The Supersymmetric Gegenbauer's Twin

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Work in progress

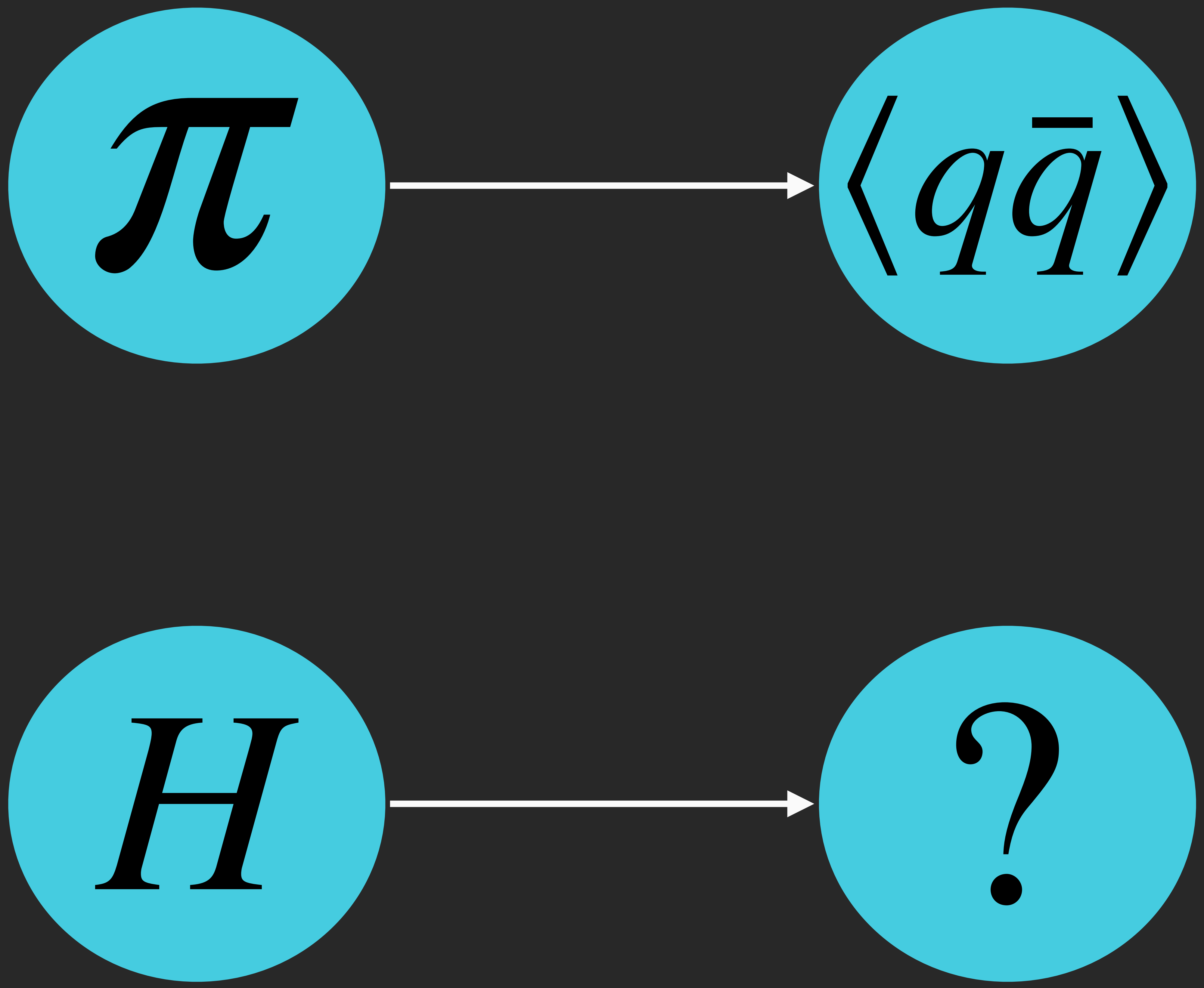
Workshop on Dark Universe
16-19 January 2024
UTOP MARINA RESORT

Gegenbauer Goldstones

In **QCD** pions are light due to an approx $SU(2)_L \times SU(2)_R$

Quark masses are the only sym breaking parameters

Gegenbauer Goldstones



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Can the higgs also be a pNGb?

Naturalness

Example: Breaking of a U(1) global symmetry

$$\mathscr{L} = \left|\partial_\mu \phi\right|^2 - \frac{\lambda}{4}\left(\left|\phi\right|^2 - \frac{f^2}{2}\right)^2$$

Associated one pNGb

An explicit sym breaking in the UV

$$V = \epsilon \frac{\lambda}{f^{q-4}}\phi^q + \mathrm{h.c}$$

Generates the potential

$$V \propto \epsilon m_\rho^2 f^2 \cos\frac{q\Pi}{f}$$

Which is **stable** as a result of symmetry

Naturalness

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UV

$$V = \epsilon \frac{\lambda}{f^{q-4}} \phi^q + \mathrm{h.c}$$

$$V \propto \epsilon m_\rho^2 f^2 \cos \frac{q\Pi}{f}$$

IR

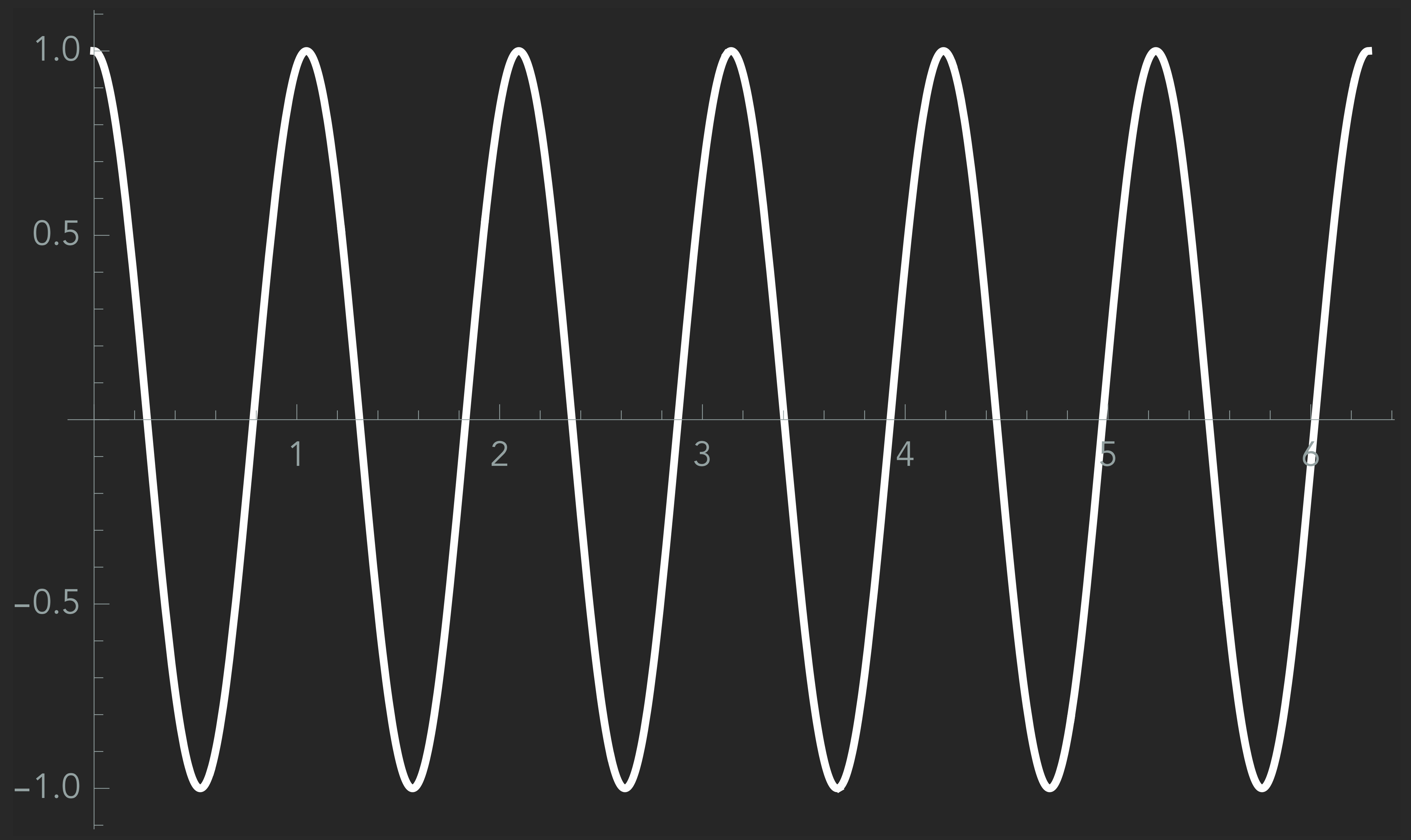
Abelian case

$$V \propto \epsilon m_\rho^2 f^2 \cos\frac{q\Pi}{f}$$

$$\langle\Pi\rangle = f\pi n/q$$

$$m^2 = \epsilon q^2 m_\rho^2$$

$$\lambda = \epsilon q^4 m_\rho^2/f^2$$



Natural small vev and mass

Non-abelian case

$SO(N+1) \longrightarrow SO(N)$

$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\lambda}{4}\left(\phi\cdot\phi - \frac{f^2}{2}\right)^2$$

N massless pNGbs

$$V = \frac{\lambda}{f^{n-4}}\epsilon_{a_1,a_2,\ldots,a_n}\phi^{a_1}\phi^{a_2}\ldots\phi^{a_n}$$

Spurion in a symmetric irrep (**Traceless**)

$$V = \epsilon m_\rho^2 f^2 G_n^{(N-1)/2}\left(\cos\Pi/f\right)$$

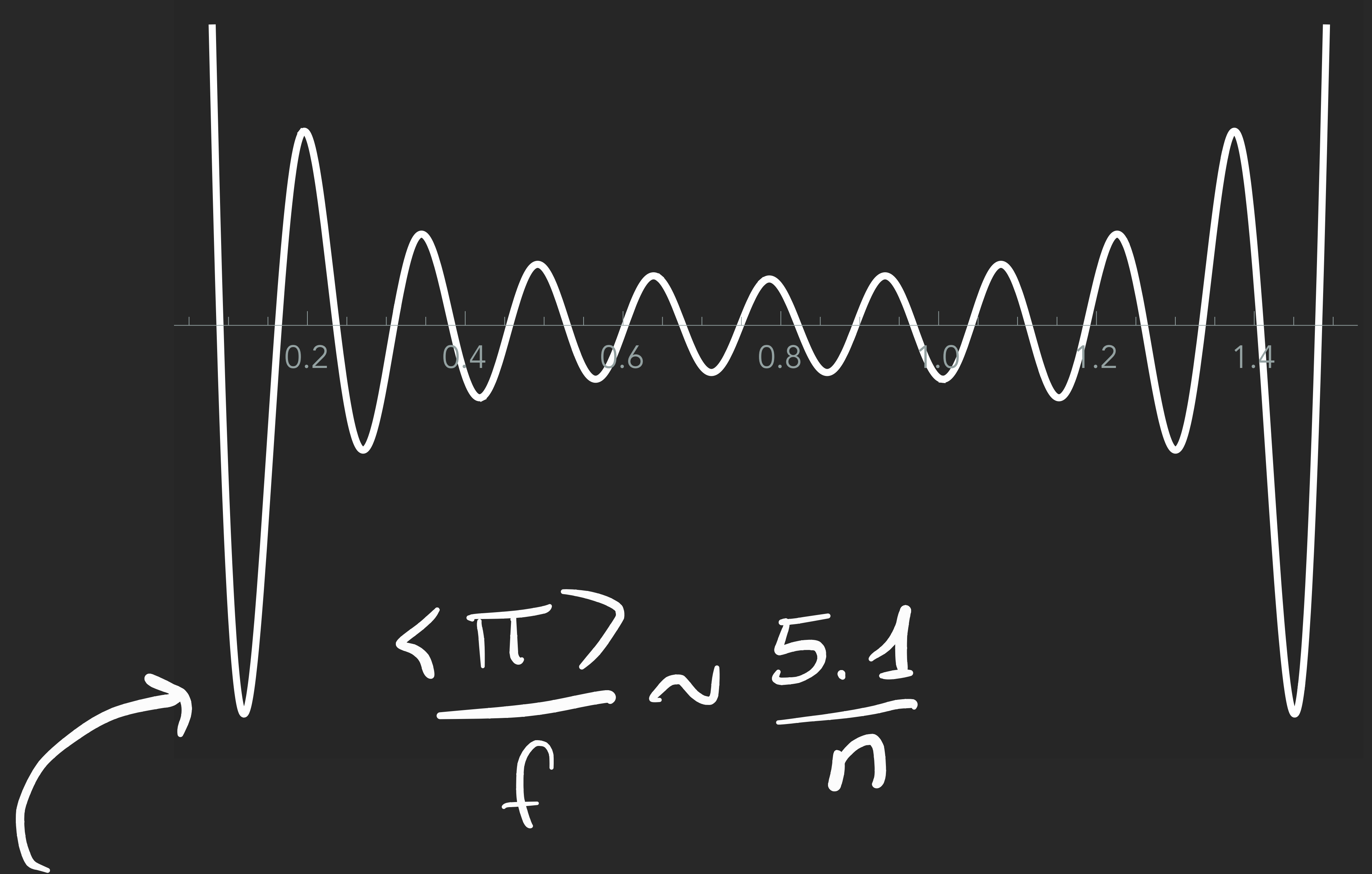
Radiatively stable

Non-abelian case

$$SO(N+1) \longrightarrow SO(N)$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{\lambda}{4}\left(\phi \cdot \phi - \frac{f^2}{2}\right)^2$$

Gegenbauer polynomial



$$V = \frac{\lambda}{f^{n-4}} \epsilon_{a_1, a_2, \ldots, a_n} \phi^{a_1} \phi^{a_2} \ldots \phi^{a_n}$$

Natural small vev

$$V = \epsilon m_\rho^2 f^2 G_n^{(N-1)/2}\left(\cos \Pi / f\right)$$

Gegenbauer as irreps

$$|t\phi - \tilde{\phi}|^{1-N} = \sum_{n=0}^{\infty} t^n K_n^{i_1 i_2 \ldots i_n}\left(\tilde{\phi}\right) \phi^{i_1}\phi^{i_2}\ldots\phi^{i_n}$$

Taylor series

Similar to angular momentum decomposition

$\tilde{\phi}$ gets a vev

$$\tilde{\phi} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ f \end{pmatrix}$$

Gegenbauer as irreps

$$|t\phi - \tilde{\phi}|^{1-N} = \sum_{n=0}^{\infty} t^n K_n^{i_1 i_2 \dots i_n}\left(\tilde{\phi}\right) \phi^{i_1}\phi^{i_2}\dots\phi^{i_n}$$

Taylor expansion on t

$$K_n^{i_1 i_2 \dots i_n}\left(\phi\right) = \frac{1}{n!}\frac{\partial^n \phi^{1-N}}{\partial\phi^{i_1}\partial\phi^{i_2}\dots\partial\phi_{i_n}}$$

$$\phi = \frac{1}{\pi}\sin\left(\frac{\pi}{f}\right)\begin{pmatrix}\pi_1\\ \pi_2\\ \vdots\\ \pi_N\\ \cot\frac{\pi}{f}\end{pmatrix}$$

$$|t\phi - \tilde{\phi}|^{1-N} = \sqrt{1 - 2t\cos\Pi/f + t^2}$$

$$= \sum_{n=0}^{\infty} t^n G_n^{(N-1)/2}\left(\cos\Pi/f\right)$$

Generating function

$$\left(1 - 2tx + t^2\right)^{-\lambda} = \sum_{n=0}^{\infty} t^N G_n^{\lambda}(x)$$

Gegenbauer as irreps

$$|t\phi - \tilde{\phi}|^{1-N} = \sum_{n=0}^{\infty} t^n K_n^{i_1 i_2 \ldots i_n}\left(\tilde{\phi}\right) \phi^{i_1}\phi^{i_2}\ldots\phi^{i_n}$$

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$$\left(1 - 2tx + t^2\right)^{-\lambda} = \sum_{n=0}^{\infty} t^N G_n^{\lambda}(x)$$

Tuning

i) Higgs vev

$$V_T \approx \frac{3y_t^2}{16\pi^2} f^2 M_T^2 \left(a_1 \cos\frac{\Pi}{f} + a_2 \cos\frac{2\Pi}{f} \right)$$

$$\langle \Pi \rangle = 0$$

$$\langle \Pi \rangle \approx n\pi f \qquad n \in \mathbb{Z}$$

Phenomenologically not viable

$$V_G = \epsilon f^2 M^2 G_n^{3/2} \left(\cos\frac{\Pi}{f} \right)$$

Large n allows for natural $v^2 \ll f^2$

ii) Higgs mass

$$V_T \approx \frac{3y_t^2}{16\pi^2} f^2 M_T^2 \left(a_1 \cos\frac{\Pi}{f} + a_2 \cos\frac{2\Pi}{f} \right)$$

$$m_h^2 \sim \frac{3y_t^2}{16\pi^2} M_T^2$$

$M_T \gtrsim 900$ **GeV**, adds additional fine tuning → **Gegenbauer's twin**

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Recap

Gegenbauer's Goldstones → Natural EW scale

Gegenbauer's Twin→ Natural Higgs mass

iii) Couplings

Radiative corrections in the Gegenbauer's Twin

$$V_t \approx \frac{3y_t^4}{64\pi^2} f^4 \left[\sin^4 \frac{h}{f} \log \frac{a}{\sin^2 \frac{h}{f}} + \cos^4 \frac{h}{f} \log \frac{a}{\cos^2 \frac{h}{f}} \right]$$

a is a UV parameter $\quad a \approx \frac{g_\star^2}{y_t^2} \quad M_\star \approx g_\star f$

For a pNGb model, $g_\star \sim 4\pi$, thus, the UV is **strongly coupled**

For the Gegenbauer Higgs, $a \lesssim 2$, the UV is perturbative → **SUSY Gegenbauer's twin**

SUSY Gegenbauer's Twin

$$W = \mu\Phi_u\Phi_d + \lambda S\Phi_u\Phi_d + MS^2$$

$$V = \left(m_{Hu}^2 + \mu^2\right)|\phi_u|^2 + \left(m_{Hd}^2 + \mu^2\right)|\phi_d|^2 - b\left(\phi_u\phi_d + h.c.\right) + \lambda^2|\phi_u\phi_d|^2$$

Z_2 **between MSSM and its mirror $\longrightarrow$ approximate global U(4) symmetry**

The Gegenbauer explicitly breaks SUSY

$$\tan\beta \equiv \frac{\mu^2 + m_{Hd}^2}{\mu^2 + m_{Hu}^2}$$

$$V_G^{(n)} = \epsilon f^2 M^2 F(\beta) G_n^{3/2}\left(\cos\left(\frac{2h}{f}\right)\right)$$

SUSY Gegenbauer's Twin

$$V = \left(m_{Hu}^2 + \mu^2\right)|\phi_u|^2 + \left(m_{Hd}^2 + \mu^2\right)|\phi_d|^2 - b\left(\phi_u\phi_d + h.c.\right) + \lambda^2|\phi_u\phi_d|^2$$

Sources of symmetry breaking

$$V_G^{(n)} = \epsilon f^2 M^2 F(\beta) G_n^{3/2}\left(\cos\left(\frac{2h}{f}\right)\right)$$ Explicit

$$V_{g',g} = \frac{g^2 + g'^2}{8}\left[\left(\left|\phi_u^A\right|^2 - \left|\phi_d^A\right|^2\right)^2 + \left(\left|\phi_u^B\right|^2 - \left|\phi_d^B\right|^2\right)^2\right]$$ D-terms

$$V_t = \frac{3y_t^4}{16\pi^2}\left(\left|\phi_u^A\right|^4 \log\frac{m_{\tilde{t}}^2}{m_t^{A2}} + \left|\phi_u^B\right|^4 \log\frac{m_{\tilde{t}}^2}{m_t^{B2}}\right)$$ Top sector

SUSY contribution to the pNGb potential

$$V_{U(4)} = \frac{3(y_t f \sin\beta)^4}{64\pi^2}\left[\sin^4\left(\frac{h}{f}\right)\log\frac{a}{\sin^2\left(\frac{h}{f}\right)} + \cos^4\left(\frac{h}{f}\right)\log\frac{a}{\cos^2\left(\frac{h}{f}\right)}\right]$$

$$a = \frac{2m_{\tilde{t}}^2}{(y_t f \sin\beta)^2} e^{\frac{2\pi^2\cos^2(2\beta)(g'^2+g^2)}{3(y_t\cos\beta)^4}}$$

$m_{\tilde{t}} \sim \mathcal{O}(f)$: we have a model that is a hybrid between a **SUSY** model and a **pNGb** model

Summary

- ✓ Gegenbauer polynomials are a class of radiatively stable potentials
- ✓ The twin Gegenbauer points to a perturbative coupled UV
- ✓ We provided a SUSY UV completion, valid up to very small scales
- ✓ Since SUSY is perturbative, IR parameters can be calculated

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On-going work: LHC-HL. Precision measurements vs direct searches

Further questions: Origin of the explicit sym. breaking?
Cosmology (baryogenesis, phase transitions…)