Trouble with geodesics in black-to-white hole bouncing scenarios

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WEI-CHEN LIN (PUSAN NATIONAL UNIVERSITY)

COLLABORATION WITH: DEOG-KI HONG AND DONG-HAN YEOM

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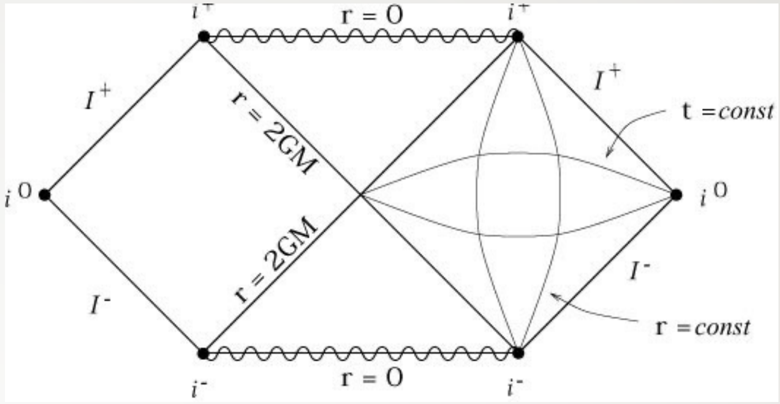
OUTLINE

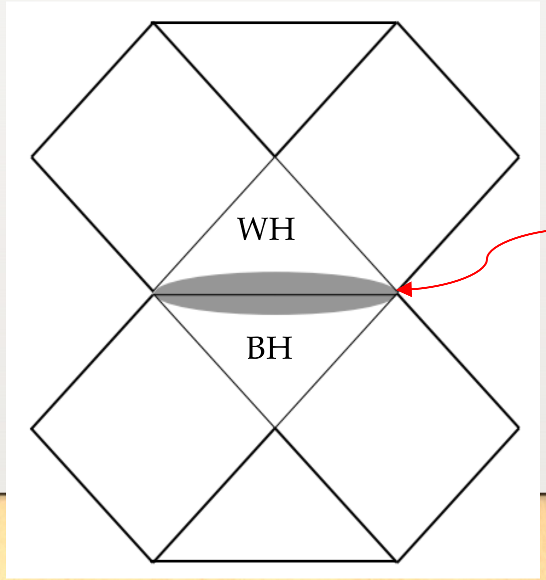
1. Introduction: What is a black-to-white hole bounce?
2. The thin shell generalization
3. Geodesic analysis – energy shift
4. Squeezing of radial geodesics & the implication
5. Possible rescues an the ongoing projects
6. Summary

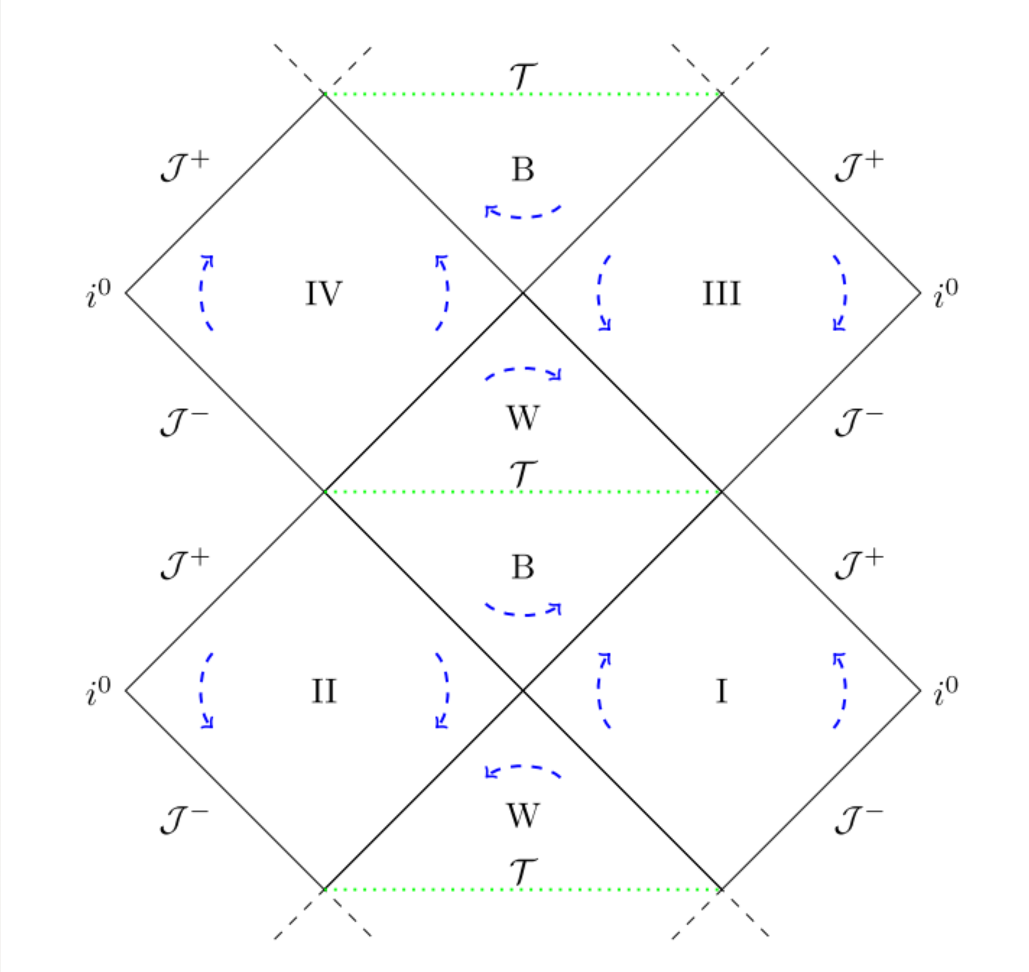
1. Introduction

What is a black-to-white hole bounce?



- Various approaches to resolve the singularity inside a black hole
- A regular black hole: a singularity-free solution that can be interpreted as a classical extension of spacetime.
- Different approaches in LQG create different black-to-white hole bouncing models.
- The Penrose diagram:





The Penrose diagram of the extended Kruskal space-time.
A figure from ``Quantum Extension of the Kruskal Space-time ’’
by Ashtekar, Olmedo, Singh, arXiv:1806.02406

A black-to-white hole bounce with mass difference:

Coriche and Singh
arXiv:1506.08015

Bodendorfer, Mele and Münch
arXiv: 1902.04542, 1911.12646, 1912.00774

A mass (de-)amplification relation:

$$M_+ = M_- \left(\frac{M_-}{m}\right)^{\beta-1} \quad ; \beta = \frac{5}{3} \text{ or } \frac{3}{5}$$

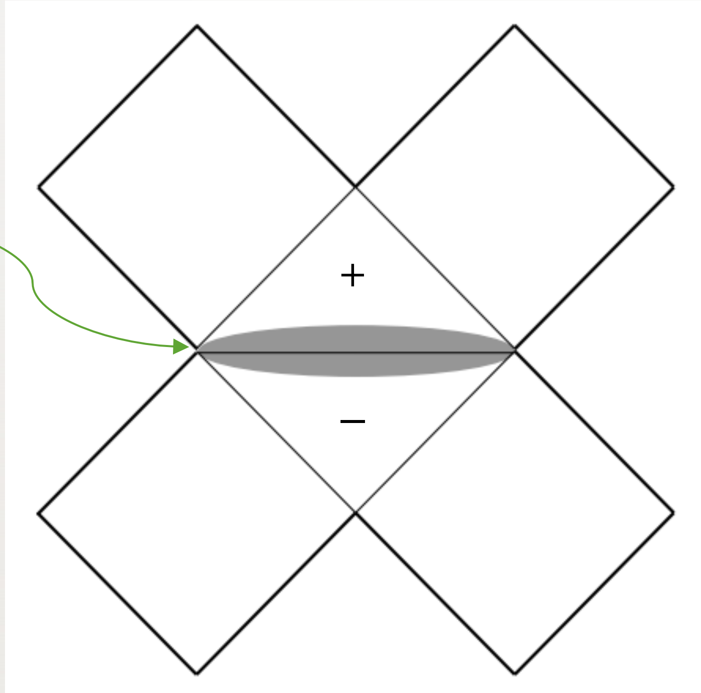
QG effect at a unique curvature scale

2. Thin-shell generalization

Israel Junction conditions:

- 1st : the induced metric must be the same on both sides of the shell
- 2nd : The shell satisfies Einstein equation

Thin shell with equation of state violating NEC



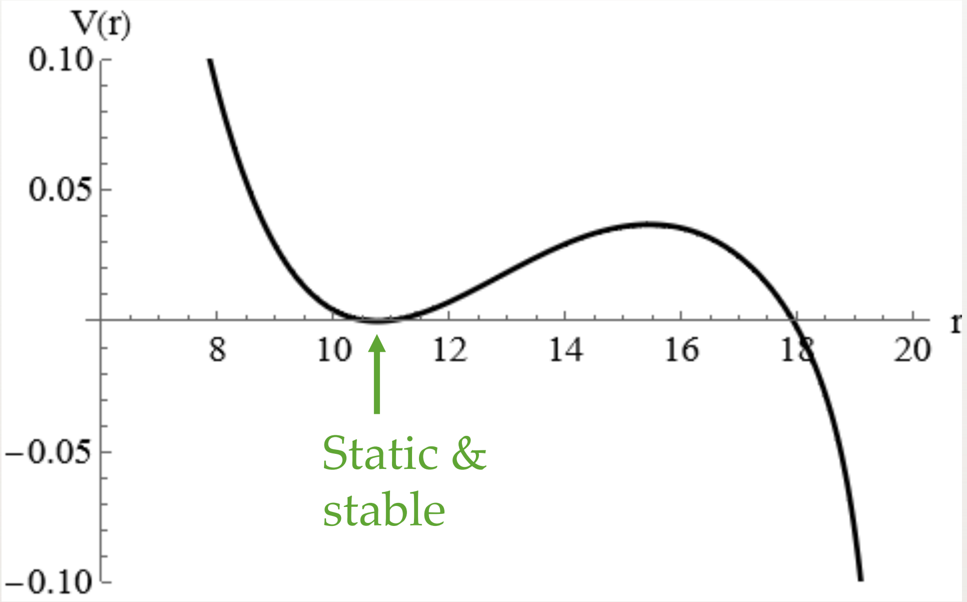
$$ds^2_\pm = -(-f_\pm)^{-1}dr^2_\pm + (-f_\pm)dt^2_\pm + r^2_\pm d\Omega^2$$

$$f_\pm = 1 - \frac{2M_\pm}{r}$$

The scenario without mass difference is studied by Brahma and Yeom [arXiv:1804.02821]

$$ds^2_{\text{shell}} = d\tau^2 + r^2(\tau)d\Omega^2$$

$$r_\pm = r(\tau)$$



$M_- = 10$, and $M_+ = 0.9M_-$

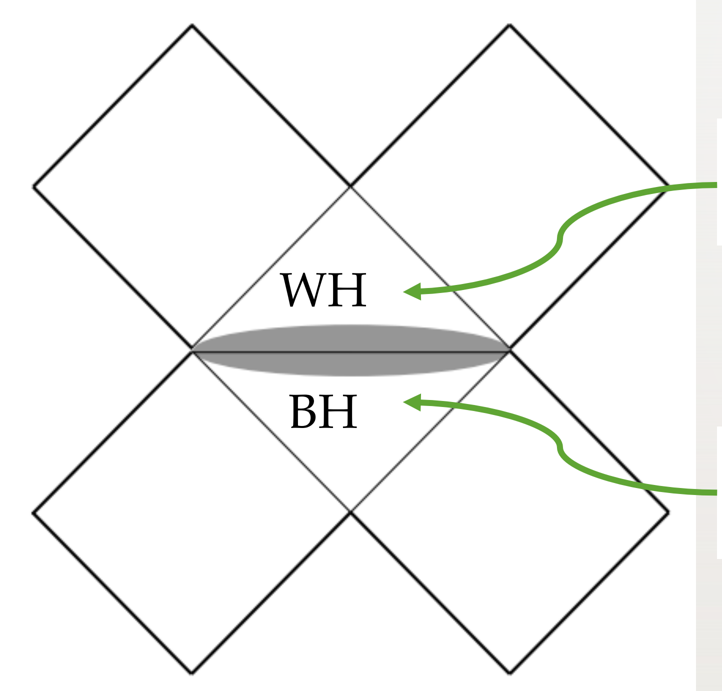
3. GEODESIC ANALYSIS

Tracking the timelike radial geodesics

Geodesics must cross the thin shell smoothly,
or equivalently speaking,
the geodesics have no cusp at the thin shell.
(a coordinate-independent way)

$$\gamma_{\pm}(b) \equiv \lim_{r_{\pm}\to b} -g_{\pm\alpha\beta}\mathcal{U}^{\alpha}_{\pm}\mathcal{V}^{\beta}_{\pm}$$

$$\gamma = \frac{1}{\sqrt{1-v_{rel}^2}}$$

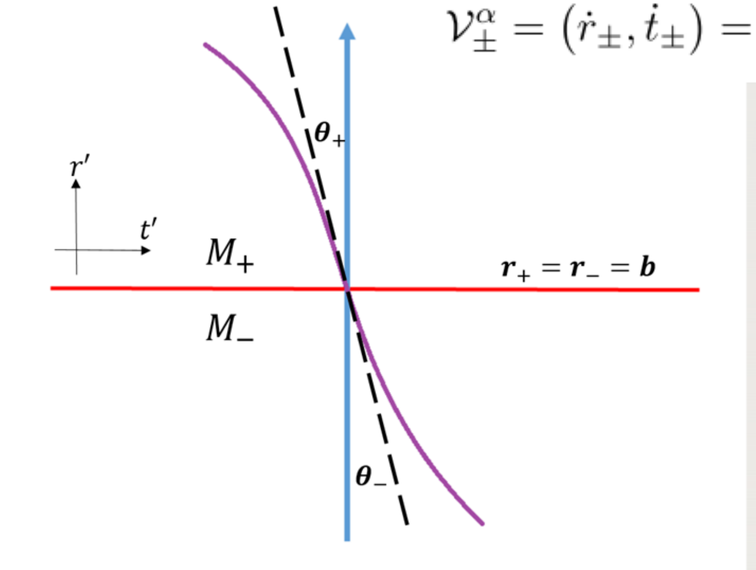


$$\mathcal{U}^{\alpha}_{+} = (\dot{r}_{+}, \dot{t}_{+}) = \left(\sqrt{E_{+}^2 - f_{+}}\,,\,\frac{E_{+}}{f_{+}}\right)$$

$$E_{\pm}^2 = 1 - 2M_{\pm}/R_{\pm}$$

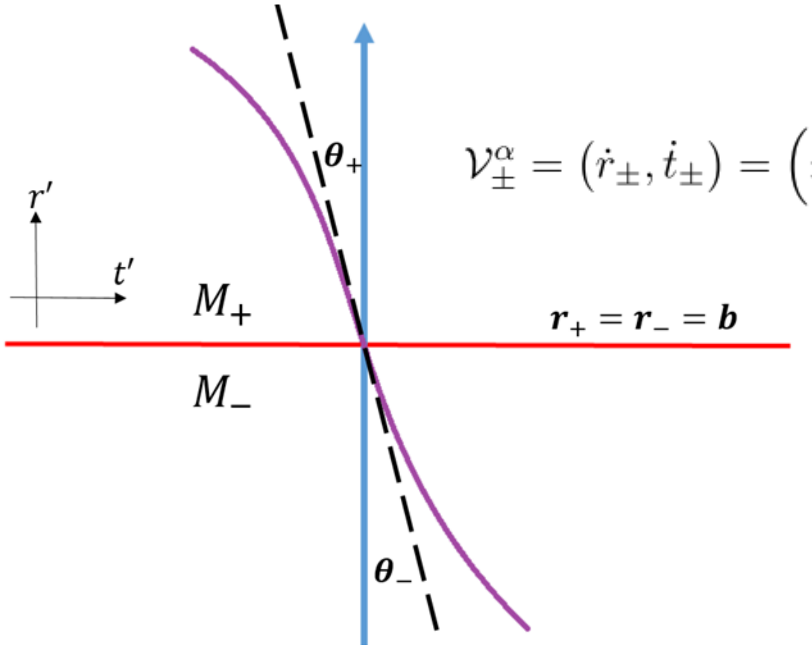
$$\mathcal{U}^{\alpha}_{-} = (\dot{r}_{-}, \dot{t}_{-}) = \left(-\sqrt{E_{-}^2 - f_{-}}\,,\,\frac{E_{-}}{f_{-}}\right)$$

$$\mathcal{V}^{\alpha}_{\pm} = (\dot{r}_{\pm}, \dot{t}_{\pm}) = \left(\pm\sqrt{-f_{\pm}}, 0\right)$$



An energy shift

$$\mathcal{U}^{\alpha}_{+} = (\dot{r}_{+}, \dot{t}_{+}) = \left(\sqrt{E_{+}^{2} - f_{+}}\,,\, \frac{E_{+}}{f_{+}}\right)$$



$$\mathcal{V}^{\alpha}_{\pm} = (\dot{r}_{\pm}, \dot{t}_{\pm}) = \left(\pm\sqrt{-f_{\pm}}, 0\right)$$

$$\mathcal{U}^{\alpha}_{-} = (\dot{r}_{-}, \dot{t}_{-}) = \left(-\sqrt{E_{-}^{2} - f_{-}}\,,\, \frac{E_{-}}{f_{-}}\right)$$

An Energy Shift

$$\gamma_{-}(b) = \gamma_{+}(b)$$

$$\gamma_{\pm}(b) \equiv \lim_{r_{\pm} \to b} -g_{\pm\alpha\beta}\,\mathcal{U}^{\alpha}_{\pm}\mathcal{V}^{\beta}_{\pm}$$

$$E_{+} = \sqrt{\frac{f_{+}(b)}{f_{-}(b)}}\,E_{-}$$

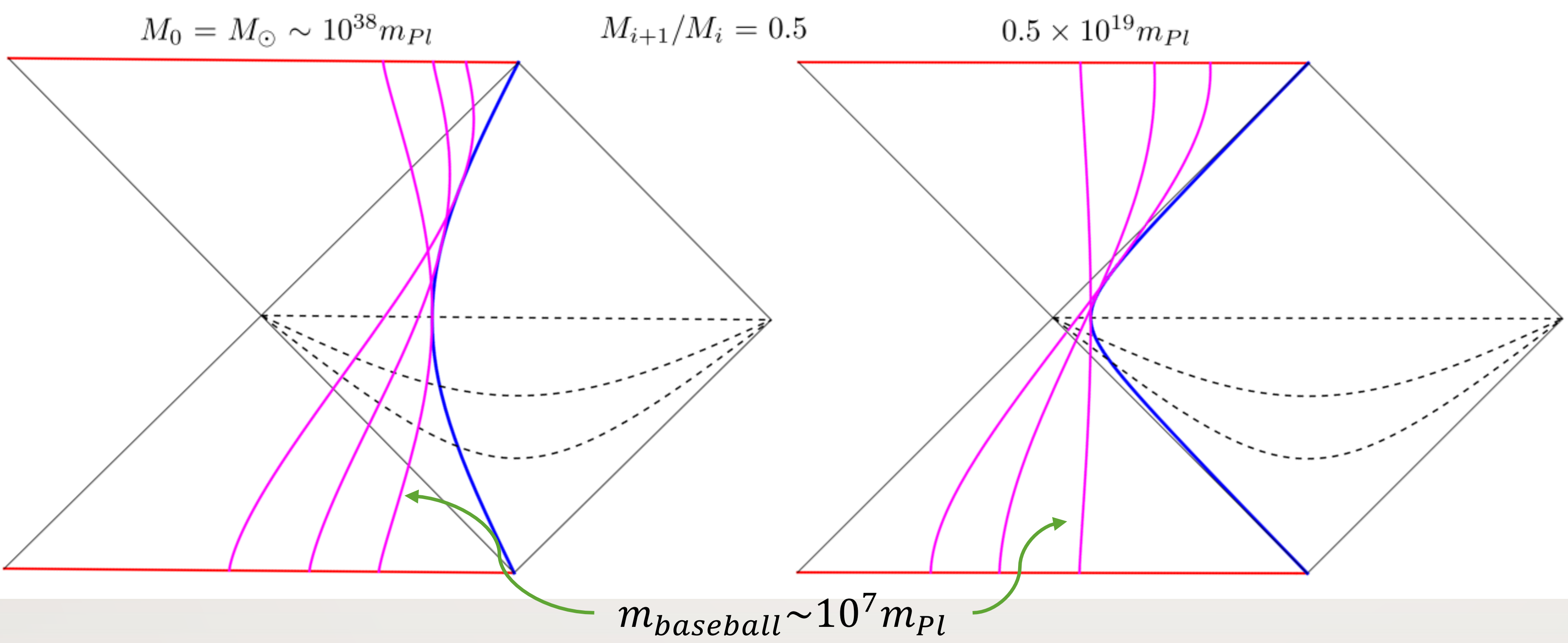
$$E_{\pm}^{2} = 1 - 2M_{\pm}/R_{\pm}$$

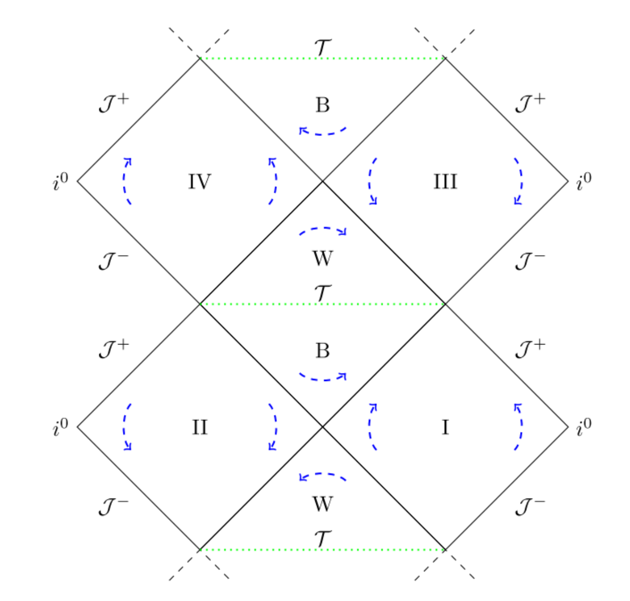
Consequence of the energy shift

$$E_+ = \sqrt{\frac{f_+(b)}{f_-(b)}} E_- \quad \Rightarrow \quad E_+ = \sqrt{\frac{M_+}{M_-}} E_- \quad \Rightarrow \quad 1 - \frac{2GM_+}{R_+} = \frac{M_+}{M_-}\left(1 - \frac{2GM_-}{R_-}\right)$$

The bounded timelike radial geodesics become
closer to the event horizon in the mass decreasing direction.

4. Squeezing of radial geodesics & the implication



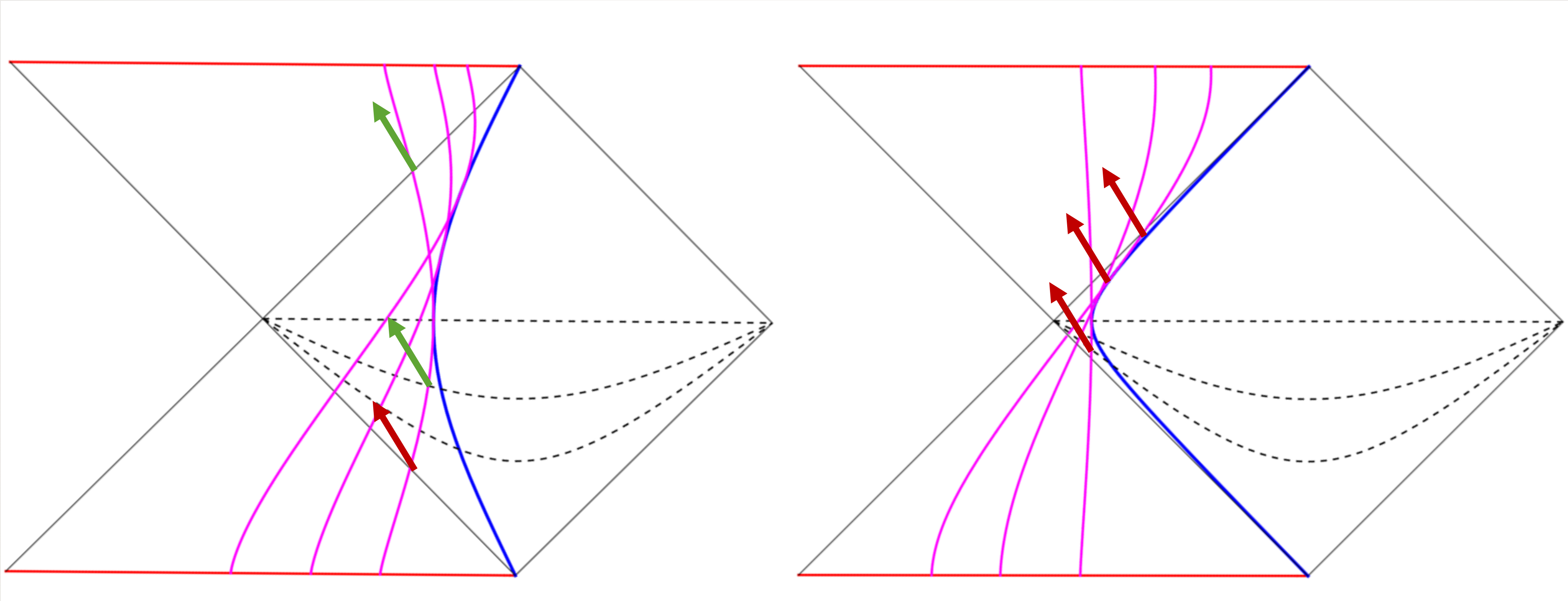


Assuming $\frac{M_{i+1}}{M_i} = \text{const.} < 1$, all bounded radial geodesics can be squeezed into the range of the stretched horizon while the black hole and white hole are still massive.

$$\gamma = -g_{\alpha\beta}\mathcal{U}^{\alpha}\mathcal{V}^{\beta} = \frac{1}{\sqrt{1 - v_{rel}^2}}$$

: $v_{rel}^2 \to 1$

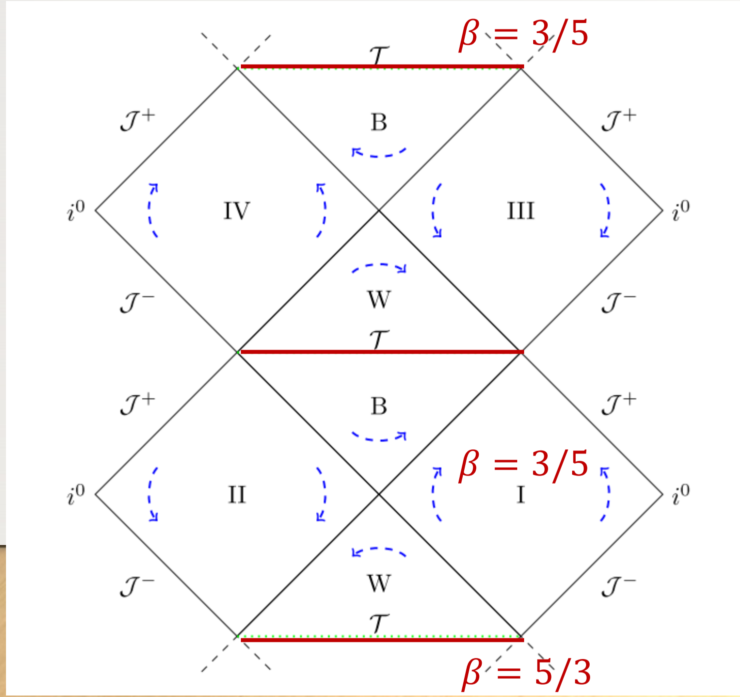
:Everything is okay!



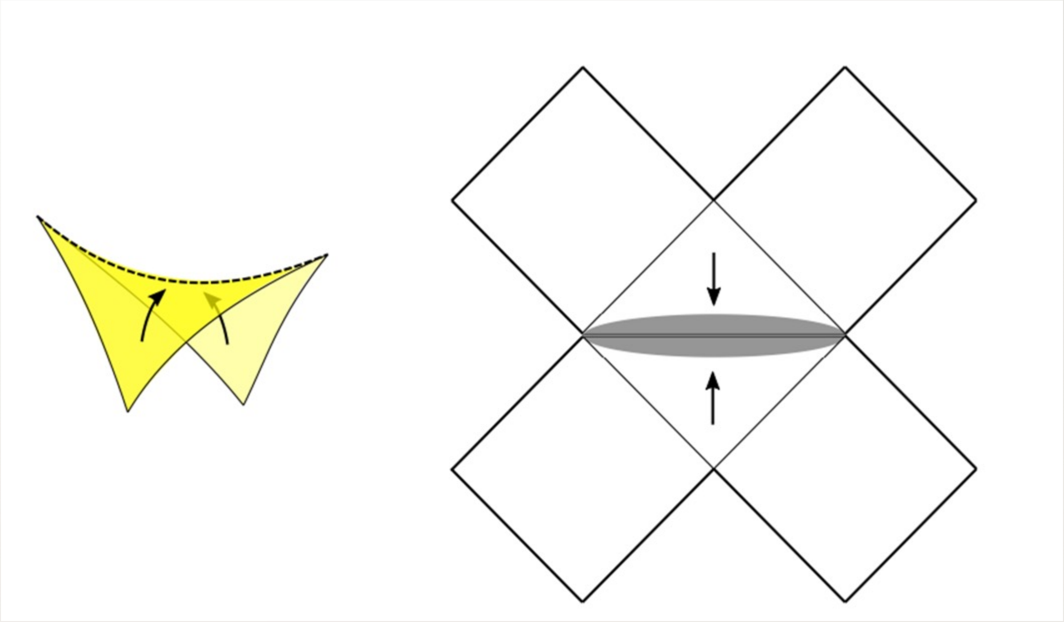
5. Possible rescues and discussion

1. β = 3/5 or 5/3 (periodically symmetric), the squeezing effect will be relaxed repeatedly.
   (Bodendorfer, Mele and Münch)

$$M_+ = M_- \left(\frac{M_-}{m}\right)^{\beta-1} \quad ; \beta = \frac{5}{3} \text{ or } \frac{3}{5}$$



2. Annihilation-to-nothing interpretation



Brahma, Chen and Yeom; arXiv:2108.05330

Ongoing projects

- A Penrose diagram without any illness at the thin shell
- Other type of spacetime constructed by a spacelike thin shell with the similar issue
- And others…

6. Summary

- When a mass difference exists in a black-to-white hole bouncing scenario, the bounded timelike radial geodesics lose energy and become closer to the event horizon in the mass decreasing direction.
- By tracing a finite amount of bouncing cycles, all bounded radial geodesics can be squeezed into the range of the stretched horizon while BH and WH are still massive.
- Those geodesics are problematic since any infalling object has a relative velocity approaching speed of light for them. This result indicates the instability of this type of bouncing scenario.

Supplementation

$$ds_{\pm}^2 = -(-f_{\pm})^{-1}dr_{\pm}^2 + (-f_{\pm})dt_{\pm}^2 + r_{\pm}^2 d\Omega^2$$

1st junction condition:
the induced metric must be the same on both sides of the shell

$$(-f_-)dt_-^2 + r_-^2 d\Omega^2 = (-f_+)dt_+^2 + r_+^2 d\Omega^2$$

$$r_+ = r_- = b$$

$$\sqrt{-f_-(b)}dt_- = \sqrt{-f_+(b)}dt_+$$

``Inflationary space-times are incomplete in past directions'' by Borde, Guth, Vilenkin;
Phys. Rev. Lett. **90**, 151301 [arXiv: gr-qc/0110012]

- Circumventing the necessity of energy conditions.
- The averaged expansion condition $H_{av} > 0$ holds along past-directed geodesics.
- A test particle (geodesic) is infinitely blueshifted within its finite proper time.
- Including cosmological cyclic models with $H_{av} > 0$.