APCTP-LDU-CCCP

WORKSHOP ON PHYSICS OF DARK COSMOS
: DARK MATTER, DARK ENERGY, AND ALL

in Busan, Oct 21-23 2022

All the covariant tensor currents of massless particles in the covariant formulation

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Motivation

In the Standard Model (SM)

Unsolved problems: Dark matter, Neutrino oscillation, Matter antimatter asymmetry, $\cdots$

But, at the LHC : No new particles and phenomena...

In this situation with no decisive evidence for new physics beyond the SM

$\downarrow$

One powerful strategy : Studying all the allowed effective interactions of particles including high-spins in a model-independent way

High-spin massless particles

Realizations

- ✓ With conventional high-spin massless fields
- ✓ With light-cone coordinates
- ✓ In SUSY

$\vdots$

Restrictions

No-go theorems

- ✓ Soft photon theorem
- ✓ LY theorem
- ✓ WW theorem

$\vdots$

Phys. Rept. **405**, 279-390 (2005)
Phys. Rev. Lett. **81**, 1562-1567 (1998)
Rev. Mod. Phys. **76**, 1 (2003)
P. A. Zyla *et al.* [Particle Data Group]
Phys. Lett. B **85**, 219-224 (1979)
Phys. Rev. D **21**, 358 (1980)
Phys. Lett. B **85**, 219-224 (1979)
Phys. Rev. **135**, B1049-B1056 (1964)
Dokl. Akad. Nauk SSSR **60** (1948) no.2, 207-209
Phys. Lett. B **96** (1980), 59-62

Motivation

No-go theorems $\rightarrow$ Complete absence of high-spin massless particles in nature?

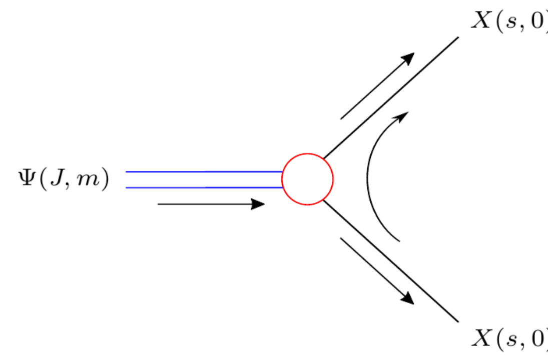
LY theorem with its generalization

✓ Spin 1 $\rightarrow \gamma\gamma$ ×

✓ Spin 1 $\rightarrow gg$ ∘

✓ Spin $J = 0, 2, \cdots \rightarrow \gamma\gamma$ or gg ∘

$\vdots$



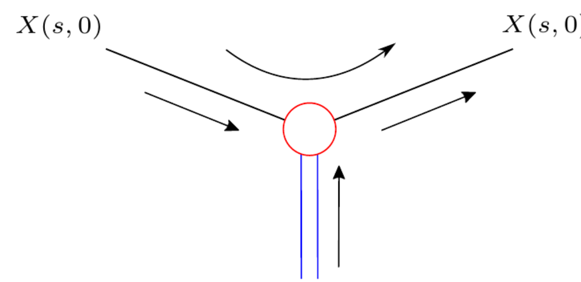
WW theorem

✓ All theories allowing a covariant conserved current J_μ

Forbid $\rightarrow$ spin-$s > 1/2$ massless particles with $Q = \int d^3\vec{x}\, J_0$

✓ All theories allowing an energy-momentum tensor $T_{\mu\nu}$

Forbid $\rightarrow$ spin-$s > 1$ massless particles with $P_\mu = \int d^3\vec{x}\, T_{0\mu}$



Exceptions

Spin-1 massless particles, γ and g with $0 = \int d^3\vec{x}\, J_0$ and Discovery of gravitational waves $\rightarrow$ a spin-2 massless particle called graviton

P. A. Zyla *et al.* [Particle Data Group]
Dokl. Akad. Nauk SSSR **60** (1948) no.2, 207-209
Phys. Rev. D **103** (2021) no.9, 096013
Phys. Lett. B **96** (1980), 59-62
Phys. Rev. Lett. **116**, no.6, 061102 (2016)

All the covariant tensor currents of massless particles in the covariant formulation

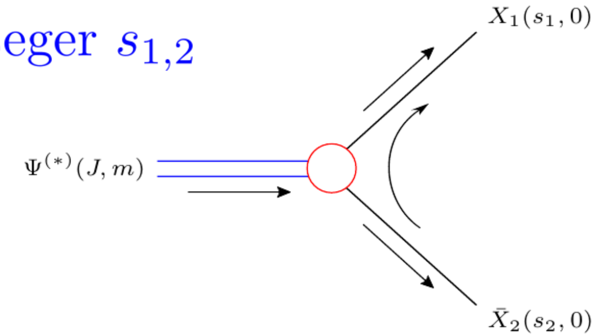
$\Theta^{[J;s_1,s_2]}_{\mu_1\cdots\mu_J}$

Extention of
Phys. Rev. D **104** (2021) no.5, 055046
Phys. Rev. D **105** (2022) no.1, 016016

Outline

Integer $s_{1,2}$

1. Algorithm
   - (a) Symmetric covariant tensor currents
   - (b) Matrix elements
   - (c) Form-factor and basic operators
   - (d) Constructing covariant three-point vertices
   - (e) How to construct covariant tensor currents



Decay matrix elements

$\langle X_1\bar{X}_2|\Theta^{[J;s_1,s_2]}_{\mu_1\cdots\mu_J}|0\rangle$

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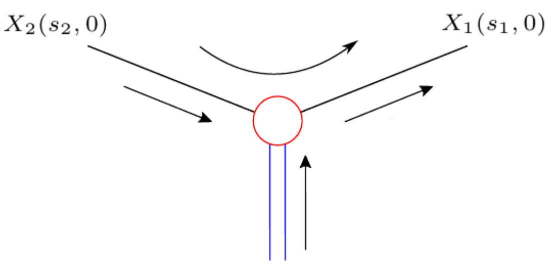
2. Crossing symmetry and identical-momentum limits
   - (a) Covariance conditions on form factors
   - (b) Revisiting the WW theorem
3. Summary

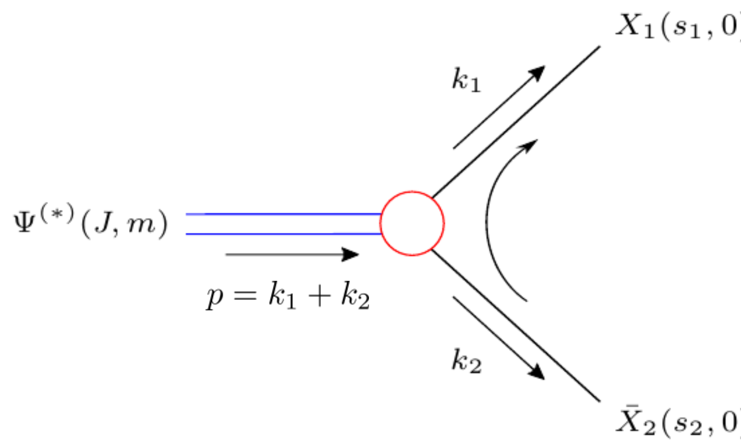


Scattering matrix elements

$\langle X_1|\Theta^{[J;s_1,s_2]}_{\mu_1\cdots\mu_J}|X_2\rangle$

Matrix elements in the covariant formulation

<Symmetric covariant tensor currents>



Matrix elements in the two-body decay

$$\langle k_1, \lambda_1; k_2, \lambda_2 | \, \Theta^{[J;s_1,s_2]}_{\mu_1 \cdots \mu_J} | \, 0 \, \rangle \equiv \Theta^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)\mu_1 \cdots \mu_J}(k_1, k_2)$$

Assumption : The tensor currents coupled to the $\Psi^{(*)}$ propagator

The Feynman propagator of $\Psi^{(*)}$

$$\Pi^{\mu_1 \cdots \mu_J \nu_1 \cdots \nu_J}(p) = \int d^4x \; e^{ip \cdot x} \; \langle 0 | T\{\Psi^{\mu_1 \cdots \mu_J}(x) \Psi^{\dagger \nu_1 \cdots \nu_J}(0)\} | 0 \rangle$$

Free field $\Psi^{\mu_1 \cdots \mu_J}$ satisfying $(\partial^2 + m^2)\Psi^{\mu_1 \cdots \mu_J} = 0$ (On-shell Ψ) $\rightarrow p^2 = m^2$

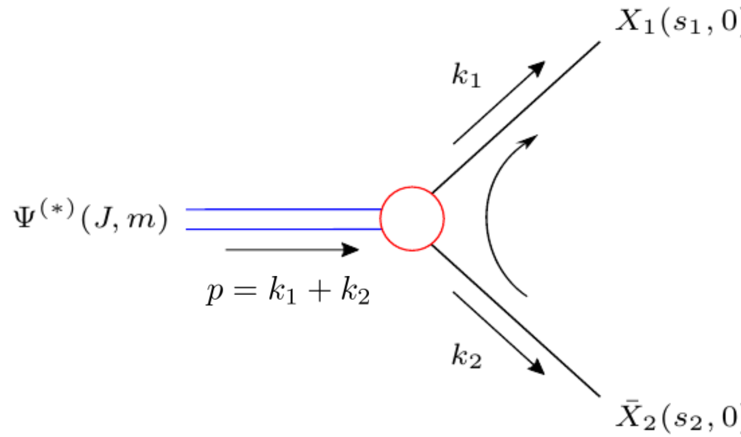
$\Psi^{\nu_1 \cdots \nu_J} \sim$ $\varepsilon^{\mu_1 \cdots \mu_J}$ and $\varepsilon^{*\mu_1 \cdots \mu_J}$

Divergence-free	$p_{\mu_i} \, \varepsilon^{\mu_1 \cdots \mu_i \cdots \mu_J}(p, \sigma)$	$= 0$
Traceless	$g_{\mu_i \mu_j} \, \varepsilon^{\mu_1 \cdots \mu_i \cdots \mu_j \cdots \mu_J}(p, \sigma)$	$= 0$
Symmetric	$\varepsilon_{\alpha \beta \mu_i \mu_j} \, \varepsilon^{\mu_1 \cdots \mu_i \cdots \mu_j \cdots \mu_J}(p, \sigma)$	$= 0$

Phys. Rev. **106**, no.2, 345 (1957) Phys. Rev. **133**, B1318-B1332 (1964) Eur. Phys. J. C **42** (2005), 375-389
Prog. Theor. Phys. **41**, 214-232 (1969)

Matrix elements in the covariant formulation

<Symmetric covariant tensor currents>



Matrix elements in the two-body decay

$$\langle k_1,\lambda_1;k_2,\lambda_2|\,\Theta^{[J;s_1,s_2]}_{\mu_1\cdots\mu_J}|\,0\,\rangle \equiv \Theta^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)\mu_1\cdots\mu_J}(k_1,k_2)$$

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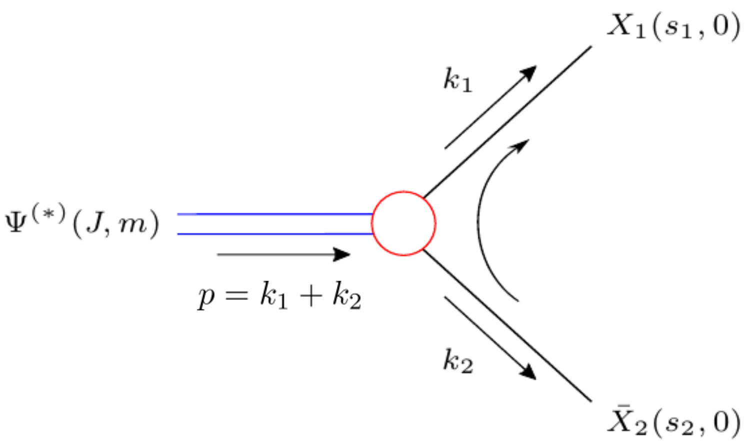
(On-shell Ψ) $\rightarrow p^2 = m^2$

Divergence-free	$p_{\mu_i}\,\Pi^{\mu_1\cdots\mu_i\cdots\mu_J\nu_1\cdots\nu_J}$	$=$	0
Traceless	$g_{\mu_i\mu_j}\,\Pi^{\mu_1\cdots\mu_i\cdots\mu_j\cdots\mu_J\nu_1\cdots\nu_J}$	$=$	0
Symmetric	$\varepsilon_{\alpha\beta\mu_i\mu_j}\,\Pi^{\mu_1\cdots\mu_i\cdots\mu_j\cdots\mu_J\nu_1\cdots\nu_J}$	$=$	0

Matrix elements in the covariant formulation

<Symmetric covariant tensor currents>



Matrix elements in the two-body decay

$$\langle k_1, \lambda_1; k_2, \lambda_2 |\, \Theta^{[J;s_1,s_2]}_{\mu_1\cdots\mu_J} |\, 0 \,\rangle \equiv \Theta^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)\mu_1\cdots\mu_J}(k_1,k_2)$$

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(Off-shell Ψ^*) $\rightarrow p^2 \neq m^2$

Divergence-free	$p_{\mu_i}\,\Pi^{\mu_1\cdots\mu_i\cdots\mu_J\nu_1\cdots\nu_J}$	$\neq$	0
Traceless	$g_{\mu_i\mu_j}\,\Pi^{\mu_1\cdots\mu_i\cdots\mu_j\cdots\mu_J\nu_1\cdots\nu_J}$	$\neq$	0
Symmetric	$\varepsilon_{\alpha\beta\mu_i\mu_j}\,\Pi^{\mu_1\cdots\mu_i\cdots\mu_j\cdots\mu_J\nu_1\cdots\nu_J}$	$=$	0

$\longrightarrow$ Symmetric

$$\varepsilon^{\alpha\beta\mu_i\mu_j}\,\Theta_{\mu_1\cdots\mu_i\cdots\mu_j\cdots\mu_J} = 0$$

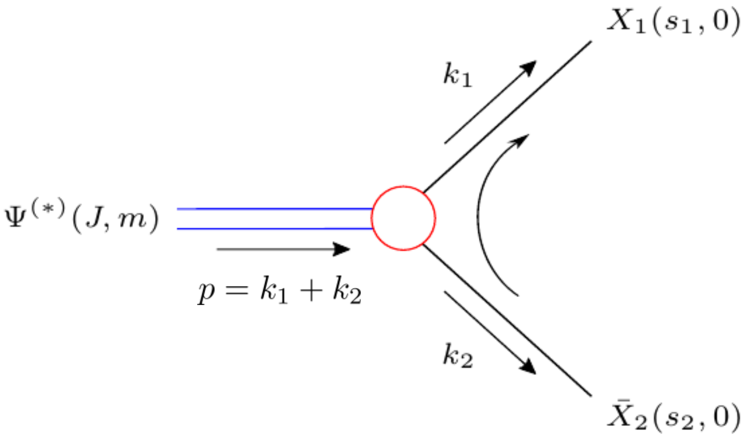
Matrix elements in the covariant formulation

<Matrix elements>

$$\Theta^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)\mu_1\cdots\mu_J}(k_1,k_2) = \varepsilon^{*\alpha_1\cdots\alpha_{n_1}}(k_1,\lambda_1)\varepsilon^{*\beta_1\cdots\beta_{n_2}}(k_2,\lambda_2)\,\Gamma^{[J;s_1,s_2]}_{\alpha_1\cdots\alpha_{n_1},\beta_1\cdots\beta_{n_2};\mu_1\cdots\mu_J}(k_1,k_2)$$

Integer $s_{1,2}$

$$\varepsilon^{*\alpha_1\cdots\alpha_{s_1}}(k_1,\pm s_1) = \varepsilon^{*\alpha_1}(k_1,\pm 1)\cdots\varepsilon^{*\alpha_{s_1}}(k_1,\pm 1)$$
$$\varepsilon^{*\beta_1\cdots\beta_{s_2}}(k_2,\pm s_2) = \varepsilon^{*\beta_1}(k_2,\pm 1)\cdots\varepsilon^{*\beta_{s_2}}(k_2,\pm 1)$$



Repeated structures
for the spin-1 polarization vector

↓

Key guideline
in constructing the Form-factor and basic operators

Matrix elements in the covariant formulation

<Form-factor and basic operators>

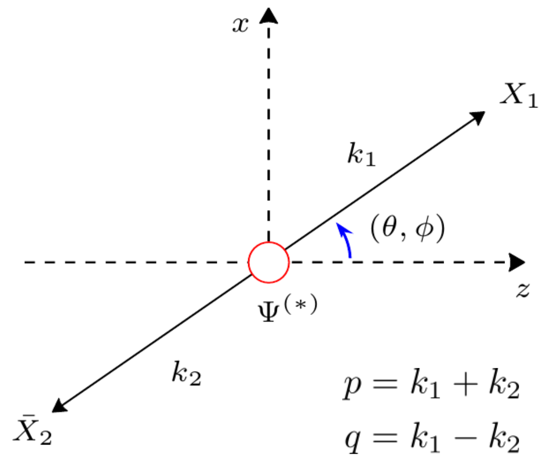
$$\Theta^{[J;0,0]}_{(0,0)\mu_1\cdots\mu_J}(k_1,k_2) = \Gamma^{[J;0,0]}_{\mu_1\cdots\mu_J}(k_1,k_2) \qquad (s_1 = s_2 = 0)$$

Symmetric covariant tensor currents $\rightarrow$ q_μ, p_μ, and $g_{\mu\nu}$

Compact square bracket operator form

$$p_{\mu_1}\cdots p_{\mu_n} \rightarrow [p]^n, \quad q_{\mu_1}\cdots q_{\mu_n} \rightarrow [q]^n, \quad g_{\mu_1\mu_2}\cdots g_{\mu_{2n-1}\mu_{2n}} \rightarrow [g]^n$$

The $\Psi^{(*)}$ rest frame (ΨRF)



$$p = k_1 + k_2$$
$$q = k_1 - k_2$$

On-shell Ψ

Divergence-free	$p_{\mu_i}\,\varepsilon^{\mu_1\cdots\mu_i\cdots\mu_J}(p,\sigma)$	$=$	0
Traceless	$g_{\mu_i\mu_j}\,\varepsilon^{\mu_1\cdots\mu_i\cdots\mu_j\cdots\mu_J}(p,\sigma)$	$=$	0
Symmetric	$\varepsilon_{\alpha\beta\mu_i\mu_j}\,\varepsilon^{\mu_1\cdots\mu_i\cdots\mu_j\cdots\mu_J}(p,\sigma)$	$=$	0

$$\left[\Theta^{[J;0,0]}_{(0,0)}(k_1,k_2)\right] = \left[F^{[J;0,0]}_{(0,0)}(k_1,k_2)\right]^J \sim A^{[J;0,0]}_{(0,0)J,0,0}[q]^J$$

Form-factor operators

$$\left[F^{[J;s_1,s_1]}_{(\lambda s_1,\mp\lambda s_2)}(k_1,k_2)\right]^n$$

Off-shell Ψ^*

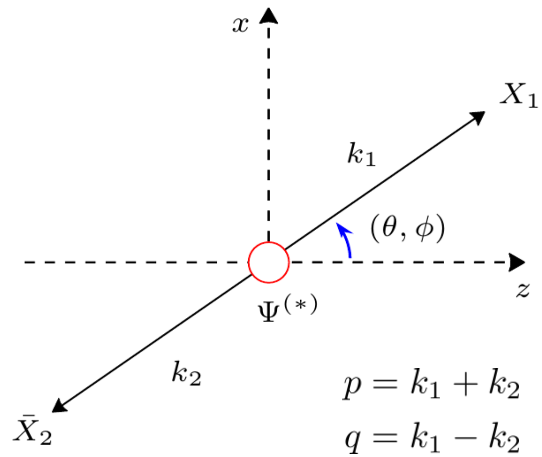
$$\left[\Theta^{[J;0,0]}_{(0,0)}(k_1,k_2)\right] = \sum_{n=0}^{J}\left[F^{[J;0,0]}_{(0,0)}(k_1,k_2)\right]^n \sim \sum_{n,m,l} A^{[J;0,0]}_{(0,0)n,m,l}[q]^n[p]^m[g]^l \ \text{ with } \quad J = n+m+2l$$

Matrix elements in the covariant formulation

<Form-factor and basic operators>

$\left[\Theta^{[J;s_1,s_1]}_{(\lambda_1,\lambda_2)}(k_1,k_2)\right]$ $\quad (s_1 \neq 0 \;\text{ or }\; s_2 \neq 0)$

The $\Psi^{(*)}$ rest frame (ΨRF)



Boosts of spin-1 polarization vectors of massless particles

$$\varepsilon_{\alpha,\beta}(k_{1,2}) \to \varepsilon_{\alpha,\beta}(k'_{1,2}) + \eta\, k'_{1\alpha,2\beta}$$

Polarization-covariant operators

$$\langle k_1\rho\alpha\mu\rangle_+ = i(k_{1\rho}g_{\alpha\mu} - k_{1\alpha}g_{\rho\mu} - k_{1\mu}g_{\rho\alpha}) \quad \to \quad F^\dagger_{1\rho\mu} = \partial_\rho X^\dagger_{1\mu} - \partial_\mu X^\dagger_{1\rho}$$
$$\langle k_2\sigma\beta\nu\rangle_+ = i(k_{2\sigma}g_{\beta\nu} - k_{2\beta}g_{\sigma\nu} - k_{2\nu}g_{\sigma\beta}) \quad \to \quad F_{2\sigma\nu} = \partial_\sigma X_{2\nu} - \partial_\nu X_{2\sigma}$$
$$\langle k_1\rho\alpha\mu\rangle_- = \varepsilon_{\gamma\rho\alpha\mu}\, k_1^\gamma \quad \to \quad -i\tilde{F}^\dagger_{1\rho\mu} = -i\varepsilon_{\rho\mu\gamma\alpha}F^{\dagger\gamma\alpha}/2$$
$$\langle k_2\sigma\beta\nu\rangle_- = \varepsilon_{\delta\sigma\beta\nu}\, k_2^\delta \quad \to \quad -i\tilde{F}_{2\sigma\nu} = -i\varepsilon_{\rho\mu\delta\beta}F^{\delta\beta}/2$$

Fundamental operators

$$\langle abcd\,\rangle_+ = i(g_{ab}g_{cd} - g_{ac}g_{bd} - g_{ad}g_{bc})$$
$$\langle abcd\,\rangle_- = \varepsilon_{abcd}$$

Contraction symbols

$$\langle abcd\rangle_i\langle efgh\rangle_j = \langle abcd\rangle_i\langle efgh\rangle_j\, g^{bf} g^{dh}$$

Matrix elements in the covariant formulation

<Form-factor and basic operators>

$$\left[\Theta^{[0;1,1]}_{(\lambda_1,\lambda_2)}(k_1,k_2)\right]$$

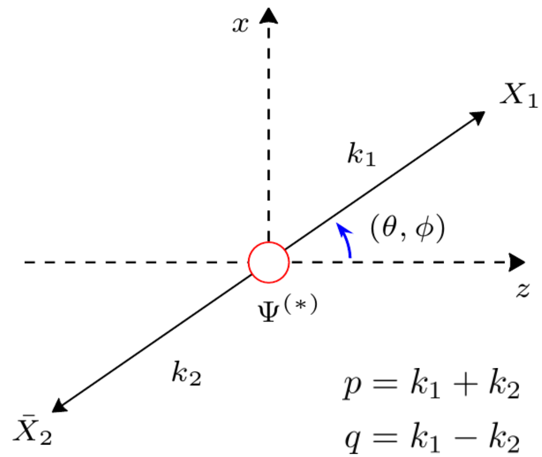
Even-parity scalar operator

$$\frac{1}{2}\langle k_1\rho\alpha\mu\rangle_\pm\langle k_2\sigma\beta\nu\rangle_\pm \overset{\text{eff}}{=} i\langle k_1k_2\alpha\beta\rangle_+ \quad\rightarrow\quad \begin{aligned}\Theta^{[0;1,1]}_{(\pm,\pm)}(k_1,k_2) &= (k_1\cdot k_2)\\ \Theta^{[0;1,1]}_{(\pm,\mp)}(k_1,k_2) &= 0\end{aligned}$$

Odd-parity scalar operator

$$\frac{1}{2}\langle k_1\rho\alpha\mu\rangle_\pm\langle k_2\sigma\beta\nu\rangle_\mp = i\langle k_1k_2\alpha\beta\rangle_- \quad\rightarrow\quad \begin{aligned}\Theta^{[0;1,1]}_{(\pm,\pm)}(k_1,k_2) &= \pm(k_1\cdot k_2)\\ \Theta^{[0;1,1]}_{(\pm,\mp)}(k_1,k_2) &= 0\end{aligned}$$

The $\Psi^{(*)}$ rest frame (ΨRF)



$$p = k_1 + k_2$$
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Polarization-covariant operators

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$$\langle k_2\sigma\beta\nu\rangle_+ = i(k_{2\sigma}g_{\beta\nu} - k_{2\beta}g_{\sigma\nu} - k_{2\nu}g_{\sigma\beta})$$
$$\langle k_1\rho\alpha\mu\rangle_- = \varepsilon_{\gamma\rho\alpha\mu}\,k_1^\gamma$$
$$\langle k_2\sigma\beta\nu\rangle_- = \varepsilon_{\delta\sigma\beta\nu}\,k_2^\delta$$

Basic bosonic scalar operators

$$S^\pm_{\alpha,\beta} = \frac{i}{2}\left[\langle k_1k_2\alpha\beta\rangle_+ \pm \langle k_1k_2\alpha\beta\rangle_-\right] \quad\rightarrow\quad \Theta^{[0;1,1]}_{(\lambda_1,\lambda_2)}(k_1,k_2) = (k_1\cdot k_2)\,\delta_{\lambda_1,\pm}\delta_{\lambda_1,\lambda_2}$$

Matrix elements in the covariant formulation

<Form-factor and basic operators>

$$[\Theta^{[1;1,0]}_{(\lambda_1,\lambda_2)}(k_1,k_2)] \quad \text{and} \quad [\Theta^{[1;0,1]}_{(\lambda_1,\lambda_2)}(k_1,k_2)]$$

Even-parity vector operators

$$i\langle k_1 k_2 \alpha\mu\rangle_+ \quad \to \quad \Theta^{[1;1,0]}_{(\pm,0)\mu}(k_1,k_2) = -(k_1\cdot k_2)\,\varepsilon^*_{\perp\mu}(k_1,\pm;k_2)$$

$$i\langle k_2 k_1 \beta\nu\rangle_+ \quad \to \quad \Theta^{[1;0,1]}_{(0,\pm)\nu}(k_1,k_2) = -(k_1\cdot k_2)\,\varepsilon^*_{\perp\nu}(k_2,\pm;k_1)$$

Odd-parity vector operators

$$i\langle k_1 k_2 \alpha\mu\rangle_- \quad \to \quad \Theta^{[1;1,0]}_{(\pm,0)\mu}(k_1,k_2) = \mp(k_1\cdot k_2)\,\varepsilon^*_{\perp\mu}(k_1,\pm;k_2)$$

$$i\langle k_2 k_1 \beta\nu\rangle_- \quad \to \quad \Theta^{[1;0,1]}_{(0,\pm)\nu}(k_1,k_2) = \mp(k_1\cdot k_2)\,\varepsilon^*_{\perp\nu}(k_2,\pm;k_1)$$

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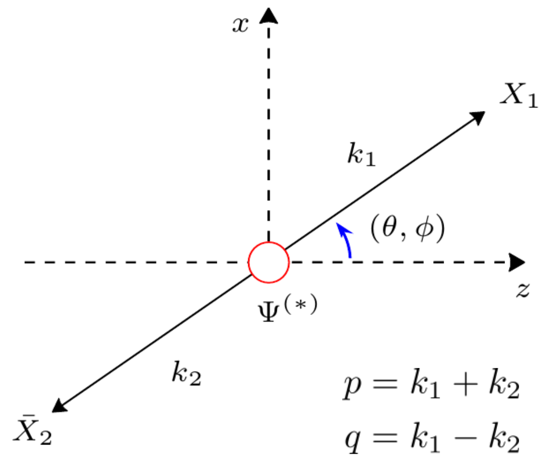

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$$\langle k_1\rho\alpha\mu\rangle_- = \varepsilon_{\gamma\rho\alpha\mu}\,k_1^\gamma$$
$$\langle k_2\sigma\beta\nu\rangle_- = \varepsilon_{\delta\sigma\beta\nu}\,k_2^\delta$$

Covariant polarization vectors

$$\varepsilon_\perp(k_{1,2},\pm;k_{2,1}) = \varepsilon(k_{1,2},\pm) - \left(\frac{k_{2,1}\cdot\varepsilon(k_{1,2},\pm)}{k_1\cdot k_2}\right)k_{1,2} \equiv \varepsilon_{1,2\perp}(\pm)$$

Basic bosonic vector operators

$$V^\pm_{1\alpha;\mu} = \frac{i}{2}\left[\langle k_1k_2\alpha\mu\rangle_+ \pm \langle k_1k_2\alpha\mu\rangle_-\right] \quad \to \quad \Theta^{[1;1,0]}_{(\lambda_1,0)\mu}(k_1,k_2) = -(k_1\cdot k_2)\,\varepsilon^*_{\perp\mu}(k_1,\pm;k_2)\,\delta_{\lambda_1,\pm}$$

$$V^\pm_{2\beta;\nu} = \frac{i}{2}\left[\langle k_2k_1\beta\nu\rangle_+ \pm \langle k_2k_1\beta\nu\rangle_-\right] \quad \to \quad \Theta^{[1;0,1]}_{(0,\lambda_2)\nu}(k_1,k_2) = -(k_1\cdot k_2)\,\varepsilon^*_{\perp\nu}(k_2,\pm;k_1)\,\delta_{\lambda_2,\pm}$$

Matrix elements in the covariant formulation

<Form-factor and basic operators>

$\left[\Theta^{[2;1,1]}_{(\lambda_1,\lambda_2)}(k_1,k_2)\right]$

Even-parity tensor operator

$$\langle k_1 \overbracket{\rho\alpha\mu\rangle_+\langle k_2\sigma}\beta\nu\rangle_+ - \langle k_1 \overbracket{\rho\alpha\mu\rangle_-\langle k_2\sigma}\beta\nu\rangle_-$$

$$\rightarrow \quad \Theta^{[2;1,1]}_{(\pm,\mp)\mu\nu}(k_1,k_2) = -(k_1\cdot k_2)\left[\varepsilon^*_{1\perp\mu}(\pm)\varepsilon^*_{2\perp\nu}(\mp) + \mu \leftrightarrow \nu\right]$$

$$\langle k_1 \overbracket{\rho\alpha\mu\rangle_-\langle k_2\sigma}\beta\nu\rangle_- \overset{\text{eff}}{=} -\langle k_1 \overbracket{\rho\alpha\nu\rangle_+\langle k_2\sigma}\beta\mu\rangle_+ + i\langle k_1 k_2\alpha\beta\rangle_+ g_{\mu\nu}$$

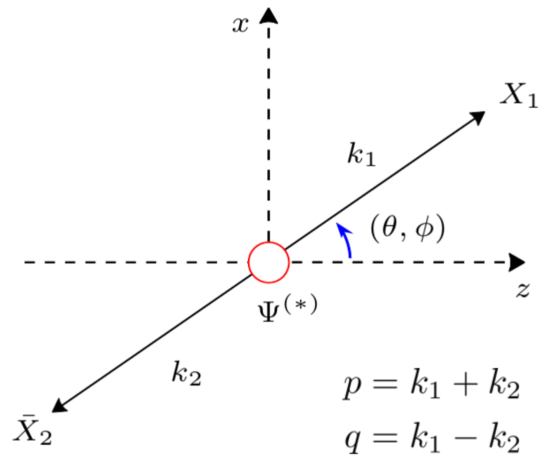
Odd-parity tensor operator

$$\langle k_1 \overbracket{\rho\alpha\mu\rangle_+\langle k_2\sigma}\beta\nu\rangle_- - \langle k_1 \overbracket{\rho\alpha\mu\rangle_-\langle k_2\sigma}\beta\nu\rangle_+$$

$$\rightarrow \quad \Theta^{[2;1,1]}_{(\pm,\mp)\mu\nu}(k_1,k_2) = \mp(k_1\cdot k_2)\left[\varepsilon^*_{1\perp\mu}(\pm)\varepsilon^*_{2\perp\nu}(\mp) + \mu \leftrightarrow \nu\right]$$

$$\langle k_1 \overbracket{\rho\alpha\mu\rangle_-\langle k_2\sigma}\beta\nu\rangle_+ \overset{\text{eff}}{=} -\langle k_1 \overbracket{\rho\alpha\nu\rangle_+\langle k_2\sigma}\beta\mu\rangle_- + i\langle k_1 k_2\alpha\beta\rangle_- g_{\mu\nu}$$

The $\Psi^{(*)}$ rest frame (ΨRF)



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$$\langle k_1\rho\alpha\mu\rangle_- = \varepsilon_{\gamma\rho\alpha\mu}\, k_1^\gamma$$
$$\langle k_2\sigma\beta\nu\rangle_- = \varepsilon_{\delta\sigma\beta\nu}\, k_2^\delta$$

Energy-momentum tensor of a spin-1 particle

Basic bosonic tensor operators

$$T^{\pm}_{\alpha,\beta;\mu\nu} = \frac{1}{2}\sum_{\tau=\pm}\tau\left[\langle k_1 \overbracket{\rho\alpha\mu\rangle_\tau\langle k_2\sigma}\beta\nu\rangle_\tau \pm \langle k_1 \overbracket{\rho\alpha\mu\rangle_\tau\langle k_2\sigma}\beta\nu\rangle_{-\tau}\right] \quad \rightarrow \quad g^{\rho\sigma}F^\dagger_{1\mu\rho}F_{2\nu\sigma} - \frac{1}{4}g_{\mu\nu}F_1^{\dagger\rho\sigma}F_{2\rho\sigma}$$

$$\rightarrow \quad \Theta^{[2;1,1]}_{(\lambda_1,\lambda_2)\mu\nu}(k_1,k_2) = -(k_1\cdot k_2)\left[\varepsilon^*_{1\perp\mu}(\pm)\varepsilon^*_{\perp\nu}(\mp) + \mu\leftrightarrow\nu\right]\delta_{\lambda_1,\pm}\,\delta_{\lambda_1,-\lambda_2}$$

Matrix elements in the covariant formulation

Compact square bracket operator form

$$S^{\pm}_{\alpha_1,\beta_1}\cdots S^{\pm}_{\alpha_n\,\beta_n} \quad\rightarrow\quad [S^{\pm}]^n$$

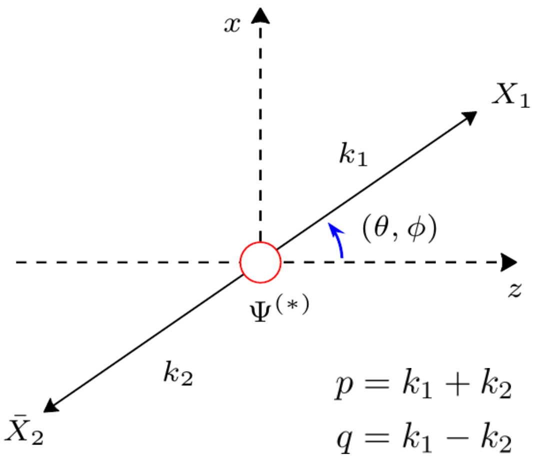
$$V^{\pm}_{1\alpha_1;\mu_1}\cdots V^{\pm}_{1\alpha_n;\mu_n} \quad\rightarrow\quad [V_1^{\pm}]^n$$

$$V^{\pm}_{2\beta_1;\mu_1}\cdots V^{\pm}_{2\beta_n;\mu_n} \quad\rightarrow\quad [V_2^{\pm}]^n$$

$$T^{\pm}_{\alpha_1,\beta_1;\mu_1\mu_2}\cdots T^{\pm}_{\alpha_n,\beta_n;\mu_{2n-1}\mu_{2n}} \quad\rightarrow\quad [T^{\pm}]^n$$

$$\Gamma^{[J;s_1,s_2]}_{\alpha_1\cdots\alpha_{n_1},\beta_1\cdots\beta_{n_2};\mu_1\cdots\mu_J} \quad\rightarrow\quad [\Gamma^{[J;s_1,s_2]}]$$

The $\Psi^{(*)}$ rest frame (ΨRF)



<Constructing covariant three-point vertices>

$$S^{\pm}_{\alpha,\beta} \rightarrow (\pm,\pm)$$
$$V^{\pm}_{1\alpha;\mu} \rightarrow (\pm,0)$$
$$V^{\pm}_{2\beta;\mu} \rightarrow (\pm,0)$$
$$T^{\pm}_{\alpha,\beta;\mu\nu} \rightarrow (\pm,\mp)$$

$$\left[\Theta^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)}(k_1,k_2)\right]$$

Ex) $\Psi^{(*)} \rightarrow X_1(k_1,+s_1) + \bar{X}_2(k_2,+s_2)$ (Integer $s_{1,2}\neq 0$ and $s_1 \geq s_2$)

$$\rightarrow \sum_{n=0}^{J_-}\left[F^{[J;s_1,s_2]}_{(+s_1,+s_2)}\right]^n [S^{+}]^{s_2}[V_1^{+}]^{s_1-s_2}$$

with $J_- = J-(s_1-s_2)$

1. $[+s_2,\ +s_2]$
2. $[+(s_1-s_2)+s_2,\ +s_2]$
3. Complement μ indices

Matrix elements in the covariant formulation

<Constructing covariant three-point vertices>

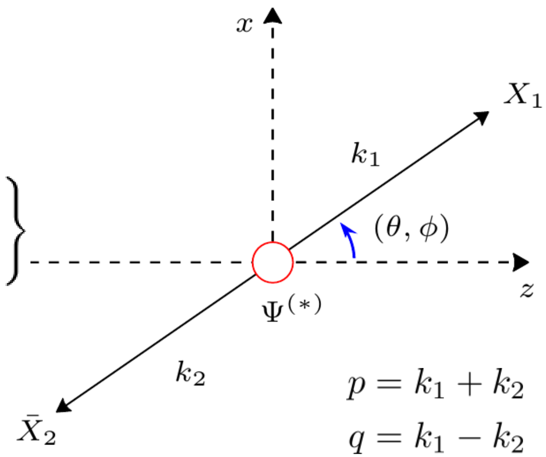
$$\left[\Gamma^{[J;s_1,s_2]}\right] = \sum_{\lambda=\pm}\Bigg\{\Theta(J_-)\sum_{n=0}^{J_-}\left[F^{[J;s_1,s_2]}_{(\lambda s_1,\lambda s_2)}\right]^n [S^\lambda]^{s_{\rm min}}\left([V_1^\lambda]^{s_1-s_{\rm min}}+[V_2^\lambda]^{s_2-s_{\rm min}}\right)$$

$$+\gamma_{s_{\rm min}}\Theta(J_+)\sum_{n=0}^{J_+}\left[F^{[J;s_1,s_2]}_{(\lambda s_1,-\lambda s_2)}\right]^n [T^\lambda]^{s_{\rm min}}\left([V_1^\lambda]^{s_1-s_{\rm min}}+[V_2^{-\lambda}]^{s_2-s_{\rm min}}\right)\Bigg\}$$

$$(J_\mp = J - |s_1 \mp s_2|),\ \ (\gamma_{s_{\rm min}} = 1-\delta_{s_{\rm min},0}\ \ \text{with}\ \ s_{\rm min} = \min[s_1,s_2])$$

First key result

The $\Psi^{(*)}$ rest frame (ΨRF)



$$p = k_1 + k_2$$
$$q = k_1 - k_2$$

$$S^{\pm}_{\alpha,\beta} \rightarrow (\pm,\pm)$$
$$V^{\pm}_{1\alpha;\mu} \rightarrow (\pm,0)$$
$$V^{\pm}_{2\beta;\mu} \rightarrow (\pm,0)$$
$$T^{\pm}_{\alpha,\beta;\mu\nu} \rightarrow (\pm,\mp)$$

Matrix elements in the covariant formulation

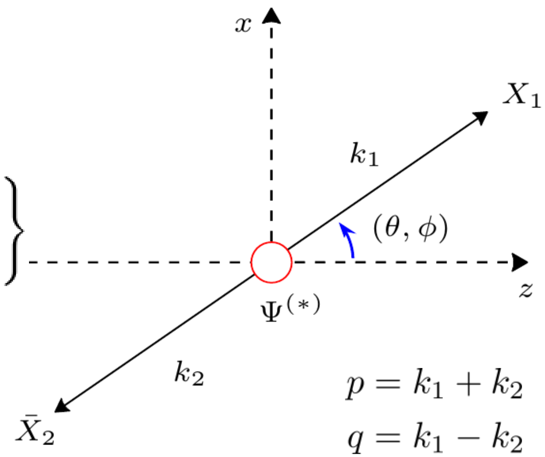
<Constructing covariant three-point vertices>

$$[\Gamma^{[J;s_1,s_2]}] = \sum_{\lambda=\pm} \Big\{ \Theta(J_-) \sum_{n=0}^{J_-} [F^{[J;s_1,s_2]}_{(\lambda s_1,\lambda s_2)}]^n [S^\lambda]^{s_{\min}} \Big([V_1^\lambda]^{s_1-s_{\min}} + [V_2^\lambda]^{s_2-s_{\min}} \Big)$$

$$+\gamma_{s_{\min}} \Theta(J_+) \sum_{n=0}^{J_+} [F^{[J;s_1,s_2]}_{(\lambda s_1,-\lambda s_2)}]^n [T^\lambda]^{s_{\min}} \Big([V_1^\lambda]^{s_1-s_{\min}} + [V_2^{-\lambda}]^{s_2-s_{\min}} \Big) \Big\}$$

$$(J_\mp = J - |s_1 \mp s_2|), \quad (\gamma_{s_{\min}} = 1 - \delta_{s_{\min},0} \ \text{with} \ s_{\min} = \min[s_1, s_2])$$

The $\Psi^{(*)}$ rest frame (ΨRF)



$p = k_1 + k_2$

$q = k_1 - k_2$

Constraints

- ✓ $\Theta_{\lambda_1,\lambda_2} = 0$ for $J < |s_1 - s_2|$
- ✓ $\Theta_{\pm s_1,\mp s_2} = 0$ for $J < |s_1 + s_2|$

$$S^{\pm}_{\alpha,\beta} \rightarrow (\pm,\pm)$$

$$V^{\pm}_{1\alpha;\mu} \rightarrow (\pm,0)$$

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Matrix elements in the covariant formulation

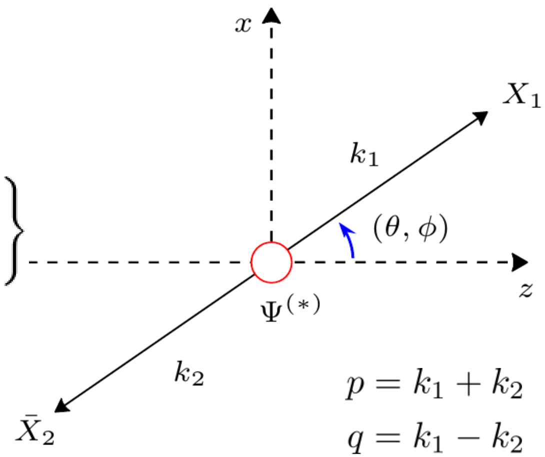
<Constructing covariant three-point vertices>

$$[\Gamma^{[J;s_1,s_2]}] = \sum_{\lambda=\pm}\Bigg\{\Theta(J_-)\sum_{n=0}^{J_-}[F^{[J;s_1,s_2]}_{(\lambda s_1,\lambda s_2)}]^n\,[S^\lambda]^{s_{\min}}\Big([V_1^\lambda]^{s_1-s_{\min}}+[V_2^\lambda]^{s_2-s_{\min}}\Big)$$

$$+\gamma_{s_{\min}}\,\Theta(J_+)\sum_{n=0}^{J_+}[F^{[J;s_1,s_2]}_{(\lambda s_1,-\lambda s_2)}]^n\,[T^\lambda]^{s_{\min}}\Big([V_1^\lambda]^{s_1-s_{\min}}+[V_2^{-\lambda}]^{s_2-s_{\min}}\Big)\Bigg\}$$

$(J_\mp = J-|s_1\mp s_2|)$, $(\gamma_{s_{\min}} = 1-\delta_{s_{\min},0}$ with $s_{\min}=\min[s_1,s_2])$

The $\Psi^{(*)}$ rest frame (ΨRF)



$p = k_1 + k_2$

$q = k_1 - k_2$

<How to construct covariant tensor currents>

Wave tensors	Momenta	Form factors
$\varepsilon^{*\alpha} \rightarrow X_1^{\dagger\alpha}$	$ik_{1\rho} \rightarrow \partial_\rho X_1^{\dagger\alpha}$	$A[(k_1\cdot k_2)] \rightarrow (\partial_\gamma X_1^{\dagger\alpha}), (\partial^\gamma X_2^\beta)$
$\varepsilon^{\beta} \rightarrow X_2^{\beta}$	$ik_{2\sigma} \rightarrow \partial_\sigma X_2^{\beta}$	

$\alpha \equiv \alpha_1\cdots\alpha_{s_1}$

$\beta \equiv \beta_1\cdots\beta_{s_2}$

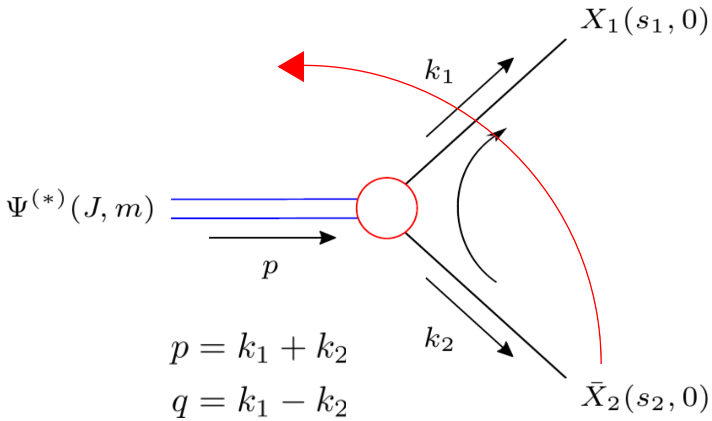
$S^\pm_{\alpha,\beta} \rightarrow (\pm,\pm)$

$V^\pm_{1\alpha;\mu} \rightarrow (\pm,0)$

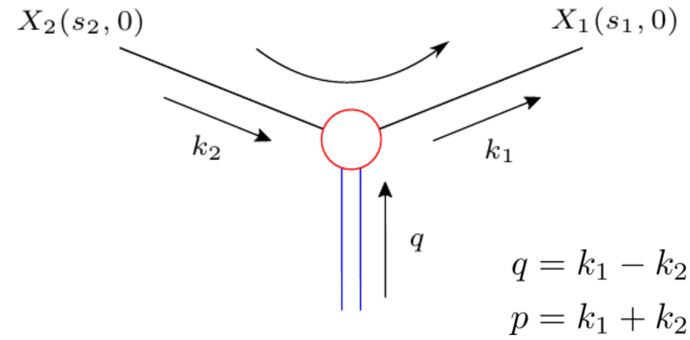
$V^\pm_{2\beta;\mu} \rightarrow (\pm,0)$

$T^\pm_{\alpha,\beta;\mu\nu} \rightarrow (\pm,\mp)$

Crossing symmetry and identical-momentum limits



$\longrightarrow$



Decay matrix elements

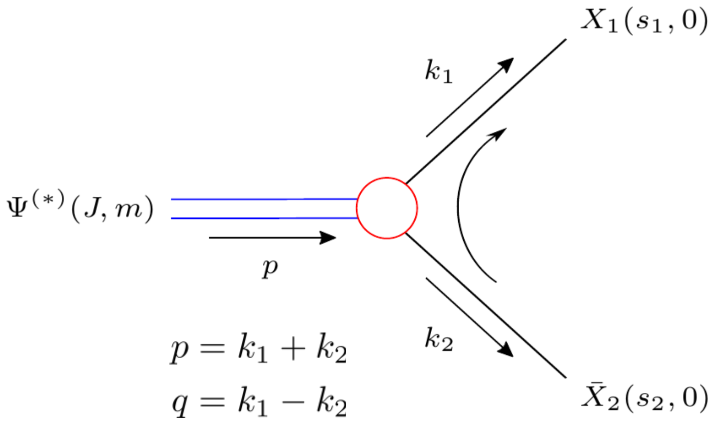
Scattering matrix elements

$$\langle k_1,\lambda_1;k_2,\lambda_2|[\Theta^{[J;s_1,s_2]}]|\,0\,\rangle = [\Theta^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)}(k_1,k_2)] \longrightarrow \langle k_1,\lambda_1|[\Theta^{[J;s_1,s_2]}]|k_2,\lambda_2\rangle = [\bar{\Theta}^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)}(k_1,k_2)]$$

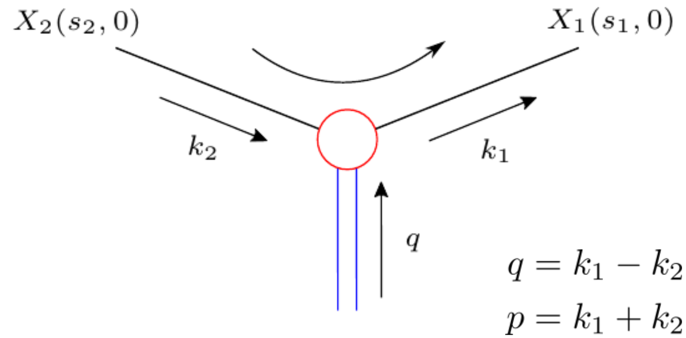
In the three-point vertices	$+k_2 \to -k_2$
In the X_2 wave tensors	$\varepsilon^*(k_2,\pm) \to -\varepsilon(k_2,\mp)$

$$[\Theta^{[J;s_1,s_2]}_{(\lambda_1,-\lambda_2)}] \longleftrightarrow [\bar{\Theta}^{[J;s_1,s_2]}_{(\lambda_1,+\lambda_2)}]$$

Crossing symmetry and identical-momentum limits



$\longrightarrow$



Decay matrix elements

$$\langle k_1,\lambda_1;k_2,\lambda_2|[\Theta^{[J;s_1,s_2]}]|\,0\,\rangle = [\Theta^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)}(k_1,k_2)]$$

$\longrightarrow$

Scattering matrix elements

$$\langle k_1,\lambda_1|[\Theta^{[J;s_1,s_2]}]|k_2,\lambda_2\rangle = [\bar{\Theta}^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)}(k_1,k_2)]$$

$$[\Theta^{[J;s_1,s_2]}_{(\lambda_1,-\lambda_2)}] \longleftrightarrow [\bar{\Theta}^{[J;s_1,s_2]}_{(\lambda_1,+\lambda_2)}]$$

Constraints

✓ $\Theta_{\lambda_1,\lambda_2} = 0$ for $J < |s_1 - s_2|$

✓ $\Theta_{\pm s_1,\mp s_2} = 0$ for $J < |s_1 + s_2|$

$\longrightarrow$

Constraints

✓ $\bar{\Theta}_{\lambda_1,\lambda_2} = 0$ for $J < |s_1 - s_2|$

✓ $\bar{\Theta}_{\pm s_1,\pm s_2} = 0$ for $J < |s_1 + s_2|$

Crossing symmetry and identical-momentum limits

$$p = k_1 + k_2$$
$$q = k_1 - k_2$$

Constraints

✓ $\bar{\Theta}_{\lambda_1,\lambda_2} = 0 \quad \text{for} \quad J < |s_1 - s_2|$

✓ $\bar{\Theta}_{\pm s_1,\pm s_2} = 0 \quad \text{for} \quad J < |s_1 + s_2|$

$+k_2 \to -k_2$ $\qquad \varepsilon^*(k_2,\pm) \to -\varepsilon(k_2,\mp)$

$$\bar{S}^{\pm}_{\alpha,\beta} = -S^{\pm}_{\alpha,\beta} \quad \to \quad \bar{\Theta}^{[0;1,1]}_{(\lambda_1,\lambda_2)}(k_1,k_2) = +(k_1\cdot k_2)\,\delta_{\lambda_1,\pm}\delta_{\lambda_1,-\lambda_2}$$

$$\bar{V}^{\pm}_{1\alpha;\mu} = -V^{\pm}_{1\alpha;\mu} \quad \to \quad \bar{\Theta}^{[1;1,0]}_{(\lambda_1,0)\mu}(k_1,k_2) = +(k_1\cdot k_2)\,\varepsilon^*_{1\perp\mu}(\pm)\,\delta_{\lambda_1,\pm}$$

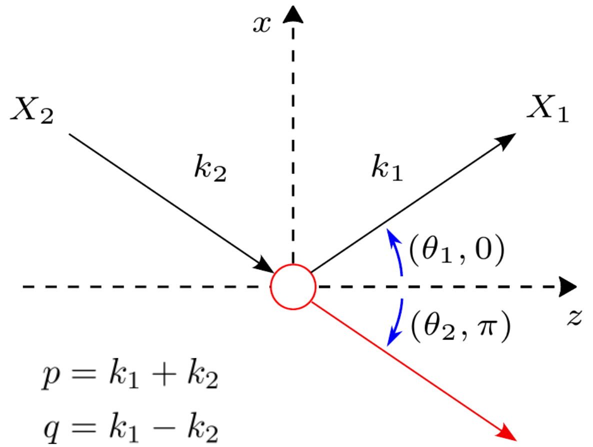
$$\bar{V}^{\mp}_{2\beta;\nu} = -V^{\pm}_{2\beta;\nu} \quad \to \quad \bar{\Theta}^{[1;0,1]}_{(0,\lambda_2)\nu}(k_1,k_2) = -(k_1\cdot k_2)\,\varepsilon_{2\perp\nu}(\mp)\,\delta_{\lambda_2,\mp}$$

$$\bar{T}^{\pm}_{\alpha,\beta;\mu\nu} = -T^{\pm}_{\alpha,\beta;\mu\nu} \quad \to \quad \bar{\Theta}^{[2;1,1]}_{(\lambda_1,\lambda_2)\mu\nu}(k_1,k_2) = -(k_1\cdot k_2)\left[\varepsilon^*_{1\perp\mu}(\pm)\varepsilon_{2\perp\nu}(\pm) + \mu\leftrightarrow\nu\right]\delta_{\lambda_1,\pm}\delta_{\lambda_1,\lambda_2}$$

$$F_{(\lambda_1,\lambda_2)} \sim (q_\mu),(p_\mu),(g_{\mu\nu}) \quad \to \quad \bar{F}_{(\lambda_1,-\lambda_2)} \sim (p_\mu),(q_\mu),(g_{\mu\nu})$$

Crossing symmetry and identical-momentum limits

$$k_1 = k^0(1, +\sin\theta_1, 0, \cos\theta_1)$$
$$k_2 = k^0(1, -\sin\theta_2, 0, \cos\theta_2)$$



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✓ $\bar\Theta_{\lambda_1,\lambda_2} = 0 \quad \text{for} \quad J < |s_1 - s_2|$

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$$\bar T^{\pm}_{\alpha,\beta;\mu\nu} = -T^{\pm}_{\alpha,\beta;\mu\nu} \quad \to \quad \bar\Theta^{[2;1,1]}_{(\lambda_1,\lambda_2)\mu\nu}(k_1,k_2) = -(k_1\cdot k_2)\left[\varepsilon^*_{1\perp\mu}(\pm)\varepsilon_{2\perp\nu}(\pm) + \mu\leftrightarrow\nu\right]\delta_{\lambda_1,\pm}\delta_{\lambda_1,\lambda_2}$$

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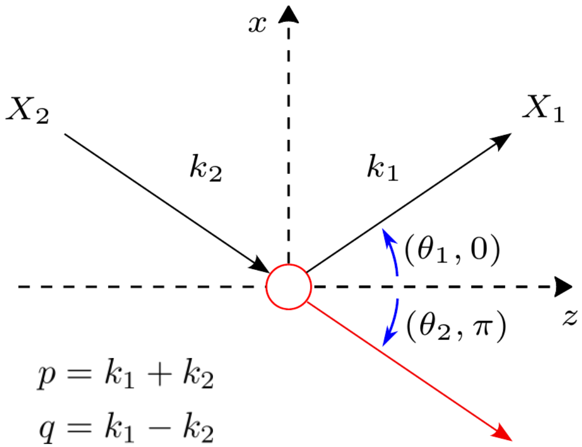
$$\xrightarrow{\theta_{1,2}\to 0} \quad k_{1,2} = k = k^0(1,0,0,1)$$



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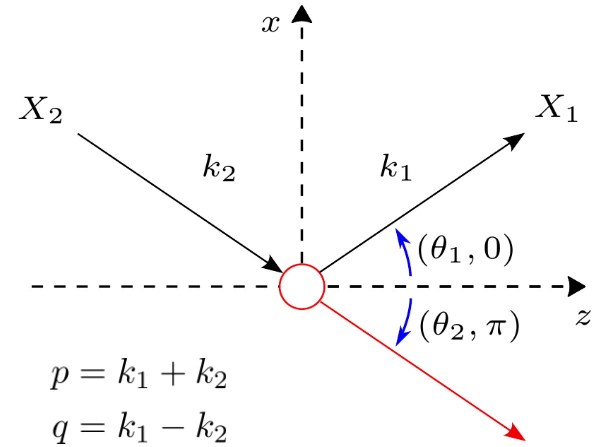

$$p = k_1 + k_2$$
$$q = k_1 - k_2$$

Assumption in the original paper

$$\lim_{\theta_{1,2}\to 0} \langle k_1, \lambda_1 | \left[\Theta^{[J;s_1,s_2]}\right] | k_2, \lambda_2\rangle = \langle k, \lambda_1 | \left[\Theta^{[J;s_1,s_2]}\right] | k, \lambda_2\rangle (-1)^{-\lambda_2}$$

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✓ $\bar{\Theta}_{\lambda_1,\lambda_2} = 0 \quad \text{for} \quad J < |s_1 - s_2|$

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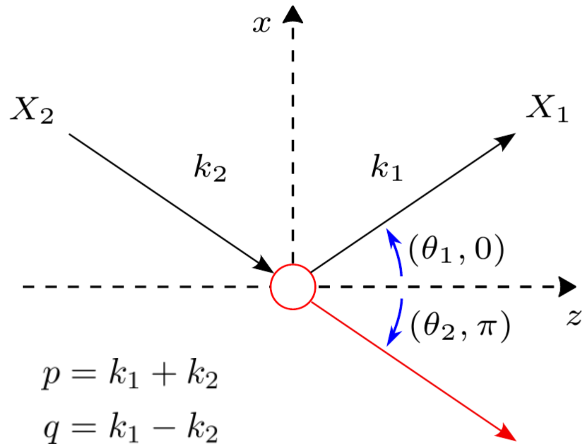

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✓ $\bar{\Theta}_{\lambda_1,\lambda_2} = 0$ for $J < |s_1 - s_2|$

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$$\bar{T}^{\pm}_{\alpha,\beta;\mu\nu} = -T^{\pm}_{\alpha,\beta;\mu\nu} \quad \to \quad \bar{\Theta}^{[2;1,1]}_{(\lambda_1,\lambda_2)\mu\nu}(k_1,k_2) = -(k_1\cdot k_2)\left[\varepsilon^*_{1\perp\mu}(\pm)\varepsilon_{2\perp\nu}(\pm) + \mu\leftrightarrow\nu\right]\delta_{\lambda_1,\pm}\delta_{\lambda_1,\lambda_2}$$
$$F_{(\lambda_1,\lambda_2)} \sim (q_\mu),(p_\mu),(g_{\mu\nu}) \quad \to \quad \bar{F}_{(\lambda_1,-\lambda_2)} \sim (p_\mu),(q_\mu),(g_{\mu\nu})$$

Involve only $(k_1\cdot k_2)$ and $\varepsilon^{(*)}_\perp$

Crossing symmetry and identical-momentum limits

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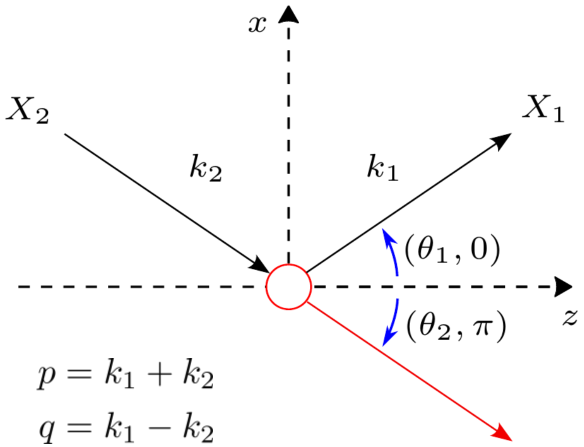
$$k_2 = k^0(1, -\sin\theta_2, 0, \cos\theta_2)$$
$$\xrightarrow{\theta_{1,2}\to 0} \quad k_{1,2} = k = k^0(1,0,0,1)$$



$p = k_1 + k_2$

$q = k_1 - k_2$

Assumption in the original paper

$$\lim_{\theta_{1,2}\to 0} \langle k_1, \lambda_1 | [\Theta^{[J;s_1,s_2]}] | k_2, \lambda_2\rangle = \langle k, \lambda_1 | [\Theta^{[J;s_1,s_2]}] | k, \lambda_2\rangle (-1)^{-\lambda_2}$$

$$\lim_{\theta_{1,2}\to 0} \sqrt{k_1\cdot k_2}\, \varepsilon^*_\perp(k_1, \pm; k_2) = \pm k$$
$$\lim_{\theta_{1,2}\to 0} \sqrt{k_1\cdot k_2}\, \varepsilon_\perp(k_2, \pm; k_1) = \pm k$$

Constraints

✓ $\bar{\Theta}_{\lambda_1,\lambda_2} = 0$ for $J < |s_1 - s_2|$

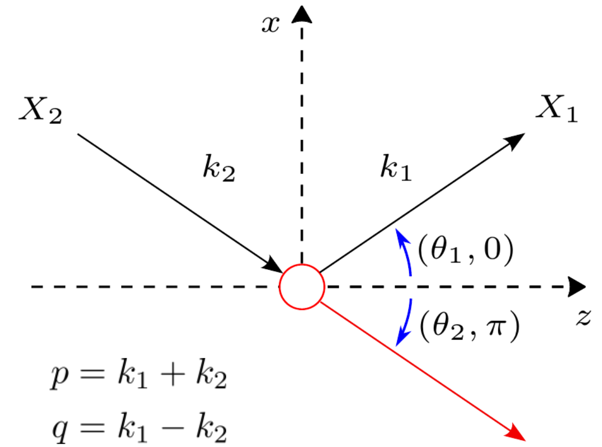
✓ $\bar{\Theta}_{\pm s_1,\pm s_2} = 0$ for $J < |s_1 + s_2|$

$+k_2 \to -k_2$ $\qquad \varepsilon^*(k_2, \pm) \to -\varepsilon(k_2, \mp)$

$$\bar{S}^\pm_{\alpha,\beta} = -S^\pm_{\alpha,\beta} \quad\to\quad \bar{\Theta}^{[0;1,1]}_{(\lambda_1,\lambda_2)}(k_1,k_2) = +(k_1\cdot k_2)\,\delta_{\lambda_1,\pm}\delta_{\lambda_1,-\lambda_2}$$
$$\bar{V}^\pm_{1\alpha;\mu} = -V^\pm_{1\alpha;\mu} \quad\to\quad \bar{\Theta}^{[1;1,0]}_{(\lambda_1,0)\mu}(k_1,k_2) = +(k_1\cdot k_2)\,\varepsilon^*_{1\perp\mu}(\pm)\,\delta_{\lambda_1,\pm}$$
$$\bar{V}^\mp_{2\beta;\nu} = -V^\pm_{2\beta;\nu} \quad\to\quad \bar{\Theta}^{[1;0,1]}_{(0,\lambda_2)\nu}(k_1,k_2) = -(k_1\cdot k_2)\,\varepsilon_{2\perp\nu}(\mp)\,\delta_{\lambda_2,\mp}$$
$$\bar{T}^\pm_{\alpha,\beta;\mu\nu} = -T^\pm_{\alpha,\beta;\mu\nu} \quad\to\quad \bar{\Theta}^{[2;1,1]}_{(\lambda_1,\lambda_2)\mu\nu}(k_1,k_2) = -(k_1\cdot k_2)\left[\varepsilon^*_{1\perp\mu}(\pm)\varepsilon_{2\perp\nu}(\pm) + \mu\leftrightarrow\nu\right]\delta_{\lambda_1,\pm}\delta_{\lambda_1,\lambda_2}$$
$$F_{(\lambda_1,\lambda_2)} \sim (q_\mu), (p_\mu), (g_{\mu\nu}) \quad\to\quad \bar{F}_{(\lambda_1,-\lambda_2)} \sim (p_\mu), (q_\mu), (g_{\mu\nu})$$

Crossing symmetry and identical-momentum limits

$$k_1 = k^0(1, +\sin\theta_1, 0, \cos\theta_1)$$
$$k_2 = k^0(1, -\sin\theta_2, 0, \cos\theta_2)$$
$$\xrightarrow{\theta_{1,2}\to 0} \quad k_{1,2} = k = k^0(1,0,0,1)$$



$p = k_1 + k_2$

$q = k_1 - k_2$

Assumption in the original paper

$$\lim_{\theta_{1,2}\to 0}\langle k_1,\lambda_1|\big[\Theta^{[J;s_1,s_2]}\big]|k_2,\lambda_2\rangle = \langle k,\lambda_1|\big[\Theta^{[J;s_1,s_2]}\big]|k,\lambda_2\rangle(-1)^{-\lambda_2}$$

$$\lim_{\theta_{1,2}\to 0}\sqrt{k_1\cdot k_2}\,\varepsilon_\perp^*(k_1,\pm;k_2) = \pm k$$
$$\lim_{\theta_{1,2}\to 0}\sqrt{k_1\cdot k_2}\,\varepsilon_\perp(k_2,\pm;k_1) = \pm k$$

Constraints

✓ $\bar\Theta_{\lambda_1,\lambda_2} = 0 \quad \text{for} \quad J < |s_1 - s_2|$

✓ $\bar\Theta_{\pm s_1,\pm s_2} = 0 \quad \text{for} \quad J < |s_1 + s_2|$

$$\bar S^{\pm}_{\alpha,\beta} = -S^{\pm}_{\alpha,\beta} \quad\to\quad \bar\Theta^{[0;1,1]}_{(\lambda_1,\lambda_2)}(k_1,k_2) = \cancel{+(k_1\cdot k_2)}\,\delta_{\lambda_1,\pm}\delta_{\lambda_1,-\lambda_2}$$
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$$\bar T^{\pm}_{\alpha,\beta;\mu\nu} = -T^{\pm}_{\alpha,\beta;\mu\nu} \quad\to\quad \bar\Theta^{[2;1,1]}_{(\lambda_1,\lambda_2)\mu\nu}(k_1,k_2) = -(k_1\cdot k_2)\big[\varepsilon^*_{1\perp\mu}(\pm)\varepsilon_{2\perp\nu}(\pm) + \mu\leftrightarrow\nu\big]\,\delta_{\lambda_1,\pm}\delta_{\lambda_1,\lambda_2} \quad\longrightarrow\quad 2k_\mu k_\nu$$
$$F_{(\lambda_1,\lambda_2)} \sim (q_\mu),(p_\mu),(g_{\mu\nu}) \quad\to\quad \bar F_{(\lambda_1,-\lambda_2)} \sim (p_\mu),\cancel{(q_\mu)},(g_{\mu\nu}) \quad\longrightarrow\quad 2(k_\mu),(g_{\mu\nu})$$

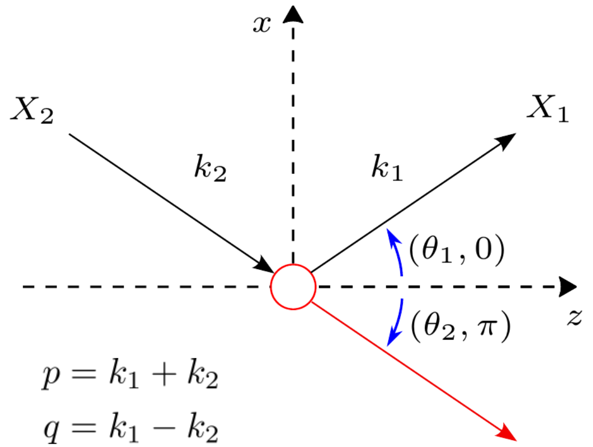
Crossing symmetry and identical-momentum limits

Form factors including $(k_1 \cdot k_2)$ in denominators

$$\bar S^{\pm}_{\alpha,\beta}/(k_1\cdot k_2) \quad\rightarrow\quad \bar\Theta^{[0;1,1]}_{(\lambda_1,\lambda_2)}(k,k) = +\,\delta_{\lambda_1,\pm}\delta_{\lambda_1,-\lambda_2},$$

$$\bar V^{\pm}_{1\alpha;\mu}/\sqrt{k_1\cdot k_2} \quad\rightarrow\quad \bar\Theta^{[1;1,0]}_{(\lambda_1,0)\mu}(k,k) = \pm k_\mu\,\delta_{\lambda_1,\pm}$$

$$\bar V^{\pm}_{2\beta;\nu}/\sqrt{k_1\cdot k_2} \quad\rightarrow\quad \bar\Theta^{[1;0,1]}_{(0,\lambda_2)\nu}(k,k) = \mp k_\nu\,\delta_{\lambda_2,\pm}$$



Constraints

- ✓ $\bar\Theta_{\lambda_1,\lambda_2} = 0 \quad \text{for} \quad J < |s_1 - s_2|$
- ✓ $\bar\Theta_{\pm s_1,\pm s_2} = 0 \quad \text{for} \quad J < |s_1 + s_2|$

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$$\bar V^{\mp}_{2\beta;\nu} = -V^{\pm}_{2\beta;\nu} \quad\rightarrow\quad \bar\Theta^{[1;0,1]}_{(0,\lambda_2)\nu}(k_1,k_2) = \cancel{-(k_1\cdot k_2)}\,\varepsilon_{2\perp\nu}(\mp)\,\delta_{\lambda_2,\mp}$$

$$\bar T^{\pm}_{\alpha,\beta;\mu\nu} = -T^{\pm}_{\alpha,\beta;\mu\nu} \quad\rightarrow\quad \bar\Theta^{[2;1,1]}_{(\lambda_1,\lambda_2)\mu\nu}(k_1,k_2) = -(k_1\cdot k_2)\left[\varepsilon^*_{1\perp\mu}(\pm)\varepsilon_{2\perp\nu}(\pm) + \mu\leftrightarrow\nu\right]\delta_{\lambda_1,\pm}\delta_{\lambda_1,\lambda_2} \quad\longrightarrow\quad 2k_\mu k_\nu$$

$$F_{(\lambda_1,\lambda_2)} \sim (q_\mu),(p_\mu),(g_{\mu\nu}) \quad\rightarrow\quad \bar F_{(\lambda_1,-\lambda_2)} \sim (p_\mu),\cancel{(q_\mu)},(g_{\mu\nu}) \quad\longrightarrow\quad 2(k_\mu),(g_{\mu\nu})$$

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$$\bar{S}^{\pm}_{\alpha,\beta}/(k_1 \cdot k_2) \quad \to \quad \bar{\Theta}^{[0;1,1]}_{(\lambda_1,\lambda_2)}(k,k) = +\,\delta_{\lambda_1,\pm}\delta_{\lambda_1,-\lambda_2},$$

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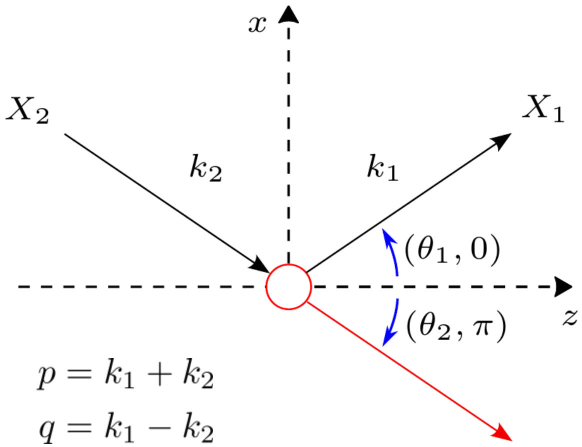
$$\bar{V}^{\pm}_{2\beta;\nu}/\sqrt{k_1 \cdot k_2} \quad \to \quad \bar{\Theta}^{[1;0,1]}_{(0,\lambda_2)\nu}(k,k) = \mp k_\nu\,\delta_{\lambda_2,\pm}$$

$\to$ *Violate the covariance of tensor currents!*



Constraints

✓ $\bar{\Theta}_{\lambda_1,\lambda_2} = 0$ for $J < |s_1 - s_2|$

✓ $\bar{\Theta}_{\pm s_1,\pm s_2} = 0$ for $J < |s_1 + s_2|$

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$$\bar{V}^{\mp}_{2\beta;\nu} = -V^{\pm}_{2\beta;\nu} \quad \to \quad \bar{\Theta}^{[1;0,1]}_{(0,\lambda_2)\nu}(k_1,k_2) = \cancel{-(k_1 \cdot k_2)}\,\varepsilon_{2\perp\nu}(\mp)\,\delta_{\lambda_2,\mp}$$

$$\bar{T}^{\pm}_{\alpha,\beta;\mu\nu} = -T^{\pm}_{\alpha,\beta;\mu\nu} \quad \to \quad \bar{\Theta}^{[2;1,1]}_{(\lambda_1,\lambda_2)\mu\nu}(k_1,k_2) = -(k_1 \cdot k_2)\left[\varepsilon^*_{1\perp\mu}(\pm)\varepsilon_{2\perp\nu}(\pm) + \mu \leftrightarrow \nu\right]\delta_{\lambda_1,\pm}\delta_{\lambda_1,\lambda_2} \quad \longrightarrow \quad 2k_\mu k_\nu$$

$$F_{(\lambda_1,\lambda_2)} \sim (q_\mu),(p_\mu),(g_{\mu\nu}) \quad \to \quad \bar{F}_{(\lambda_1,-\lambda_2)} \sim (p_\mu),\cancel{(q_\mu)},(g_{\mu\nu}) \quad \longrightarrow \quad 2(k_\mu),(g_{\mu\nu})$$

Crossing symmetry and identical-momentum limits

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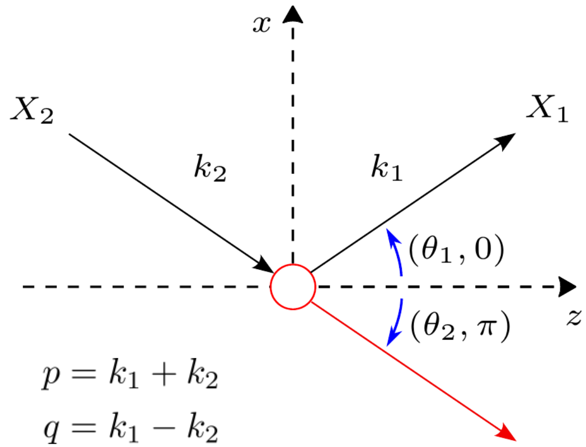
$$\bar{S}^{\pm}_{\alpha,\beta}/(k_1 \cdot k_2) \quad \rightarrow \quad \bar{\Theta}^{[0;1,1]}_{(\lambda_1,\lambda_2)}(k,k) = +\,\delta_{\lambda_1,\pm}\delta_{\lambda_1,-\lambda_2},$$

$$\bar{V}^{\pm}_{1\alpha;\mu}/\sqrt{k_1 \cdot k_2} \quad \rightarrow \quad \bar{\Theta}^{[1;1,0]}_{(\lambda_1,0)\mu}(k,k) = \pm k_\mu\,\delta_{\lambda_1,\pm}$$

$$\bar{V}^{\mp}_{2\beta;\nu}/\sqrt{k_1 \cdot k_2} \quad \rightarrow \quad \bar{\Theta}^{[1;0,1]}_{(0,\lambda_2)\nu}(k,k) = \pm k_\nu\,\delta_{\lambda_2,\mp}$$

$\rightarrow$ *Violate the covariance of tensor currents!*



Ex) $\bar{S}^{\pm}_{\alpha,\beta}/(k_1 \cdot k_2) \rightarrow$ A scalar current Θ

$$\langle k,\pm|R^\dagger_z(\varphi)\,\Theta\,R_z(\varphi)|k,\mp\rangle \;\rightarrow\; \langle k,\pm|\,\Theta\,|k,\mp\rangle = e^{\pm 2i\varphi}\langle k,\pm|\,\Theta\,|k,\mp\rangle = 0$$

$\rightarrow$ *Contradiction!*

Constraints

✓ $\bar{\Theta}_{\lambda_1,\lambda_2} = 0 \quad$ for $\quad J < |s_1 - s_2|$

✓ $\bar{\Theta}_{\pm s_1,\pm s_2} = 0 \quad$ for $\quad J < |s_1 + s_2|$

$$\bar{S}^{\pm}_{\alpha,\beta} = -S^{\pm}_{\alpha,\beta} \quad \rightarrow \quad \bar{\Theta}^{[0;1,1]}_{(\lambda_1,\lambda_2)}(k_1,k_2) = \cancel{+(k_1 \cdot k_2)}\,\delta_{\lambda_1,\pm}\delta_{\lambda_1,-\lambda_2}$$

$$\bar{V}^{\pm}_{1\alpha;\mu} = -V^{\pm}_{1\alpha;\mu} \quad \rightarrow \quad \bar{\Theta}^{[1;1,0]}_{(\lambda_1,0)\mu}(k_1,k_2) = \cancel{+(k_1 \cdot k_2)}\,\varepsilon^*_{1\perp\mu}(\pm)\,\delta_{\lambda_1,\pm}$$

$$\bar{V}^{\mp}_{2\beta;\nu} = -V^{\pm}_{2\beta;\nu} \quad \rightarrow \quad \bar{\Theta}^{[1;0,1]}_{(0,\lambda_2)\nu}(k_1,k_2) = \cancel{-(k_1 \cdot k_2)}\,\varepsilon_{2\perp\nu}(\mp)\,\delta_{\lambda_2,\mp}$$

$$\bar{T}^{\pm}_{\alpha,\beta;\mu\nu} = -T^{\pm}_{\alpha,\beta;\mu\nu} \quad \rightarrow \quad \bar{\Theta}^{[2;1,1]}_{(\lambda_1,\lambda_2)\mu\nu}(k_1,k_2) = -(k_1 \cdot k_2)\left[\varepsilon^*_{1\perp\mu}(\pm)\varepsilon_{2\perp\nu}(\pm) + \mu \leftrightarrow \nu\right]\delta_{\lambda_1,\pm}\delta_{\lambda_1,\lambda_2} \quad \longrightarrow \quad 2k_\mu k_\nu$$

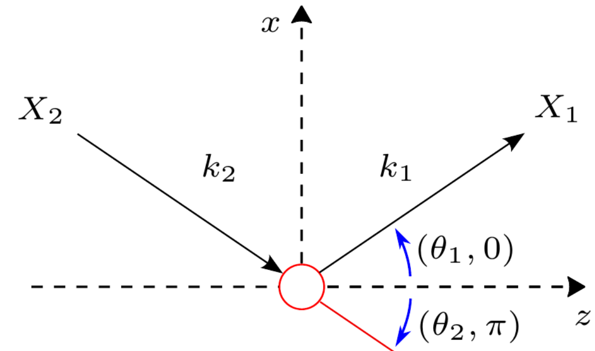
$$F_{(\lambda_1,\lambda_2)} \sim (q_\mu),(p_\mu),(g_{\mu\nu}) \quad \rightarrow \quad \bar{F}_{(\lambda_1,-\lambda_2)} \sim (p_\mu),\cancel{(q_\mu)},(g_{\mu\nu}) \quad \longrightarrow \quad 2(k_\mu),(g_{\mu\nu})$$

Crossing symmetry and identical-momentum limits

Covariant vertices

$$[\bar{\Gamma}^{[J;s_1,s_2]}] = \sum_{\lambda=\pm}\Big\{\Theta(J_-)\sum_{n=0}^{J_-}[\bar{F}^{[J;s_1,s_2]}_{(\lambda s_1,-\lambda s_2)}]^n\,\boxed{[\bar{S}^{\lambda}]^{s_{\min}}\left([\bar{V}_1^{\lambda}]^{s_1-s_{\min}}+[\bar{V}_2^{\lambda}]^{s_2-s_{\min}}\right)}$$

$$+\gamma_{s_{\min}}\,\Theta(J_+)\sum_{n=0}^{J_+}[\bar{F}^{[J;s_1,s_2]}_{(\lambda s_1,\lambda s_2)}]^n\,[\bar{T}^{\lambda}]^{s_{\min}}\boxed{\left([\bar{V}_1^{\lambda}]^{s_1-s_{\min}}+[\bar{V}_2^{-\lambda}]^{s_2-s_{\min}}\right)}\Big\}$$



$$p = k_1 + k_2$$

$$q = k_1 - k_2$$

<Covariance conditions on form factors>

$$\lim_{k_{1,2}\to k}\bar{A}^{[J;s_1,s_2]}_{(\lambda s_1,-\lambda s_2)n,m,l}\times(k_1\cdot k_2)^{\frac{s_1+s_2}{2}} = 0$$

$$\lim_{k_{1,2}\to k}\bar{A}^{[J;s_1,s_2]}_{(\lambda s_1,+\lambda s_2)n,m,l}\times(k_1\cdot k_2)^{\frac{|s_1-s_2|}{2}} = 0$$

Second key result

Constraints

✓ $\bar{\Theta}_{\lambda_1,\lambda_2} = 0$ for $J < |s_1 - s_2|$

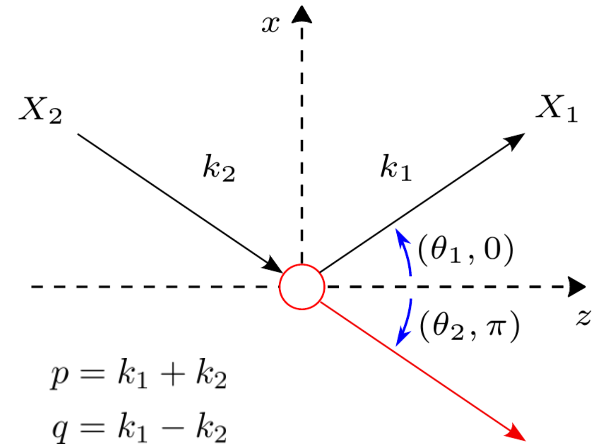
✓ $\bar{\Theta}_{\pm s_1,\pm s_2} = 0$ for $J < |s_1 + s_2|$

Crossing symmetry and identical-momentum limits

Covariant vertices

$$[\bar{\Gamma}^{[J;s_1,s_2]}] = \sum_{\lambda=\pm} \Bigg\{ \Theta(J_-) \sum_{n=0}^{J_-} [\bar{F}^{[J;s_1,s_2]}_{(\lambda s_1,-\lambda s_2)}]^n [\bar{S}^\lambda]^{s_{\min}} \Big([\bar{V}_1^\lambda]^{s_1-s_{\min}} + [\bar{V}_2^\lambda]^{s_2-s_{\min}} \Big)$$

$$+ \gamma_{s_{\min}} \Theta(J_+) \sum_{n=0}^{J_+} [\bar{F}^{[J;s_1,s_2]}_{(\lambda s_1,\lambda s_2)}]^n [\bar{T}^\lambda]^{s_{\min}} \Big([\bar{V}_1^\lambda]^{s_1-s_{\min}} + [\bar{V}_2^{-\lambda}]^{s_2-s_{\min}} \Big) \Bigg\}$$



$p = k_1 + k_2$

$q = k_1 - k_2$

<Revisiting the WW theorem>

Conserved tensor currents

$$q^{\mu_i} \bar{\Theta}^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)\,\mu_1\cdots\mu_i\cdots\mu_J}(k_1,k_2) = 0$$

$$q^\mu \{V^\pm_{1,2}, T^\pm\}_\mu = 0$$

$$\bar{F} \sim (q_\mu), (p_\mu), (g_{\mu\nu}) \quad \to \quad \bar{F} \sim (p_\mu), (q^2 g_{\mu\nu} - q_\mu q_\nu)$$

Constraints

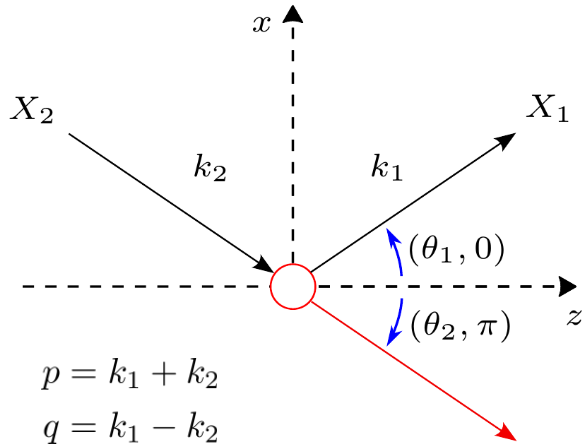
✓ $\bar{\Theta}_{\lambda_1,\lambda_2} = 0$ for $J < |s_1 - s_2|$

✓ $\bar{\Theta}_{\pm s_1,\pm s_2} = 0$ for $J < |s_1 + s_2|$

Crossing symmetry and identical-momentum limits

Covariant vertices

$$[\bar{\Gamma}^{[J;s_1,s_2]}] = \sum_{\lambda=\pm}\left\{\Theta(J_-)\sum_{n=0}^{J_-}[\bar{F}^{[J;s_1,s_2]}_{(\lambda s_1,-\lambda s_2)}]^n [\bar{S}^\lambda]^{s_{\rm min}}\left([\bar{V}_1^\lambda]^{s_1-s_{\rm min}}+[\bar{V}_2^\lambda]^{s_2-s_{\rm min}}\right)\right.$$
$$\left.+\gamma_{s_{\rm min}}\Theta(J_+)\sum_{n=0}^{J_+}[\bar{F}^{[J;s_1,s_2]}_{(\lambda s_1,\lambda s_2)}]^n[\bar{T}^\lambda]^{s_{\rm min}}\left([\bar{V}_1^\lambda]^{s_1-s_{\rm min}}+[\bar{V}_2^{-\lambda}]^{s_2-s_{\rm min}}\right)\right\}$$



$p = k_1 + k_2$

$q = k_1 - k_2$

<Revisiting the WW theorem>

Conserved tensor currents

$$q^{\mu_i}\,\bar{\Theta}^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)\,\mu_1\cdots\mu_i\cdots\mu_J}(k_1,k_2) = 0$$

$$q^\mu\{V^\pm_{1,2}, T^\pm\}_\mu = 0$$

$$\bar{F}\sim(q_\mu),(p_\mu),(g_{\mu\nu}) \quad\rightarrow\quad \bar{F}\sim(p_\mu),(q^2 g_{\mu\nu}-q_\mu q_\nu)$$

In the identical-momentum limit $(k_1 - k_2)\rightarrow 0$

$$\bar{F}\sim(p_\mu),(q^2 g_{\mu\nu}-q_\mu q_\nu) \quad\rightarrow\quad \bar{F}\sim 2(k_\mu)$$

Constraints for $k_1 - k_2 = 0$

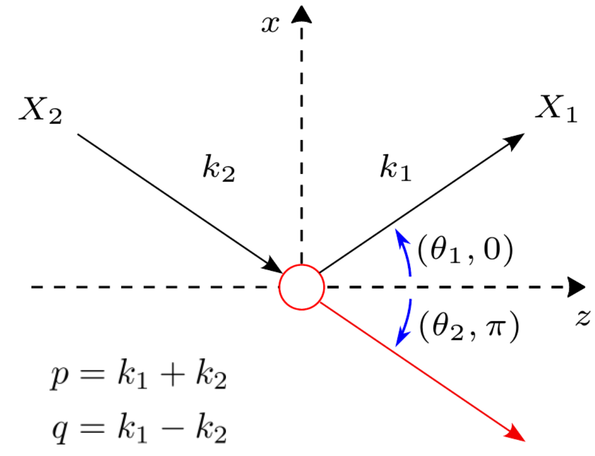
- ✓ $\bar{\Theta}_{\pm s_1,\mp s_2} = 0$ Always
- ✓ $\bar{\Theta}_{\pm s_1,\pm s_2} = 0$ for $J < |s_1 + s_2|$
- ✓ $\bar{\Theta}_{\pm s_1,\pm s_2} = 0$ for $s_1 \neq s_2$

Crossing symmetry and identical-momentum limits

Covariant vertices for $k_1 - k_2 = 0$

$$[\bar{\Gamma}^{[J;s,s]}] = 2^{J-2s} \sum_{\lambda=\pm} \left\{ \Theta(J-2s)\, \bar{A}^{[J;s,s]}_{(\lambda s, \lambda s)}\, [\,k\,]^{J-2s}\, [\bar{T}^{\lambda}]^{s} \right\}$$

$$\longrightarrow \quad [\Theta^{[J;s,s]}_{\pm,\pm}(k,k)] = \Theta(J-2s)\, 2^{J-s}\, \bar{A}^{[J;s,s]}_{(\pm,\pm)}\, [\,k\,]^{J}$$



<Revisiting the WW theorem>

Conserved tensor currents

$$q^{\mu_i}\, \bar{\Theta}^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)\,\mu_1\cdots\mu_i\cdots\mu_J}(k_1,k_2) = 0$$

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Constraints for $k_1 - k_2 = 0$

- ✓ $\bar{\Theta}_{\pm s_1, \mp s_2} = 0$ Always
- ✓ $\bar{\Theta}_{\pm s_1, \pm s_2} = 0$ for $J < |s_1 + s_2|$
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In the identical-momentum limit $(k_1 - k_2) \to 0$

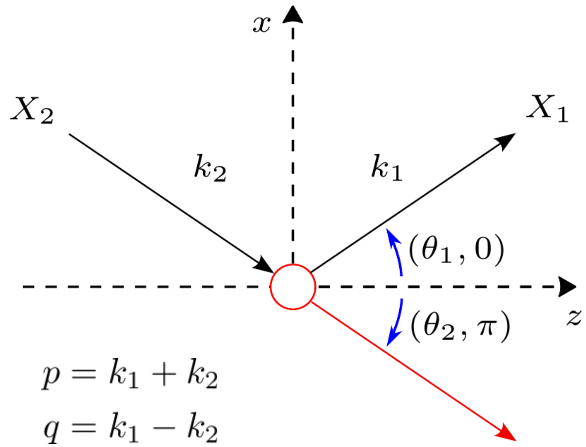
$$\bar{F} \sim (p_\mu), (q^2 g_{\mu\nu} - q_\mu q_\nu) \quad \to \quad \bar{F} \sim 2(k_\mu)$$

Crossing symmetry and identical-momentum limits

Covariant vertices for $k_1 - k_2 = 0$

$$\left[\bar\Gamma^{[J;s,s]}\right] = 2^{J-2s} \sum_{\lambda=\pm} \left\{ \Theta(J-2s)\, \bar A^{[J;s,s]}_{(\lambda s,\lambda s)} \,[\,k\,]^{J-2s}\, [\,\bar T^{\lambda}]^{s} \right\}$$

$$\longrightarrow \quad \left[\Theta^{[J;s,s]}_{\pm,\pm}(k,k)\right] = \Theta(J-2s)\, 2^{J-s}\, \bar A^{[J;s,s]}_{(\pm,\pm)}\, [\,k\,]^{J}$$



$p = k_1 + k_2$

$q = k_1 - k_2$

<Revisiting the WW theorem>

$$\int d^3\vec x\, \frac{\langle k,\lambda|\Theta_\mu(x)|k',\lambda'\rangle}{\sqrt{\langle k,\lambda|k,\lambda\rangle}\sqrt{\langle k',\lambda'|k',\lambda'\rangle}} = \frac{\langle k,\lambda|\Theta_\mu(0)|k,\lambda\rangle}{2k^0}\delta_{\lambda,\lambda'},$$

with $\langle k,\lambda|k',\lambda'\rangle = 2k^0(2\pi)^3\delta^3(\vec k - \vec k')\delta_{\lambda,\lambda'}$

Take $\bar A_{(\pm,\pm)} = Q$ or 2^{s-1} for J_μ or $T_{\mu\nu}$

$$\frac{\langle k,\pm|J_0|k,\pm\rangle}{2k^0} = \Theta(1-2s)\, Q \qquad \frac{\langle k,\pm|T_{0\mu}|k,\pm\rangle}{2k^0} = \Theta(2-2s)\, k_\mu$$

Constraints for $k_1 - k_2 = 0$

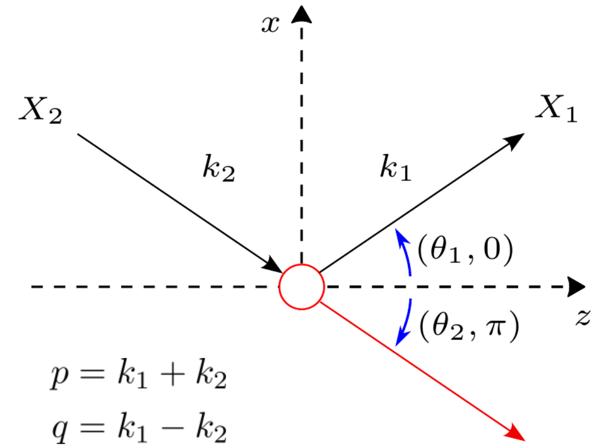
- ✓ $\bar\Theta_{\pm s_1,\mp s_2} = 0$ Always
- ✓ $\bar\Theta_{\pm s_1,\pm s_2} = 0$ for $J < |s_1 + s_2|$
- ✓ $\bar\Theta_{\pm s_1,\pm s_2} = 0$ for $s_1 \neq s_2$

Crossing symmetry and identical-momentum limits

Covariant vertices for $k_1 - k_2 = 0$

$$[\bar{\Gamma}^{[J;s,s]}] = 2^{J-2s} \sum_{\lambda=\pm} \left\{ \Theta(J-2s)\, \bar{A}^{[J;s,s]}_{(\lambda s,\lambda s)}\, [\, k\,]^{J-2s}\, [\,\bar{T}^{\lambda}]^{s} \right\}$$

$$\longrightarrow \quad [\Theta^{[J;s,s]}_{\pm,\pm}(k,k)] = \Theta(J-2s)\, 2^{J-s}\, \bar{A}^{[J;s,s]}_{(\pm,\pm)}\, [\, k\,]^{J}$$



$p = k_1 + k_2$

$q = k_1 - k_2$

<Revisiting the WW theorem>

$$\int d^3\vec{x}\, \frac{\langle k,\lambda|\Theta_\mu(x)|k',\lambda'\rangle}{\sqrt{\langle k,\lambda|k,\lambda\rangle}\sqrt{\langle k',\lambda'|k',\lambda'\rangle}} = \frac{\langle k,\lambda|\Theta_\mu(0)|k,\lambda\rangle}{2k^0}\delta_{\lambda,\lambda'},$$

with $\langle k,\lambda|k',\lambda'\rangle = 2k^0(2\pi)^3\delta^3(\vec{k}-\vec{k}')\delta_{\lambda,\lambda'}$

Take $\bar{A}_{(\pm,\pm)} = Q$ or 2^{s-1} for J_μ or $T_{\mu\nu}$

$$\frac{\langle k,\pm|J_0|k,\pm\rangle}{2k^0} = \Theta(1-2s)\, Q \qquad \frac{\langle k,\pm|T_{0\mu}|k,\pm\rangle}{2k^0} = \Theta(2-2s)\, k_\mu$$

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Invariant roles of basic operators in all the momentum transfers

$\bar{S}^{\pm}_{\alpha,\beta} \rightarrow (\pm,\mp)$

$\bar{V}^{\pm}_{1\alpha;\mu} \rightarrow (\pm,0)$

$\bar{V}^{\mp}_{2\beta;\mu} \rightarrow (\mp,0)$

$T^{\pm}_{\alpha,\beta;\mu\nu} \rightarrow (\pm,\pm)$

Assumption

$$\lim_{\theta_{1,2}\to k} \langle k_1,\lambda_1|[\Theta^{[J;s_1,s_2]}]|k_2,\lambda_2\rangle = \langle k,\lambda_1|[\Theta^{[J;s_1,s_2]}]|k,\lambda_2\rangle(-1)^{-\lambda_2}$$

is correct in the quantum field theory including conventional high-spin fields

Third key result

Summary

- ✓ An efficient algorithm in the covariant formulation. $\Theta^{[J;s_1,s_2]}_{\mu_1\cdots\mu_J}$
- ✓ Covariance constraints on form factors.
- ✓ Validity of the continuity assumption.
- ✓ Covariant three-point vertices for all off-shell cases.
- ✓ Covariant four-point vertices.
- ✓ Program for generating covariant vertices and Lagrangian operators.
- ✓ Algorithm $\leftrightarrow$ Spinor helicity formalism.

Thank you

Back up

Three-point vertices for identical particles, X_1 and $\bar{X}_2$ ($X_1 = \bar{X}_2$)

$$\Big[\langle X_1; k_1, \lambda_1 | \langle \bar{X}_2; k_2, \lambda_2 |\Big] \Theta_{\mu_1 \cdots \mu_J} | 0 \rangle = \Big[\langle \bar{X}_2; k_1, \lambda_1 | \langle X_1; k_2, \lambda_2 |\Big] \Theta_{\mu_1 \cdots \mu_J} | 0 \rangle$$

$$\downarrow \quad k_1 \leftrightarrow k_2$$

$$\varepsilon^{*\alpha}(k_1, \lambda_1) \varepsilon^{*\beta}(k_2, \lambda_2)\, \Gamma_{\alpha,\beta;\mu}(k_1, k_2) = \varepsilon^{*\alpha}(k_2, \lambda_2) \varepsilon^{*\beta}(k_1, \lambda_1)\, \Gamma_{\alpha,\beta;\mu}(k_2, k_1)$$

$$\downarrow \quad \alpha \leftrightarrow \beta$$

Identical particle condition

$$\Gamma_{\alpha,\beta;\mu}(p, q) = \Gamma_{\beta,\alpha;\mu}(p, -q)$$

Selction rules

1. $(J = 1, 3, \cdots) \not\to (0 + 0)$
2. $(1) \to$ ~~$(0+0)$~~, $(1/2 + 1/2)$ ~~$(1+1)$~~, ~~$(\cdots)$~~

$\vdots$

$$
\begin{aligned}
B^{\pm}(0,5.2) &\rightarrow K^{*}(1,0.9)^{\pm}+\gamma(1,0)\\
H(0,125) &\rightarrow \gamma(1,0)+\gamma(1,0)\\
H(0,125) &\rightarrow g(1,0)+g(1,0)\\
H(0,125) &\rightarrow Z(1,91)+\gamma(1,0)\\
H(0,125) &\rightarrow Z^{*}(1,\text{virtual})+Z(1,91)\\
t(1/2,173) &\rightarrow b(1/2,4)+W^{+}(1,80)\\
\tau(1/2,1.7) &\rightarrow \pi(0,0.15)+\nu_{\tau}(1/2,0)\\
\tau(1/2,1.7) &\rightarrow \rho(1,0.77)+\nu_{\tau}(1/2,0)\\
Z(1,91) &\rightarrow \tau(1/2,1.7)+\bar{\tau}(1/2,1.7)\\
V^{*}(1,\text{virtual}) &\rightarrow W^{-}(1,80)+W^{+}(1,80)\\
J/\psi(1,3) &\rightarrow a_2(1320)(2,1.3)+\rho(1,0.77)\\
J/\psi(1,3) &\rightarrow f_4(2050)(4,2)+\gamma(1,0)
\end{aligned}
$$