RG study of U(1) models for muon $g-2$ and W boson mass

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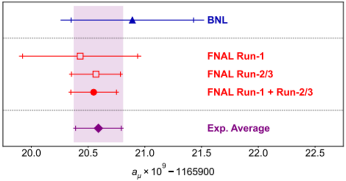
Chung-Ang University

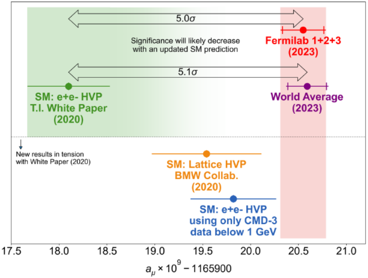
18/01/2024

Muon $g-2_\mu$ anomaly

Muon $g-2$ collaboration



J. Mott, Fermilab

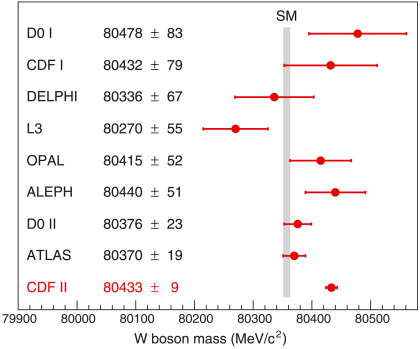


W boson mass

$M_w^{CDFII} = 80.4335$ GeV ± 8.4 MeV

CDF collaboration

$\longrightarrow 7\sigma$ discrepancy with SM estimate $M_w^{SM} = 80.357$ GeV ± 6 MeV



Content of the model

New ingredients

H.M. Lee, K. Yamashita

- $U(1)'$ gauge group
- Enlarged scalar sector
- Vector-like lepton

	q_L	u_R	d_R	l_L	e_R	H	H'	E_L	E_R	ϕ
$U(1)'$	0	0	0	0	0	0	$+2$	-2	-2	-2

$$\mathcal{L} = -\frac{1}{4}W^a_{\mu\nu}W^{a\,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} - \frac{\sin\xi}{2}F'_{\mu\nu}B^{\mu\nu} + |D_\mu\phi|^2 + |D_\mu H'|^2 + |D_\mu H|^2 - V(\phi, H', H) + \mathcal{L}_{\rm fermions}$$

with

$$\mathcal{L}_{\rm fermions} = \sum_{i={\rm SM},E} i\bar\psi_i\gamma^\mu D_\mu\psi_i - y_d\bar q_L d_R H - y_u\bar q_L u_R\widetilde{H} - y_l\bar l_L e_R H - M_E\bar E E - \lambda_E\phi\bar E_L e_R - y_E\bar l_L E_R H' + {\rm h.c.}$$

Potential

$$
\begin{aligned}
V(\phi, H, H') &= \mu_1^2 H^\dagger H + \mu_2^2 H'^\dagger H' - (\mu_3 \phi H^\dagger H' + \mathrm{h.c.}) \\
&+ \lambda_1 (H^\dagger H)^2 + \lambda_2 (H'^\dagger H')^2 + \lambda_3 (H^\dagger H)(H'^\dagger H') \\
&+ \mu_\phi^2 \phi^* \phi + \lambda_\phi (\phi^* \phi)^2 + \lambda_{H\phi} H^\dagger H \phi^* \phi + \lambda_{H'\phi} H'^\dagger H' \phi^* \phi
\end{aligned}
$$

$+\lambda_4 (H^\dagger H')(H'^\dagger H)$: no changes for analysis

Rich vacuum structure: “normal”, CP-breaking, charge breaking minima

In 2HDM:

- At tree-level, minima of a different nature cannot simultaneously coexist
- stability of a 2HDM vacuum against tunneling to another vacuum of a different nature P.M. Ferreira, R. Santos, A. Barroso / I.P. Ivanov

"Panic vacuum": τ determines exclusion V. Branchina, F. Contino, P.M. Ferreira

Stability conditions

Stability: Copositive definite quartic coupling matrix $\mathcal{M}_\lambda$

$$\mathcal{M}_\lambda = \begin{pmatrix} \lambda_1 & \frac{\lambda_3}{2} & \frac{\lambda_{H\phi}}{2} \\ \frac{\lambda_3}{2} & \lambda_2 & \frac{\lambda_{H'\phi}}{2} \\ \frac{\lambda_{H\phi}}{2} & \frac{\lambda_{H'\phi}}{2} & \lambda_\phi \end{pmatrix}$$

$$\begin{cases} \lambda_1 \geq 0, \qquad \lambda_2 \geq 0, \qquad \lambda_\phi \geq 0 \\ \lambda_3 + 2\sqrt{\lambda_1\lambda_2} \geq 0, \qquad \lambda_{H\phi} + 2\sqrt{\lambda_1\lambda_\phi} \geq 0, \qquad \lambda_{H'\phi} + 2\sqrt{\lambda_2\lambda_\phi} \geq 0 \end{cases}$$

and

$$\sqrt{\lambda_1\lambda_2\lambda_\phi} + \frac{\lambda_3}{2}\sqrt{\lambda_\phi} + \frac{\lambda_{H\phi}}{2}\sqrt{\lambda_2} + \frac{\lambda_{H'\phi}}{2}\sqrt{\lambda_1} + \frac{1}{2}\sqrt{(\lambda_3 + 2\sqrt{\lambda_1\lambda_2})(\lambda_{H\phi} + 2\sqrt{\lambda_1\lambda_\phi})(\lambda_{H'\phi} + 2\sqrt{\lambda_2\lambda_\phi})} \geq 0.$$

We will restrict in the analysis to background configurations with only a CP-even, neutral background

Before going to the RG analysis, let's see how the model addresses $g-2_\mu$ and m_w

Lepton seesaw

In a CP-even, neutral vacuum ($v_1 = v\cos\beta$, $v_2 = v\sin\beta$, v_ϕ)

$$\mathcal{L}_m = -M_E\bar{E}E - m_0\bar{e}e - \left(m_R\bar{E}_L e_R + m_L\bar{e}_L E_R + h.c.\right)$$

$m_0 = \frac{1}{\sqrt{2}}y_l v_1, \quad m_R = \frac{1}{\sqrt{2}}\lambda_E v_\phi, \quad m_L = \frac{1}{\sqrt{2}}y_E v_2$

Squared mass matrix

$$\mathcal{M}_L^\dagger\mathcal{M}_L = \begin{pmatrix} m_0^2 + m_R^2 & m_0 m_L + m_R M_E \\ m_0 m_L + m_R M_E & m_L^2 + M_E^2 \end{pmatrix}$$

Assuming $M_E \gg m_0, m_R, m_L$, eigenvalues reduce to:

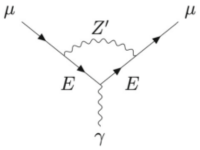
$$m_1 \sim M_E, \qquad m_2 \sim m_0 - \frac{m_R m_L}{M_E}$$

Bi-unitary diagonalization

- $\sin(2\theta_R) \sim \frac{2m_R}{M_E}$ $\sin(2\theta_L) \sim \frac{2m_L}{M_E}$

$g-2_\mu$

Dominant new physics contribution



Depends, among others, on $M_E/m_{Z'}$. Easily readable limits:

$$\Delta a_\mu \simeq \begin{cases} \frac{g_{Z'}^2 M_E m_\mu}{16\pi^2 m_{Z'}^2}(c_V^2 - c_A^2), & M_E \gg m_{Z'} (\gg m_\mu), \\ \frac{g_{Z'}^2 M_E m_\mu}{4\pi^2 m_{Z'}^2}(c_V^2 - c_A^2), & m_\mu \ll M_E \ll m_{Z'}. \end{cases}$$

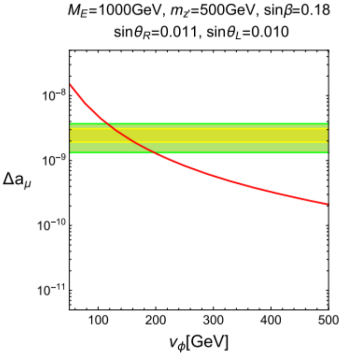
$$c_V = \frac{1}{2}(\sin 2\theta_R + \sin 2\theta_L) \qquad > \qquad c_A = \frac{1}{2}(\sin 2\theta_R - \sin 2\theta_L)$$

$g-2_\mu$

In the $M_E \gg m_0, m_R, m_L$ limit

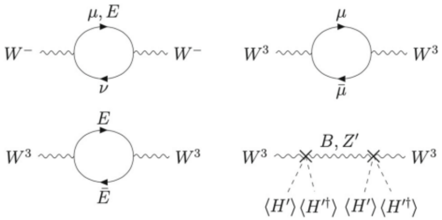
$$c_V^2 - c_A^2 \sim 4\frac{m_\mu}{M_E}$$

Whatever the $M_E - m_{Z'}$ hierarchy, Δa_μ is $(\sim)$ M_E independent!



W boson

New physics contribution



$$\Delta M_W \simeq \frac{1}{2} M_W \frac{s_W^2}{c_W^2 - s_W^2}\left(1 - \cos\theta_L - (\Delta r)_{\rm new}\right),$$

related to corrections to ρ parameter

$$(\Delta r)_{\rm new} = -\frac{c_W^2}{s_W^2}(\underbrace{\Delta\rho_L}_{\text{Loop}} + \underbrace{\Delta\rho_H}_{\text{Tree}}).$$

W boson

For $M_E \gtrsim M_Z$

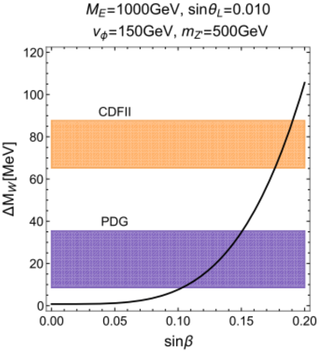
$$\Delta\rho_L \simeq \frac{\alpha M_E^2}{16\pi s_W^2 c_W^2 M_Z^2}\,\sin^4\theta_L$$

From $Z - Z'$ mass mixing

$$\Delta\rho_H \simeq \begin{cases} \frac{16 s_W^2 g_{Z'}^2}{g_Y^2}\frac{M_Z^2}{m_{Z'}^2}\,\sin^4\beta, & m_{Z'} \gg M_Z, \\ -\frac{16 s_W^2 g_{Z'}^2}{g_Y^2}\,\sin^4\beta, & m_{Z'} \ll M_Z. \end{cases}$$

$\Longrightarrow$ To have a positive correction: $m_{Z'} \gg M_Z$

Results for W boson



UV-fate of the model

Set-up: Wilsonian RG

CP-even neutral background configuration

Perturbative setting ... a few manipulations

Using a hard cutoff Λ

- Operating with $\Lambda \frac{d}{d\Lambda}$ we connect to Wilsonian RGEs in UV regime $k^2 \gg m_k^2$ of the flow

Ex: $4D$ scalar theory in LPA $S_k[\phi] = \int d^4x \frac{1}{2}(\partial\phi)^2 + U_k(\phi)$

- Wegner-Houghton equation F.J. Wegner, A. Houghton

$$k\frac{\partial}{\partial k}U_k(\phi) = -\frac{k^4}{16\pi^2}\log\left(\frac{k^2+U_k''(\phi)}{k^2+U_k''(0)}\right)$$

ϕ^4

ϕ^4 truncation: $U_k(\phi) = \frac{1}{2} m_k^2 \phi^2 + \frac{\lambda_k}{4!}\phi^4$

$$\begin{cases} k\frac{\partial}{\partial_k} m_k^2 = -\frac{k^4}{16\pi^2}\frac{\lambda_k}{k^2+m_k^2} \\ k\frac{\partial}{\partial_k}\lambda_k = \frac{3k^4}{16\pi^2}\frac{\lambda_k^2}{(k^2+m_k^2)^2} \end{cases}$$

UV-regime $k^2 \gg m_k^2$

$$\begin{cases} k\frac{\partial}{\partial_k} m_k^2 = -\frac{\lambda_k}{16\pi^2}k^2 + \frac{\lambda_k}{16\pi^2}m_k^2 \\ k\frac{\partial}{\partial_k}\lambda_k = \frac{3\lambda_k^2}{16\pi^2} \end{cases}$$

Other regimes:

- IR stable regime $m_k^2 > 0$, $m_k^2 \gg k^2 \to$ Running $\sim$ frozen
- IR unstable regime $m_k^2 < 0$, when $m_k^2 + k^2 \to 0^+$ instability, system enters a spinodal region

J. Alexandre, V. Branchina, J. Polonyi

Quadratic running and physical tuning

Callan-Symanzik $\rightarrow$ renormalized running

- Obtained forcing system close to critical surface of Gaussian FP

Full running: approximate solution (freezing $\lambda = \lambda_\Lambda$) C.B., V. Branchina, F. Contino

Excellent approximation (see numerical): λ_k runs very slowly

$$m_\mu^2 = \left(\frac{\mu}{\Lambda}\right)^{\frac{\lambda}{16\pi^2}} \underbrace{\left(m_\Lambda^2 + \frac{\lambda\Lambda^2}{32\pi^2-\lambda}\right)}_{\text{“Physical” Fine-tuning}} - \underbrace{\frac{\lambda\mu^2}{32\pi^2-\lambda}}_{\text{Quadratic running}}$$

1. Quadratic until $\left(m_\Lambda^2 + \frac{\lambda\Lambda^2}{32\pi^2-\lambda}\right) \sim \frac{\lambda k^2}{32\pi^2-\lambda} \rightarrow (\sim)$ frozen
2. **Fine-tuning** in the Wilsonian framework:

   determines when & at which value the running freezes

   To get $m_{\mu_F}^2 \sim 125$ GeV $\longleftrightarrow m_\Lambda^2 + \frac{\lambda\Lambda^2}{32\pi^2-\lambda} \sim \mathcal{O}(100)$ GeV
3. Shows progressive **dynamical dressing** of the mass from quantum fluctuations: it connects $m_{\mu_F}^2$ and m_Λ^2

Subtraction: explicit example

$$m_\mu^2 = \left(\frac{\mu}{\Lambda}\right)^{\frac{\lambda}{16\pi^2}} \left(m_\Lambda^2 + \frac{\lambda\Lambda^2}{32\pi^2-\lambda}\right) - \frac{\lambda\mu^2}{32\pi^2-\lambda}$$

Define $\widetilde{m}_\mu^2 = m_\mu^2 + \frac{\lambda\mu^2}{32\pi^2-\lambda} \Rightarrow \widetilde{m}_\mu^2 = \left(\frac{\mu}{\Lambda}\right)^{\frac{\lambda}{16\pi^2}} \widetilde{m}_\Lambda^2$ $\gamma \equiv \lambda/16\pi^2$ 1/ mass anomalous dimension

$$\mu\frac{d}{d\mu}\widetilde{m}_\mu^2 = \gamma\widetilde{m}_\mu^2$$

- Renormalized mass m_r^2 = deviation from criticality!
- m_r^2 multiplicative renormalization $\leftrightarrow$ universality of d.f.c.
  $\leftrightarrow$ d.f.c. protected: runs slowly, $\sim$ the same at all scales

Quasi FP-behaviour $\frac{m_k^2}{k^2} = -\frac{\lambda}{32\pi^2-\lambda}$
NH problem: How is critical region reached?

Standard Model and Higgs mass

$$\mu \frac{d}{d\mu} m_H^2 = \frac{\alpha(\mu)}{16\pi^2}\mu^2 + \gamma(\mu) m_H^2$$

$$16\pi^2\alpha(\mu) = 12y_t - 12\lambda - \tfrac{3}{2}g_1^2 - \tfrac{9}{2}g_2^2, \qquad 16\pi^2\gamma(\mu) = 6y_t + 12\lambda - \tfrac{3}{2}g_1^2 - \tfrac{9}{2}g_2^2$$

Excellent analytic approximate solution

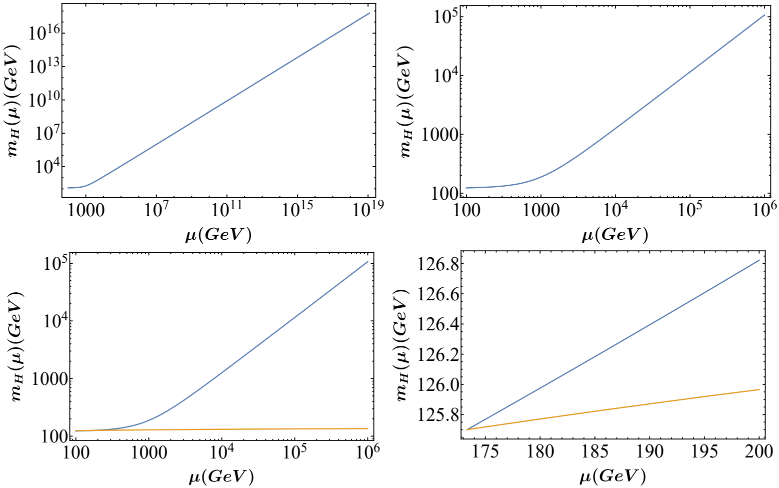
$$m_\mu^2 = \left(\frac{\mu}{\Lambda}\right)^\gamma \underbrace{\left(m_H^2(\mu) - \frac{\alpha\Lambda^2}{2-\gamma}\right)}_{\text{“Physical” Fine-tuning}} + \underbrace{\frac{\alpha\mu^2}{2-\gamma}}_{\text{Quadratic running}}$$

Critical mass: $m_{H,\,\mathrm{cr}}^2(\mu) = \frac{\alpha\mu^2}{2-\gamma}$, Subtracted mass: $\widetilde{m}_H^2(\mu) = m_H^2(\mu) - \frac{\alpha\mu^2}{2-\gamma}$

$$\mu\frac{d}{d\mu}\widetilde{m}_H^2(\mu) = \gamma\widetilde{m}_H^2(\mu)$$

Numerical solution and analytical approximation

additive $\rightarrow$ **multiplicative** renormalization



Back to the model

Two things we are going to use

- When running, check the whole "physical tuning": verify model-independence
- Threshold corrections: implement freezing for $m_k^2 \gtrsim k^2$

Further example: T.E. Clark, B. Haeri, S.T. Love / M. Maggiore

$$S_k = \int d^4x \frac{1}{2}(\partial\phi)^2 + \bar{\psi}\partial\!\!\!/\psi + \frac{1}{2}m_k^2\phi^2 + \frac{\lambda_k}{4!}\phi^4 + M_k\bar{\psi}\psi + g_k\bar{\psi}\psi\phi$$

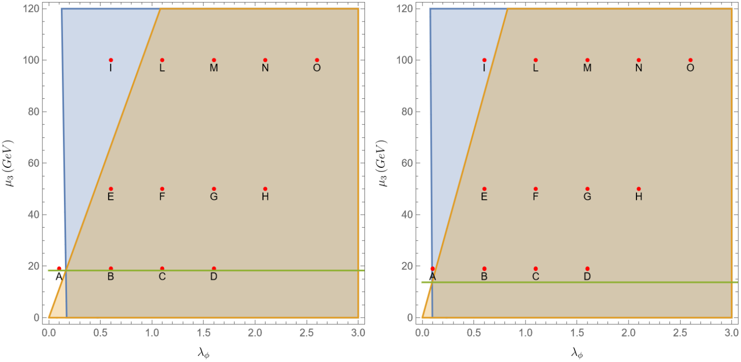
$$\begin{aligned}
k\frac{\partial}{\partial k}m_k^2 &= -\frac{k^4}{16\pi^2}\frac{\lambda_k}{k^2+m_k^2} + \frac{k^4}{2\pi^2}\frac{g_k^2}{k^2+M_k^2} - \frac{k^4}{\pi^2}\frac{g_k^2 M_k^2}{k^2+M_k^2} \\
k\frac{\partial}{\partial k}\lambda_k &= \frac{3k^4}{16\pi^2}\frac{\lambda_k^2}{(k^2+m_k^2)^2} - \frac{k^4}{\pi^2}\frac{3g_k^4(k^4-6k^2M_k^2+M_k^4)}{(k^2+M_k^2)^4} \\
k\frac{\partial}{\partial k}M_k &= \frac{k^4}{8\pi^2}\frac{g_k^2 M_k}{(k^2+m_k^2)(k^2+M_k^2)} \\
k\frac{\partial}{\partial k}g_k &= \frac{k^4}{8\pi^2}\frac{g_k^3(k^2-M_k^2)}{(k^2+m_k^2)(k^2+M_k^2)^2}
\end{aligned}$$

First tentative flow

- Running using the previous results as IR boundary conditions
  1. Decoupling ϕ scenario $\rightarrow \lambda_{H\phi}$ and $\lambda_{H'\phi}$ fixed;
  2. Aligned scenario: 2×2 scalar $\mathcal{M}^2$ diagonalized with angle β

Argument: Symmetry enhancement A. Pilaftsis

In this scenario $\lambda_1 = \lambda_2 = \frac{\lambda_3}{2} = \lambda_{SM}$: only 2 free parameters



Peculiarity: t couples only to $h_1 \rightarrow y_t$ enhanced

Results decoupled and aligned scenario

Surprise: Extremely low Landau pole!

$v_\phi = \sqrt{2} \times 150$ GeV			
λ_ϕ	μ_3 (GeV)	Singularity (GeV)	Perturbativity (GeV)
0.1	19	3.09×10^4	1.05×10^4
0.6	19	2.96×10^4	1.02×10^4
1.1	19	2.73×10^4	9.69×10^3
1.6	19	2.40×10^4	9.13×10^3
0.6	50	2.83×10^4	9.55×10^3
1.1	50	2.60×10^4	9.08×10^3
1.6	50	2.28×10^4	8.64×10^3
2.1	50	1.90×10^4	7.92×10^3
0.6	99	2.61×10^4	8.59×10^3
1.1	99	2.37×10^4	8.06×10^3
1.6	99	2.07×10^4	7.55×10^3
2.1	99	1.73×10^4	7.00×10^3
2.6	99	1.40×10^4	6.42×10^3

Results decoupled and aligned scenario 2

Same...

$v_\phi = \sqrt{2} \times 200$ GeV			
λ_ϕ	μ_3 (GeV)	Singularity (GeV)	Perturbativity (GeV)
0.1	19	6.36×10^5	2.23×10^5
0.6	19	5.40×10^5	1.94×10^5
1.1	19	3.57×10^5	1.54×10^5
1.6	19	1.73×10^5	8.72×10^4
0.6	50	4.84×10^5	1.72×10^5
1.1	50	3.17×10^5	1.35×10^5
1.6	50	1.56×10^5	7.91×10^4
2.1	50	6.16×10^4	3.09×10^4
0.6	100	3.89×10^5	1.36×10^5
1.1	100	2.51×10^5	1.05×10^5
1.6	100	1.28×10^5	6.53×10^4
2.1	100	5.41×10^4	2.73×10^4
2.6	100	2.60×10^4	1.31×10^4

Landau pole

TeV scale Landau pole is unacceptable

Problem: too many and too large bosonic contributions

- Small y_E, λ_E necessary to seesaw limit (Also constrained by Z decay widths; values taken are close to upper bound)
- $m_{z'} = 500$ GeV $\to g_{Z'} \sim 1.15\,(v_\phi = 150$ GeV), $g_{Z'} \sim 0.87\,(v_\phi = 200$ GeV)

$+$ a bunch of other bosonic contributions...

$$\mu \frac{d}{d\mu} g_{z'}(\mu) = \frac{28}{48\pi^2} g_{z'}^3(\mu) \to g_{z'}(\mu) = \frac{1}{\sqrt{\frac{1}{g_{z'}^2(m_{z'})} - \frac{7}{6\pi^2}\log\left(\frac{\mu}{m_{z'}}\right)}}$$

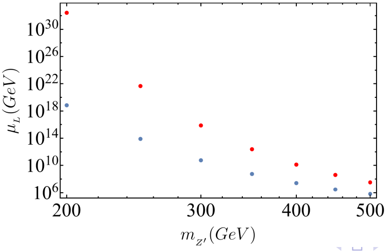
and solving for $g_{z'}(m_{z'}) = 1.15$ we find a Landau pole at

$$\mu_L = m_{z'} e^{\frac{6\pi^2}{7 g_{z'}(m_{z'})}} = 2.99 \times 10^5 \text{ GeV}$$

(For $g_{z'}(m_{z'}) = 0.87 \to 3.57 \times 10^7$ GeV)

Lowering $m_{Z'}$

$v_\phi = \sqrt{2} \times 200$ GeV, $\lambda_\phi = 0.1$, $\mu_3 = 19$ GeV		
$m_{Z'}$ (GeV) $\to g_{Z'}$	Singularity (GeV)	Perturbativity (GeV)
$500 \to 0.87$	6.36×10^5	2.23×10^5
$450 \to 0.79$	2.94×10^6	1.06×10^6
$400 \to 0.70$	2.49×10^7	9.28×10^6
$350 \to 0.61$	5.49×10^8	2.14×10^8
$300 \to 0.52$	5.68×10^{10}	2.37×10^{10}
$250 \to 0.44$	7.89×10^{13}	3.68×10^{13}
$200 \to 0.35$	6.87×10^{18}	3.97×10^{18}



So, the Landau pole problem can be solved lowering $m_{z'}$ to $m_{z'} \lesssim 200$ GeV to have an at least Planckian pole.

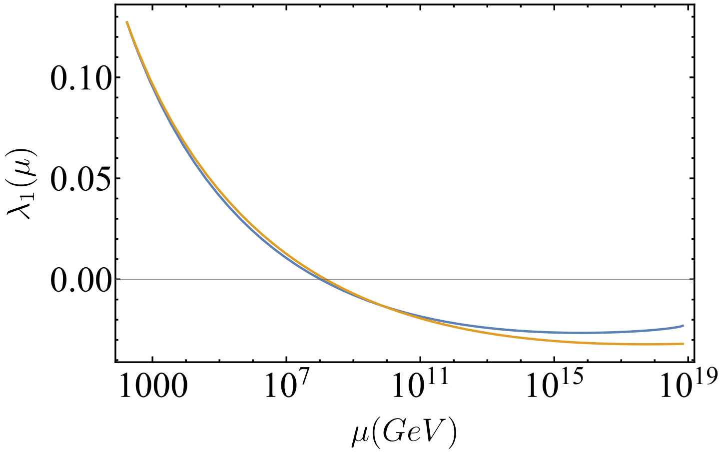
It means abandoning the idea of a simultaneous solution

Note: WGC with several U(1) gauge fields $\rightarrow \Lambda \lesssim \min_i g_i M_p$

C. Cheung, G. Remmen / K. Benakli, C.B., G. Lafforgue-Marmet

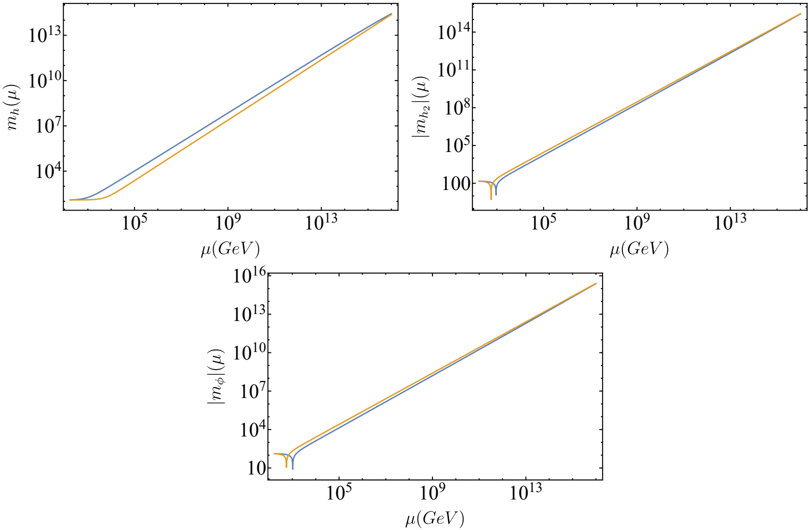
Stability

Slightly worse than SM



The other stability conditions are all respected for $\lambda_1(\mu) \geq 0$

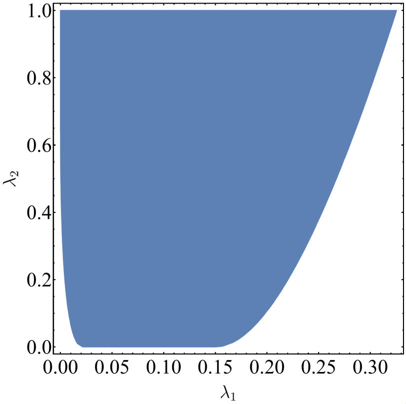
"Physical tuning" scenario



Breaking the alignment

Address stability through small breaking of alignment

- Relax $\lambda_1 = \lambda_2 = \lambda_3/2$; 5 free parameters
- Impose $\lambda_- = m_h^2$ on lowest $\mathcal{M}_s^2$ eigenvalue
  $\to \lambda_3 = \lambda_3(\lambda_1, \lambda_2, \lambda_\phi, \mu_3)$: 2 solutions
- Of course, growing $\lambda_1 \to$ stabilize; $|\lambda_3|$ grows, pole decreases

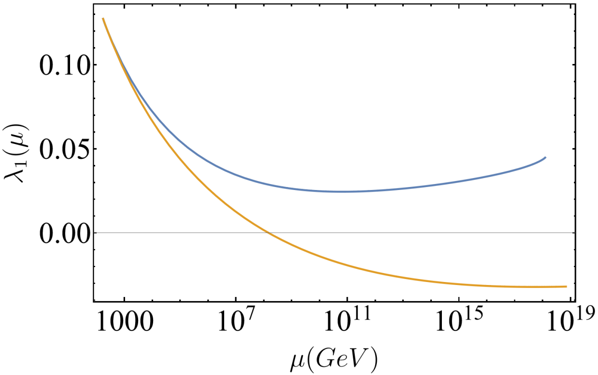


Aligned back again

... But ... surprise!

Second λ_3 solution: model can be aligned without $\lambda_1 = \lambda_2 = \lambda_3/2$!
With this second solution

- Stable potential
- Landau pole slightly lower: 1.25×10^{18} GeV



Conclusions

RG study of the model reveals

- Impossibility of explaining simultaneously $g-2_\mu$ and m_W without occurring in a low Landau pole
- Planckian Landau pole requires a sufficiently light $g_{Z'} \lesssim 0.3$
- A stable potential can be found in the alignment limit
- Verifies the physical tuning scenario
- In general, attention must be paid with these BSM models: keeping the couplings at bay requires precise balance
- Although naively additional bosons $\to$ improved stability, attention must be paid when enlarging the scalar sector requires larger y_t
- Related to recent remarks, we have also seen the connection between threshold corrections in perturbative setting and Wilsonian RG S. Kanemura, Y. Mura