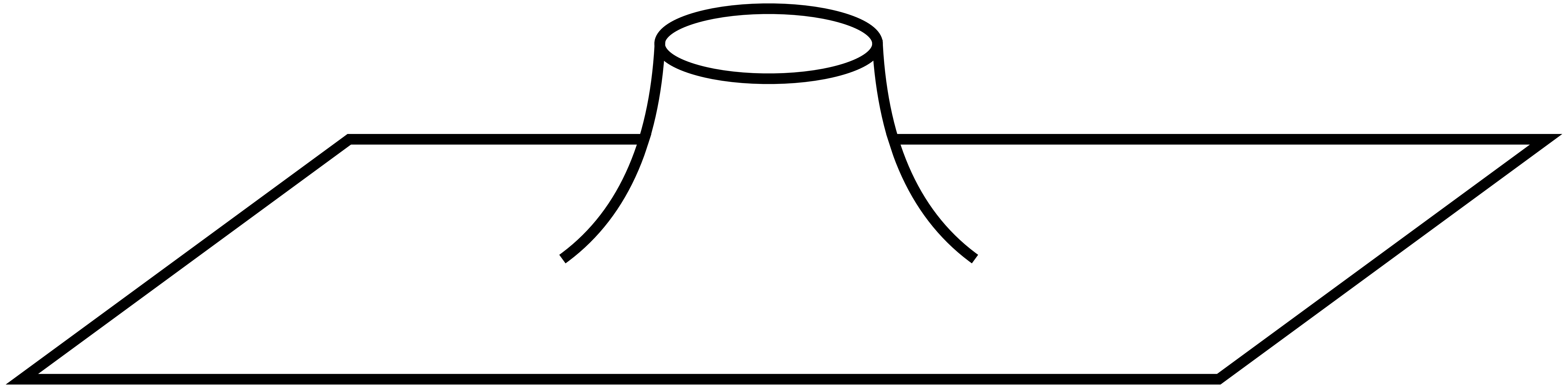
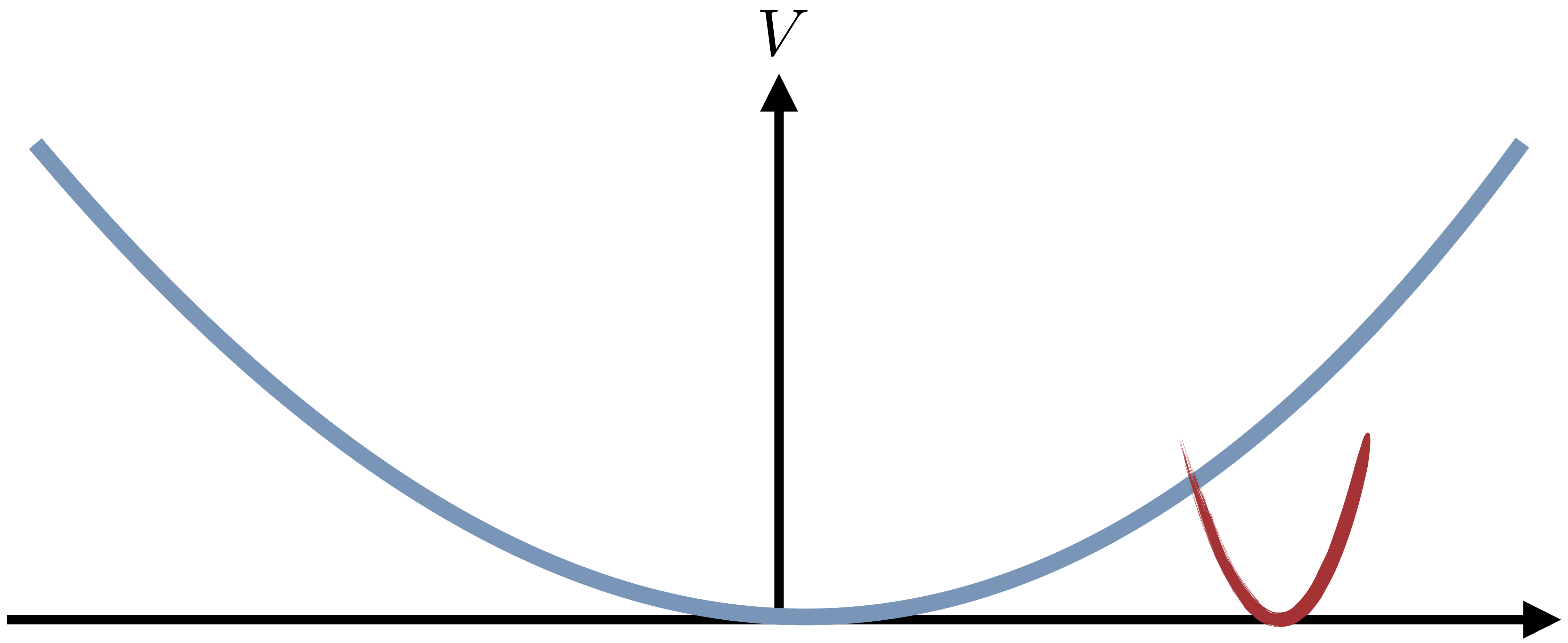
An Effective Approach for Axion Wormholes





Dhong Yeon Cheong (Yonsei University)

(with K. Hamaguchi, Y. Kanazawa, S.M. Lee, N. Nagata, S.C. Park, C.S. Shin)

DYC, K. Hamaguchi, Y. Kanazawa, S.M. Lee, N. Nagata, S.C. Park, [2210.11330]
DYC, S.C. Park, C.S. Shin [2310.11260]

Workshop on Dark Universe
Jan. 18th. 2024

연세대학교
YONSEI UNIVERSITY

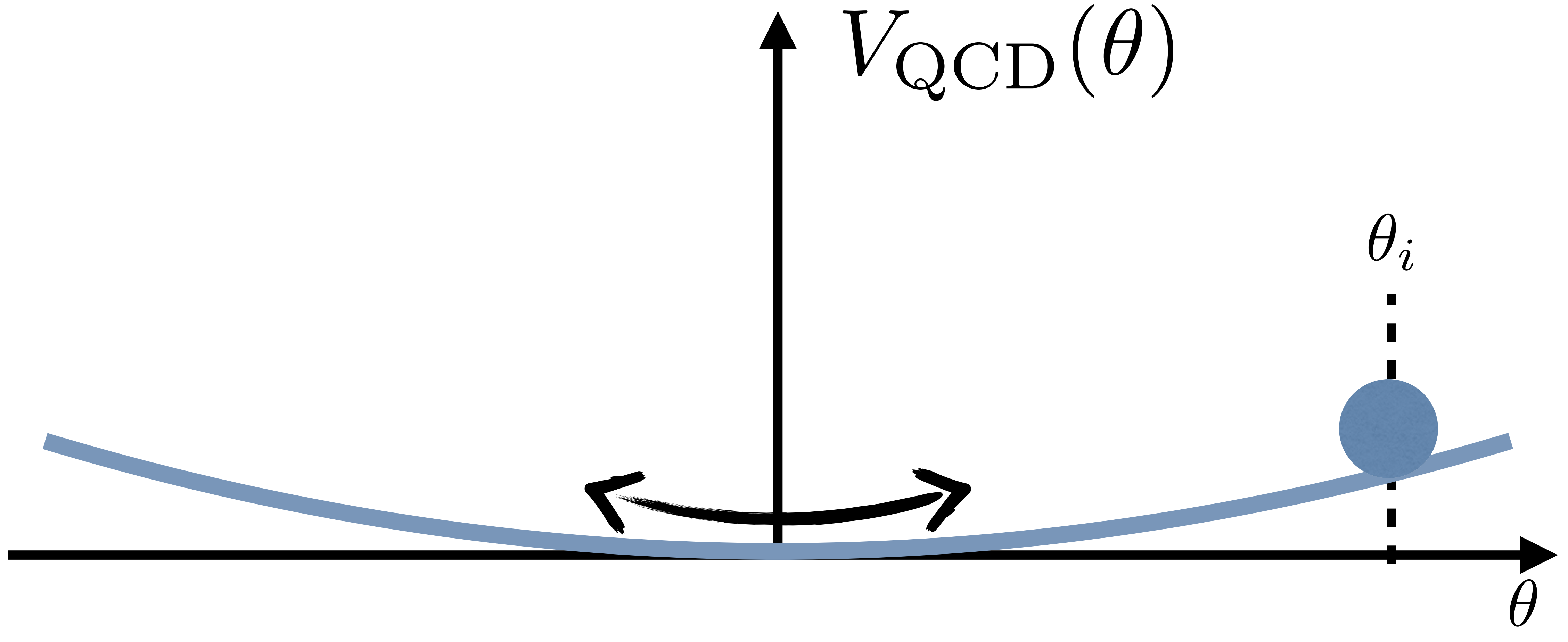
Motivation : Axions

(QCD) Axions : *"Solution for the Strong CP Problem"*

[R. D. Peccei, H. R. Quinn, (1977)]
[F. Wilczek, (1978)]
[S. Weinberg, (1978)]
[J. E. Kim, (1979)] ...

$$\boxed{\mathcal{L} \supset \left(\theta_0 + \frac{a}{f_a}\right) \frac{g_s^2}{32\pi^2} G\tilde{G}}$$

Axions : *"Dark Matter Candidate"*



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Predictions rely on contributions to the axion potential

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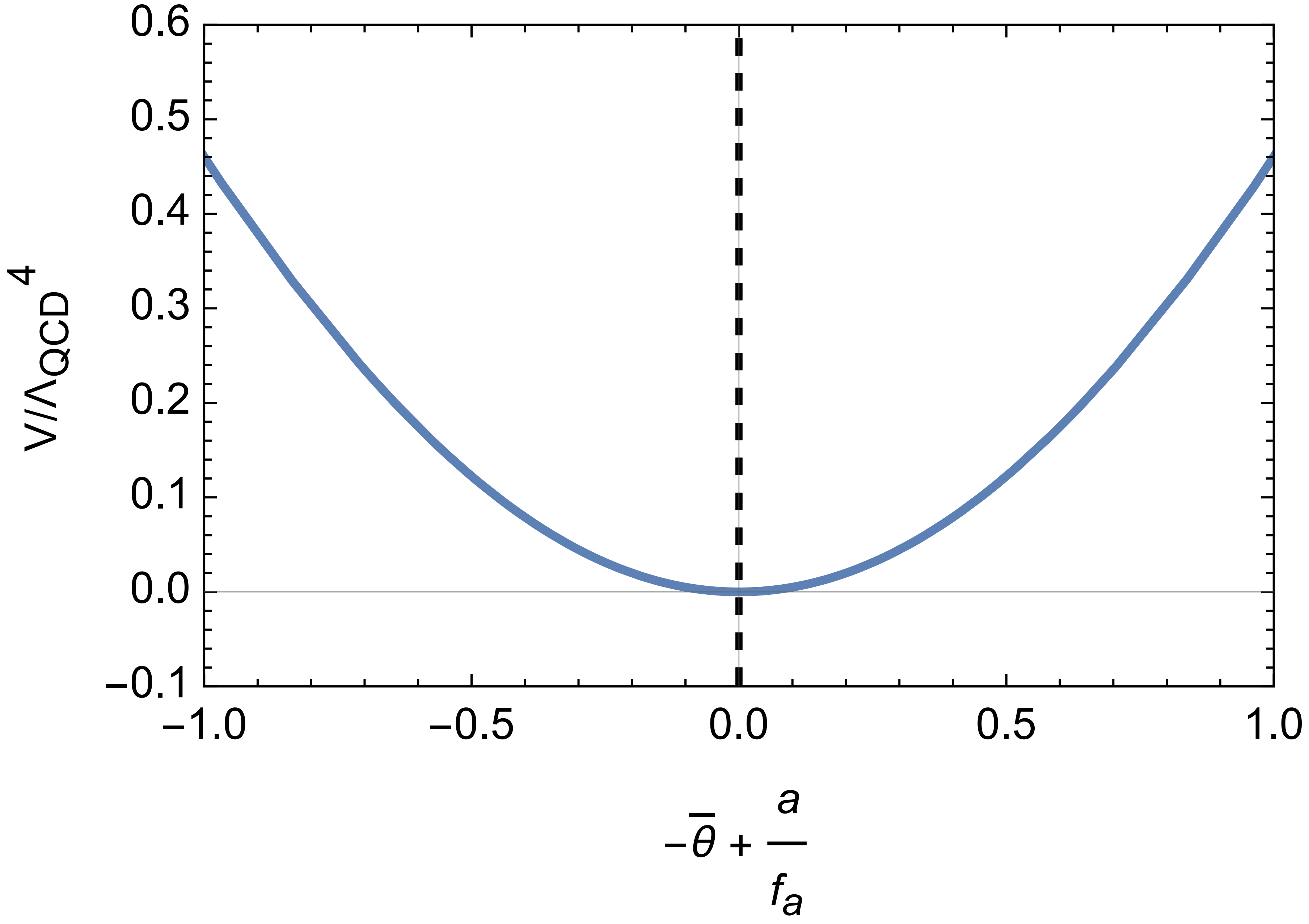

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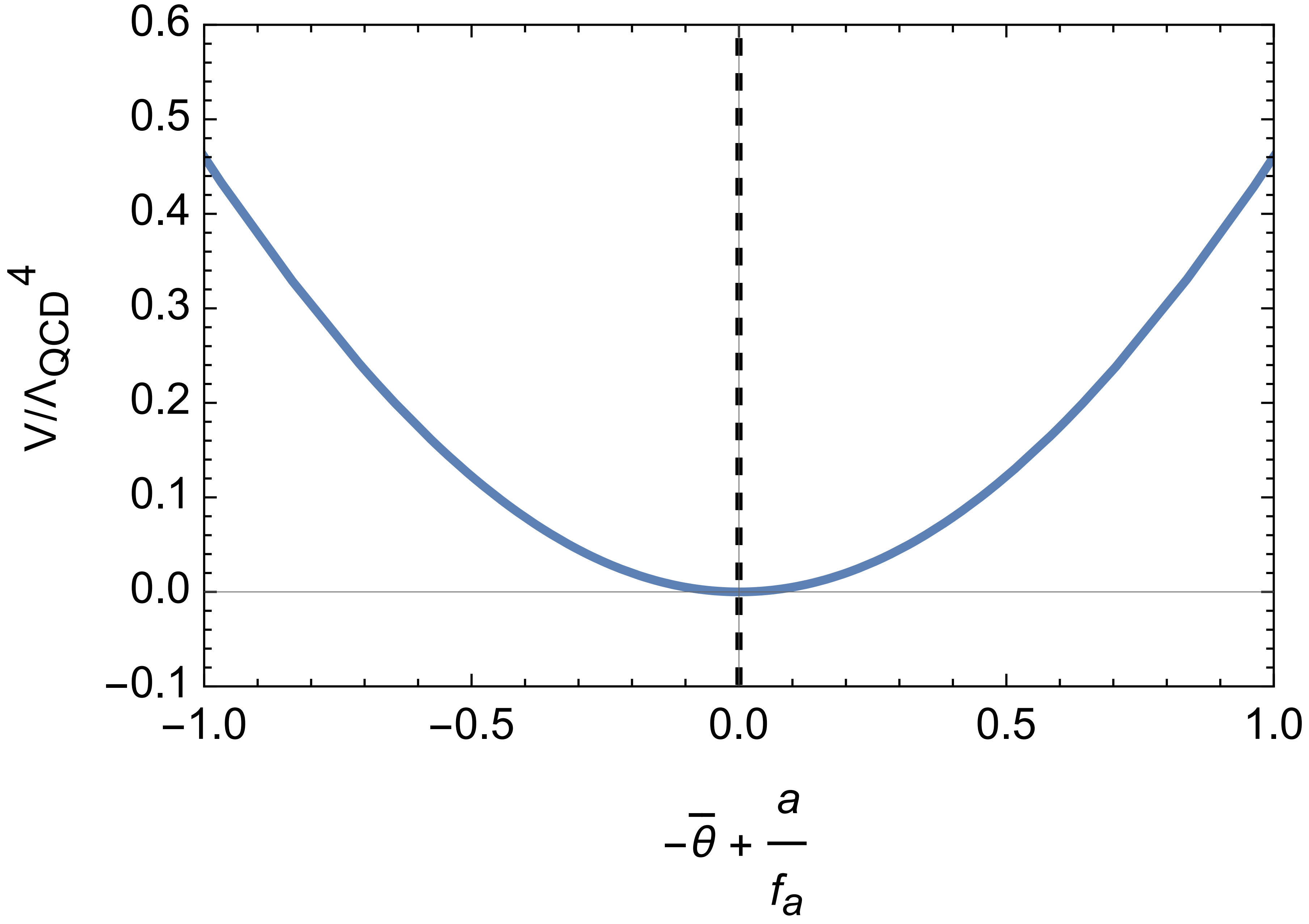

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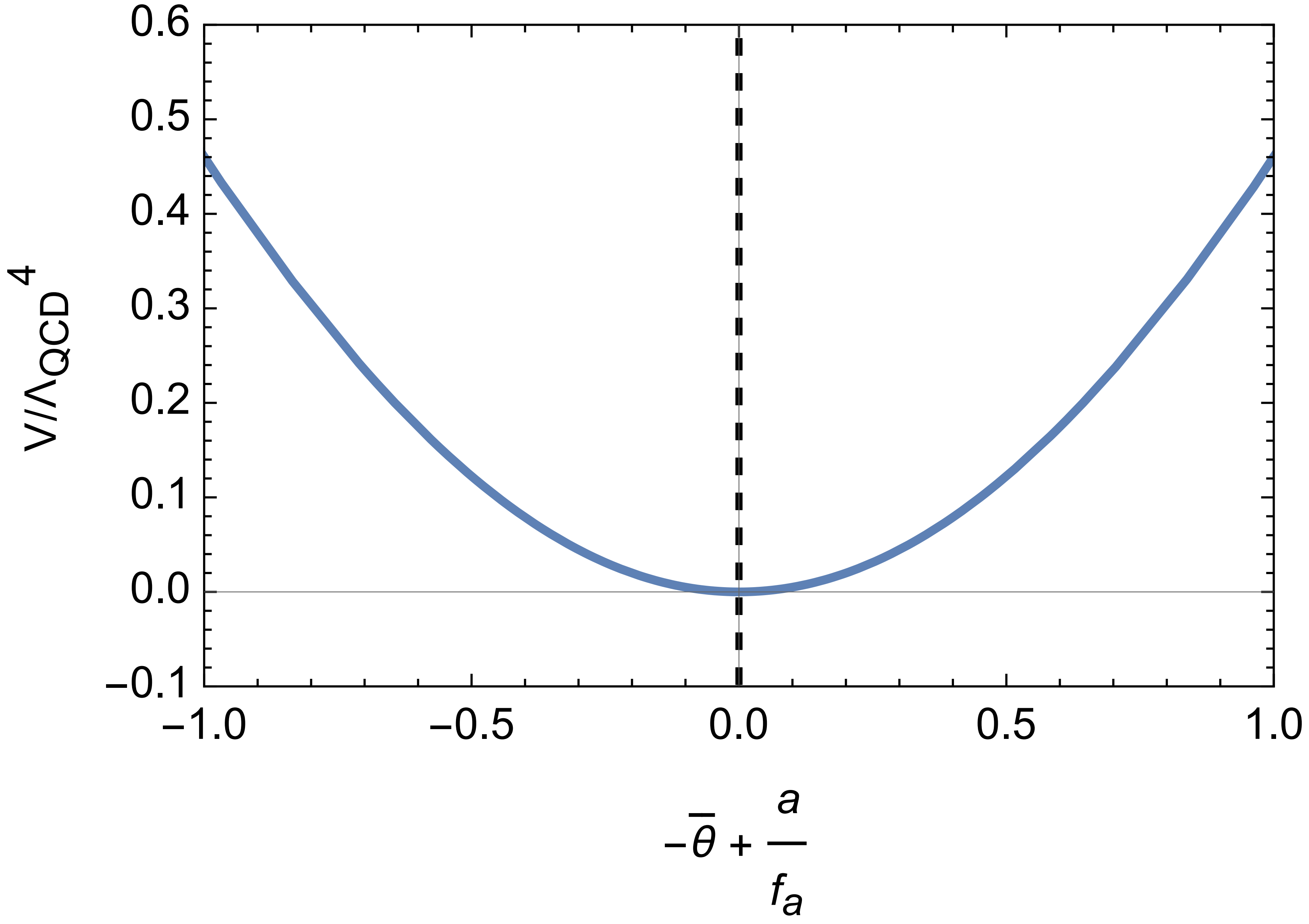

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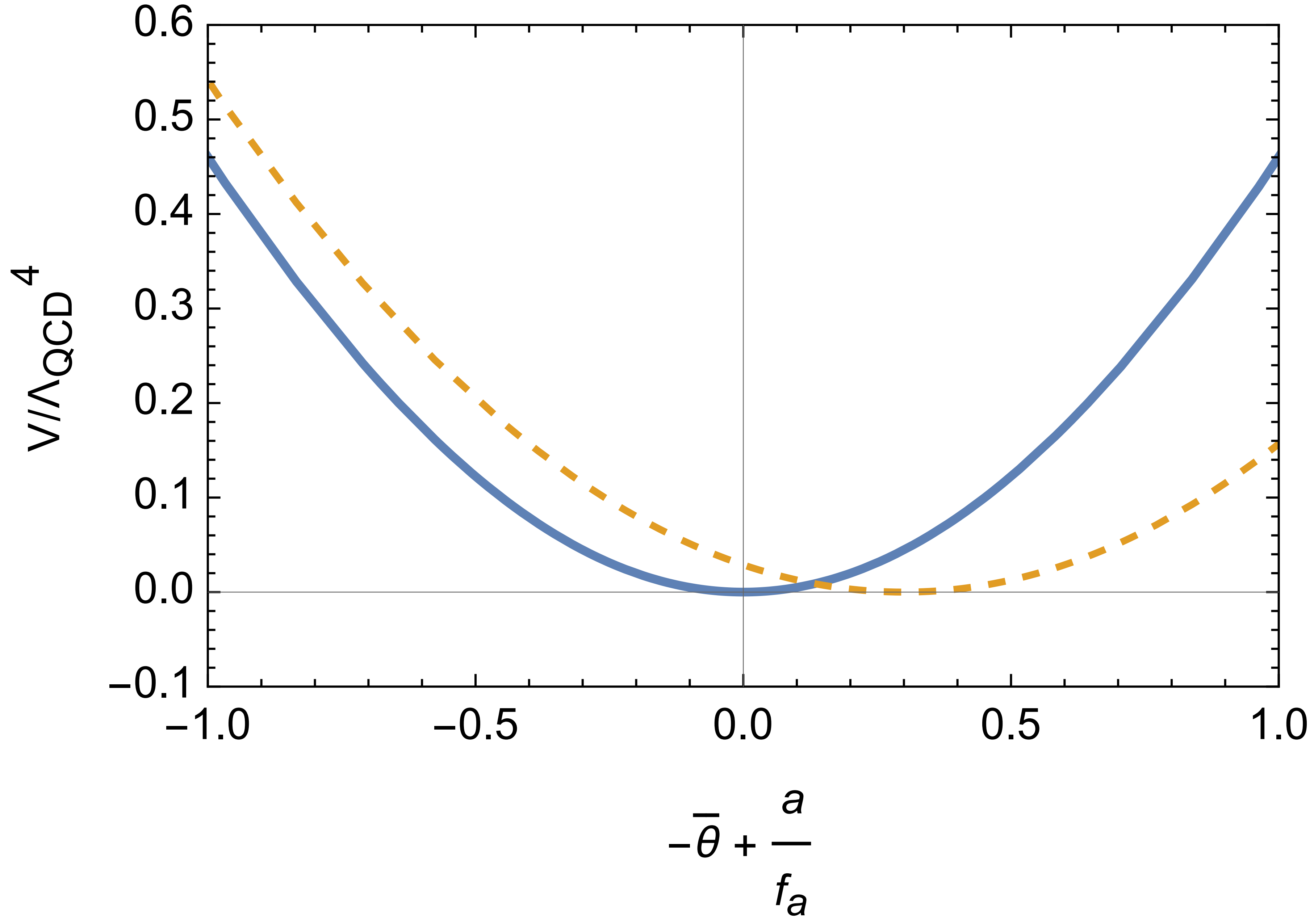

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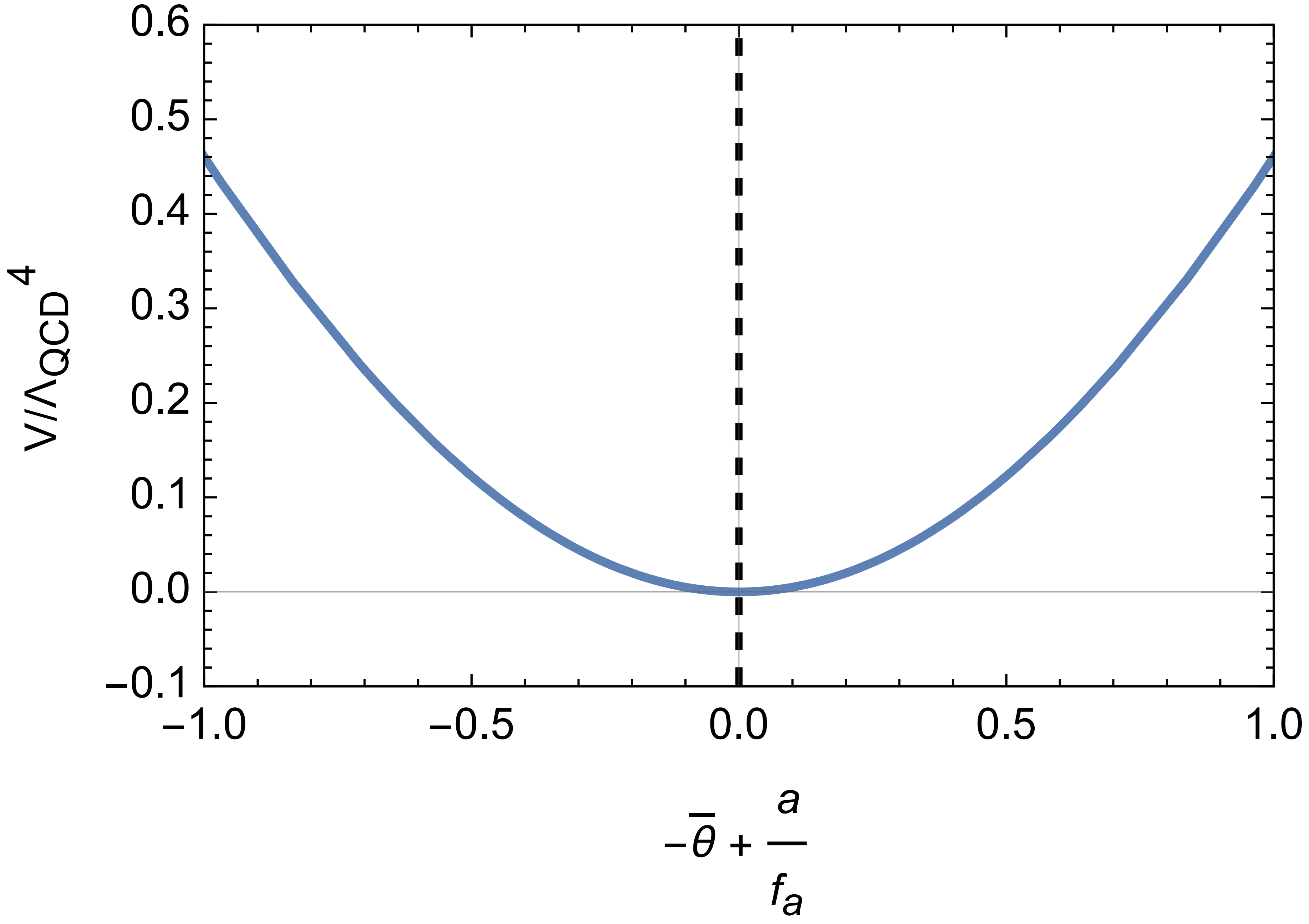

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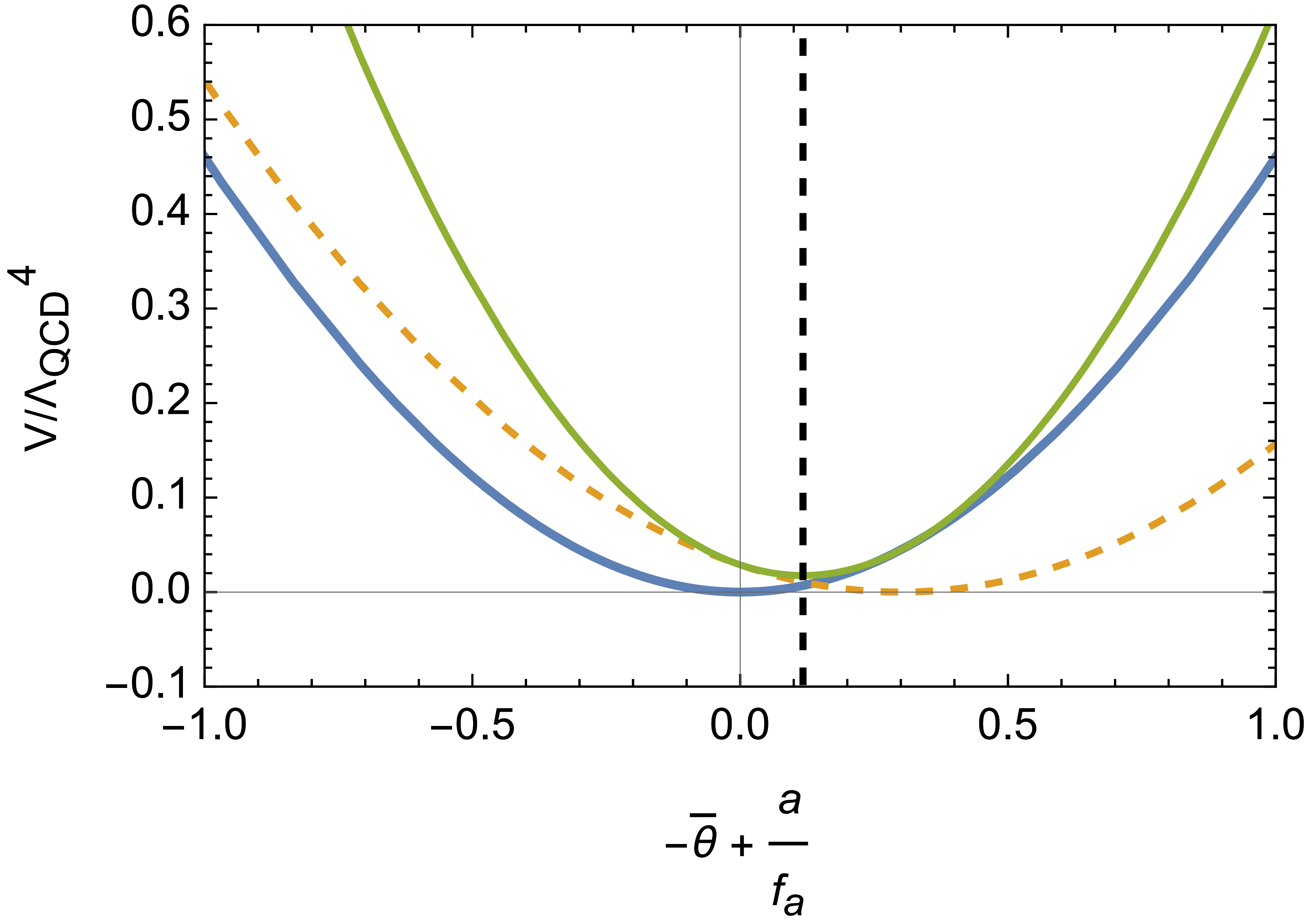

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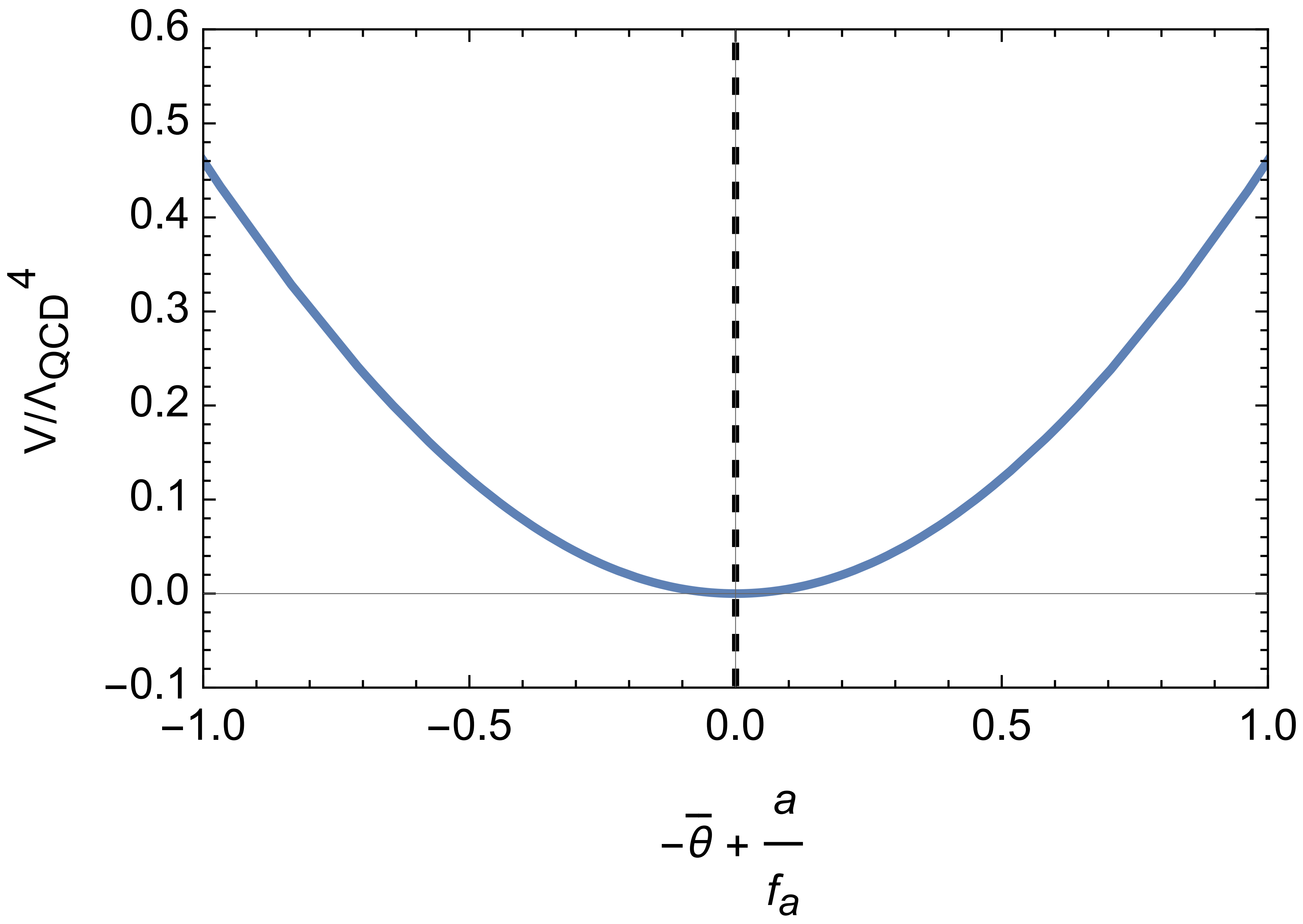

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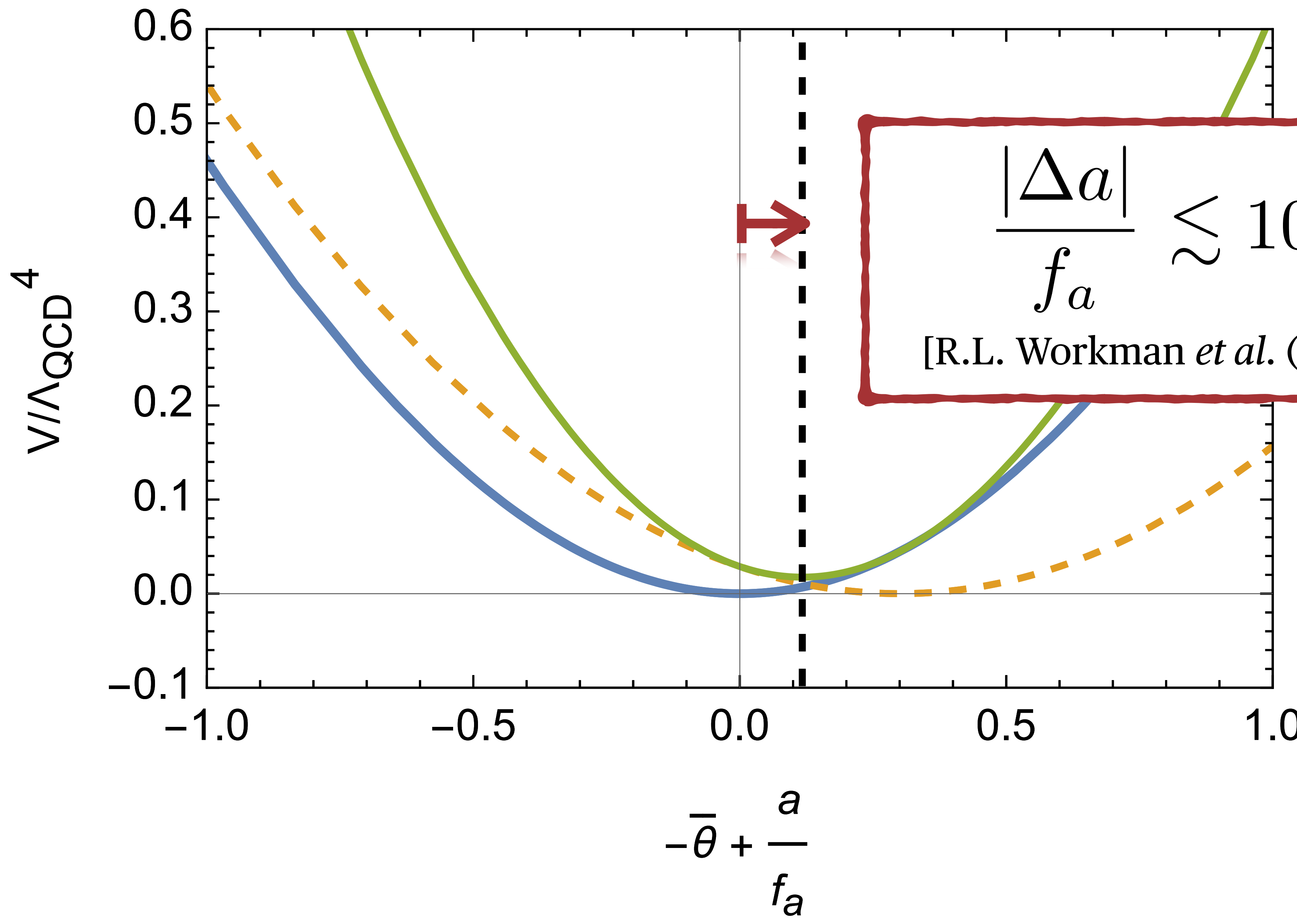

$$\frac{|\Delta a|}{f_a} \lesssim 10^{-10}$$

[R.L. Workman *et al.* (PDG) (2022)]

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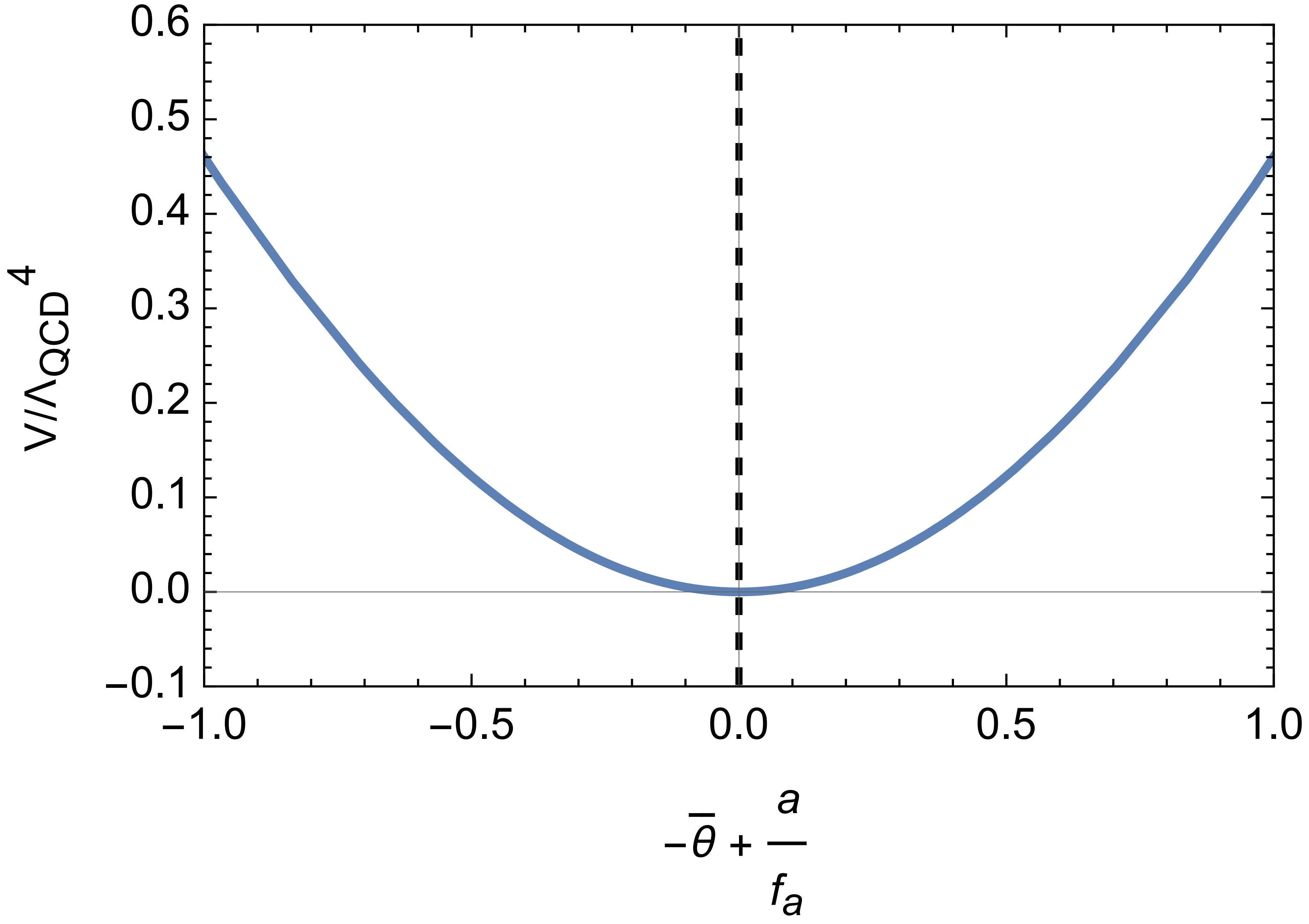

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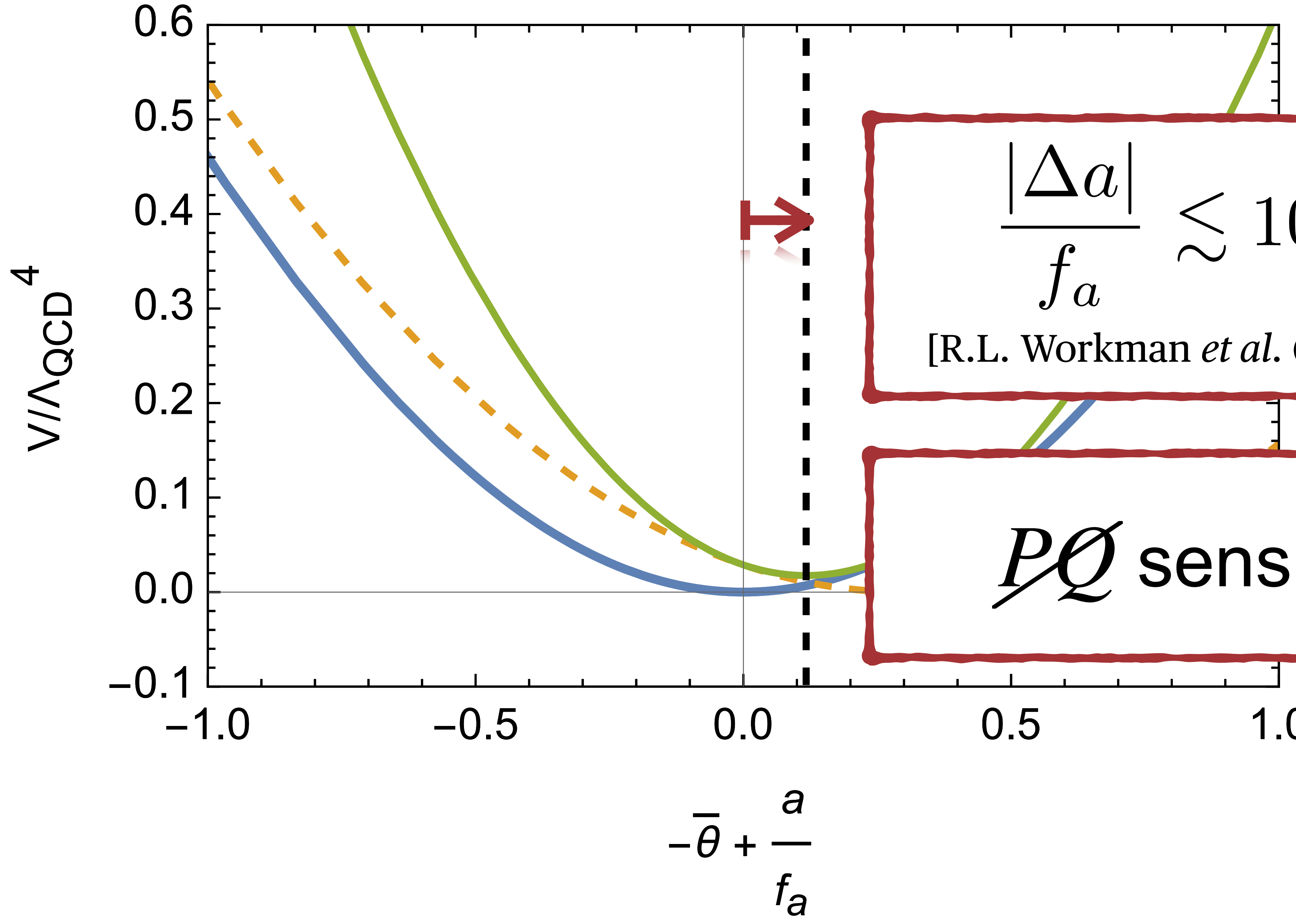

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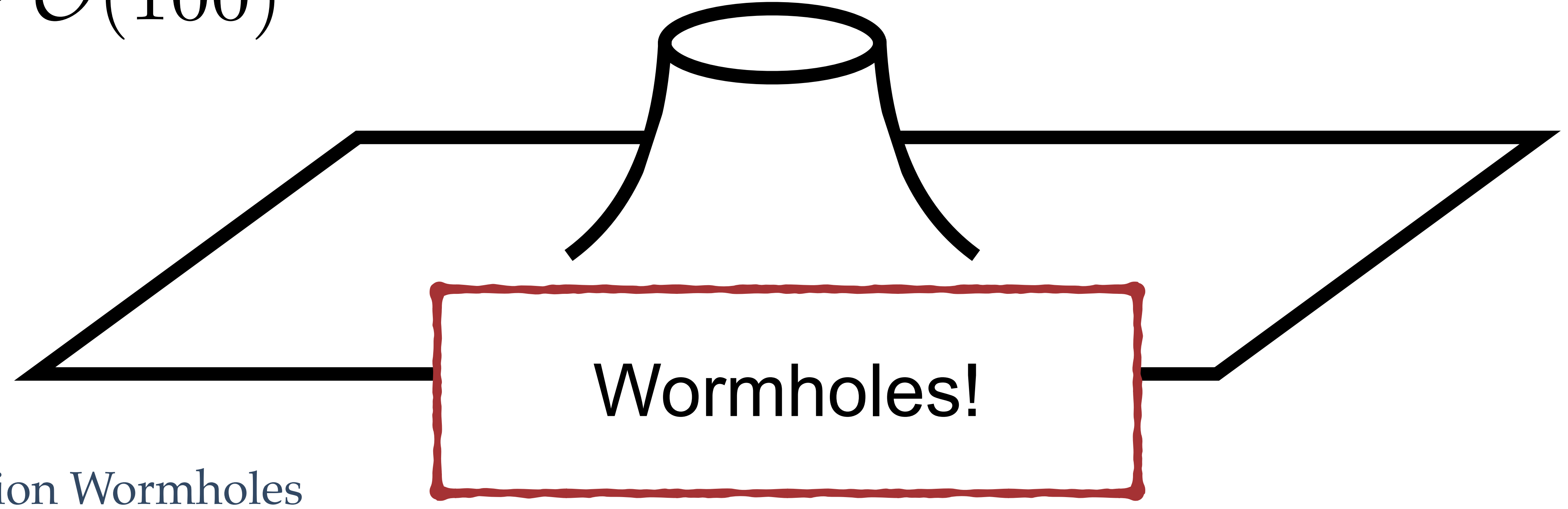

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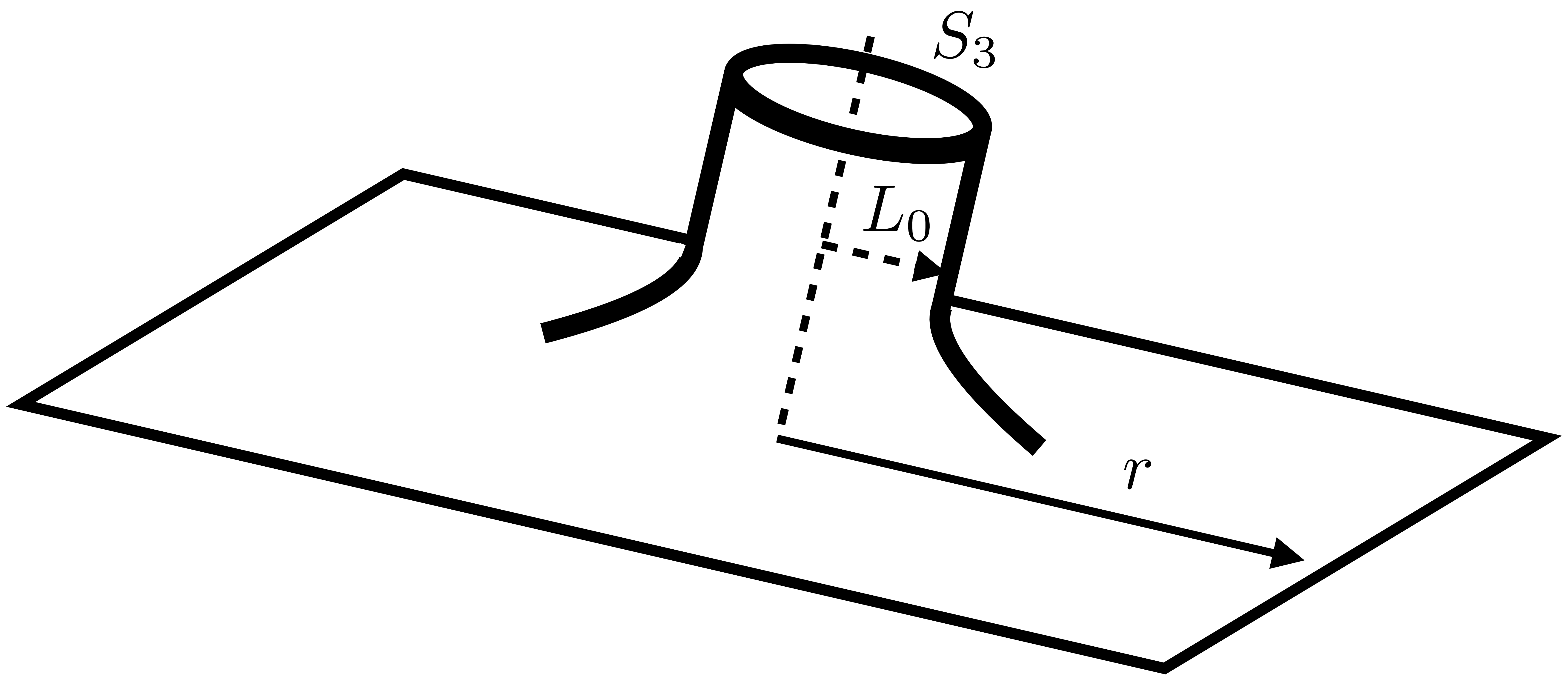

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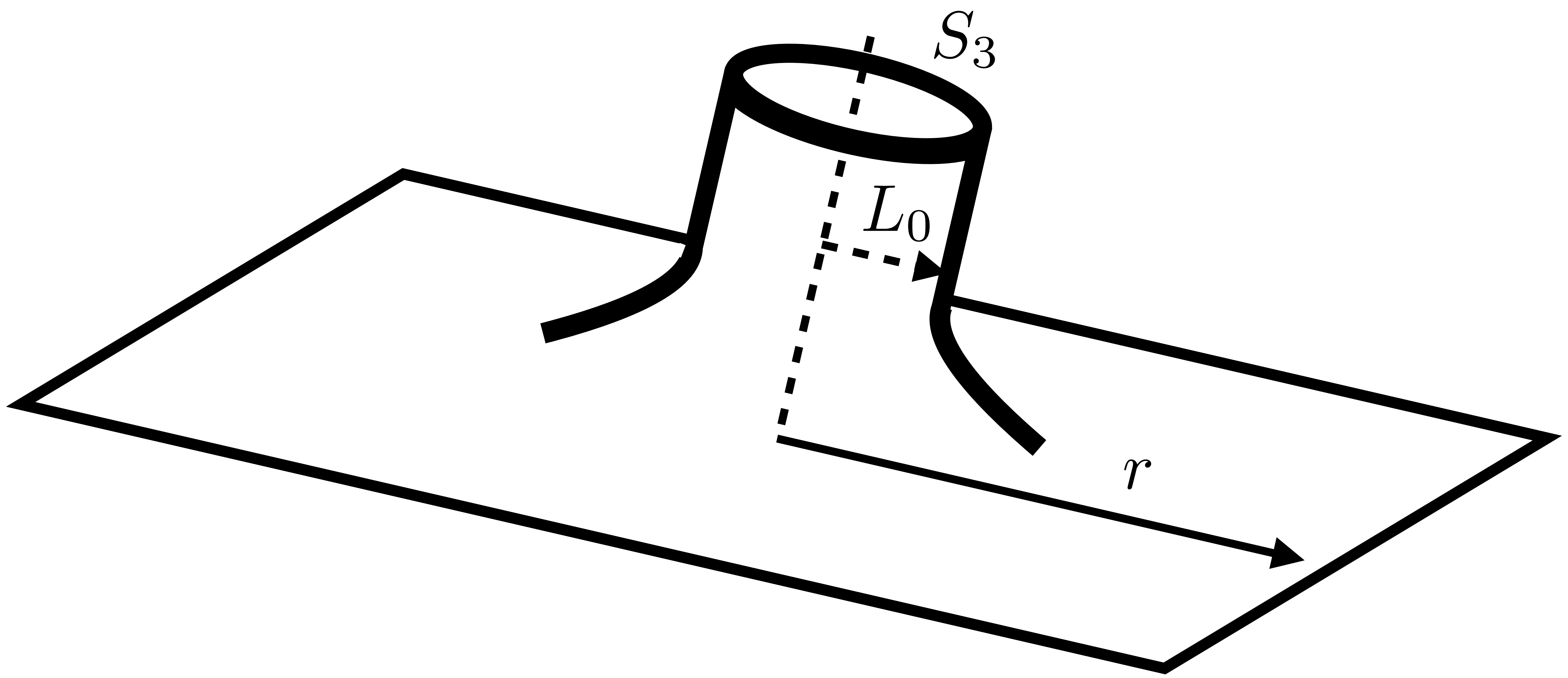

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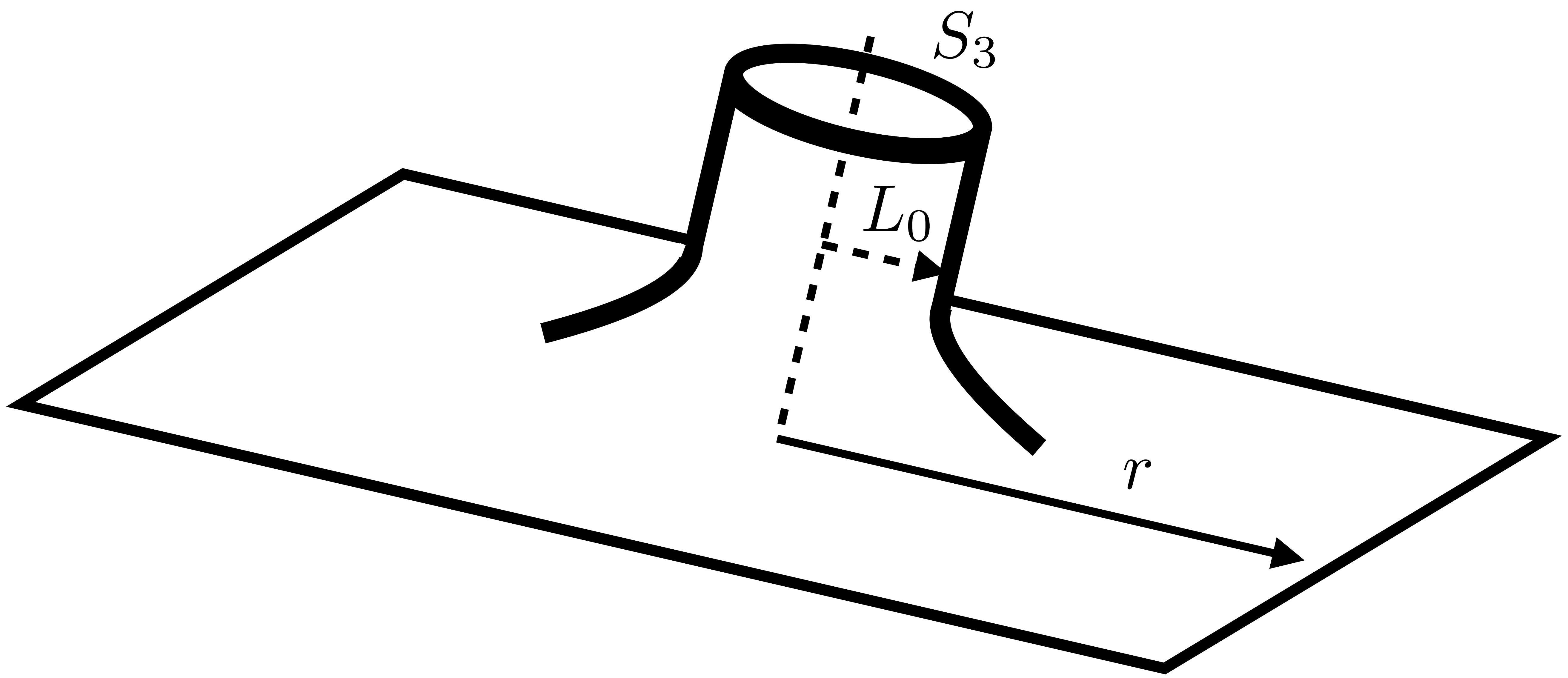

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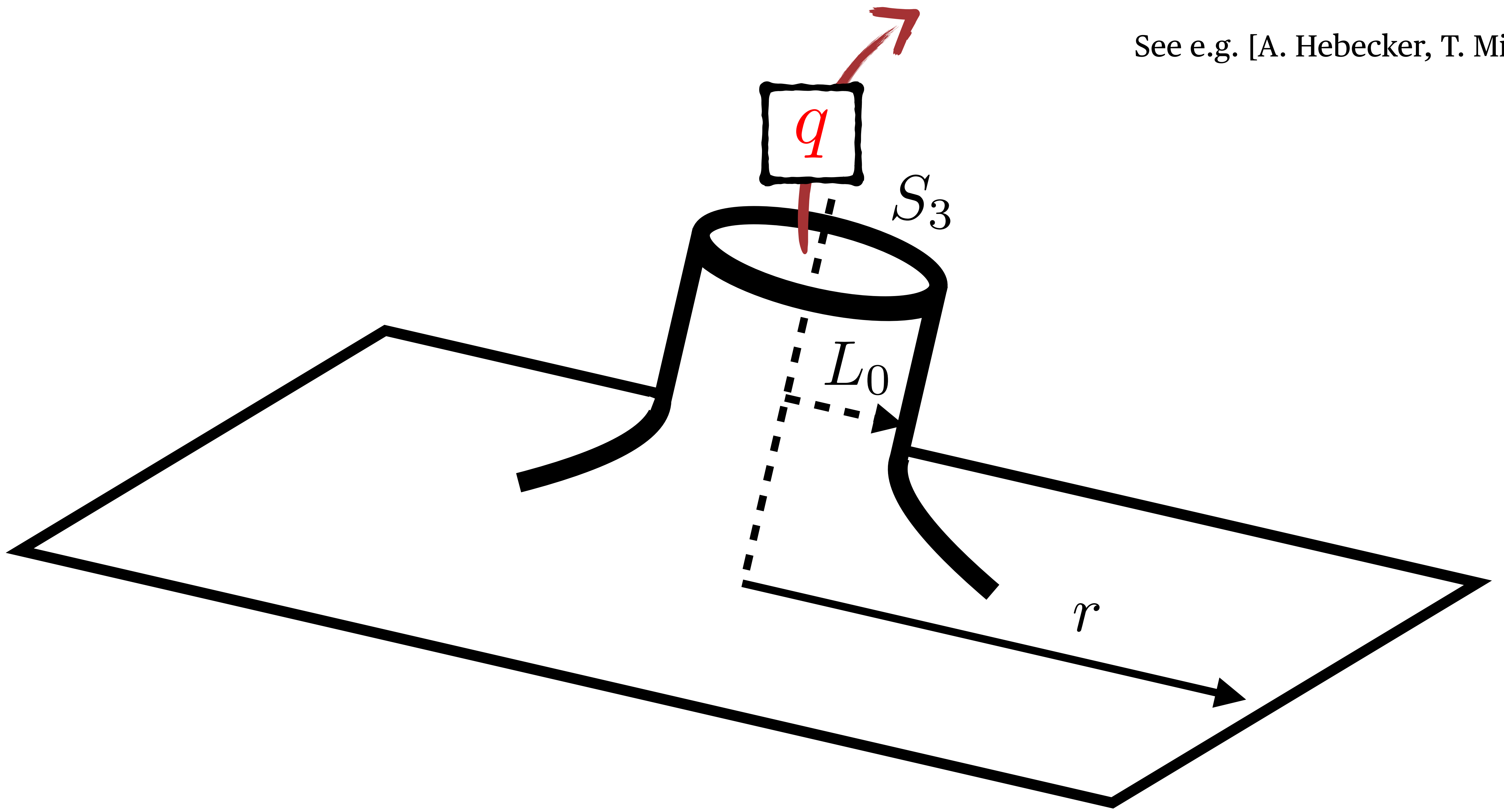

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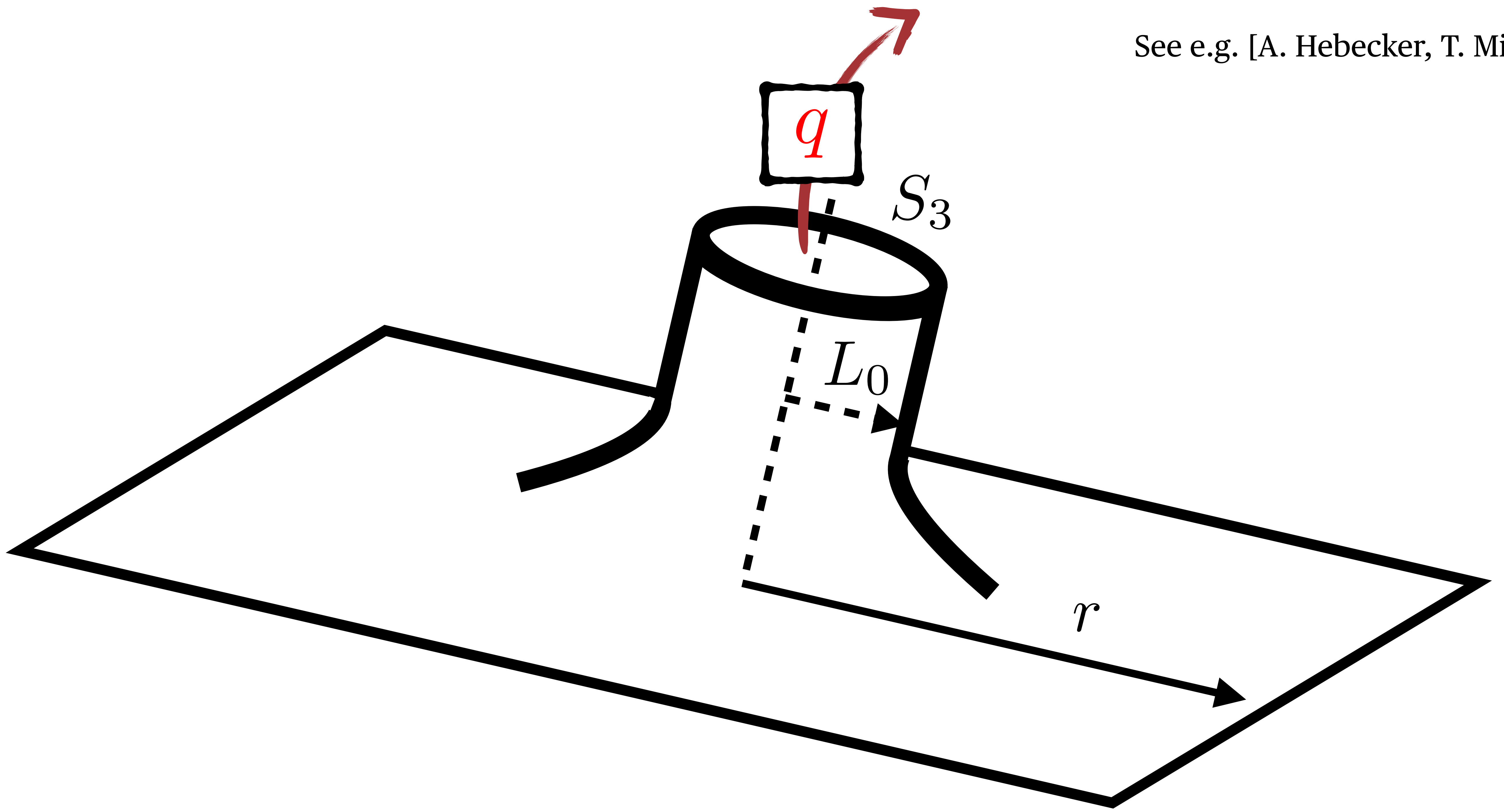

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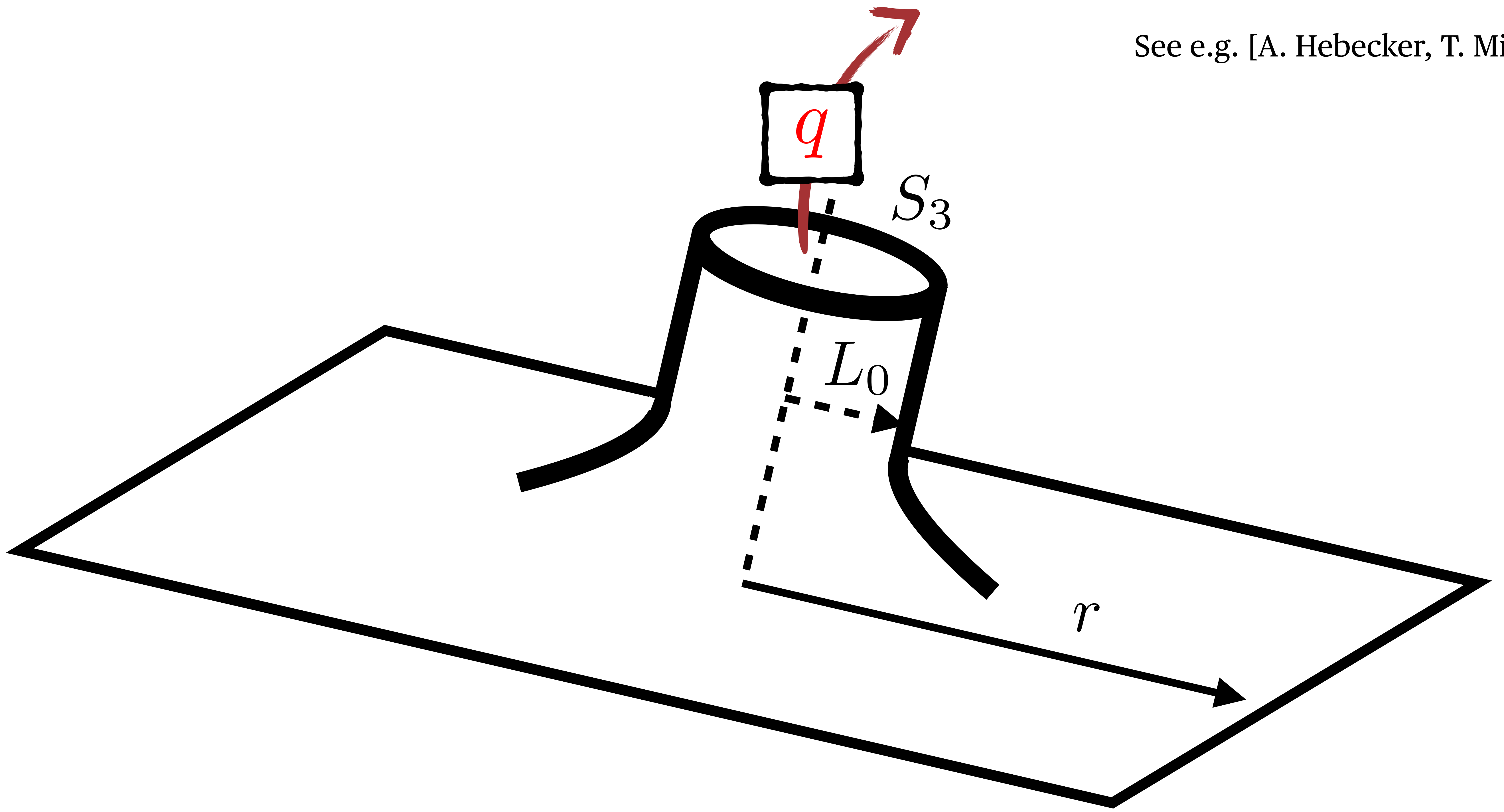

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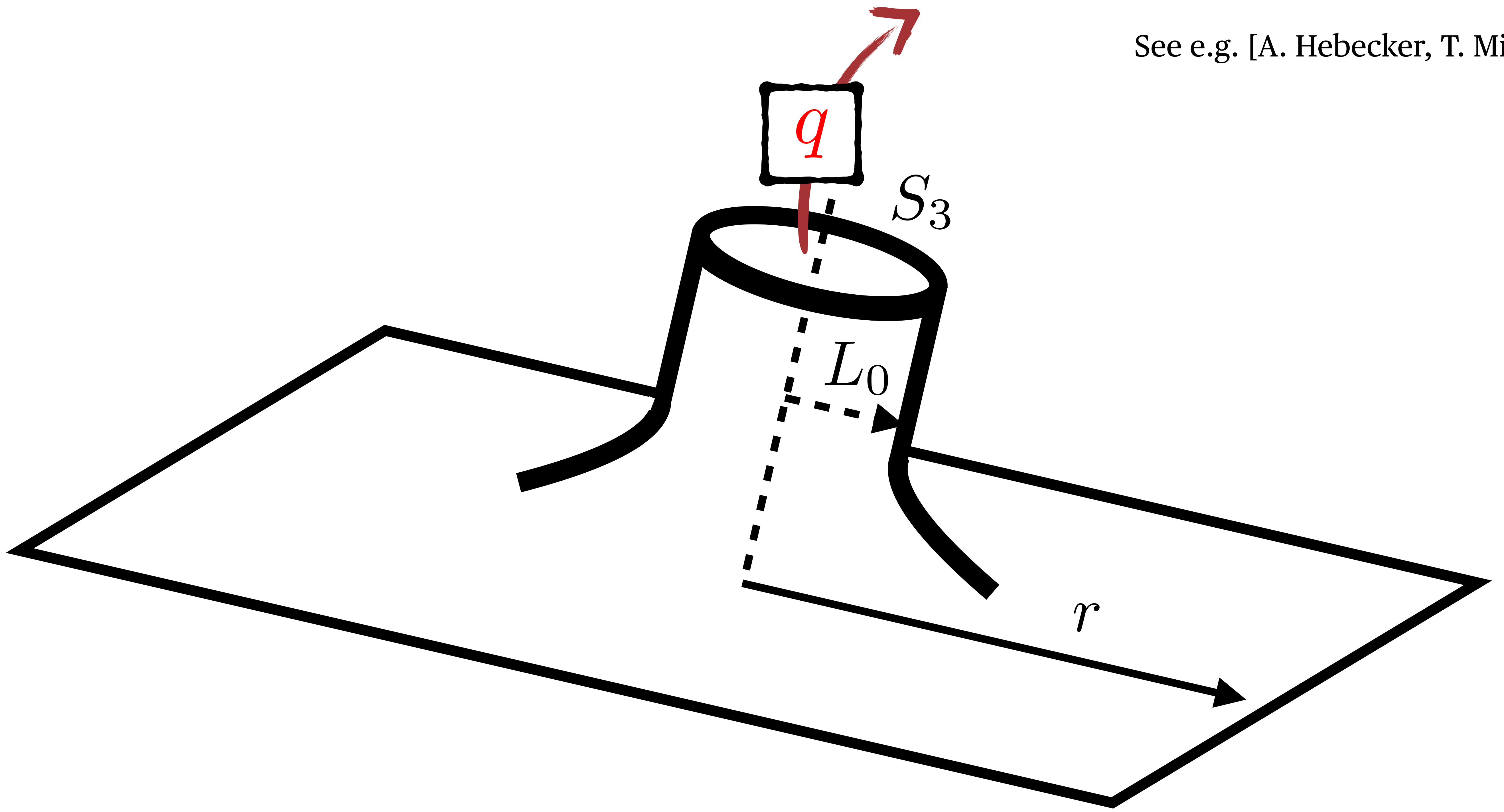

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Instanton contributions : $\exp(-\mathcal{S}) = \exp(-\mathcal{S}_{\rm wh} + i n_I \theta)$

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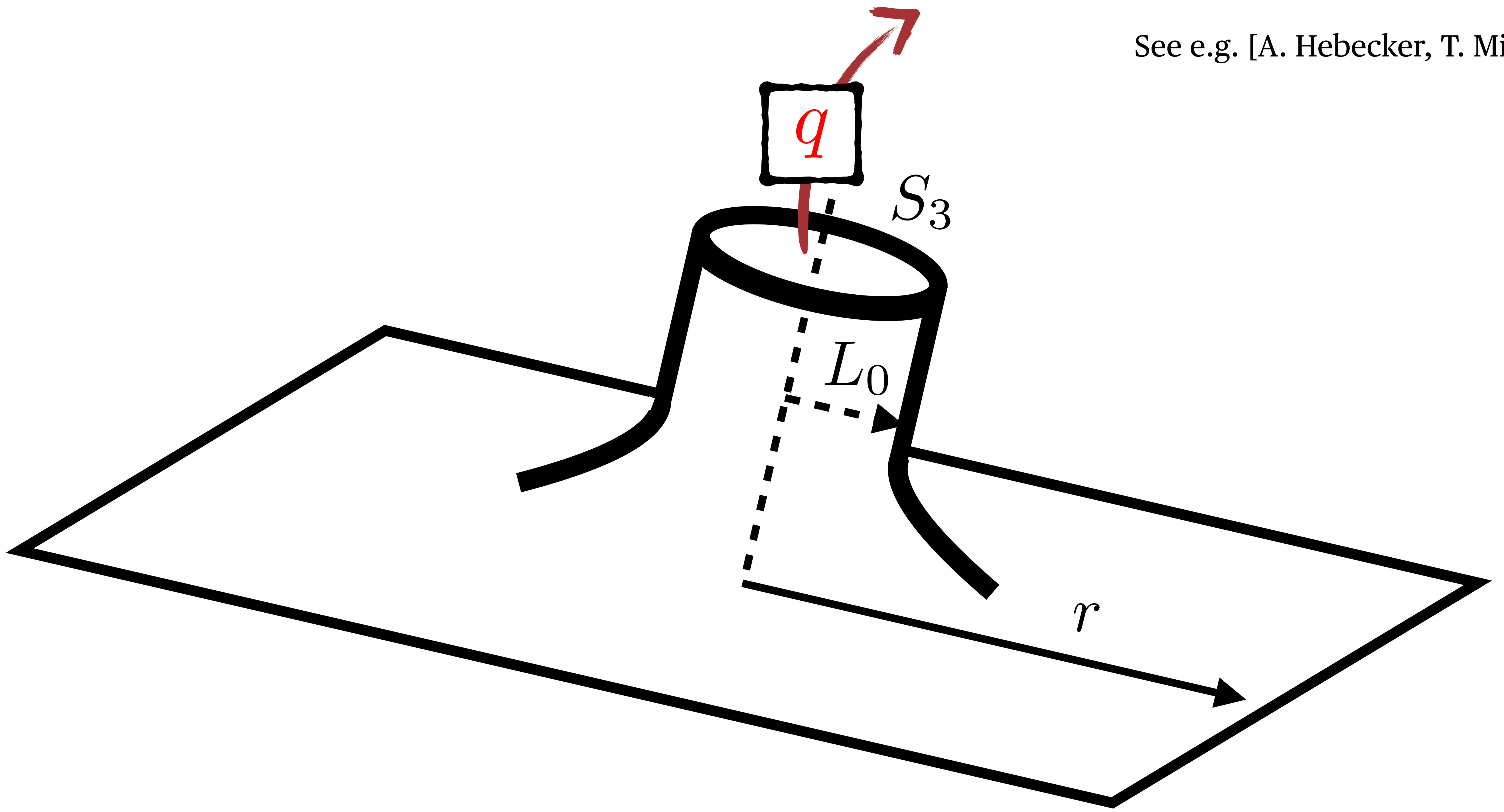

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Instanton contributions : $\exp\left(-\mathcal{S}\right) = \exp(-\mathcal{S}_{\rm wh} + i n_I \theta) \longrightarrow \Delta V^{\rm wh}_{\cancel{\rm PQ}} \simeq \frac{1}{L_0^4}\exp(-S_{\rm wh})\cos(n_I\theta + \delta)$

Axion Wormholes - Giddings-Strominger-Lee Wormholes

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+ GHY term

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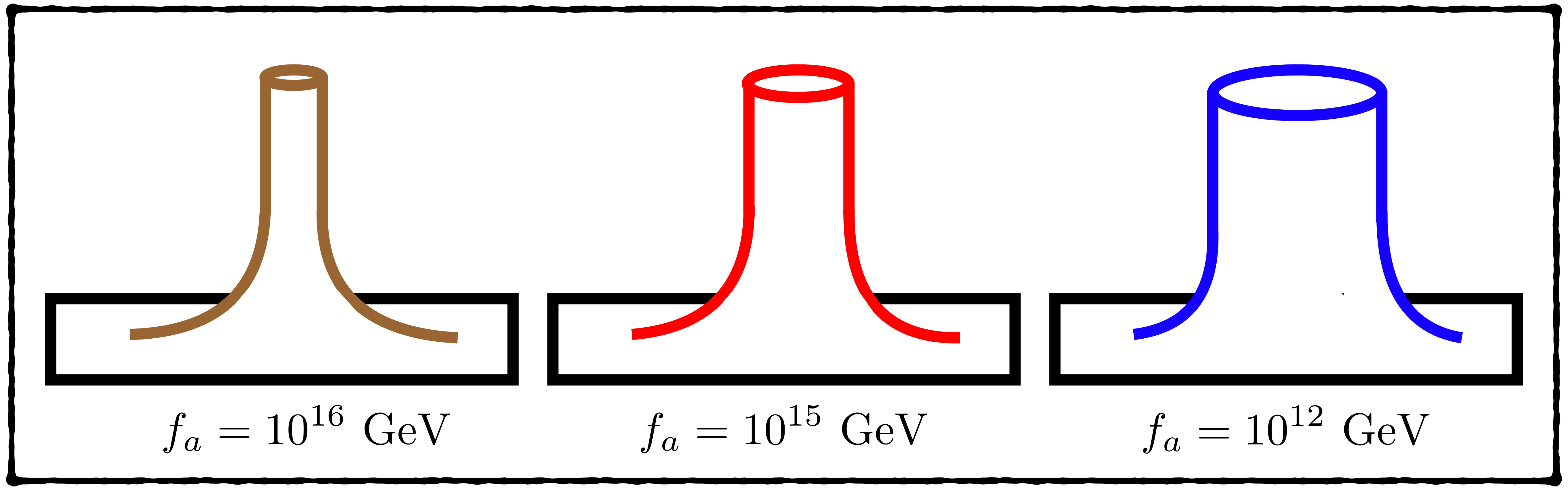

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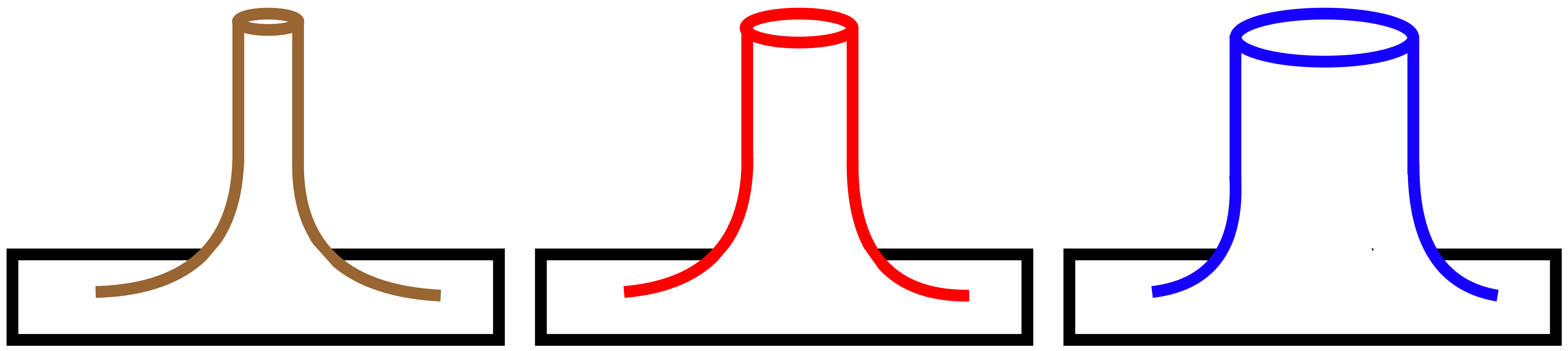

$f_a = 10^{16}$ GeV $\quad f_a = 10^{15}$ GeV $\quad f_a = 10^{12}$ GeV

$\mathcal{S}_{\rm wh} \sim \mathcal{O}(100)$ when $f_a \lesssim 10^{16}$ GeV

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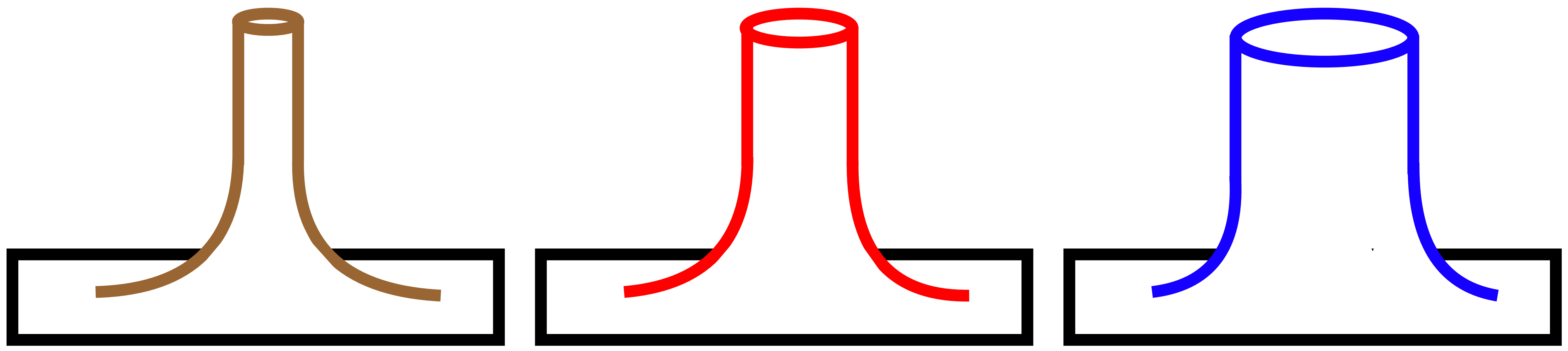

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But, this conclusion will alter heavily with the structure of the theory!

Axion Wormholes - Generalization

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Generic scenarios contain a mixture of scalars and axions

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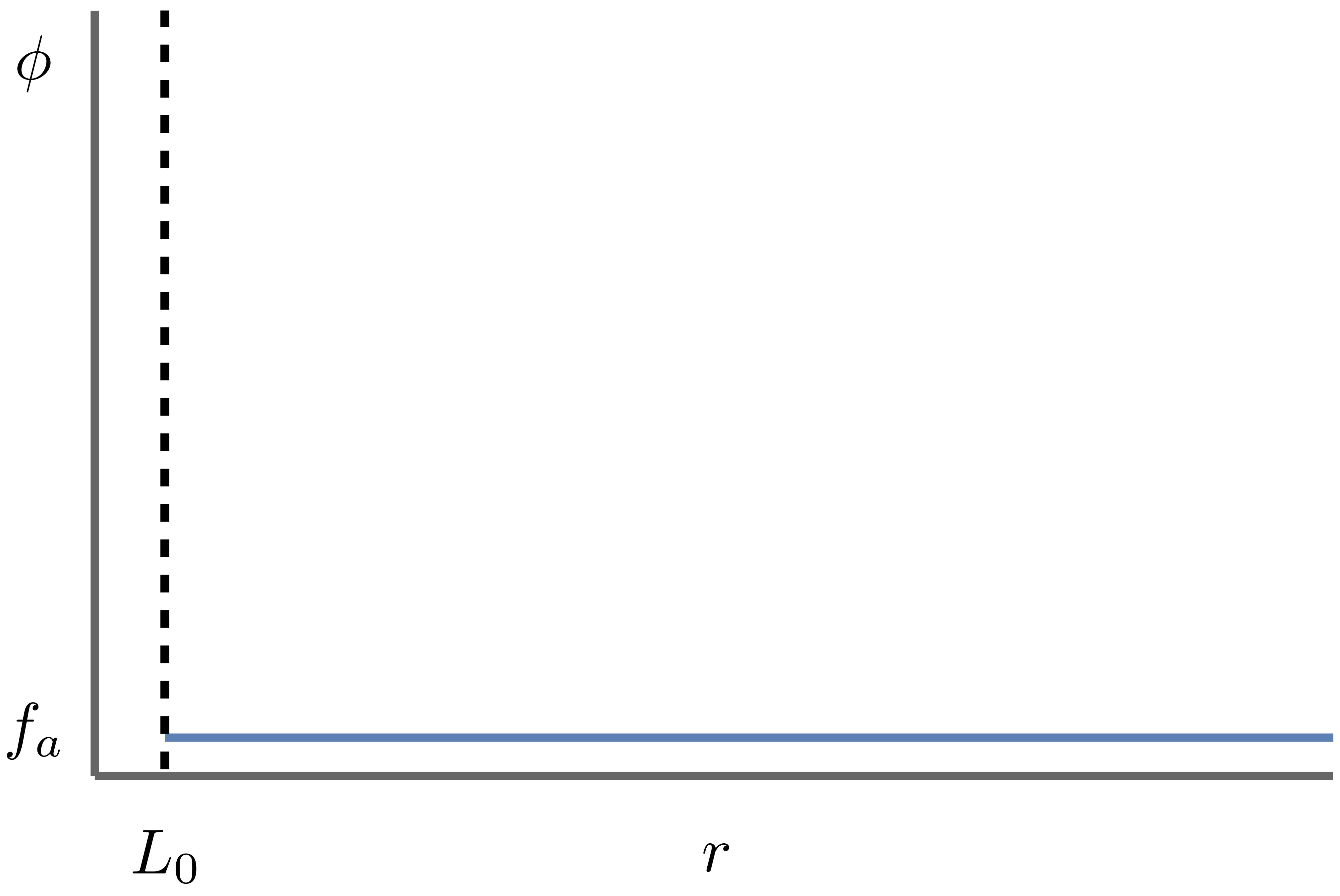

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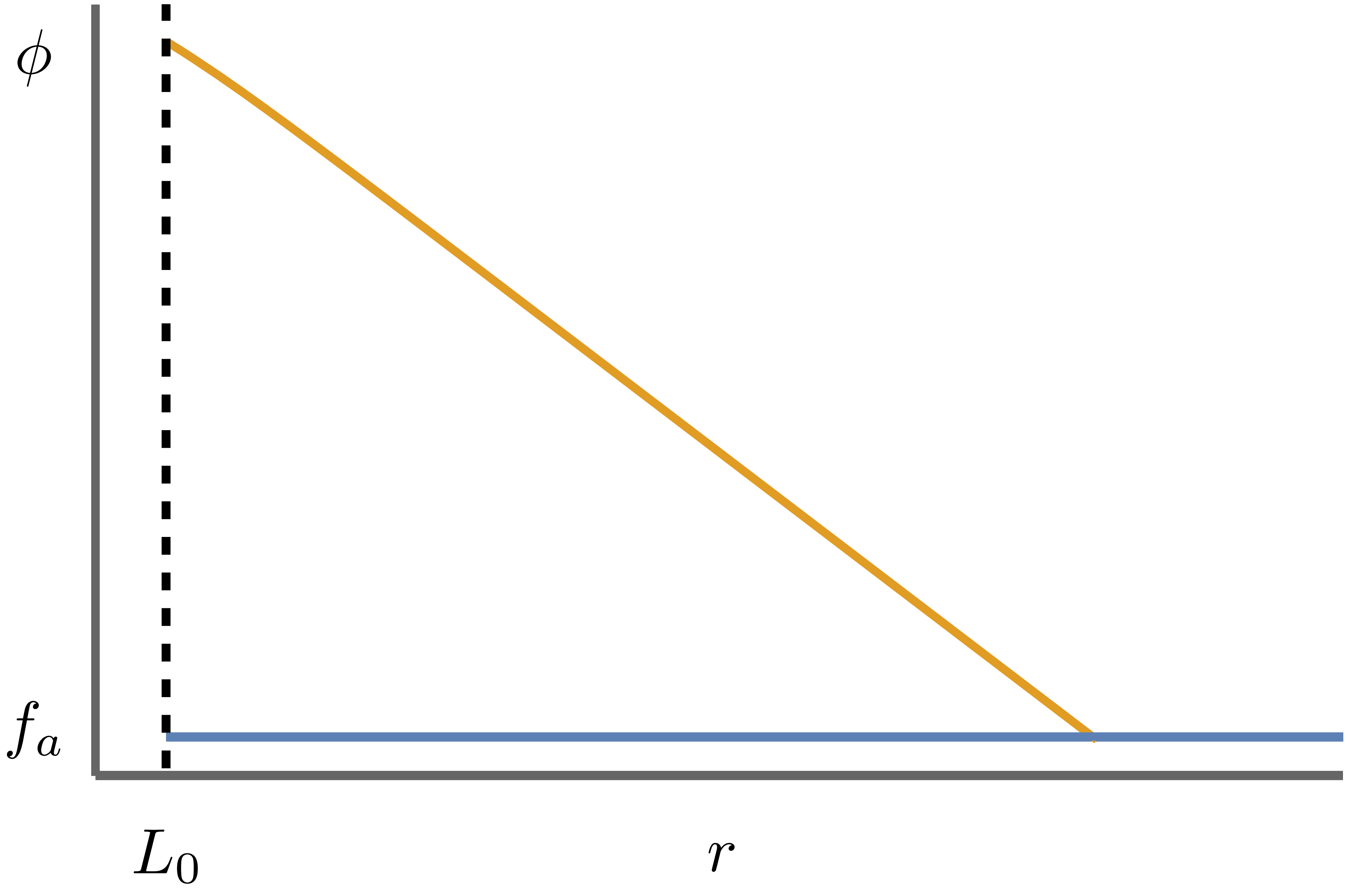

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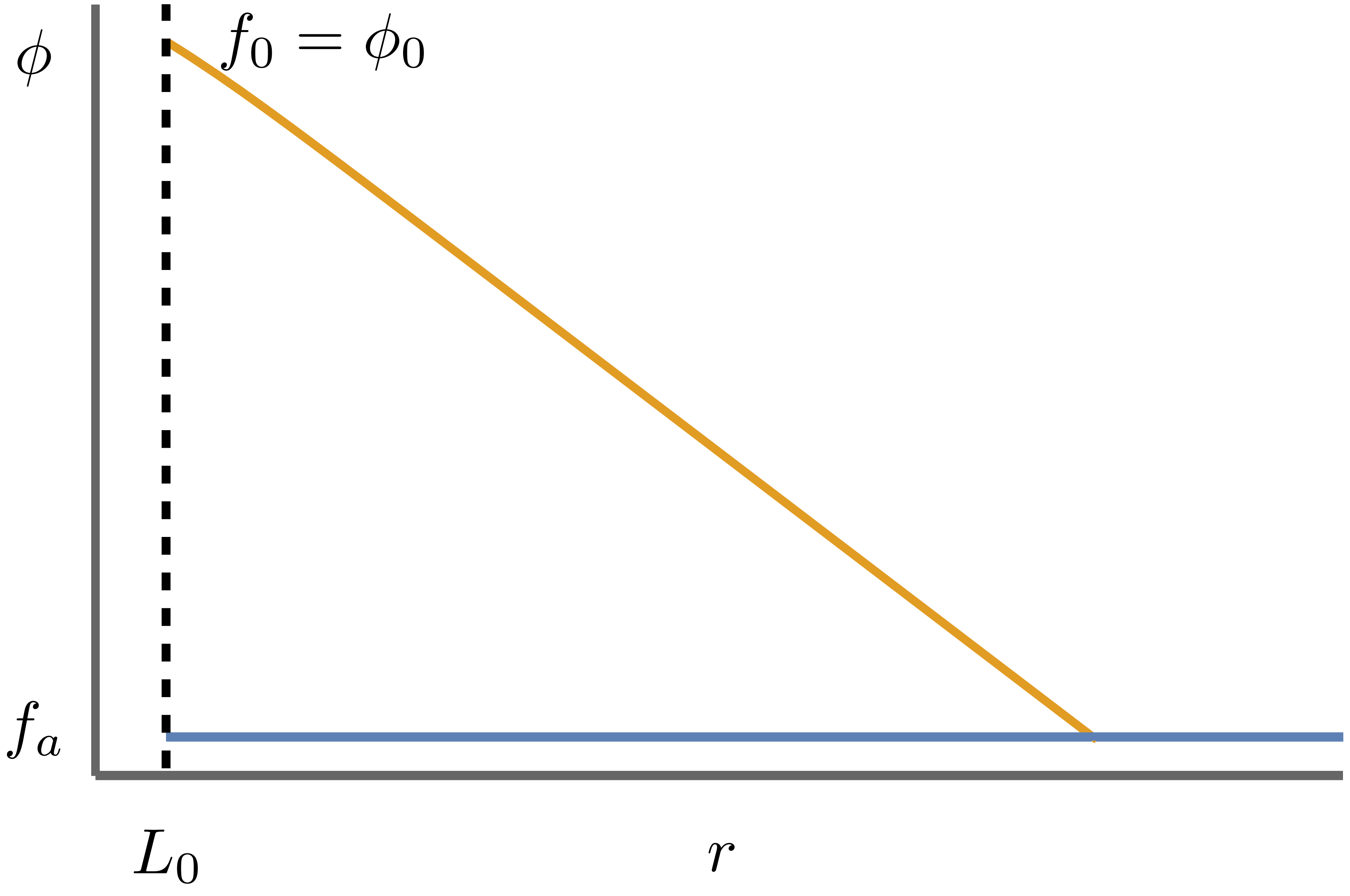

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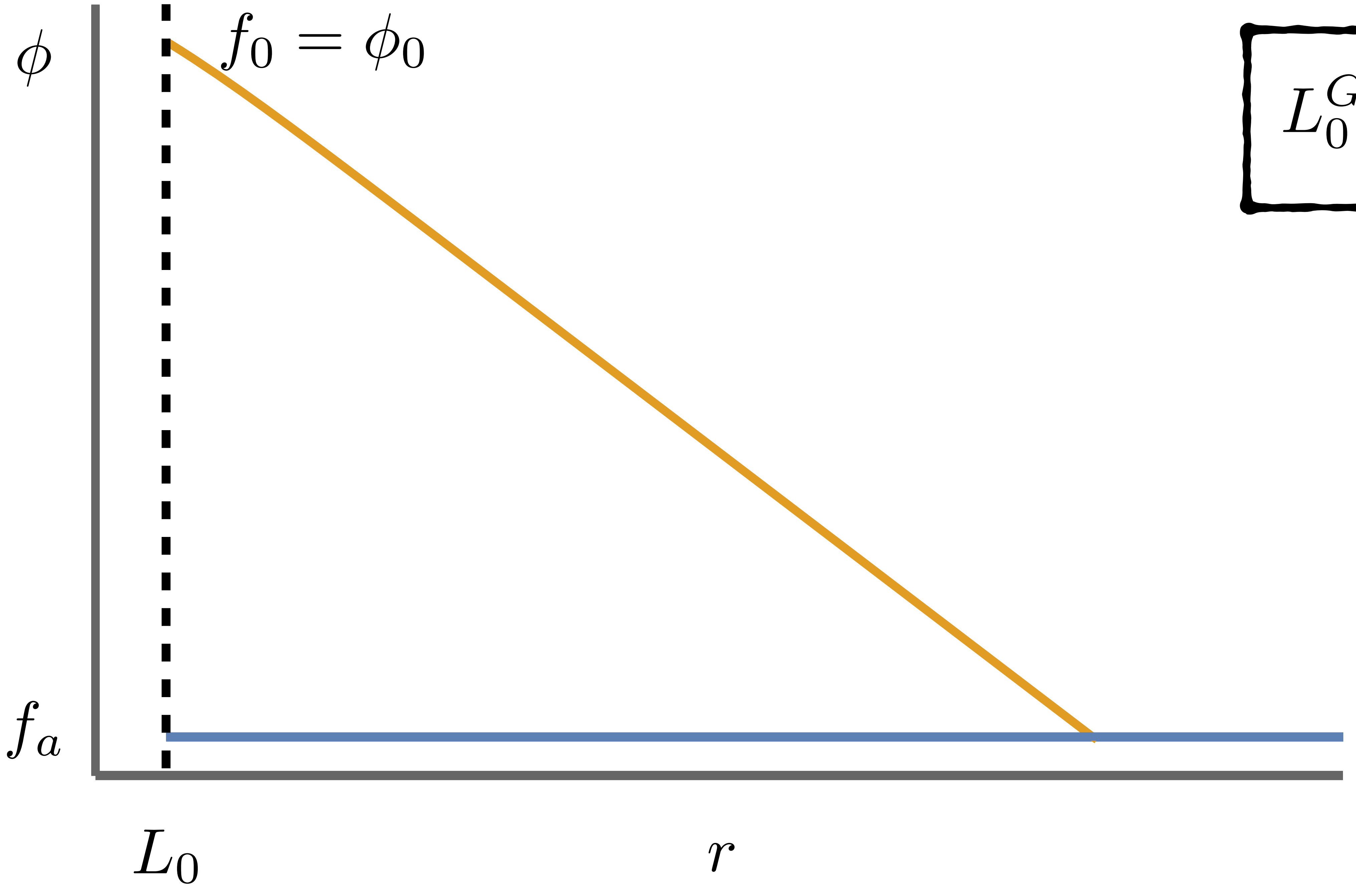
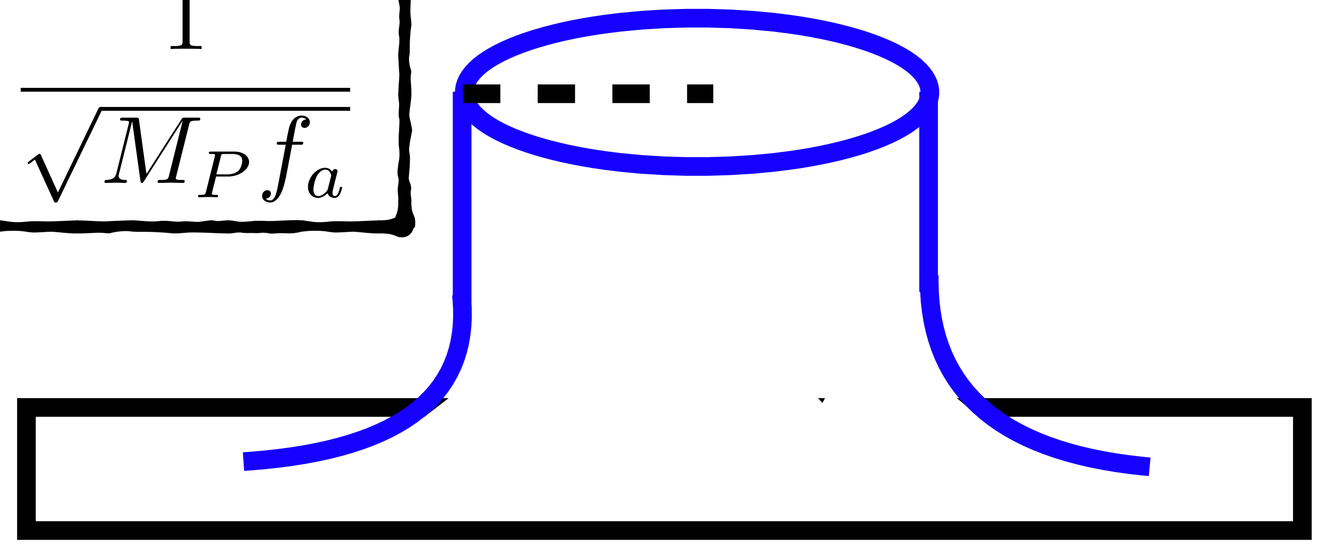


$$\boxed{L_0^{GSL} \sim \frac{1}{\sqrt{M_P f_a}}}$$



$$\boxed{\mathcal{S}_{\rm wh}^{\rm GSL} \sim \frac{M_P}{f_a}}$$

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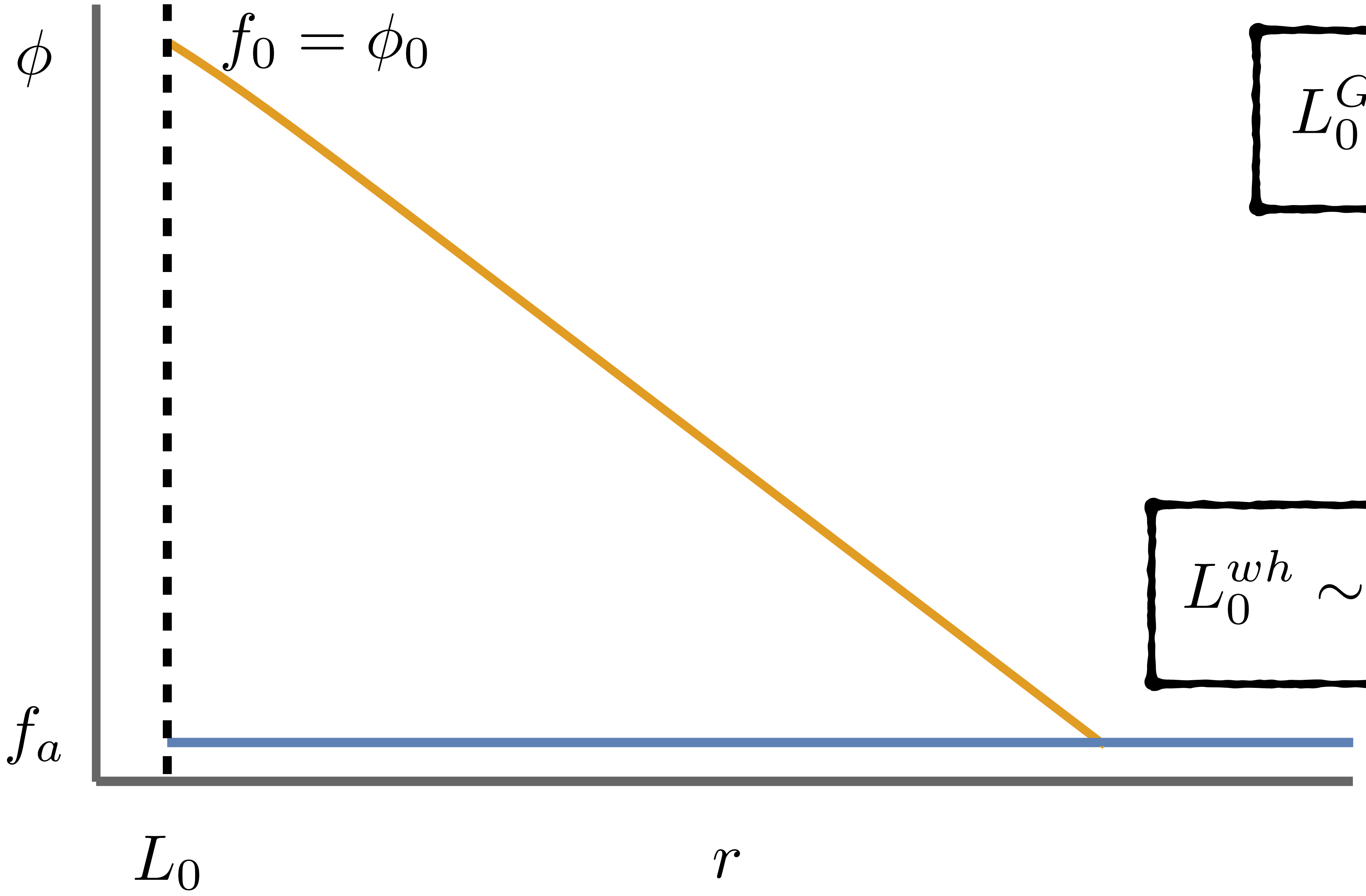

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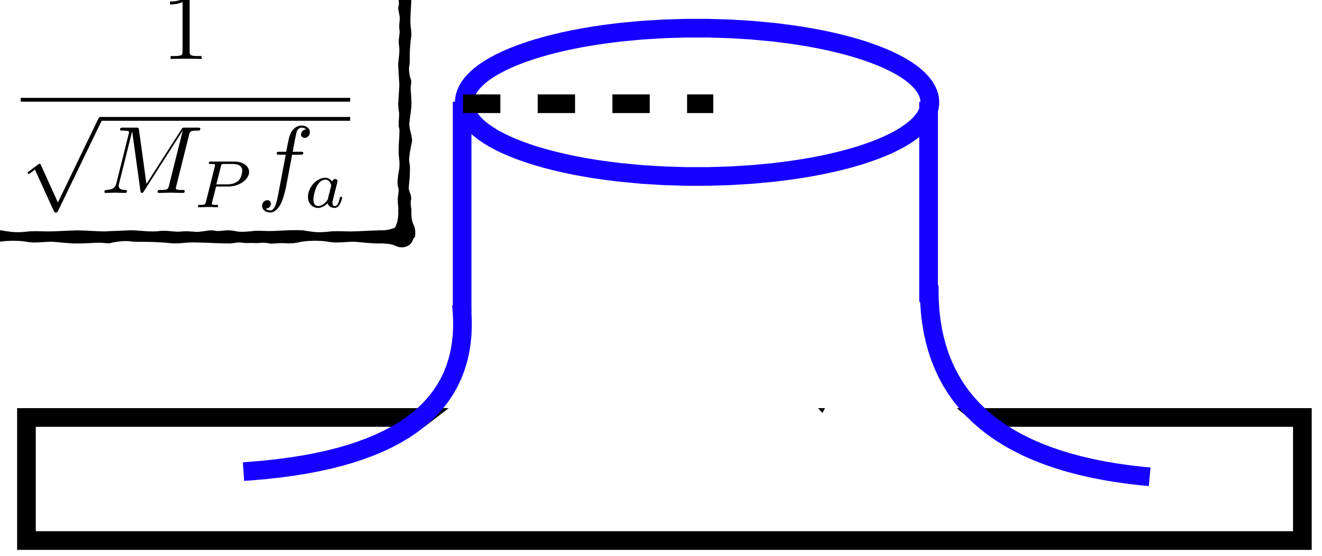

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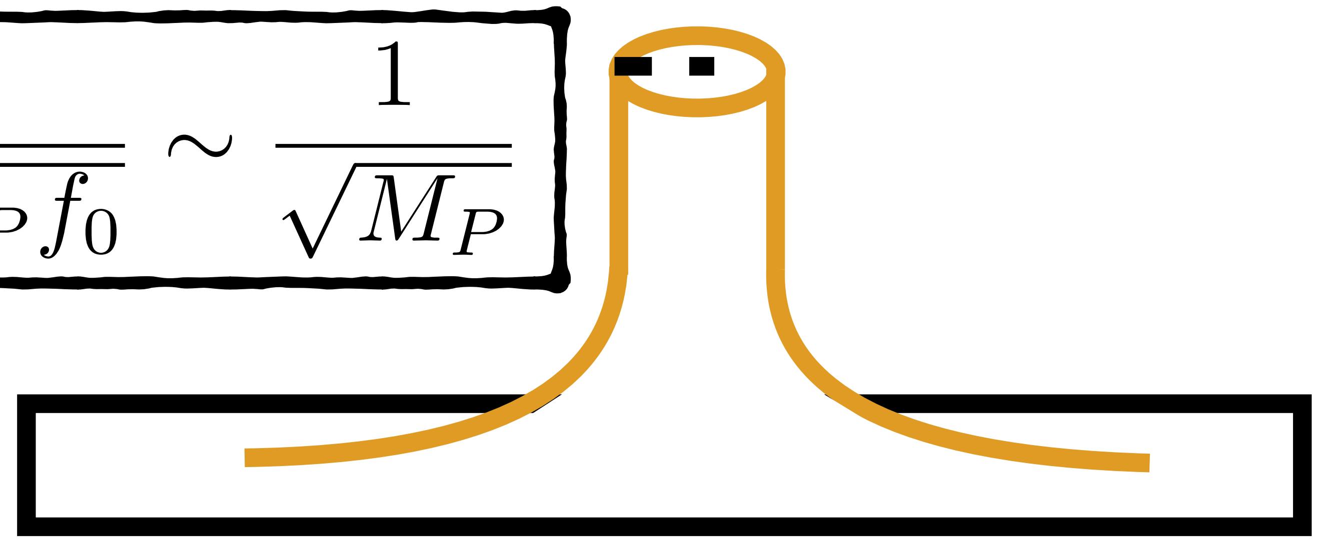

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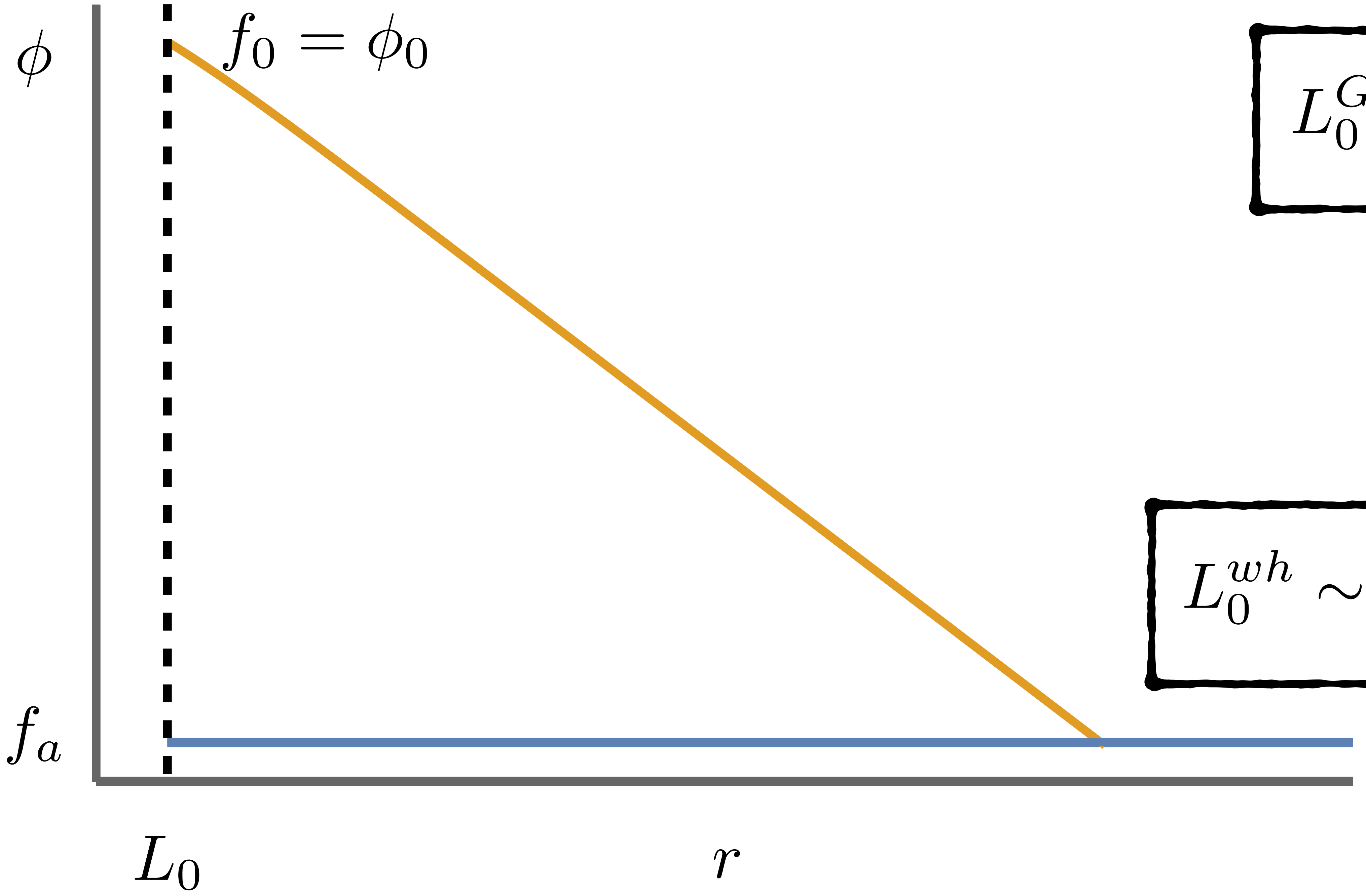

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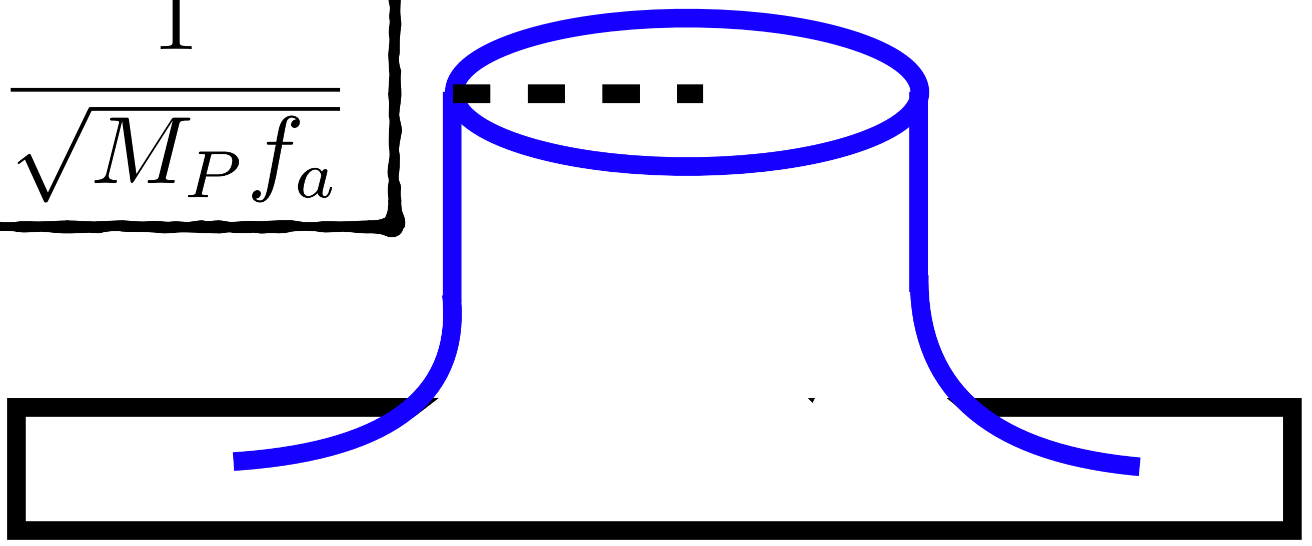

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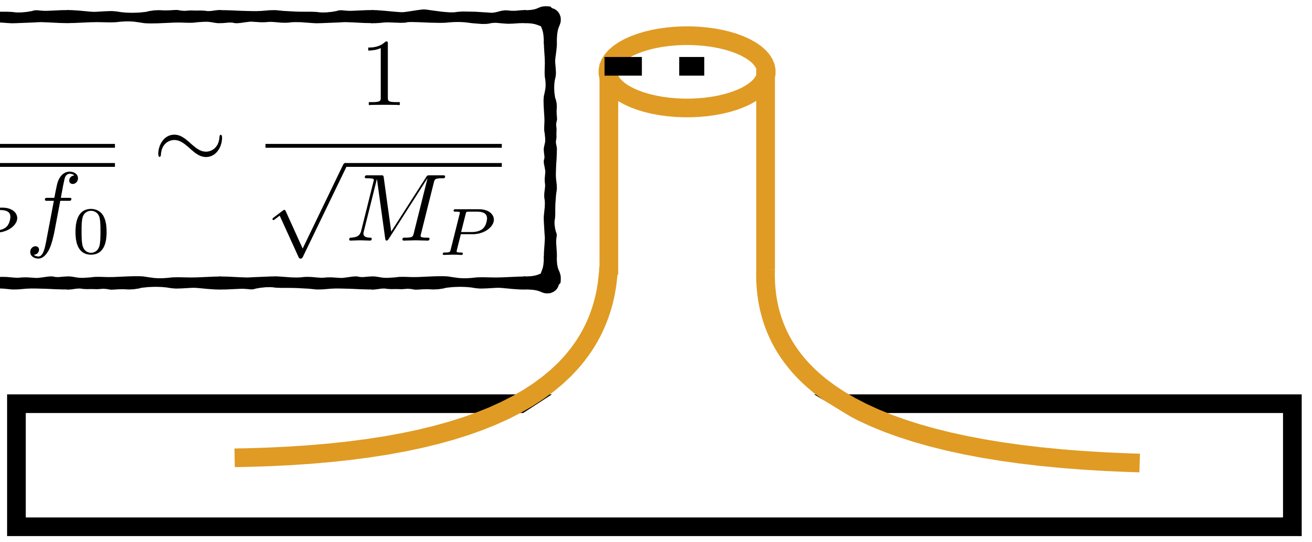

$$\mathcal{S}_{\rm wh} \sim \frac{M_P}{f_0} + \ln\frac{M_P}{f_a}$$

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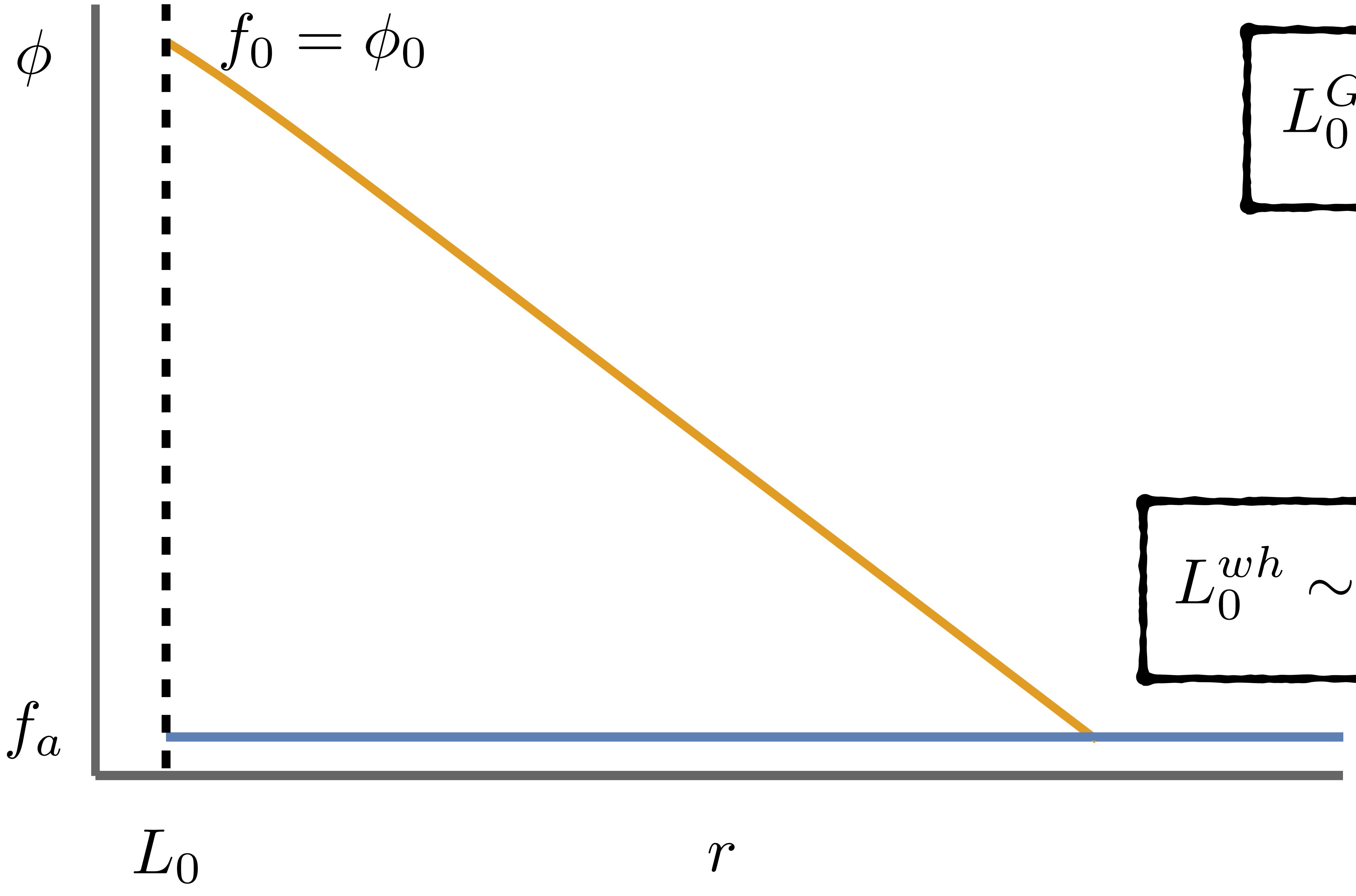

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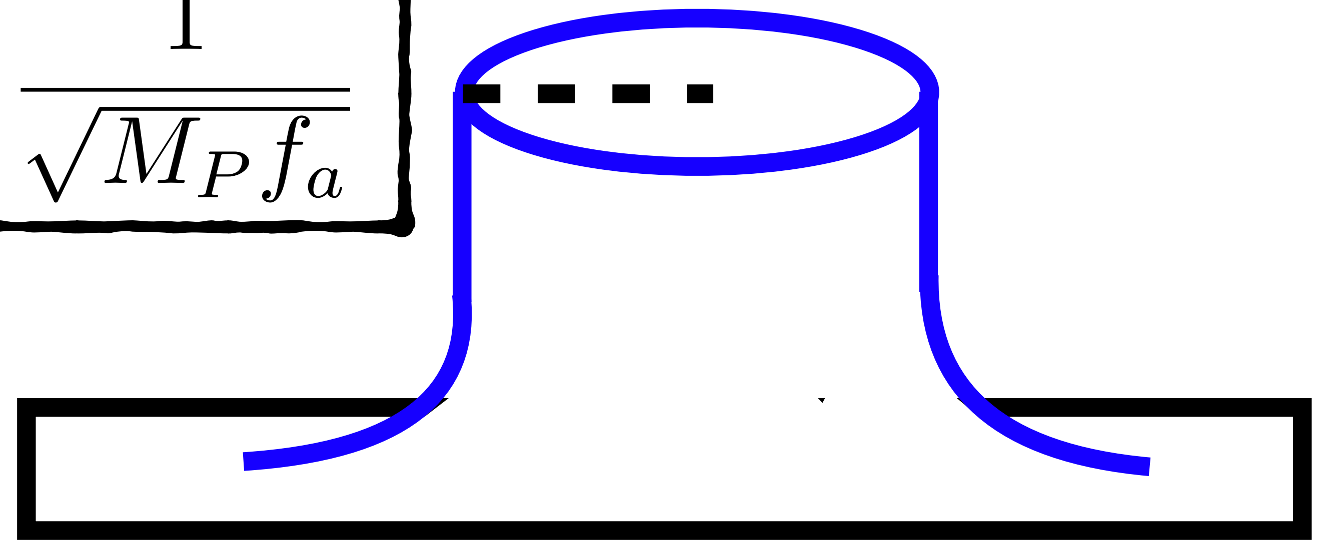

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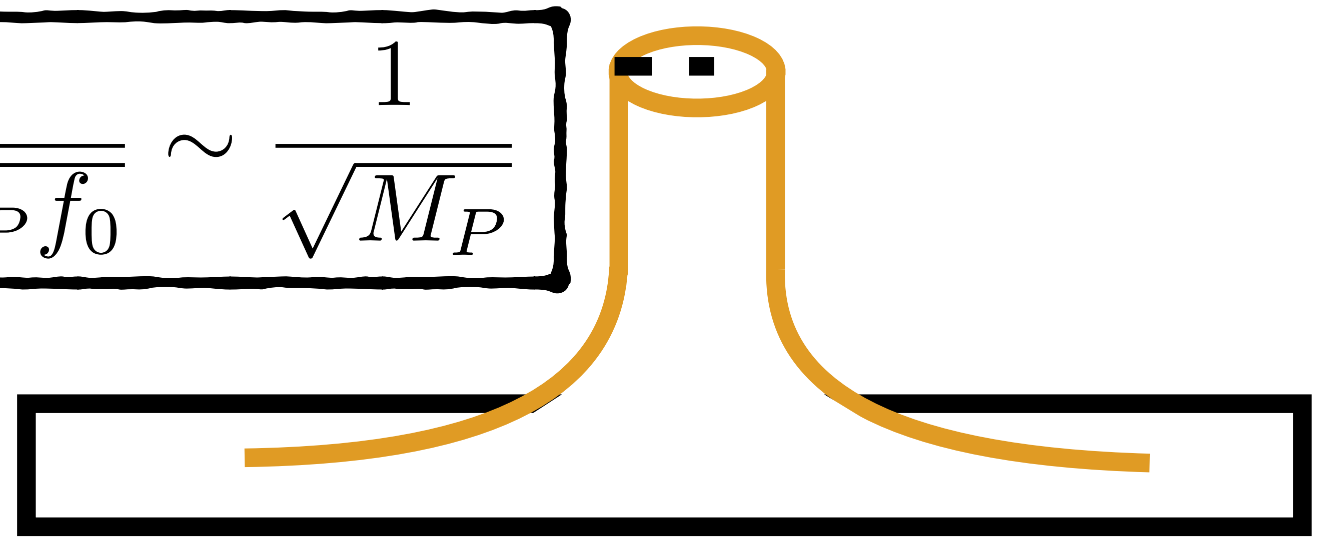

$$\mathcal{S}_{\rm wh} \sim \frac{M_P}{f_0} + \ln\frac{M_P}{f_a} \sim \mathcal{O}(1) + \ln\frac{M_P}{f_a}$$

Axion Wormholes - Effective Approach

“Is there an effective approach to compute axion wormhole properties?”

- Analytic framework for axion wormholes

- Phenomenological implications readout

Axion Wormholes - Effective Approach

[DYC, S.C. Park, C.S. Shin, 2310.11260]

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PQ Charge quantization : $\displaystyle\int_{S_3} H_{E3} = q_e = n_I \in \mathbb{N}$

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$$\tau(r) = \frac{1}{4\pi^2 L_0^2}\arctan\left(\sqrt{r^4/L_0^4 - 1}\right) \qquad p_A = G_{AB}(\phi)\frac{d\phi^B}{d\tau}$$

Master Equations

$$\mathcal{S}_{wh}[n,\phi] = \int_0^{\tau_\infty} d\tau (F^{-2}(\varphi))^{IJ} n_I n_J = 3\pi^3 M_P^2 L_0^2 + \int_{\phi_0}^{\phi} d\varphi^A\, p_A(\varphi)$$

$$L_0^4 = \frac{(F^{-2}(\phi_0))^{IJ} n_I n_J}{6(2\pi^2)^2 M_P^2} \qquad G_{AB}(\phi)\frac{d\phi^A}{d\tau}\frac{d\phi^B}{d\tau} = (F^{-2}(\phi))^{IJ} n_I n_J - (F^{-2}(\phi_0))^{IJ} n_I n_J$$

Axion Wormholes - Effective Approach

[DYC, S.C. Park, C.S. Shin, 2310.11260]

$$\mathcal{S}_E = \int d^4x\sqrt{g}\left(-\frac{M_P^2}{2}R + \frac{1}{2}G_{\alpha\beta}(\phi)\partial_\mu\phi^\alpha\partial^\mu\phi^\beta + \frac{1}{12}(F^{-2}(\phi))^{IJ}H_{I\mu\nu\rho}H_J^{\mu\nu\rho}\right)$$

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Case Studies - Single Axion

[DYC, S.C. Park, C.S. Shin, 2310.11260]

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Single axions associated with single $U(1)_{\text{PQ}}$ $\quad G_{\alpha\beta} = G(\phi)\ , \quad F^2(\phi)_{IJ} = F^2(\phi)$

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$$\int_\phi^{\phi_0}\frac{d\varphi}{M_P}\frac{\sqrt{G(\varphi)}}{\sqrt{F^2(\phi_0)/F^2(\varphi)-1}} = \frac{\pi\sqrt{6}}{4}.$$

Case Studies - Single Axion

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[DYC, K. Hamaguchi, K. Hamaguchi, Y. Kanazawa, S.M. Lee, N. Nagata, S.C. Park, (2022)]

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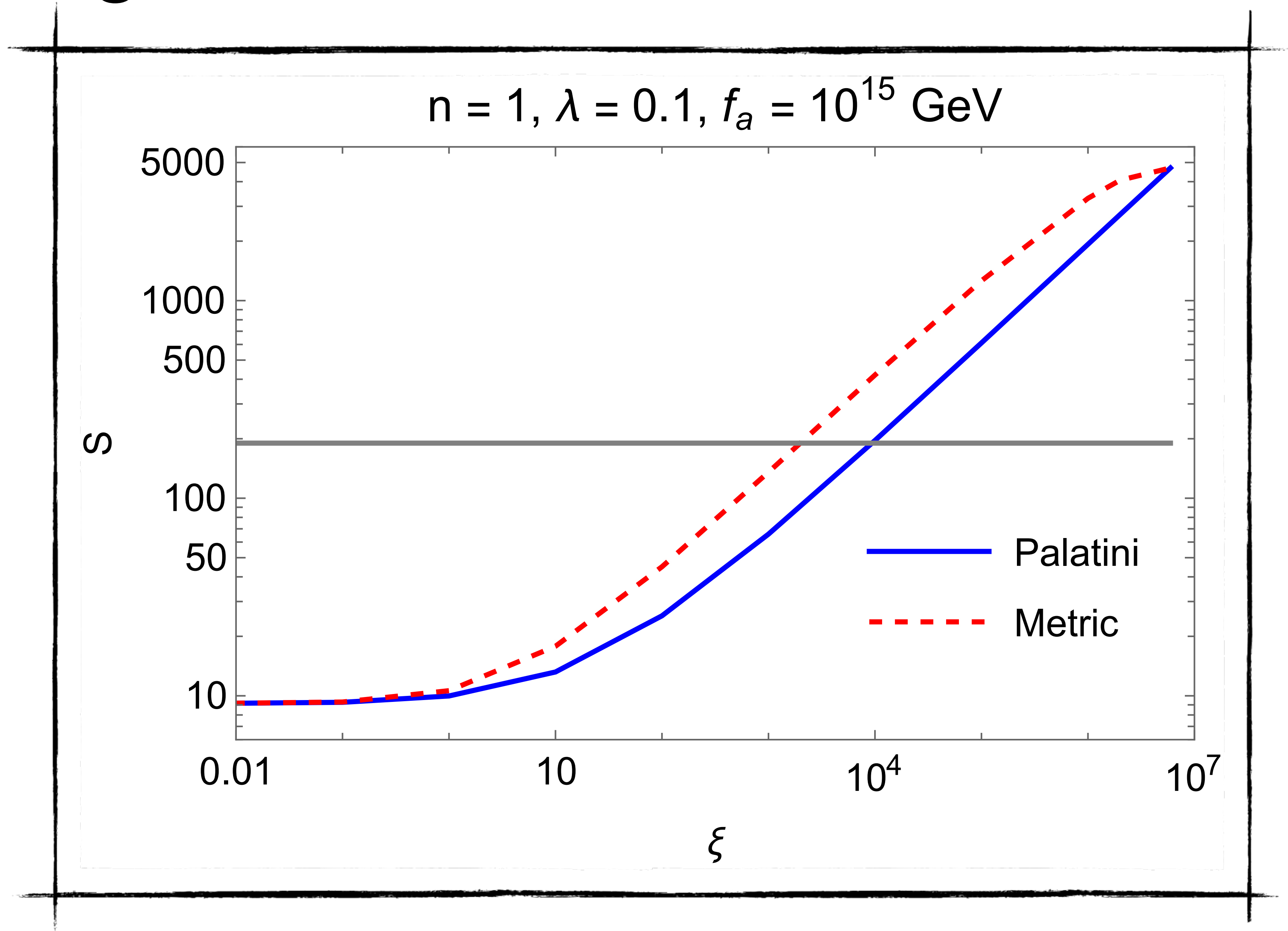

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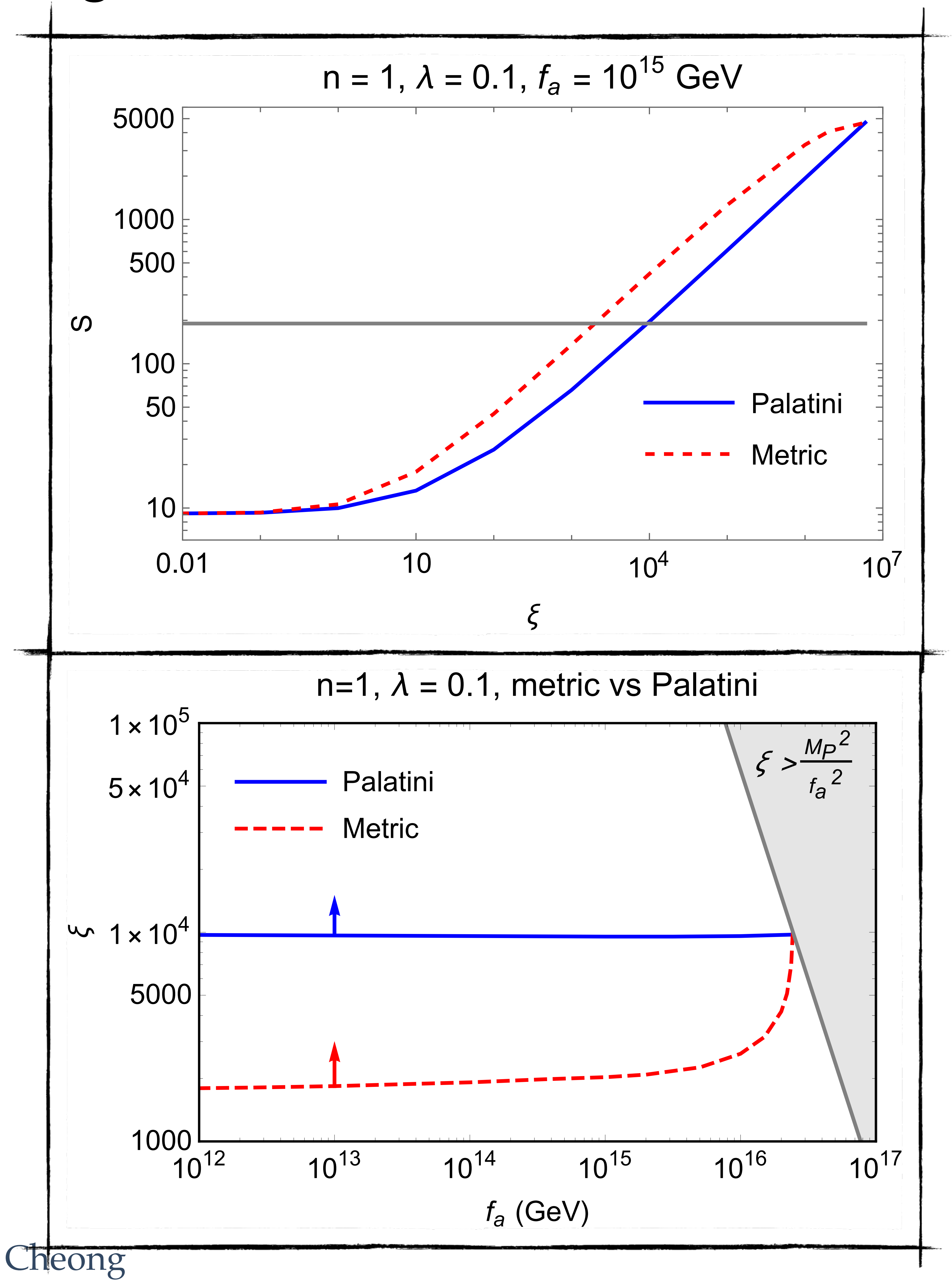

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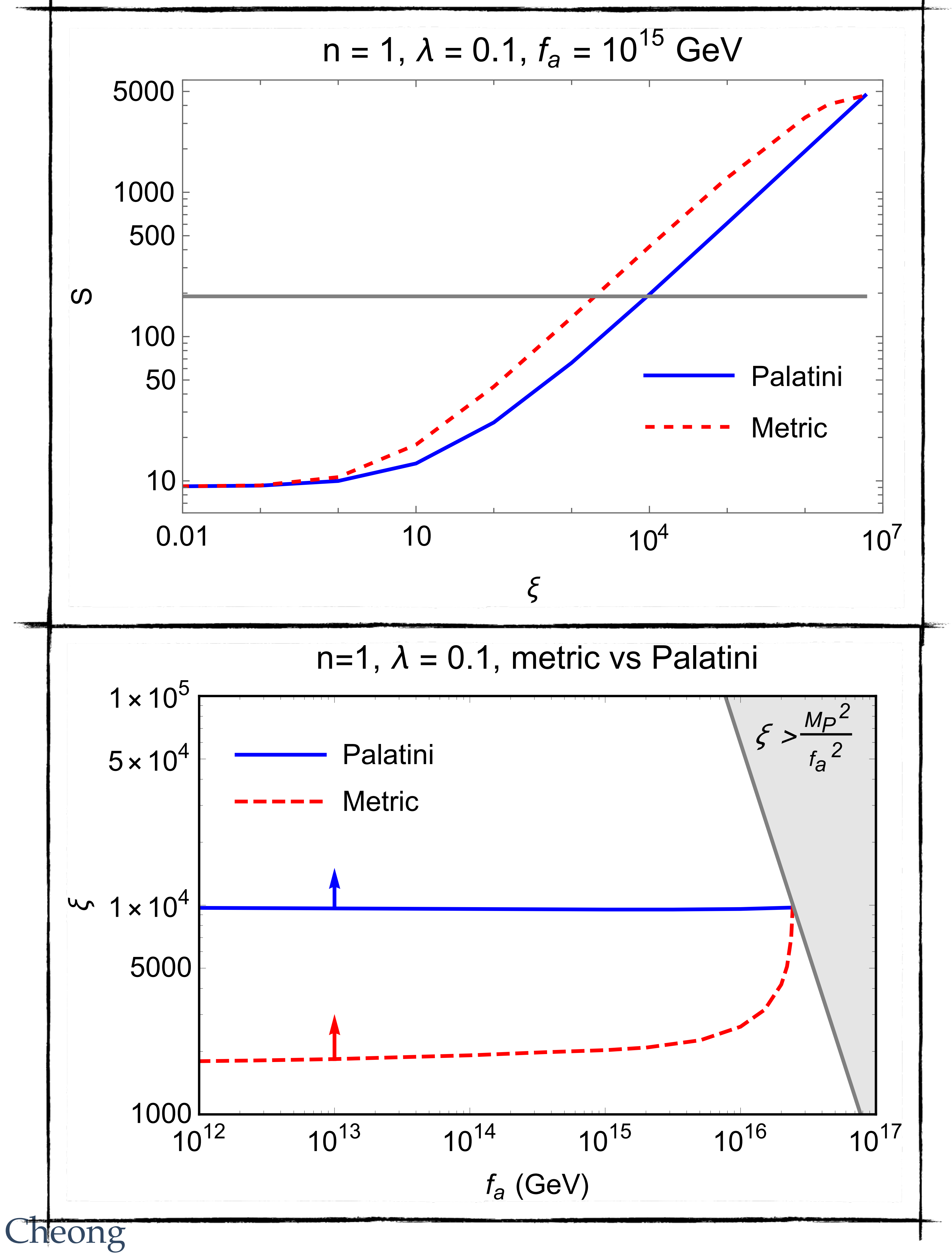

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$$\boxed{\mathcal{S}_{\rm wh} \gtrsim 190 \text{ when } \xi \gtrsim 5\times 10^3 \text{ (metric) / } 10^4 \text{ (Palatini)}}$$

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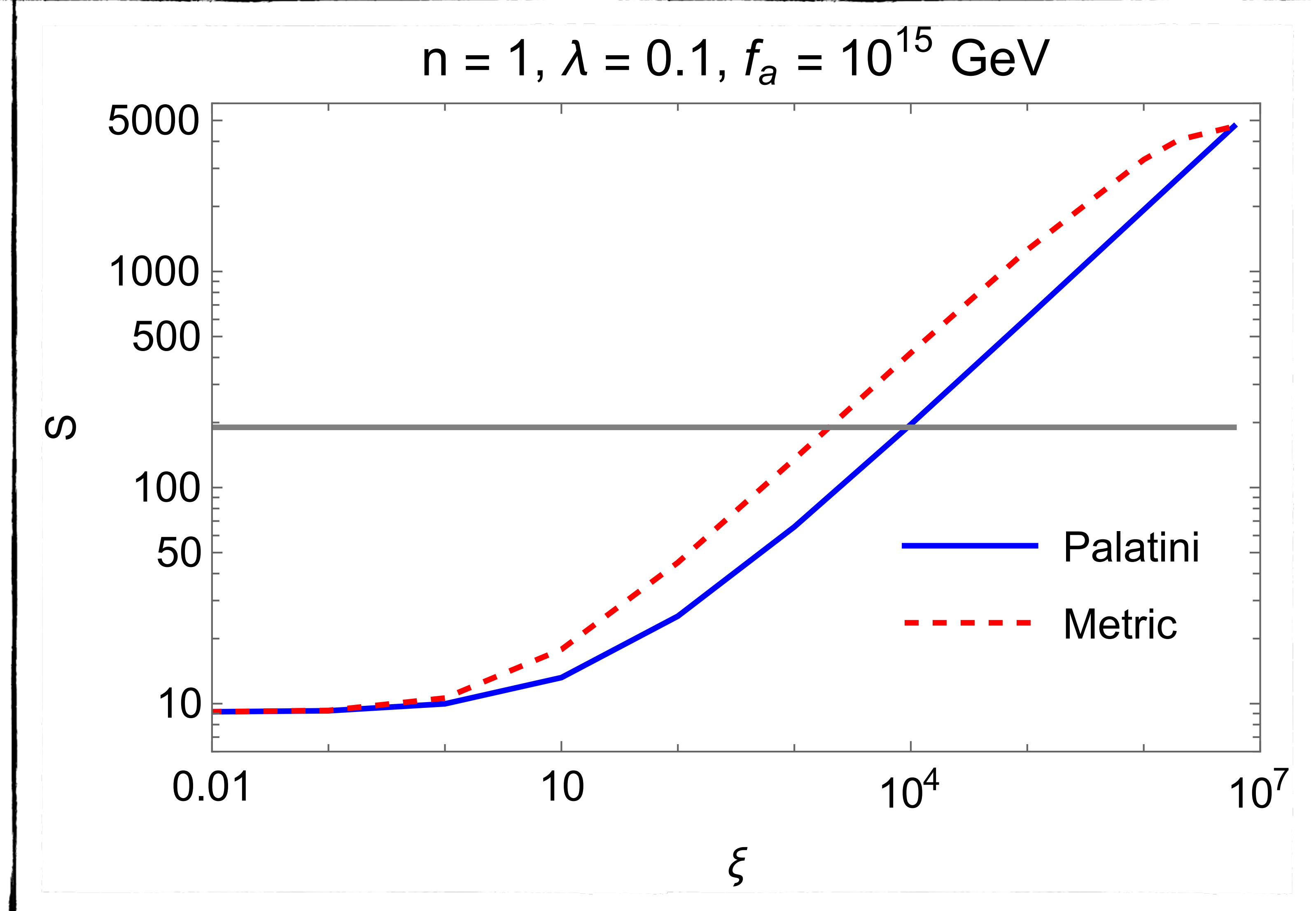

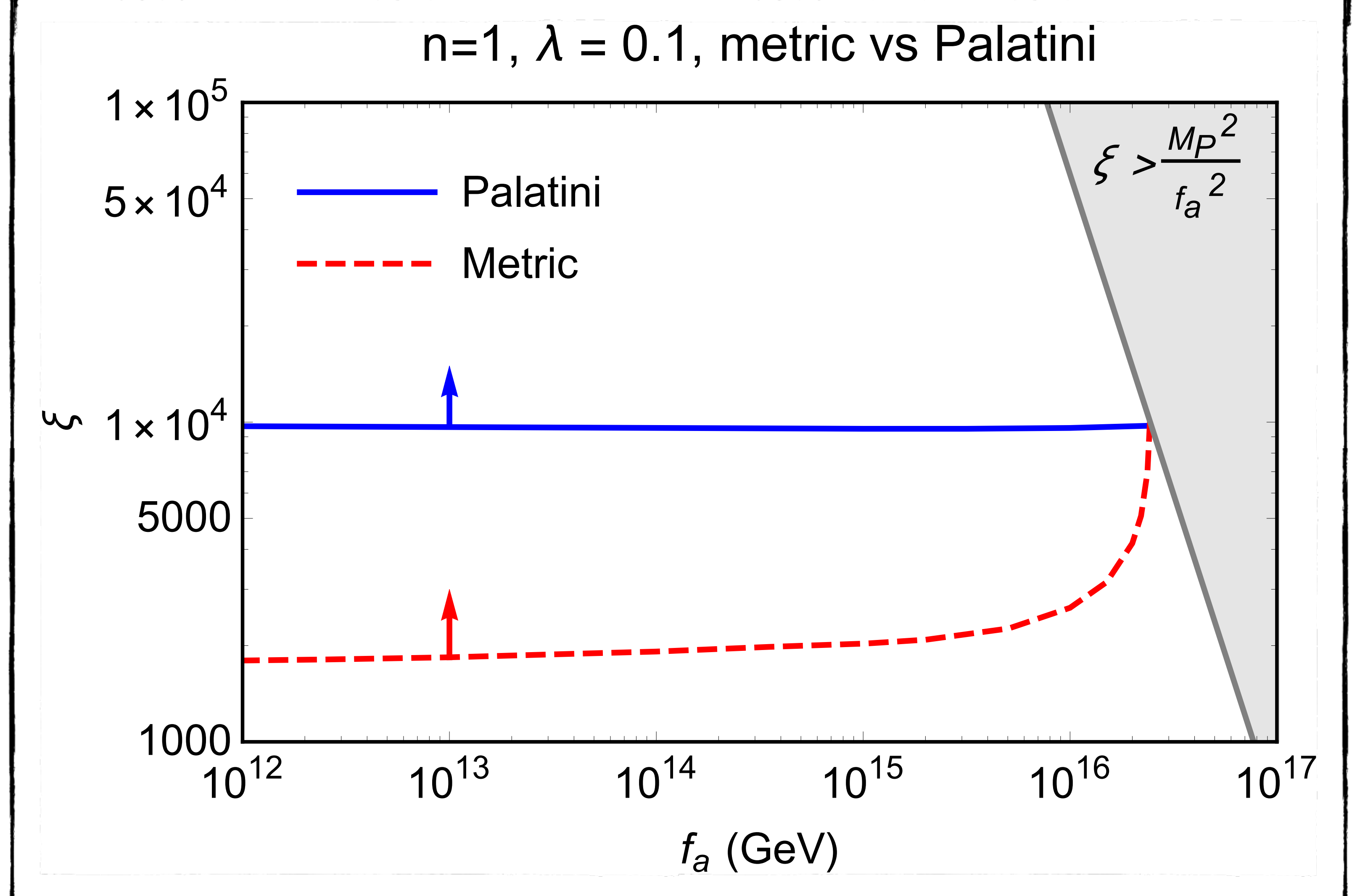

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Q) Origin for different predictions?

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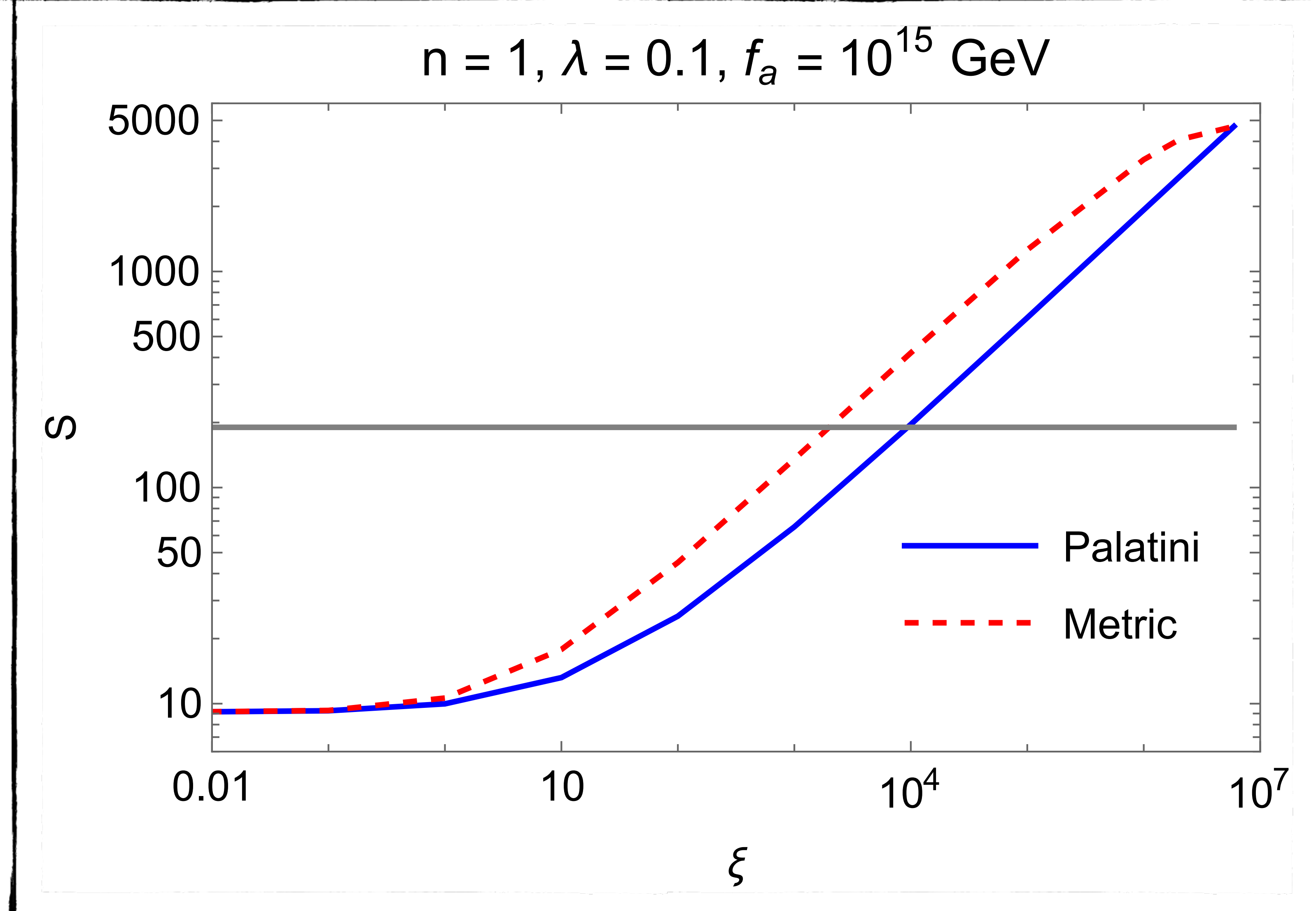

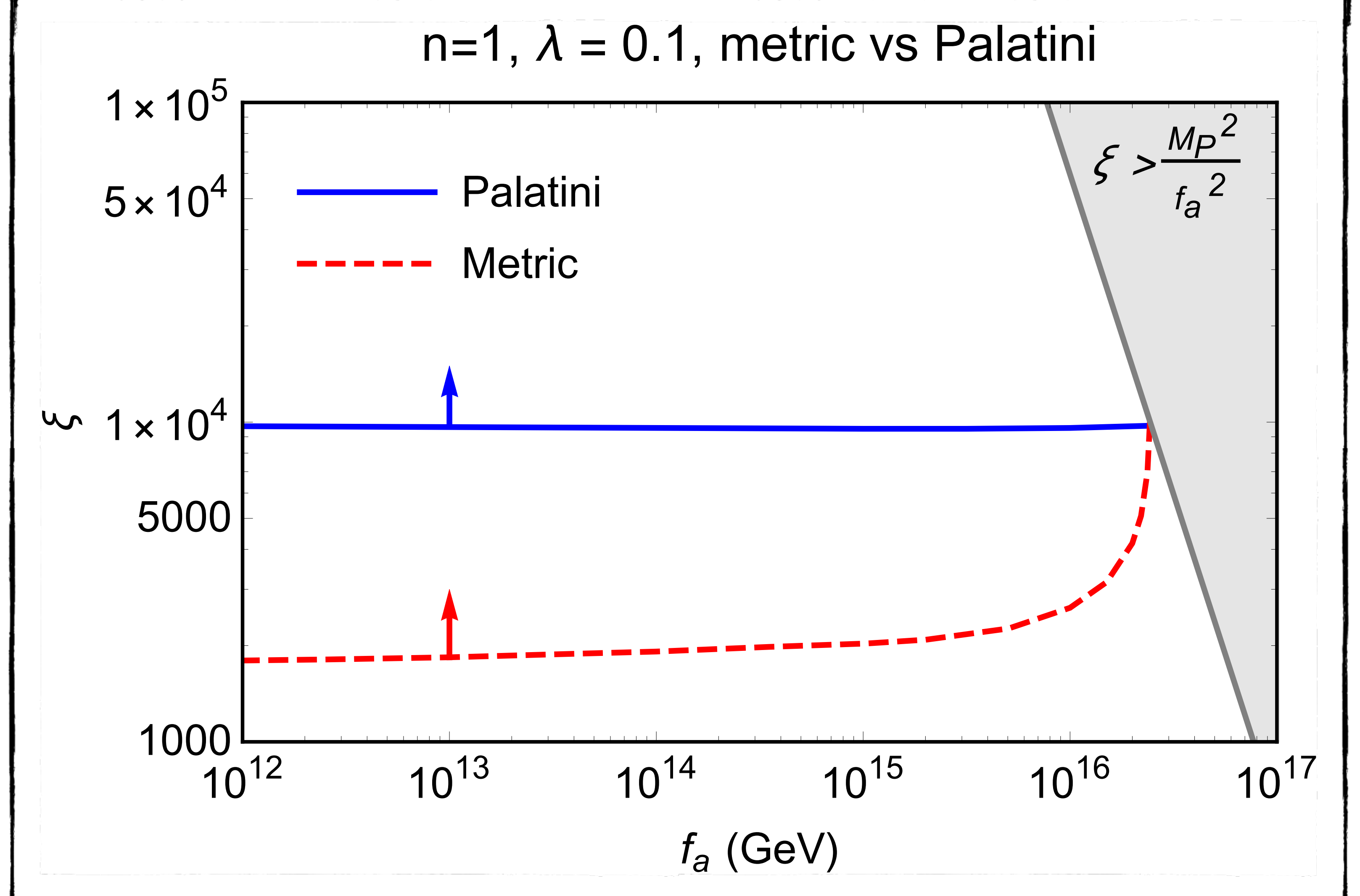

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<u>**UV/IR** structure of the wormhole!</u>

$$\boxed{\mathcal{S}_{wh}[n,\phi] \simeq n\left(\mathcal{S}_{wh}^{\mathbf{UV}}[\xi]+\ln\frac{\Lambda_{\rm eff}[\xi]}{\phi}\right)}$$

Case Studies - Single Axion

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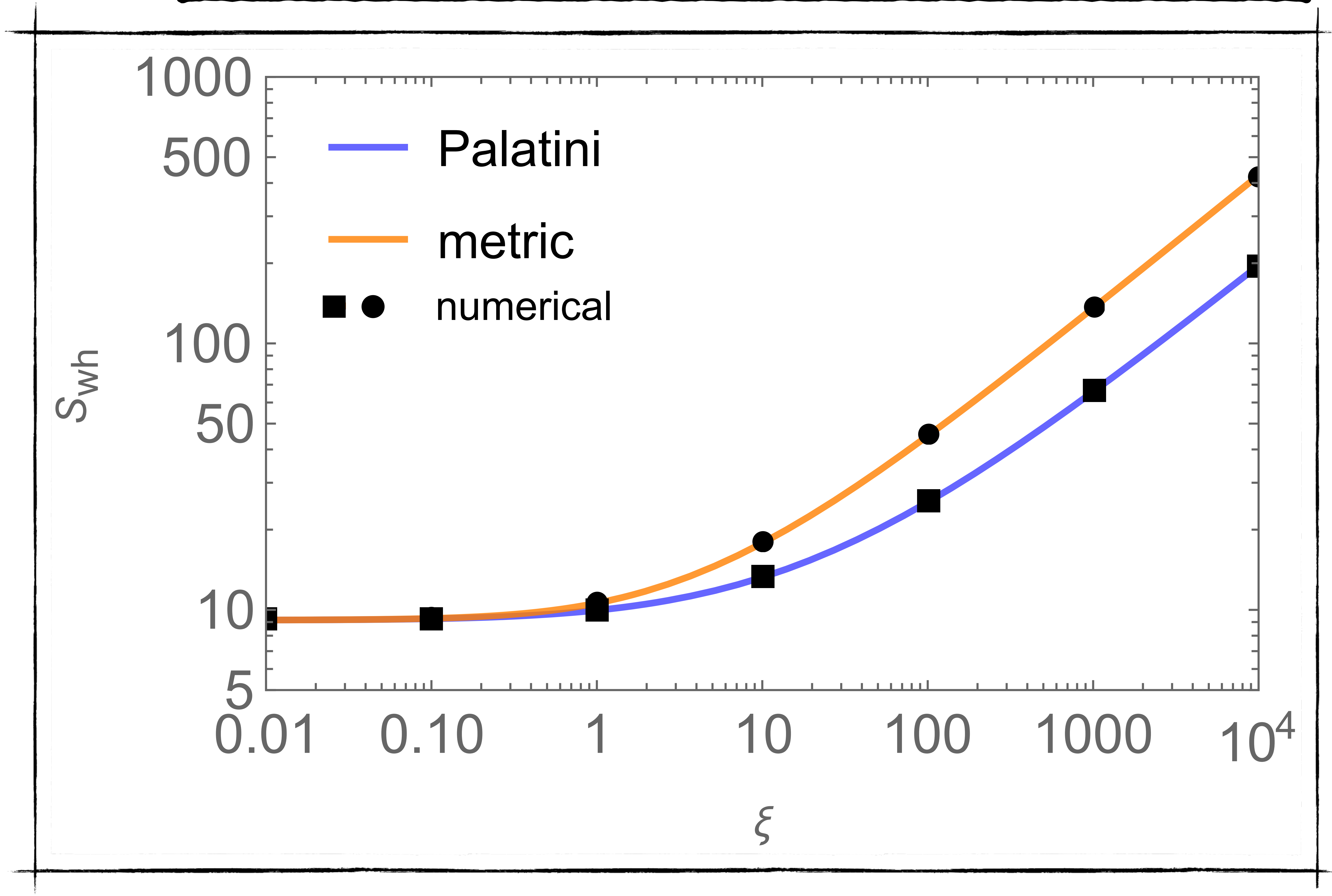

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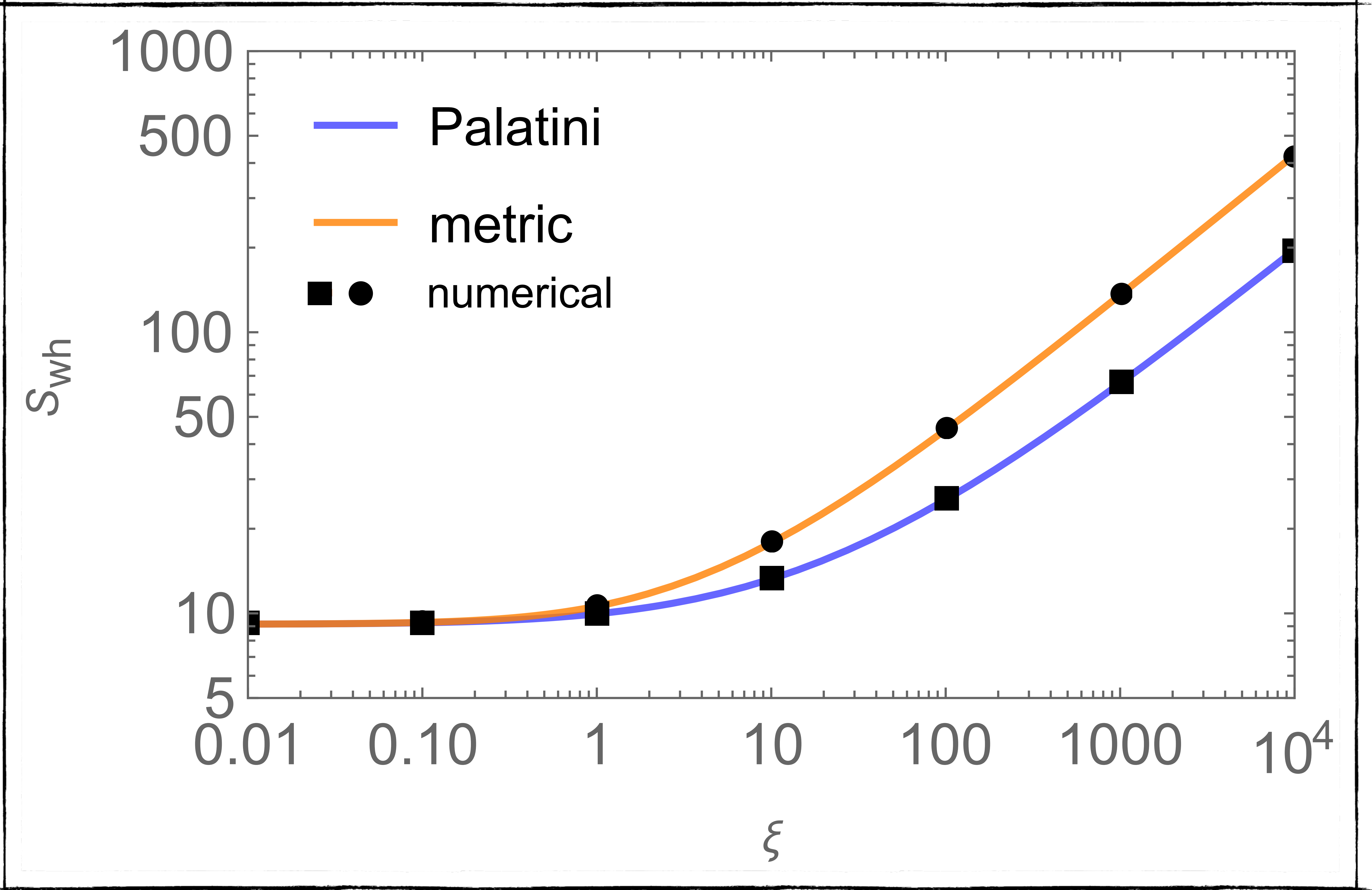

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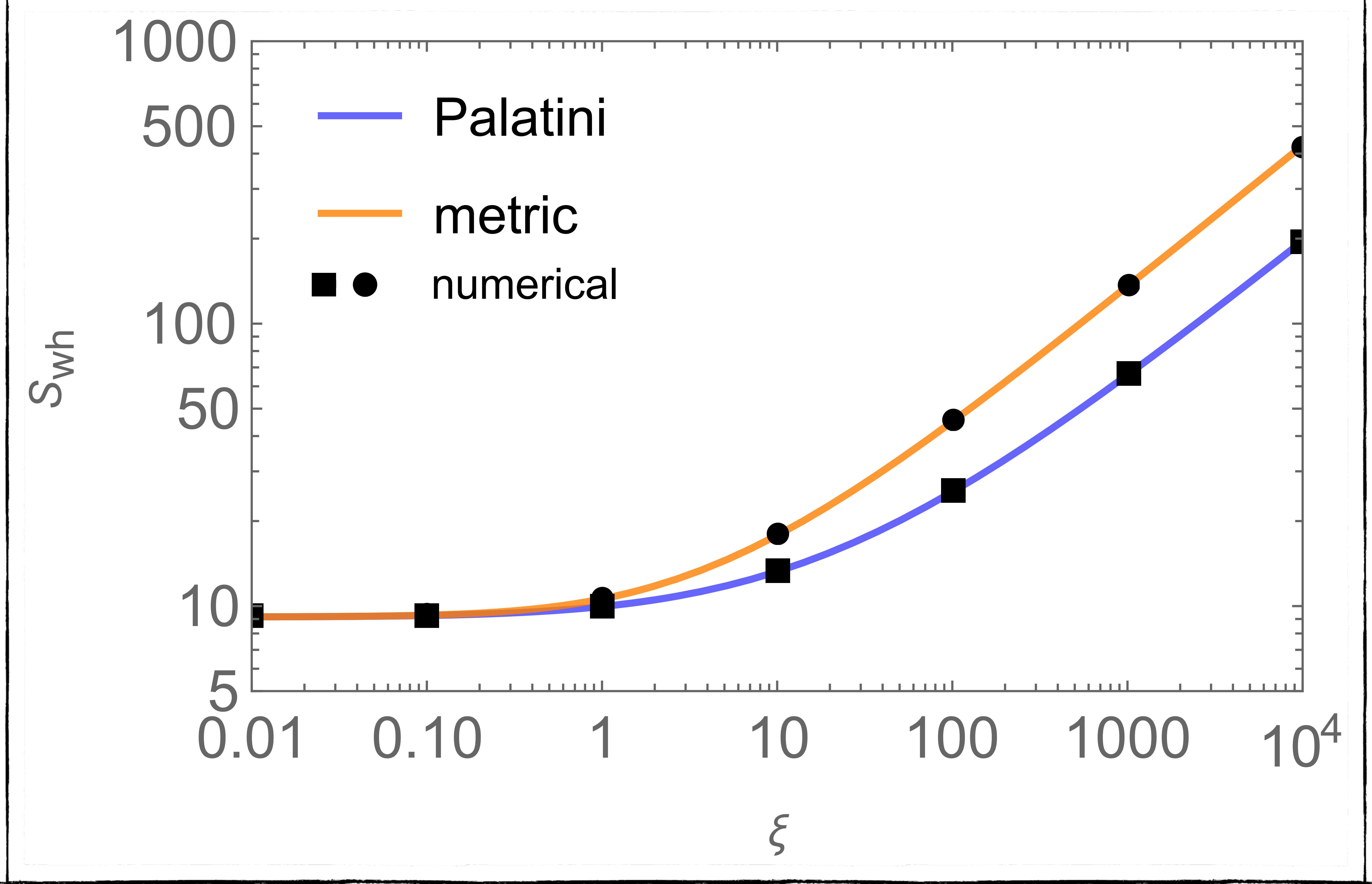

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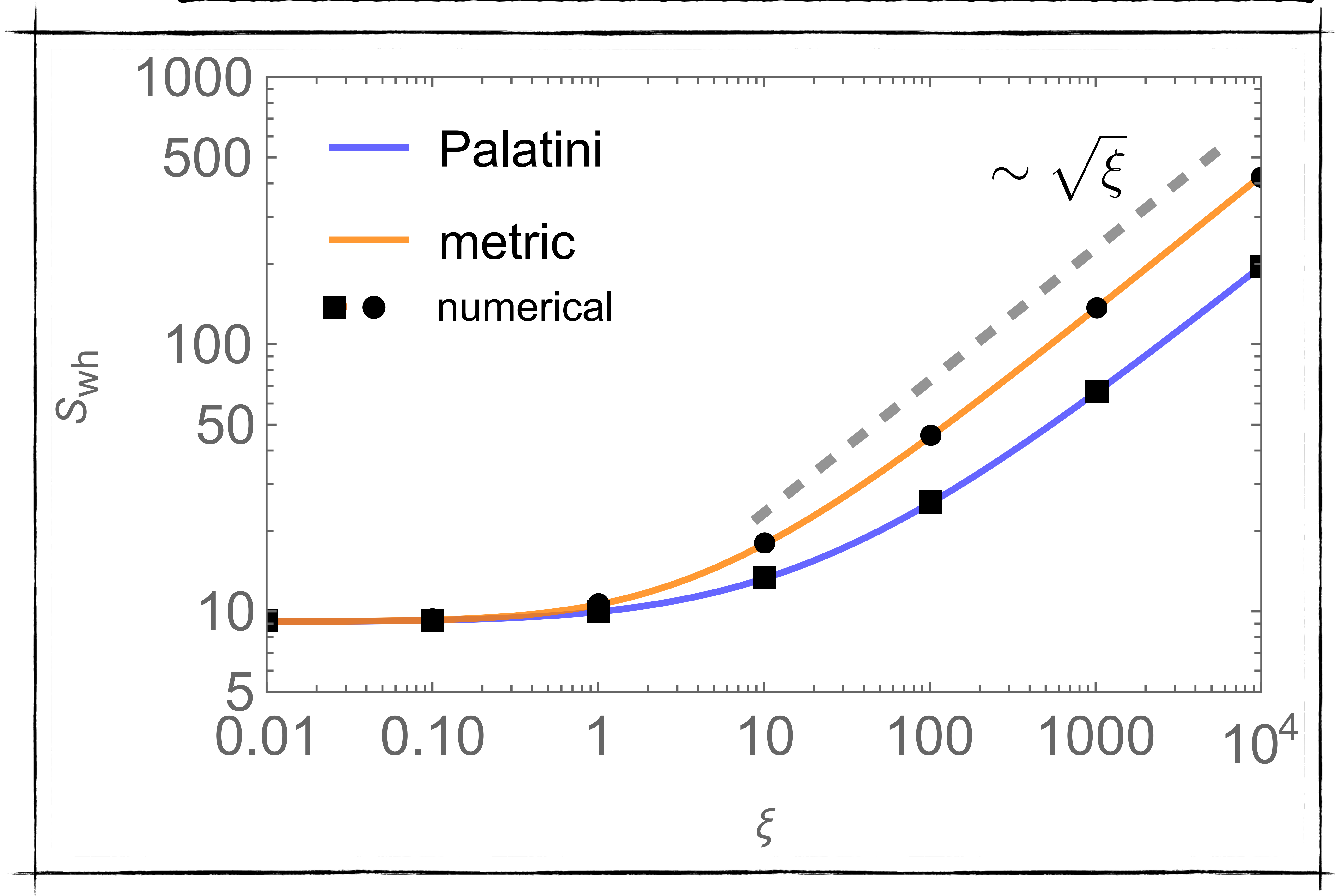

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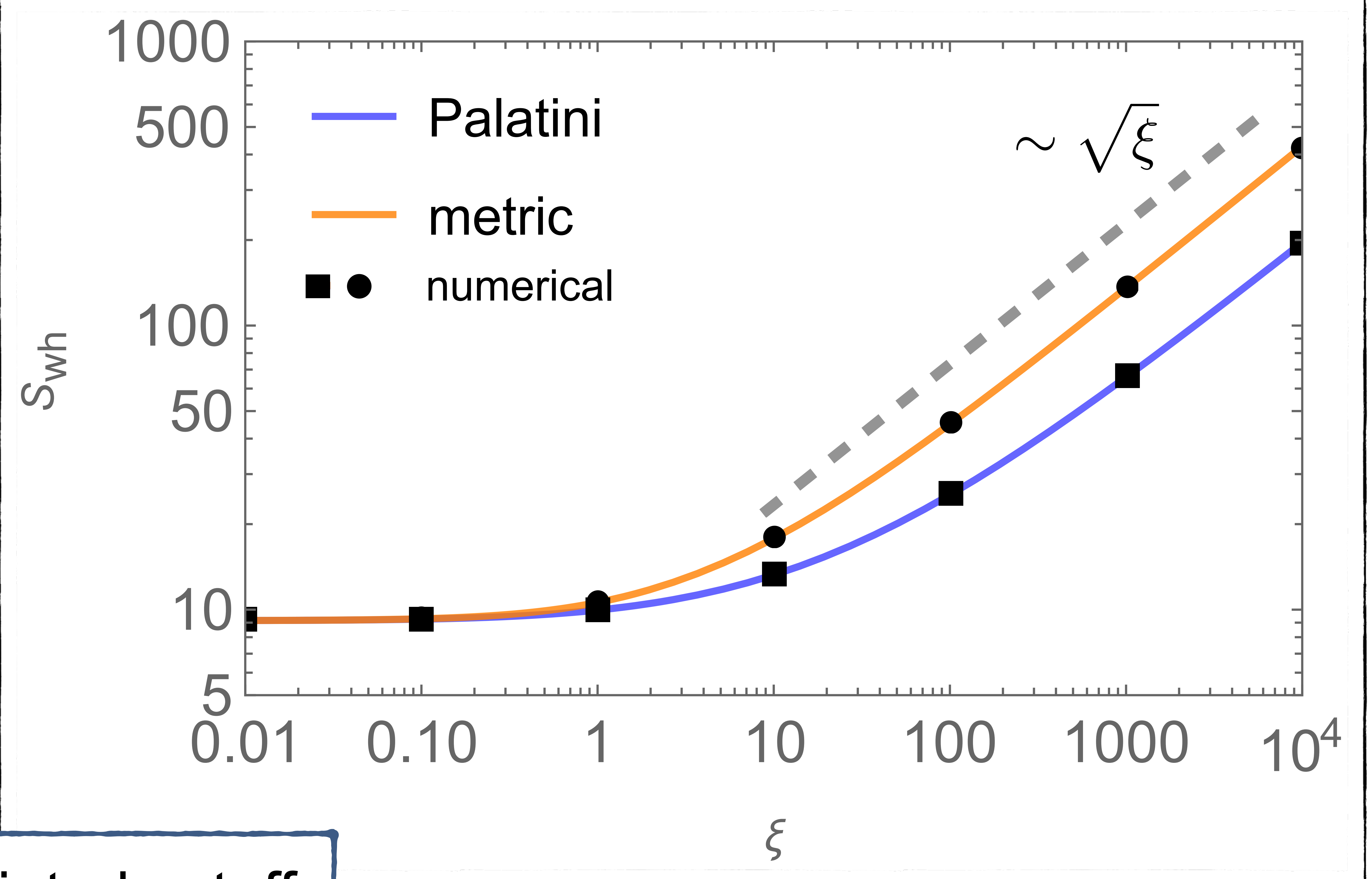

Associated cutoff

Case Studies - Single Axion

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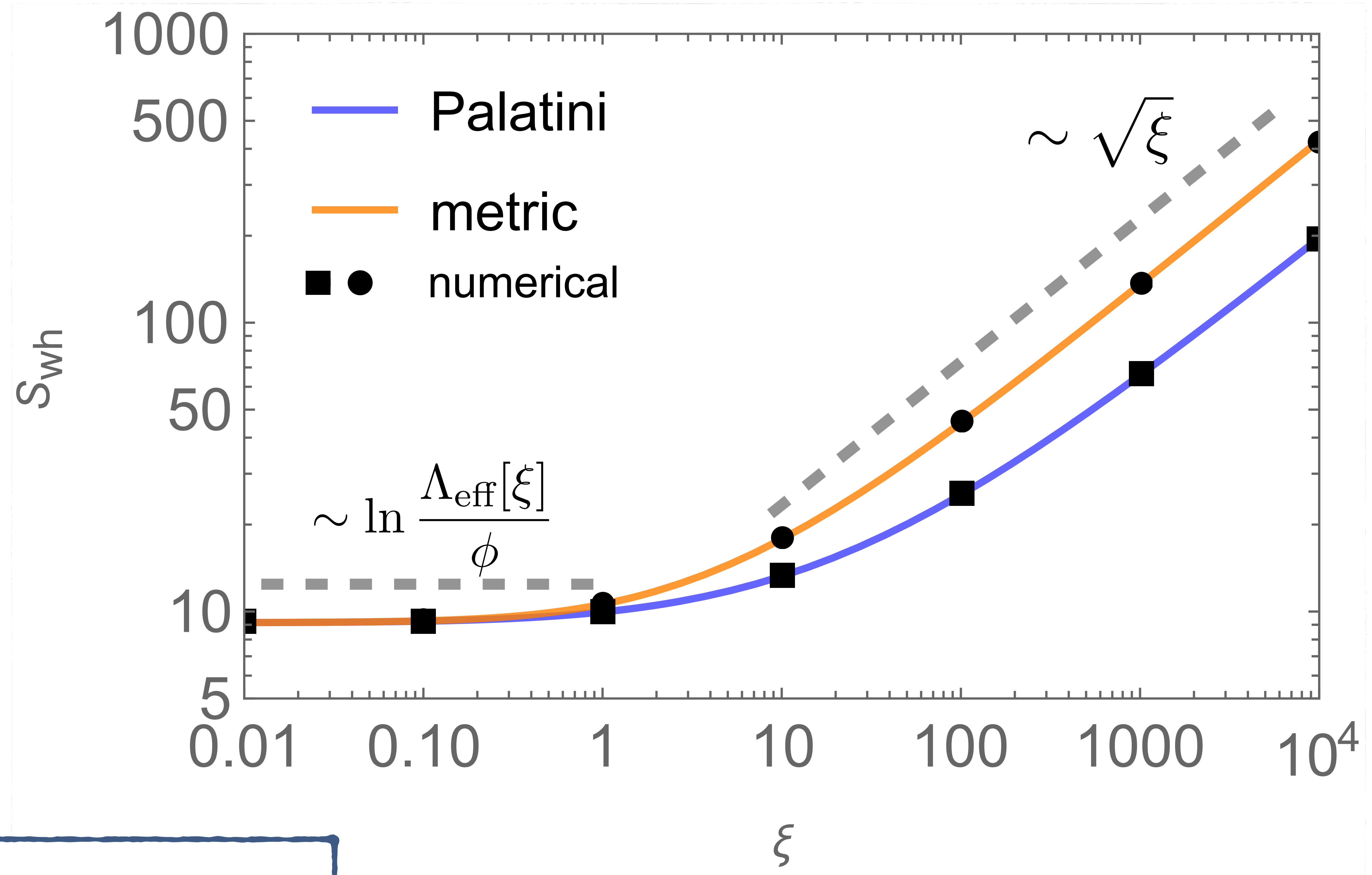

Case Studies - Single Axion + R^2

[DYC, S.C. Park, C.S. Shin, 2310.11260]

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Scenarios with additional massless scalars

$$\mathcal{S} = \int d^4x\sqrt{g}\left(-\frac{M_P^2}{2}R + \frac{1}{2}Z(\chi)\left(\partial_\mu\phi\partial^\mu\phi + F^2(\phi)\partial_\mu\theta\partial^\mu\theta\right) + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi\right)$$

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Example) Nonminimally coupled PQ scalar with R^2 gravity

$$\mathcal{S} = \int d^4x\sqrt{g}\left[-\left(\frac{M_P^2}{2} + \xi_\phi|\Phi|^2\right)R - \frac{\xi_s}{4}R^2 + \partial_\mu\Phi\partial^\mu\Phi^* - V(|\Phi|)\right]$$

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$$\boxed{Z(\chi) = e^{2\beta\chi/M_P}\Big|_{\beta=-\frac{1}{\sqrt{6}}}, \quad F^2(\phi) = \phi^2}$$

Case Studies - Single Axion + R^2

[DYC, S.C. Park, C.S. Shin, 2310.11260]

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$$\Downarrow$$

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[-\frac{M_P^2}{2}R + \frac{1}{2}e^{-\sqrt{\frac{2}{3}}\frac{\chi}{M_P}}\left(\partial_\mu\phi\partial^\mu\phi + \phi^2\partial_\mu\theta\partial^\mu\theta\right) + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - U(\chi,\phi)\right]$$

$$\boxed{Z(\chi) = e^{2\beta\chi/M_P}\Big|_{\beta=-\frac{1}{\sqrt{6}}}, \quad F^2(\phi) = \phi^2}$$

$$U(\chi,\phi) = \frac{M_P^2}{4\xi_s}\left[1 - \left(1 + \frac{\xi_\phi\phi^2}{M_P^2}\right)e^{-\sqrt{\frac{2}{3}}\frac{\chi}{M_P}}\right]^2 + e^{-2\sqrt{\frac{2}{3}}\frac{\chi}{M_P}}V(\phi)$$

Case Studies - Single Axion + R^2

[DYC, S.C. Park, C.S. Shin, 2310.11260]

Scenarios with additional massless scalars

$$\mathcal{S} = \int d^4x\sqrt{g}\left(-\frac{M_P^2}{2}R + \frac{1}{2}Z(\chi)\left(\partial_\mu\phi\partial^\mu\phi + F^2(\phi)\partial_\mu\theta\partial^\mu\theta\right) + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi\right)$$

Example) Nonminimally coupled PQ scalar with R^2 gravity

$$\mathcal{S} = \int d^4x\sqrt{g}\left[-\left(\frac{M_P^2}{2} + \xi_\phi|\Phi|^2\right)R - \frac{\xi_s}{4}R^2 + \partial_\mu\Phi\partial^\mu\Phi^* - V(|\Phi|)\right]$$

$$\Downarrow$$

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Case Studies - Single Axion + R^2

[DYC, S.C. Park, C.S. Shin, 2310.11260]

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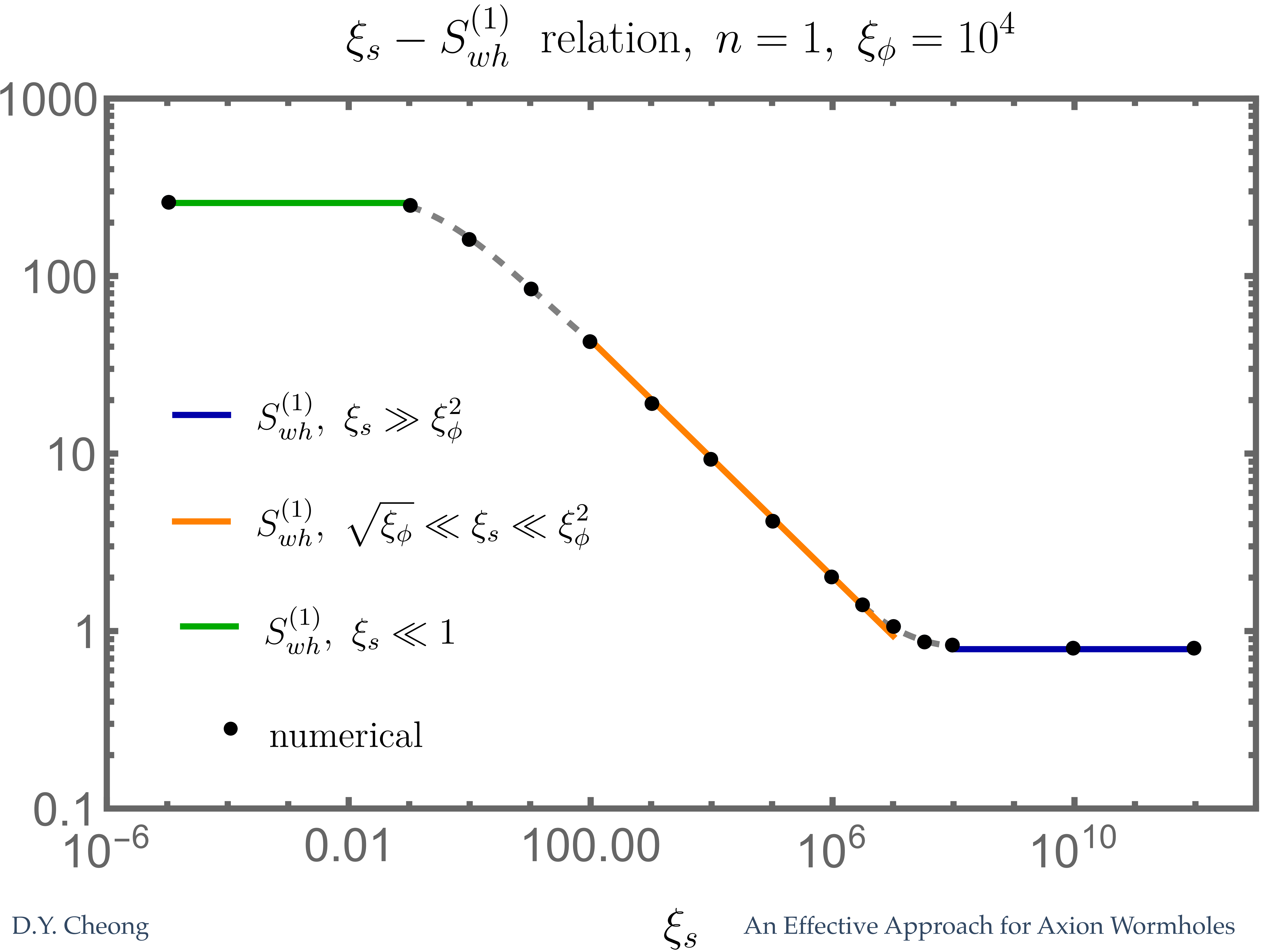
$\boxed{\textit{Effect determined by } \xi_s}$

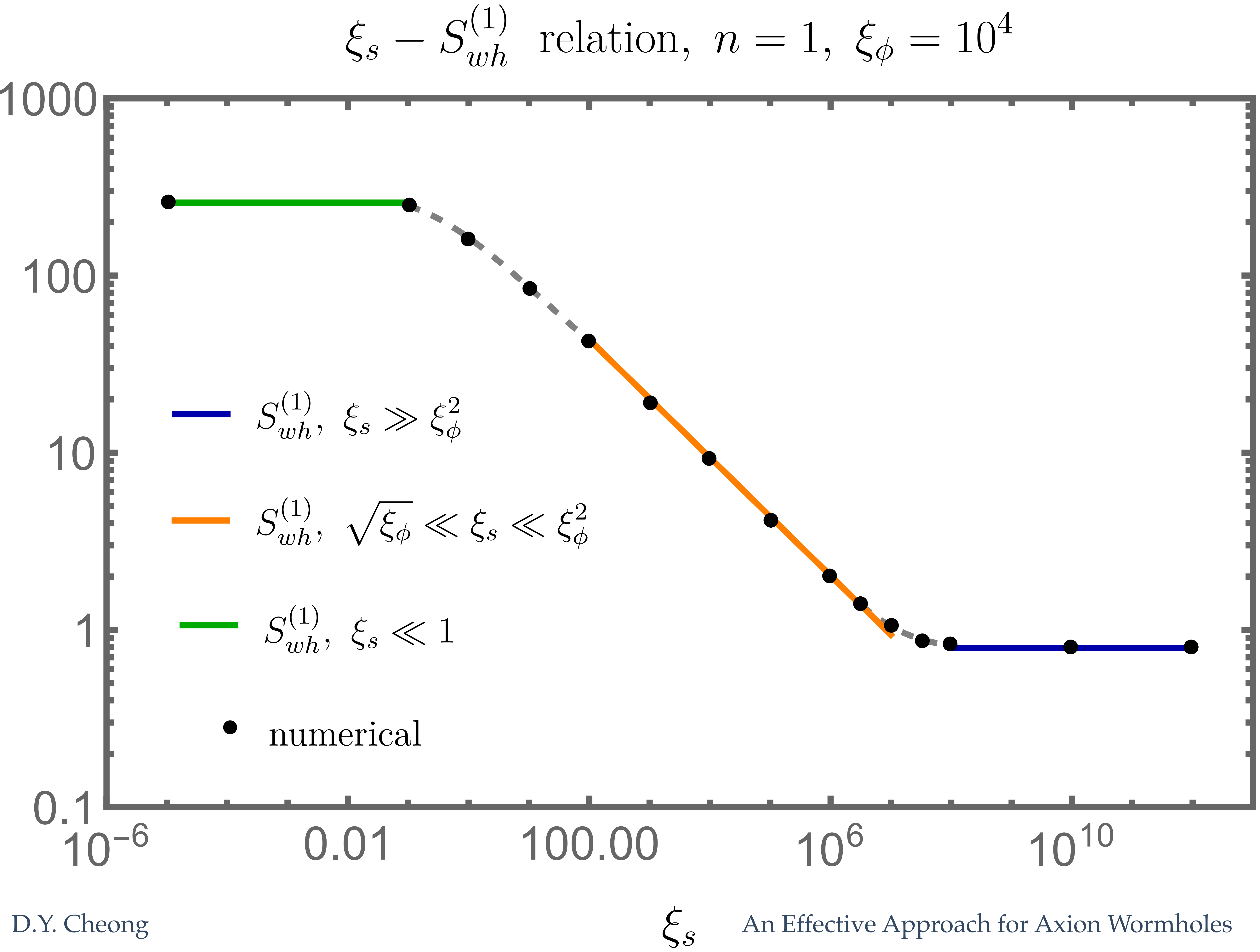
Case Studies - Single Axion + R^2

[DYC, S.C. Park, C.S. Shin, 2310.11260]



Case Studies - Single Axion + R^2

[DYC, S.C. Park, C.S. Shin, 2310.11260]

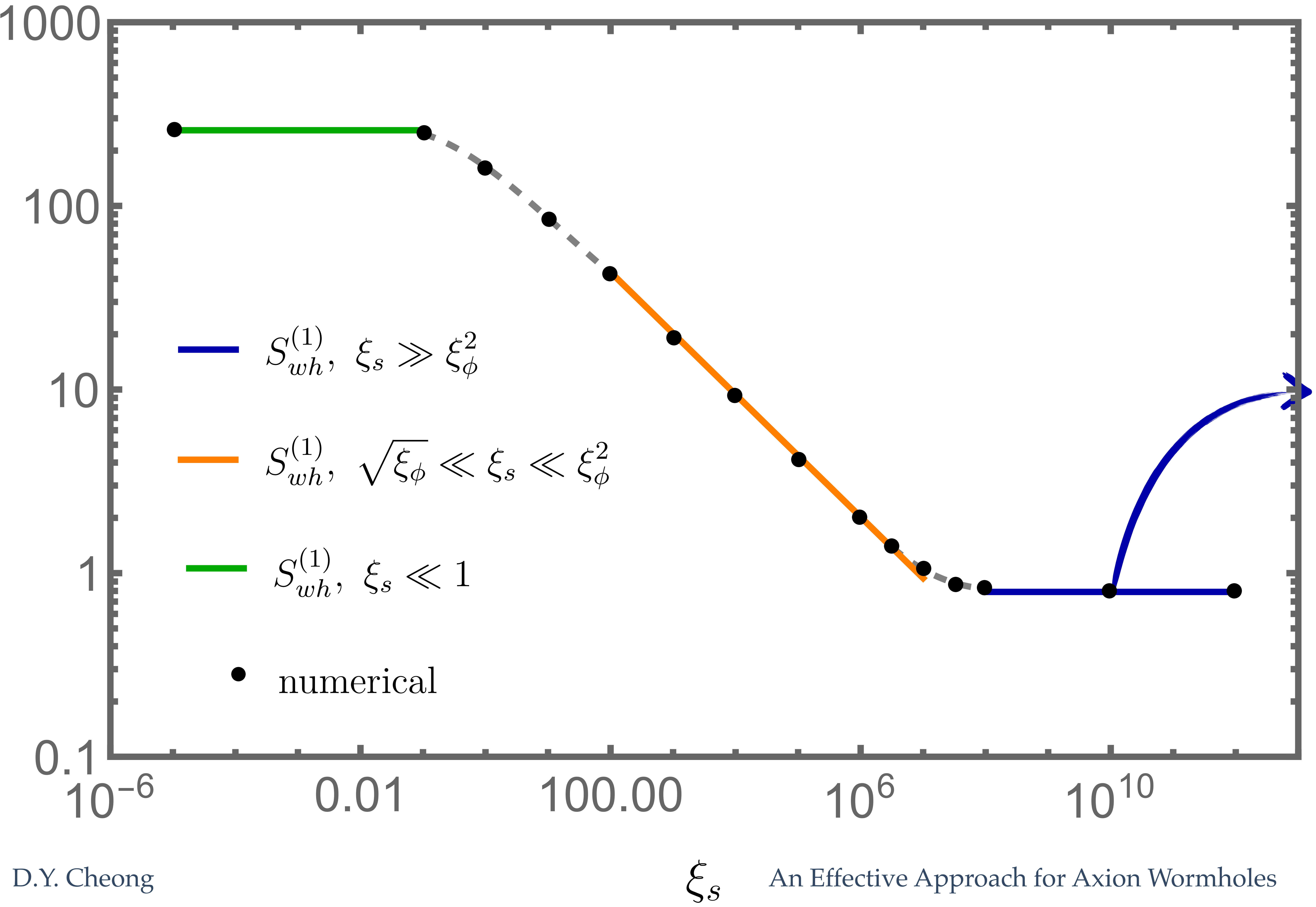


$$U(\chi,\phi) = \frac{M_P^2}{4\xi_s}\left[1 - \left(1 + \frac{\xi_\phi \phi^2}{M_P^2}\right) e^{-\sqrt{\frac{2}{3}}\frac{\chi}{M_P}}\right]^2 \simeq \frac{\xi_\phi^2}{4\xi_s}\phi^4 e^{-2\sqrt{\frac{2}{3}}\frac{\chi}{M_P}}$$

Case Studies - Single Axion + R^2

[DYC, S.C. Park, C.S. Shin, 2310.11260]

$$\xi_s - S^{(1)}_{wh} \ \text{relation}, \ n=1, \ \xi_\phi = 10^4$$



$$U(\chi,\phi) = \frac{M_P^2}{4\xi_s}\left[1-\left(1+\frac{\xi_\phi\phi^2}{M_P^2}\right)e^{-\sqrt{\frac{2}{3}}\frac{\chi}{M_P}}\right]^2$$
$$\simeq \frac{\xi_\phi^2}{4\xi_s}\phi^4 e^{-2\sqrt{\frac{2}{3}}\frac{\chi}{M_P}}$$

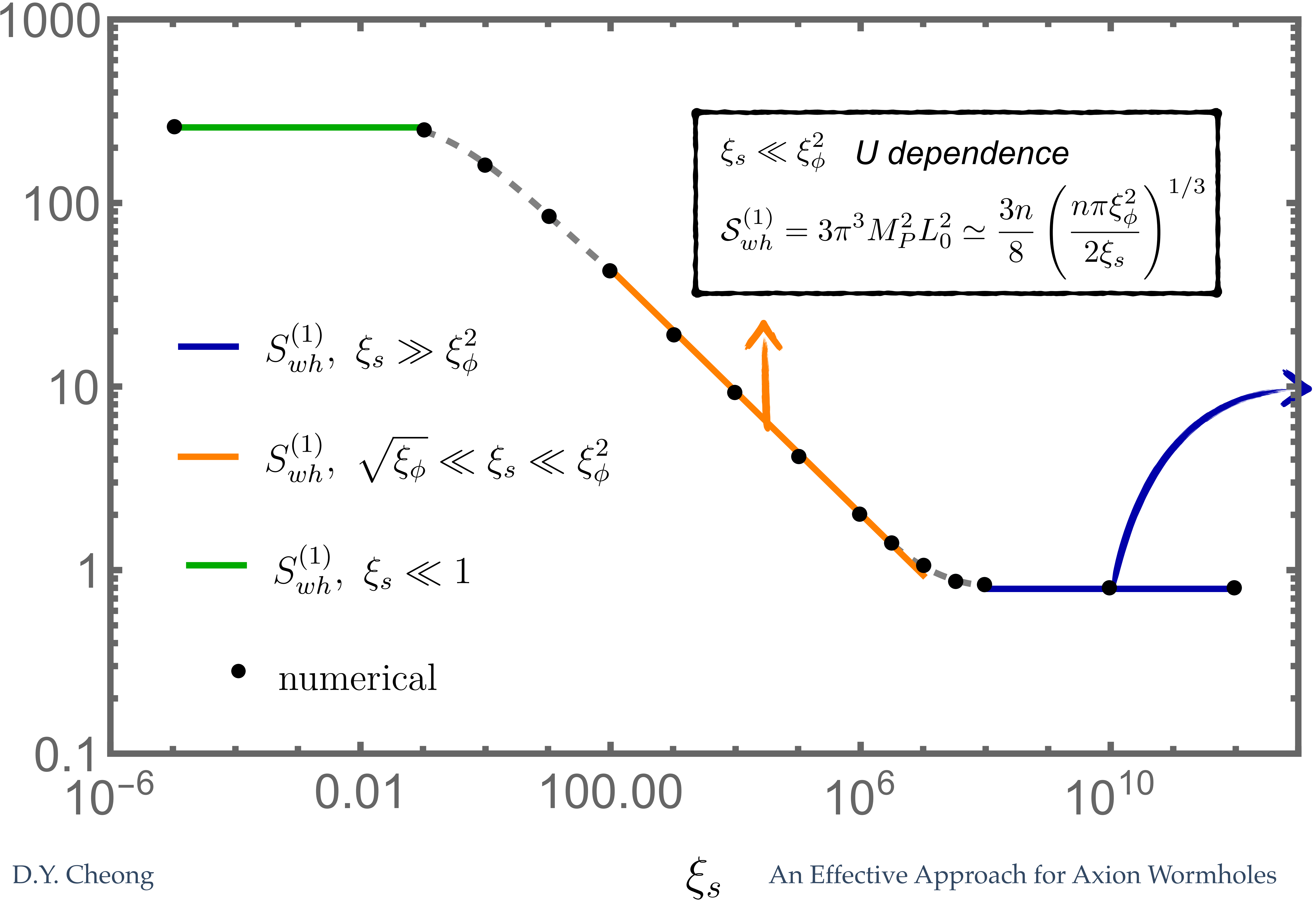
$\xi_s \gg \xi_\phi^2$ ***negligible U, dilaton case***

$$\mathcal{S}_{wh}[n,\phi,\chi] = n\int_\phi^{\phi_0}\frac{d\phi}{\phi}\frac{1}{\sqrt{1-\phi^2/\phi_0^2}}$$
$$\simeq n\ln\left(\frac{2M_P\sin(\frac{\pi\sqrt{6}}{4}\beta)/\beta}{e^{\beta\chi}\phi}\right)$$
$$\sim \mathcal{O}(1)$$

Case Studies - Single Axion + R^2

[DYC, S.C. Park, C.S. Shin, 2310.11260]

$$\xi_s - S_{wh}^{(1)} \;\; \text{relation}, \;\; n=1, \;\; \xi_\phi = 10^4$$



$$U(\chi,\phi) = \frac{M_P^2}{4\xi_s}\left[1-\left(1+\frac{\xi_\phi\phi^2}{M_P^2}\right)e^{-\sqrt{\frac{2}{3}}\frac{\chi}{M_P}}\right]^2$$
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$$\sim \mathcal{O}(1)$$

Summary

Axion wormholes provide a great framework to compute quantum gravity effects on global symmetry breaking

Specifications of wormhole properties highly depend on details of the associated theory

We present an "effective" approach for these axion wormholes, identifying the structure of the action and obtaining powerful analytic control

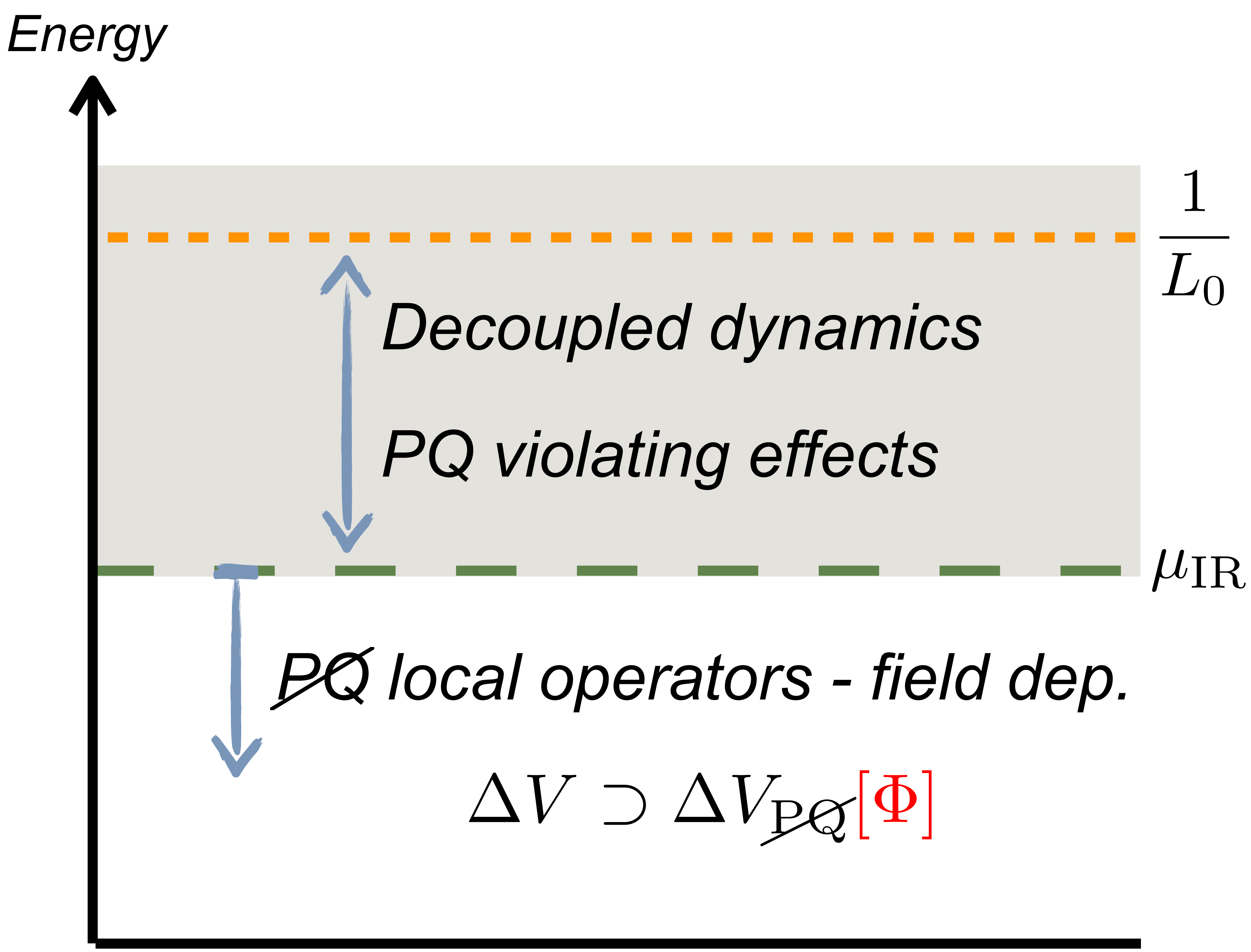
$$\boxed{\begin{aligned}\mathcal{S}_{wh}[n,\phi] &= 3\pi^3 M_P^2 L_0^2 + \int_{\phi_0}^{\phi} d\varphi^A \, p_A(\varphi) \\ &= \mathcal{S}_{\mathbf{UV}} + \mathcal{S}_{\mathbf{IR}}\end{aligned}}$$

Thank you!

Backup Slides

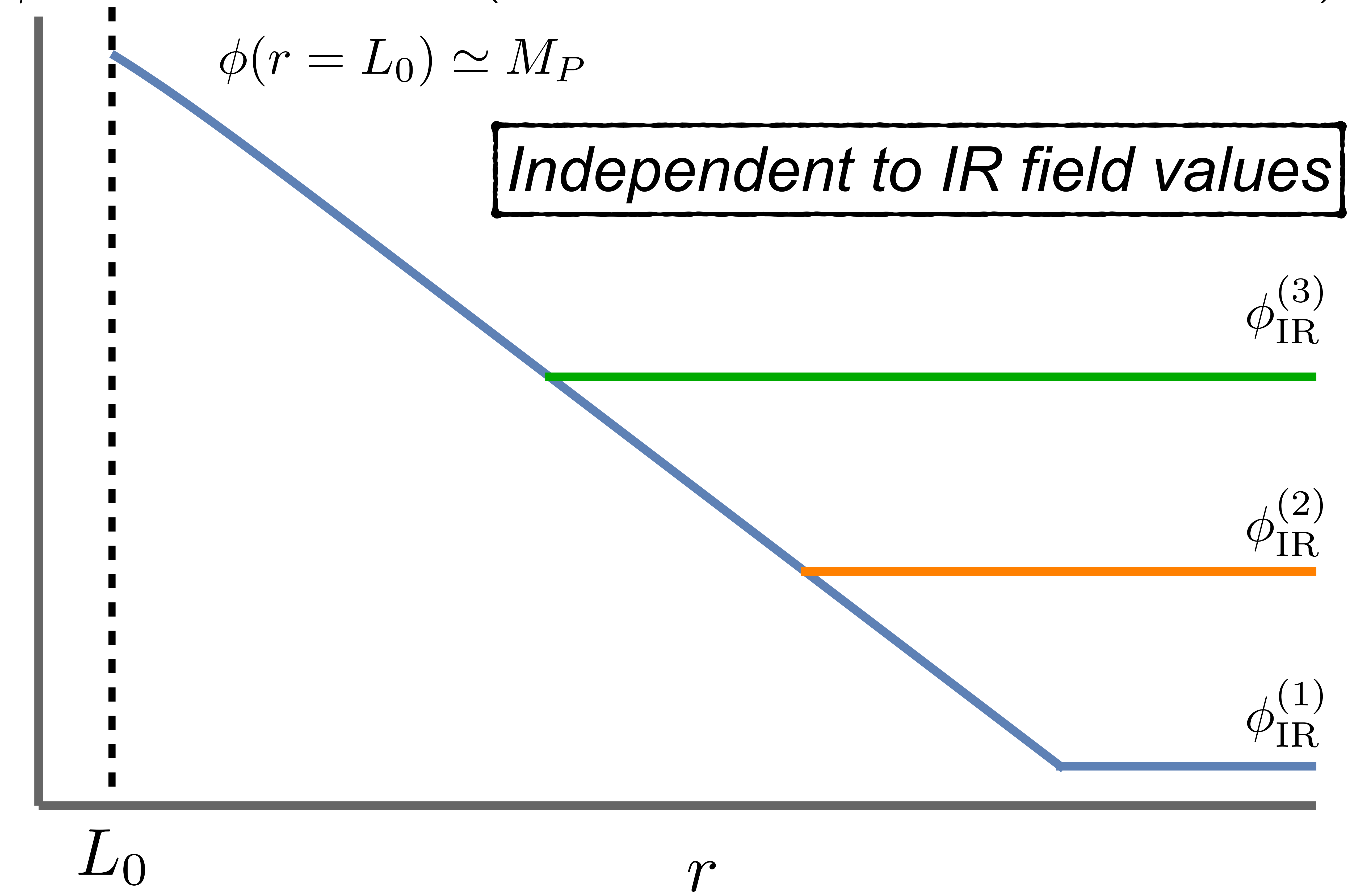
Backup Slides

Scale separation within the wormhole



Example) Dynamical Radial Mode $\Phi(x) = \frac{\phi(x)}{\sqrt{2}} e^{i\theta(x)}$

$$\mathcal{S} = \int d^4x \sqrt{g} \left(-\frac{M_P^2}{2} R + \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} \phi^2 (\partial_\mu \theta)^2 + \ldots \right)$$



Axion Wormholes - Effective Approach

[DYC, S.C. Park, C.S. Shin, 2310.11260]

Wormhole properties "well determined" through the massless limit associated with IR field values

$$S = \int d^4x\sqrt{-g}\left(-\frac{M_P^2}{2}R + \frac{1}{2}G_{\alpha\beta}(\phi)\partial_\mu\phi^\alpha\partial^\mu\phi^\beta + \frac{1}{2}(F^2(\phi))_{IJ}\partial_\mu\theta^I\partial^\mu\theta^J\right)$$

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$\updownarrow$

Dualization

$$H_{I\mu\nu\rho} = (F^2(\phi))_{IJ}\partial_\gamma\theta g^{\gamma\sigma}\epsilon_{\sigma\mu\nu\rho}$$

Wick Rotation

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Wick Rotation

$$\mathcal{S}_E = \int d^4x\sqrt{g}\left(-\frac{M_P^2}{2}R + \frac{1}{2}G_{\alpha\beta}(\phi)\partial_\mu\phi^\alpha\partial^\mu\phi^\beta + \frac{1}{12}(F^{-2}(\phi))^{IJ}H_{I\mu\nu\rho}H_J^{\mu\nu\rho}\right) - i\left(\int d^4x\sqrt{g}\frac{1}{6}\theta^I\epsilon^{\mu\nu\rho\sigma}\partial_\mu H_{I\nu\rho\sigma}\right)$$

Axion Wormholes - Effective Approach

[DYC, S.C. Park, C.S. Shin, 2310.11260]

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$$\mathcal{S}_E=\underbrace{\int d^4x\sqrt{g}\left(-\frac{M_P^2}{2}R+\frac{1}{2}G_{\alpha\beta}(\phi)\partial_\mu\phi^\alpha\partial^\mu\phi^\beta+\frac{1}{12}(F^{-2}(\phi))^{IJ}H_{I\mu\nu\rho}H_J^{\mu\nu\rho}\right)}_{\mathcal{S}_{\rm wh}[n_I,\phi]}-i\left(\int d^4x\sqrt{g}\frac{1}{6}\theta^I\epsilon^{\mu\nu\rho\sigma}\partial_\mu H_{I\nu\rho\sigma}\right)$$

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Wick Rotation

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Axion Wormholes - Effective Approach

[DYC, S.C. Park, C.S. Shin, 2310.11260]

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$$\boxed{\int_{S_3} H_{E3} = \int_{S^3} f^2 \star d\theta_E = q_e \qquad q_e = n_I \in \mathbb{N}}$$

Axion Wormholes - Effective Approach

[DYC, S.C. Park, C.S. Shin, 2310.11260]

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Wick Rotation

$$\mathcal{S}_E = \underbrace{\int d^4x\sqrt{g}\left(-\frac{M_P^2}{2}R + \frac{1}{2}G_{\alpha\beta}(\phi)\partial_\mu\phi^\alpha\partial^\mu\phi^\beta + \frac{1}{12}(F^{-2}(\phi))^{IJ}H_{I\mu\nu\rho}H_J^{\mu\nu\rho}\right)}_{\mathcal{S}_{\rm wh}[n_I,\phi]} \underbrace{-i\left(\int d^4x\sqrt{g}\frac{1}{6}\theta^I\epsilon^{\mu\nu\rho\sigma}\partial_\mu H_{I\nu\rho\sigma}\right)}_{-in_I\theta^I}$$

$$\int_{S_3} H_{E3} = \int_{S^3} f^2 \star d\theta_E = q_e$$

$$q_e = n_I \in \mathbb{N}$$

$$\mathcal{S}_E[\text{wormhole}] = \mathcal{S}_{\rm wh}\left[n_I,\phi\right] - in_I\theta^I$$

Backup Slides- Axion Quality Problem and non-minimal gravity

[**DYC**, K. Hamaguchi, Y. Kanazawa, S.M.Lee, N. Nagata, S.C.Park, 2210.11330]

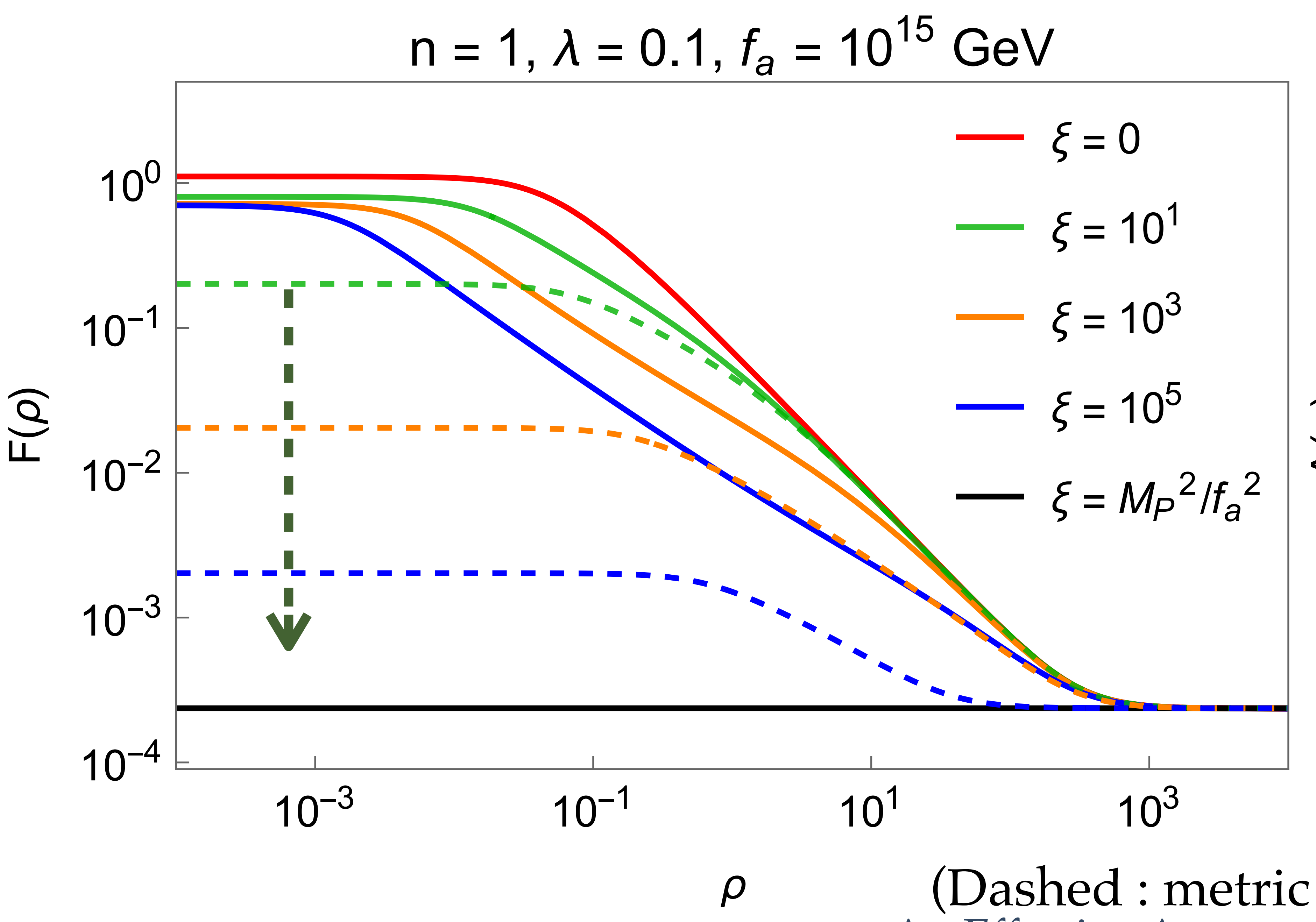
Stationary Solutions $\zeta = 0,\ 1$ (metric, Palatini)

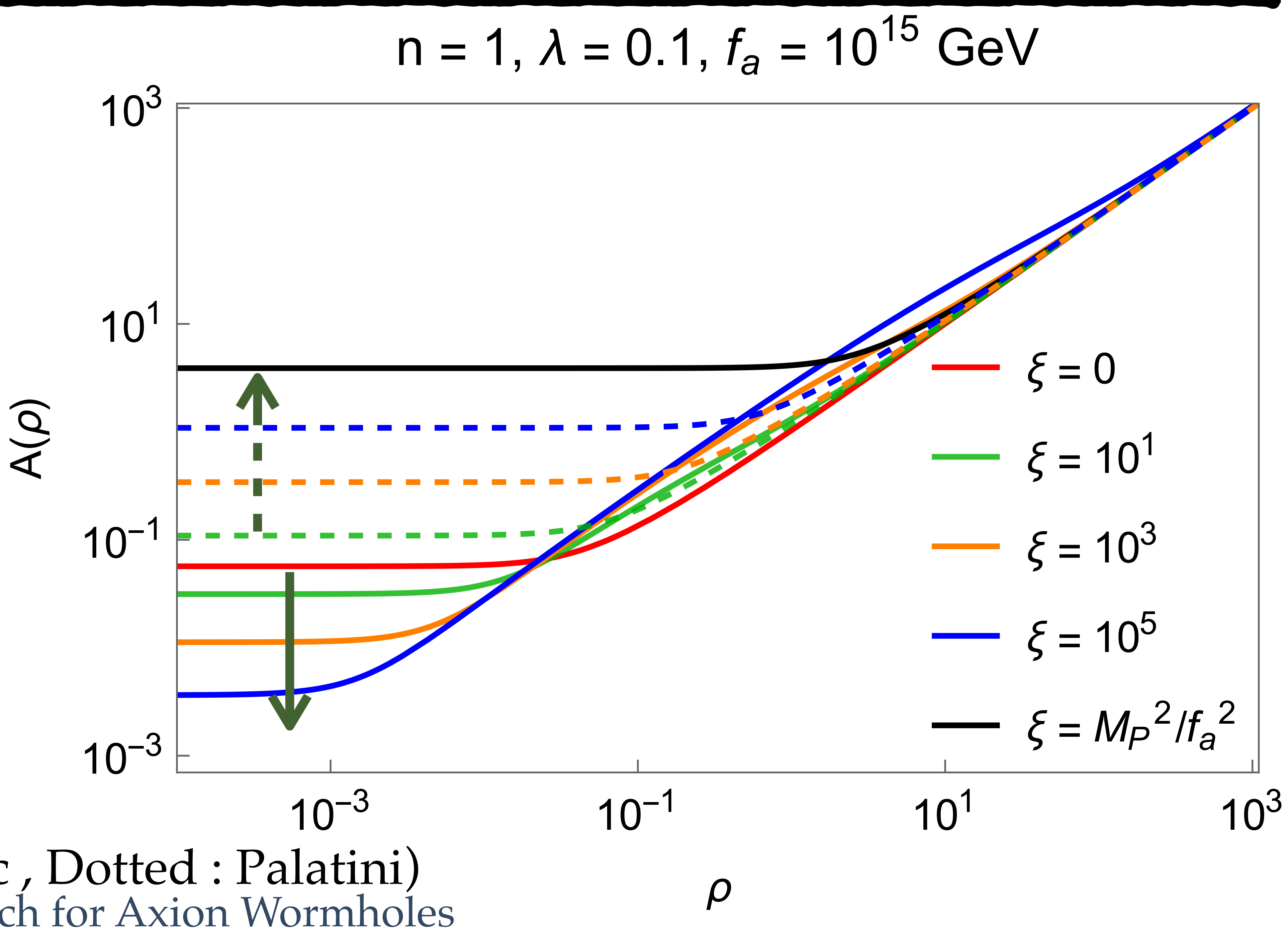
$$\Omega^2\left[R'^2 - 1 + \zeta\left(2RR'\omega' + R^2\omega'^2\right)\right] = -\frac{R^2}{3M_P^2}\left[-\frac{1}{2}f'^2 + V(f) + \frac{n^2}{8\pi^4 f^2 R^6}\right]$$

$$f'' + 3\frac{R'}{R}f' - \frac{dV}{df} + \frac{n^2}{4\pi^4 f^3 R^6} = 6\xi f\left[\frac{R''}{R} + \frac{R'^2}{R^2} - \frac{1}{R^2} + \zeta(\omega'^2 + \omega'' + 3\frac{R'}{R}\omega')\right]$$

Dim-Less parameters $\rho \equiv \sqrt{3\lambda}M_P r,\qquad A \equiv \sqrt{3\lambda}M_P R,\qquad F \equiv f/\sqrt{3}M_P.$

Boundary Conditions $R'(0) = 0,\qquad f'(0) = 0,\qquad f(\infty) = f_a.$





(Dashed : metric , Dotted : Palatini)