Explanation of binary orbit anomaly

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What is binary orbit anomaly?

- Wide binary stars observed by the Gaia satellite
- v_{2D}, s_{2D} and masses (total mass 1 - 1.6 $M_\odot$)
- Consistent with Newtonian gravity ($s_{2D} < 2000$ au)
- Deviation from Newtonian gravity ($s_{2D} > 2000$ au)
  $G \to \gamma G$ with $\gamma = 1.5 \pm \sigma_\gamma$, with $0.06 < \sigma_\gamma < 0.2$

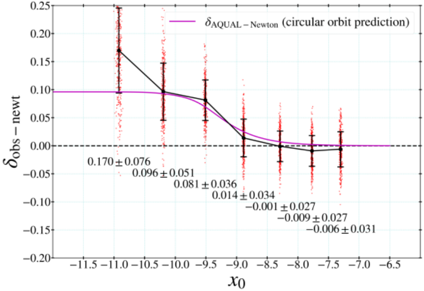
  Chae (2305.04613, 2309.10404),
  Hernandez (2304.07322), Hernandez et al. (2309.10995)

Why cannot dark matter explain binary orbit anomaly?

- Dark matter can explain galaxy rotation curves, but why can't it explain the binary orbit anomaly?
- Far smaller scale than the galactic scale
- $10^{-22}\text{kg/m}^3 \times \frac{4}{3}\pi\ (10^{4.5}\ \text{au})^3 = 2 \times 10^{-5} M_{\odot}$

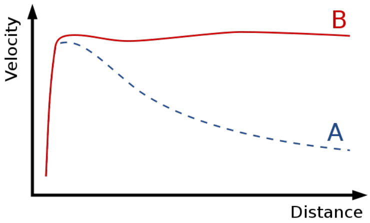
Possible explanation by MOND

- Chae assumes our galaxy has a vertical acceleration of $g_0/3$ where g_0 is the acceleration of the Sun with respect to the center of our galaxy.



- Let's find out the explanation by Verlinde's emergent gravity.
- Before doing so, we have to explain MOND.

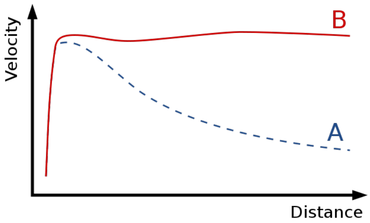
Tully-Fisher relation



$$v = b\sqrt{\sqrt{M}}, \qquad M : \text{total mass of galaxy}$$

Can we explain Tully-Fisher relation by the Newtonian gravity?

Tully-Fisher relation



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Can we explain Tully-Fisher relation by the Newtonian gravity?
Approximation: M at the center of galaxy

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \qquad \rightarrow \qquad v = \sqrt{\frac{GM}{r}}$$

Either the Newtonian gravity is wrong or we need Dark Matter.

Modified Newtonian Dynamics (MOND)

- a_M: Milgrom's constant, 1.2×10^{-10}m/s^2
  g_B: the Newtonian gravity due to the baryonic matter
- Newtonian regime

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- MOND regime (sub-Newtonian regime)

$$g_{MOND} = \sqrt{g_B a_M}, \qquad g_B \ll a_M$$

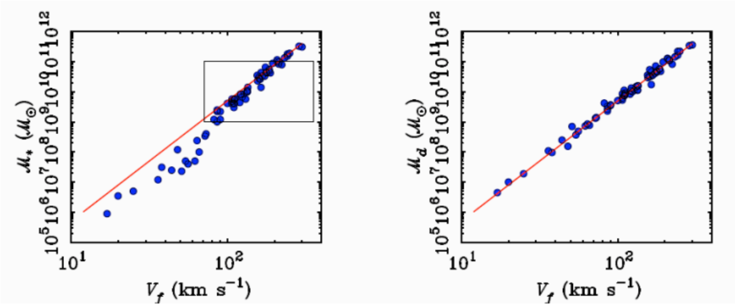
$$g_B = \frac{GM}{r^2}$$

$$g_{MOND} = \sqrt{\frac{GMa_M}{r^2}} = \frac{\sqrt{GMa_M}}{r} = \frac{v^2}{r}$$

- Tully-Fisher relation

$$v = \sqrt{\sqrt{GMa_M}}$$

Baryonic Tully-Fisher relation



S. S. McGaugh, "The Baryonic Tully-Fisher relation of galaxies with extended rotation curves and the stellar mass of rotating galaxies," Astrophys. J. **632**, 859-871 (2005)

Problems with MOND, why is Verlinde gravity more promising?

- We know what the gravitational accelerations are when g_B is much larger than a_M or much smaller than a_M, but do not know them when g_B is between.
- We may make up functions that reduce to $g_{MOND} = g_B$ for $g_B \gg a_M$ and $g_{MOND} = \sqrt{g_B a_M}$ for $g_B \ll a_M$, but too many functions satisfy these two conditions, and they all fit the galaxy rotation curves equally well.
- MOND may be able to fit the data, but it remains silent on how their equations can be derived.

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- MOND may be able to fit the data, but it remains silent on how their equations can be derived.
- Verlinde came up with "emergent gravity" that can explain Tully-Fisher relation equally well.
- Used "theory of elasticity" to derive equations.
- u_i: displacement
- $\epsilon_{ij} \equiv \frac{1}{2}(\nabla_i u_j + \nabla_j u_i)$: strain tensor

The entropic gravity

- The entropic force

$$F\Delta x = T\Delta S \qquad \rightarrow \qquad F = T\frac{\partial S}{\partial x}$$

- Verlinde, 2010, entropy graident leads to gravitational force
- The Bekenstein-Hawking entropy

$$S = \frac{A}{4G\hbar}$$

k: the Boltzmann constant, A: area

- Explained the Newtonian gravity and general relativity

Emergent gravity

- Verlinde, 2016, Emergent gravity
- Assume de Sitter Universe.

$$ds^2 = -\left(1 - \frac{r^2}{L^2}\right) dt^2 + \frac{dr^2}{1 - \frac{r^2}{L^2}} + r^2 d\Omega^2. \qquad (1)$$

Horizon at $r = L$.

$$S(L) = \frac{4\pi L^2}{4G\hbar}$$

- In addition to the Bekenstein-Hawking entropy, assume a Volume entropy

$$S_{Volume}(r) = \frac{S(L)}{V(L)} V(r) = \left(\frac{4\pi L^2/4G\hbar}{\frac{4}{3}\pi L^3}\right) \frac{4}{3}\pi r^3$$

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- $a_0 = \frac{c^2}{L} = cH_0$: acceleration scale
- Volume entropy $\propto r^3$, Area entropy $\propto r^2$
- Volume entropy relatively more important for large scale

Reduction of the area entropy

- Recall the area entropy

$$S = \frac{A}{4G\hbar} = \frac{4\pi r^2}{4G\hbar}$$

- If M is at the center, it decreases the length r and therefore A.

$$S_M(r) = -\frac{2\pi Mr}{\hbar}$$

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- $\epsilon_M(r) = \frac{|S_M(r)|}{S_{Volume}(r)} > 1$, nothing of the volume entropy is left. Volume entropy not important. The Newtonian regime

$$\epsilon_M(r) = \frac{GM/r^2}{a_0/2} > 1 \qquad \text{Compare it with} \quad g_B \gg a_M$$

Reduction of the area entropy

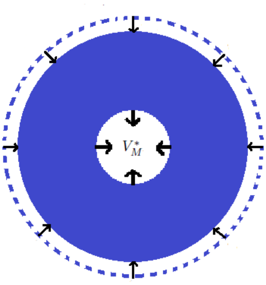
- $\epsilon_M(r) = \dfrac{|S_M(r)|}{S_{Volume}(r)} < 1$, some volume entropy is left. Volume entropy is important. Sub-Newtonian regime

$$\epsilon_M(r) = \frac{GM/r^2}{a_0/2} = \frac{g}{a_0/2} < 1 \qquad \text{Compare it with} \quad g_B \ll a_M$$

- $S_M(r)$ “removes” the volume $V_M^*(r)$ from the ball of radius r.

Elastic inclusion

Let me explain "elastic inclusion" first, which has nothing to do with Verlinde's emergent gravity per se.



V_M^* is removed from $r = 0$.
Displacement:

$$u(r) = -\frac{V_M^*}{A(r)} = -\frac{V_M^*}{4\pi r^2}$$

Strain tensor:

$$\epsilon_{rr}(r) = \frac{\partial u(r)}{\partial r} = \frac{V_M}{V(r)} = \epsilon_M(r)$$

$\epsilon < 1$ in the theory of elasticity

$$\int_{\bar{R}} \epsilon^2 dV = \int_{V_M}^{\infty} \left(\frac{V_M}{V}\right)^2 dV = -\frac{V_M^2}{V}\bigg|_{V_M}^{\infty} = V_M$$

Verlinde's derivation of Tully-Fisher relation

- In Verlinde's emergent gravity, V_M depends on r.

$$\int_0^r \epsilon^2(r')A(r')dr' = V_M(r)$$

$$\epsilon^2(r) = \frac{1}{A(r)}\frac{dV_M(r)}{dr} = \frac{2}{3a_0}\frac{GM}{r^2}$$

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- The sub-Newtonian regime ($\epsilon < 1$)
  g_D: gravity due to "apparent dark matter"

$$\epsilon = \frac{g_D}{a_0/2} \qquad \rightarrow \qquad \left(\frac{g_D}{a_0/2}\right)^2 = \frac{2g_B}{3a_0}$$

$$g_D^2 = \frac{a_0}{6}g_B$$

$$g = g_B + g_D$$

$g^2 = g_B^2 + g_D^2$

- Integration range not up to ∞

$$\int \epsilon^2(r) dV = \int_{V_M}^{\frac{4}{3}\pi r^3} \left(\frac{V_M}{V}\right)^2 dV = -\frac{V_M^2}{V}\Bigg|_{V_M}^{\frac{4}{3}\pi r^3} = V_M - \frac{V_M^2}{\frac{4}{3}\pi r^3}.$$

- An extra term proportional to V_M^2

$$\epsilon^2(r) = \frac{1}{A(r)} \frac{9V_M^2}{4\pi r^4}.$$

$$\epsilon^2(r) = \frac{1}{4\pi r^2}\left(\frac{dV_M(r)}{dr} + \frac{9V_M^2(r)}{4\pi r^4}\right).$$

$$\epsilon = \frac{g}{a_0/2}$$

$$g^2 = \frac{a_0}{6} g_B + g_B^2 = g_D^2 + g_B^2$$

Comparison with binary orbit data

g_B (m/s^2)	$g_{\rm obs}/g_{\rm pred}$	g/g_B
$10^{-10.85}$	$1.74^{+0.49}_{-0.38}$	2.39
$10^{-10.15}$	$1.37^{+0.25}_{-0.21}$	1.39
$10^{-9.45}$	$1.30^{+0.16}_{-0.14}$	1.09
$10^{-8.8}$	$1.05^{+0.12}_{-0.11}$	1.02
$10^{-8.2}$	$1.00^{+0.09}_{-0.08}$	1.01
$10^{-7.65}$	$0.97^{+0.09}_{-0.08}$	1.00
$10^{-7.15}$	$0.98^{+0.10}_{-0.09}$	1.00

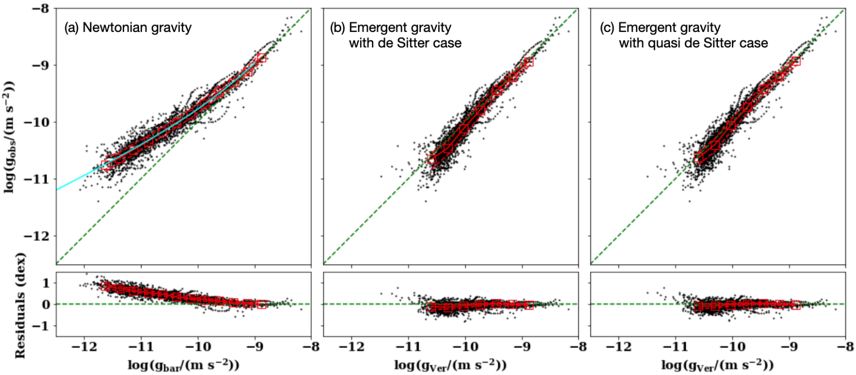
Table: Comparison between $g_{\rm obs}/g_{\rm pred}$ and its theoretical ratio.

Conclusion

- Binary orbit is a good testbed for MOND and Verlinde's emergent gravity as the dark matter cannot significantly affect their orbit, unlike the galaxy rotation curves.
- Binary orbits do not agree with Newtonian gravity but with Verlinde's emergent gravity.
- There is no need to assume the vertical acceleration of our galaxy, as MOND proponents propose.

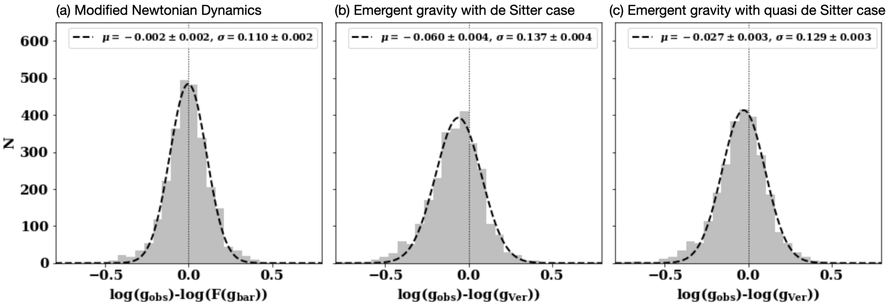
Supplemental Materials

Galaxy rotation curves



Comparison between the predicted accelerations (x-axis) and the observed accelerations (y-axis)

Galaxy rotation curves



Difference between the observed accelerations and the predicted accelerations