can improve the current constraints on NSI parameters $\varepsilon_{\mu\tau}$ and $| \varepsilon_{\mu\mu} - \varepsilon_{\tau\tau} |$.

Constraints on $\varepsilon_{e\mu}$, $\varepsilon_{e\tau}$ and $|\varepsilon_{ee} - \epsilon_{\mu\mu}|$

- In the energy range of 1 10 GeV, the effect of δ_{CP} becomes more significant because of the θ_{13} resonance effect.
- Correlations between NSI parameters and δ_{CP} can affect the oscillation probabilities.

Constraints on δ_{CP} vs. $\epsilon_{e\mu}$ with DUNE and KNO



Figure: 1 σ , 2 σ and 3 σ C.L. contours in the $\delta_{CP} - \varepsilon_{e\mu}$ plane expected from 10 years of running of the DUNE and KNO

Assuming δ_{CP} is maximal, no CPV is excluded by DUNE at 1σ C.L.

HK can exclude no CPV at 2σ C.L..

Constraints on δ_{CP} vs. $\epsilon_{e\mu}$ with HK+KNO and ORCA



HK + KNO Can exclude no CPV at 3σ C.L..

ORCA can exclude no CPV at 1σ C.L.

Constraints on δ_{CP} vs. $\epsilon_{e\mu}$ with Long Baseline experiments



P. Bakhti and M. R, "Sensitivities of future reactor and long-baseline neutrino experiments to NSI," Phys. Rev. D **103** (2021) no.7, 075003 [arXiv:2010.12849 [hep-ph]].



Figure: 1σ , 2σ and 3σ C.L. contours in the $\delta_{CP} - \varepsilon_{e\tau}$ expected from 10 years of running of DUNE (left) and HK or KNO (right). We have assumed the true value of $\delta_{CP} = 270^{\circ}$. Normal neutrino mass hierarchy is assumed to be true.

• DUNE excludes no CPV at 1σ C.L..

• HK (KNO) excludes no CPV at 2σ C.L..



Figure: 1σ , 2σ and 3σ C.L. contours in the $\delta_{CP} - \varepsilon_{e\tau}$ expected from 10 years of running of HK+KNO (left) and ORCA (right) experiments.

• HK+KNO excludes no CPV at 3σ C.L..

Constraints on δ_{CP} vs. $\epsilon_{e\tau}$ with Long Baseline experiments



P. Bakhti and M. R, "Sensitivities of future reactor and long-baseline neutrino experiments to NSI," Phys. Rev. D **103** (2021) no.7, 075003 [arXiv:2010.12849 [hep-ph]].



Figure: 1σ , 2σ and 3σ C.L. contours in the $\delta_{CP} - |\varepsilon_{ee} - \varepsilon_{\mu\mu}|$ for DUNE (left) and HK (right).

DUNE can exclude no CPV at 1σ C.L. while HK exclude no CPV at 2σ C.L.



Figure: 1 σ , 2 σ and 3 σ C.L. contours in the $\delta_{CP} - |\varepsilon_{ee} - \varepsilon_{\mu\mu}|$ for KNO (left) and ORCA (right).

KNO+HK can exclude no CPV at 1σ C.L. while ORCA is not sensitive to the determination of CP phase due to its rather high threshold energy (3 GeV). However, if the energy threshold of ORCA reduces to lower energies, its sensitivity to the determination of CP phase will increase significantly.

Constraints on δ_{CP} vs. $\epsilon_{e\tau}$ with Long Baseline experiments



P. Bakhti and M. R, "Sensitivities of future reactor and long-baseline neutrino experiments to NSI," Phys. Rev. D **103** (2021) no.7, 075003 [arXiv:2010.12849 [hep-ph]].

- We have investigated the impact of the NSI parameters $\epsilon_{\mu\tau}$ and $|\epsilon_{\mu\mu} \epsilon_{\tau\tau}|$ on the future atmospheric neutrino experiments DUNE, HK, ORCA and KNO using 10 years of simulated data.
- Our analysis shows that ORCA and HK+KNO can improve the current constraints on $\epsilon_{\mu\tau}$ and $|\epsilon_{\mu\mu} \epsilon_{\tau\tau}|$ by two orders of magnitudes.
- HK+KNO can improve the constraints on $\epsilon_{e\mu}$, $\epsilon_{e\tau}$ and $|\epsilon_{\mu\mu} \epsilon_{ee}|$ by one order of magnitude.
- HK+KNO can determine δ_{CP} in presence of NSI with 3σ C.L.

Future prospects

- Future atmospheric neutrino experiments will open a new window towards determining the possible NSI parameters.
- We expect the future improvements of the tau-neutrino detection efficiency in the experiments such as DUNE and lowering E_{th} in gigantic size experiments such as ORCA are crucial in probing the NSI parameters and the CP violation phase.
- The analysis assuming the unknown mass ordering and combining with the long-baseline data would provide more interesting results.

Thank you for your attention.

Neutrino Oscillation (conversion) in Matter (Simple scheme of two neutrinos:)

• Evolution equation for two neutrinos in matter:

$$i\frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + V_{CC} & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$
$$V_{CC} = \sqrt{2} G_F N_e \,,$$

• After the diagonalization of the effective hamiltonian in matter, we find the following mass eigenstates:

$$\nu_1^m = \nu_e \cos \theta_m + \nu_\mu \sin \theta_m$$
$$\nu_2^m = -\nu_e \sin \theta_m + \nu_\mu \cos \theta_m \,,$$

Neutrino Oscillation in Matter

• After the diagonalization of the effective hamiltonian in matter, we find the following mass eigenstates:

$$\nu_1^m = \nu_e \cos \theta_m + \nu_\mu \sin \theta_m$$

$$\nu_2^m = -\nu_e \sin \theta_m + \nu_\mu \cos \theta_m ,$$

• where θ_m is the mixing angle in matter, related with the vacuum mixing parameters by

$$\tan 2\theta_m = \frac{\frac{\Delta m^2}{2E} \sin 2\theta}{\frac{\Delta m^2}{2E} \cos 2\theta - \sqrt{2} G_F N_e}.$$



Figure: Honda et al, Phys.Rev.D75:043006, 2007

DUNE



http://lbnf.fnal.gov/

- Deep Underground Neutrino Experiment (DUNE)
- $\bullet~1300~{\rm km}$ baseline and 250 ${\rm MeV} < E < 8~{\rm GeV}$ peak around 3 GeV
- Atmosphric neutrino, Solar neutrino and SN neutrino
- Liquid Argon Time-Projection Chamber(LArTPC) with 40 kton fiducial mass

Hyper Kamiokande



Figure: https://www.hyperk.org/

- Hyper-Kamiokande (HK) the next generation of large-scale water Cherenkov detectors is planned to be an order of magnitude bigger than its predecessor, Super-Kamiokande (SK).
- We assume fiducial mass of HK: 200 kton

Korea Neutrino Observatory (KNO)



Figure: http://www.kno.or.kr/

- The Korean Neutrino Observatory (KNO) is proposed as a next generation underground neutrino detector in Korea.
- The detector mass is under discussion but in this work we assume fiducial mass of KNO: 200 kton.

Korea Neutrino Observatory (KNO)

• Detects J-PARC neutrino beam (both detectors in Korea and Japan at the same time).

Backup



Figure: Abe et al, 1611.06118

- Higher mass density and longer baseline \rightarrow better sensitivity to neutrino mass hierarchy and non-standard neutrino interactions
- More sensitive to leptonic CP violation
- Better sensitivity to neutrinos of astronomical origin.

ORCA



Figure: www.km3net.org

- Cubic Kilometre Neutrino Telescope or KM3NeT at the bottom of Mediterranean Sea will host a megaton scale detector (ORCA) for neutrino oscillation studies of atmospheric neutrinos.
- $\bullet~6$ Mton fiducial mass, Energy threshold of 3 GeV, 50% efficiency for ν detection with energies lower than 10 GeV
- will consist of 115 strings in a 20 m triangular grid.

The effective NSI parameter relevant for the neutrino propagation through matter

$$\varepsilon_{\alpha\beta} \equiv \sum_{f=e,u,d} \left(\varepsilon_{\alpha\beta}^{fL} + \varepsilon_{\alpha\beta}^{fR} \right) \; \frac{N_f}{N_e} \equiv \sum_{f=e,u,d} \varepsilon_{\alpha\beta}^f \; \frac{N_f}{N_e} \,, \tag{5}$$

where N_f is the number density of fermion f.

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• In the approximation of a neutral Earth, the number densities of electrons, protons, and neutrons are identical, $N_u \approx N_d \approx 3N_e$. Thus,

$$\varepsilon_{\alpha\beta} \approx \varepsilon_{\alpha\beta}^e + 3\,\varepsilon_{\alpha\beta}^u + 3\,\varepsilon_{\alpha\beta}^d \,. \tag{6}$$