How fast does the WAGGO ?

Yonsei-Konkuk-Sogang Mini-workshop

Jorinde van de Vis 07/02/2025

Program

Friday 07/02

- Introduction & motivation for wall velocity JvdV
- Equilibrium thermodynamics, example computation, nucleation, matrix elements PS
- Example of `DRalgo` and `WallGoMatrix` model files PS

Saturday 08/02

- `WallGo` source code JvdV
- Worked out example for `WallGo` JvdV & PS:
  Matrix elements
  Example base file
  Collision model
  Model file
- Hands-on session:
  start of implementation of `WallGo` model

Monday 10/02

- Discussion of configuration parameters and convergence JvdV
- Hands-on session:
  implementation of `WallGo` model & Q&A

Program

Friday 07/02

- Introduction & motivation for wall velocity JvdV
- Equilibrium thermodynamics, example computation, nucleation, matrix elements PS
- Example of `DRalgo` and `WallGoMatrix` model files PS

Saturday 08/02

- `WallGo` source code JvdV
- Worked out example for `WallGo` JvdV & PS:
  Matrix elements
  Example base file
  Collision model
  Model file
- Hands-on session:
  start of implementation of `WallGo` model

Monday 10/02

- Discussion of configuration parameters and convergence JvdV
- Hands-on session: implementation of `WallGo` model & Q&A

Choose your favorite model and make sure you know its effective potential and nucleation temperature *before* Saturday

Wall velocity: motivation

What is the bubble wall velocity?

- Cosmological phase transition: field with temperature-dependent effective potential transitions from high-temperature vacuum to low-temperature vacuum

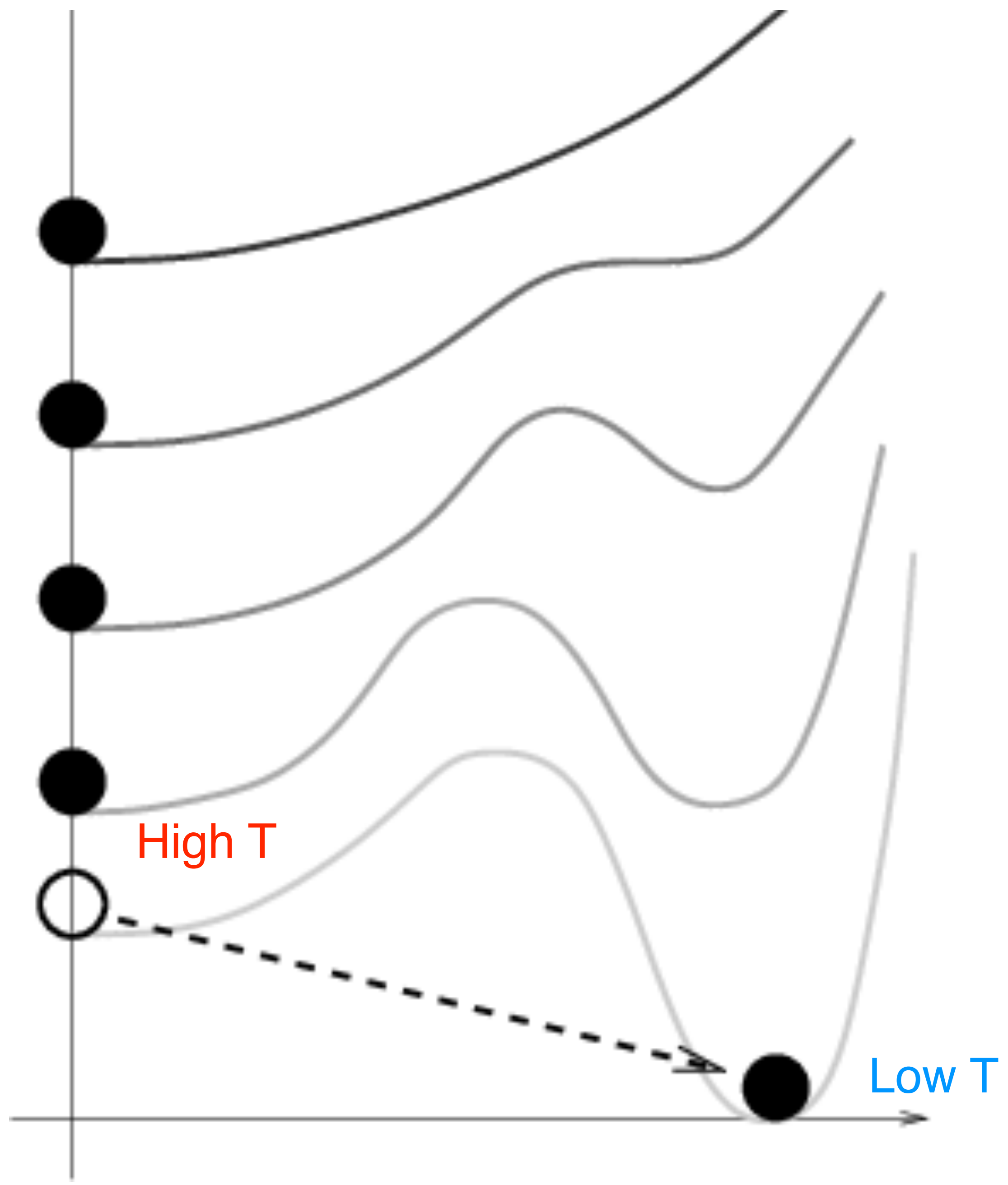
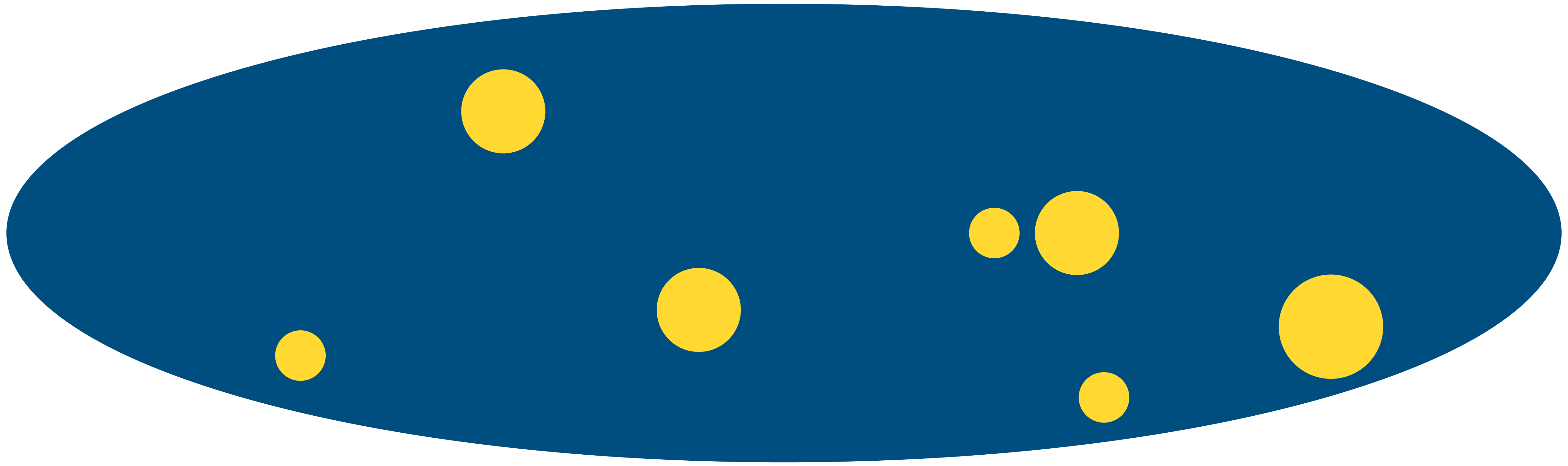


Figure: Rubakov, 2015

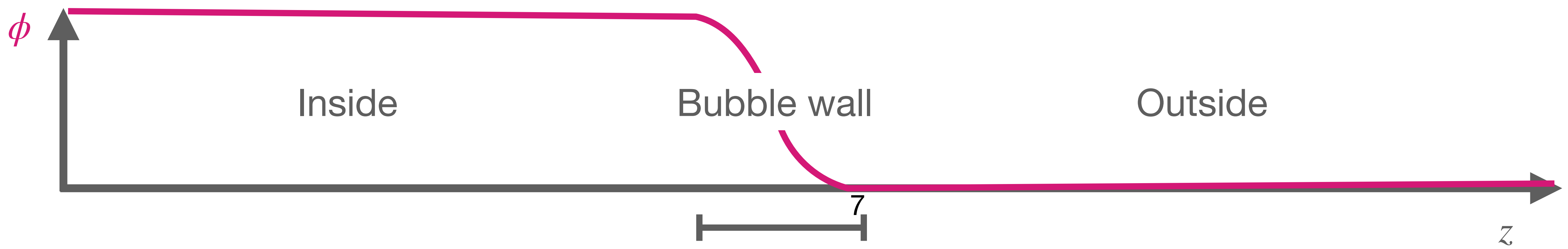
What is the bubble wall velocity?

- Cosmological phase transition: field with temperature-dependent effective potential transitions from high-temperature vacuum to low-temperature vacuum
- The phase transition proceeds via the nucleation of bubbles of the low-temperature vacuum
- The bubbles expand and collide until they fill the entire universe



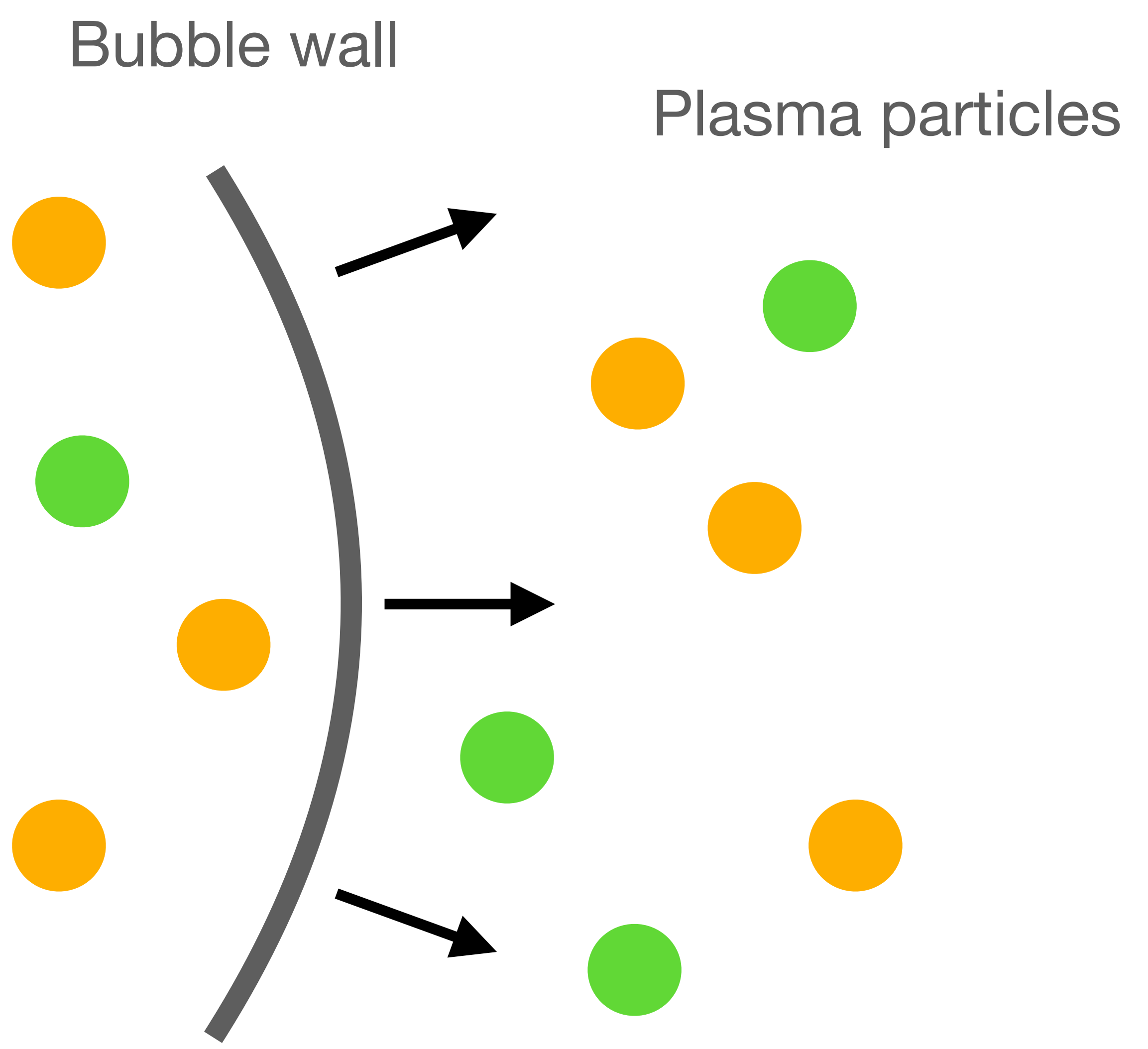
What is the bubble wall velocity?

- Cosmological phase transition: field with temperature-dependent effective potential transitions from high-temperature vacuum to low-temperature vacuum
- The phase transition proceeds via the nucleation of bubbles of the low-temperature vacuum
- The bubbles expand and collide until they fill the entire universe
- The *bubble wall* is the region where the field(s) interpolate between the high- and low-temperature vacuum



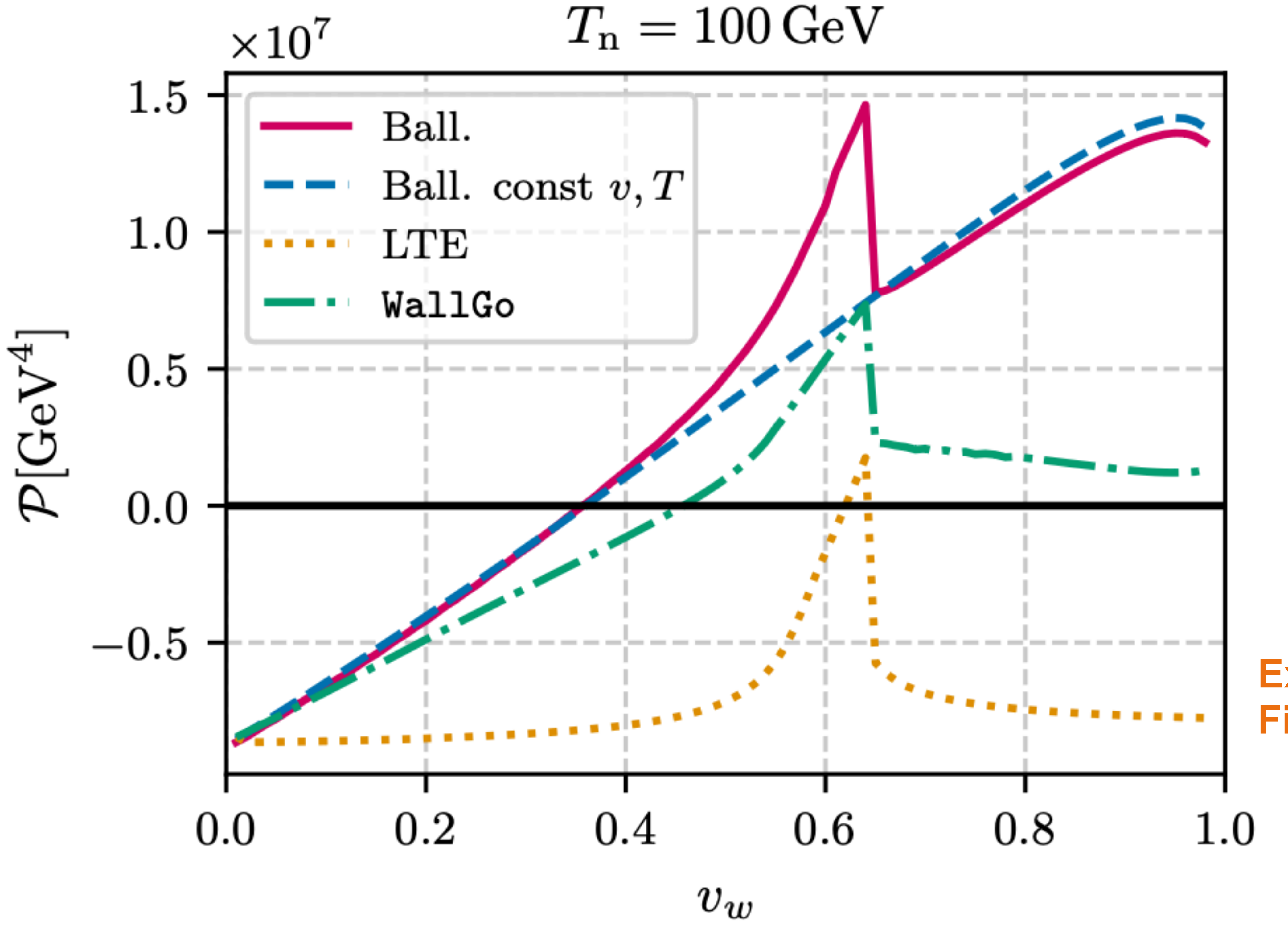
What is the bubble wall velocity?

- The release of vacuum energy provides an outward pressure: the wall accelerates initially
- Plasma particles provide friction by reflections and by gaining mass by entering the bubble
- Heating of the plasma in front of the bubble causes hydrodynamic backreaction



What is the bubble wall velocity?

- When $|P_{\text{outward}}| = |P_{\text{inward}}|$ the wall reaches a terminal velocity v_w



Example of a pressure profile
Fig. Ai, Laurent, JvdV 2024

What is the bubble wall velocity?

- When $|P_{\rm outward}| = |P_{\rm inward}|$ the wall reaches a terminal velocity v_w
- This typically happens long before the bubbles collide:
  $t_{\rm formation} \ll t_{{\rm const} v_w} \ll t_{\rm collision}$
- If the pressures never balance, the wall keeps accelerating until collision: runaway wall
- Runaway bubbles typically occur in very strong PTs; they are not the focus of this workshop

Why do we want to know the bubble wall velocity?

- First order phase transitions have many interesting possible phenomenological consequences:
  - Generation of the matter-antimatter asymmetry in electroweak baryogenesis or alternative scenarios

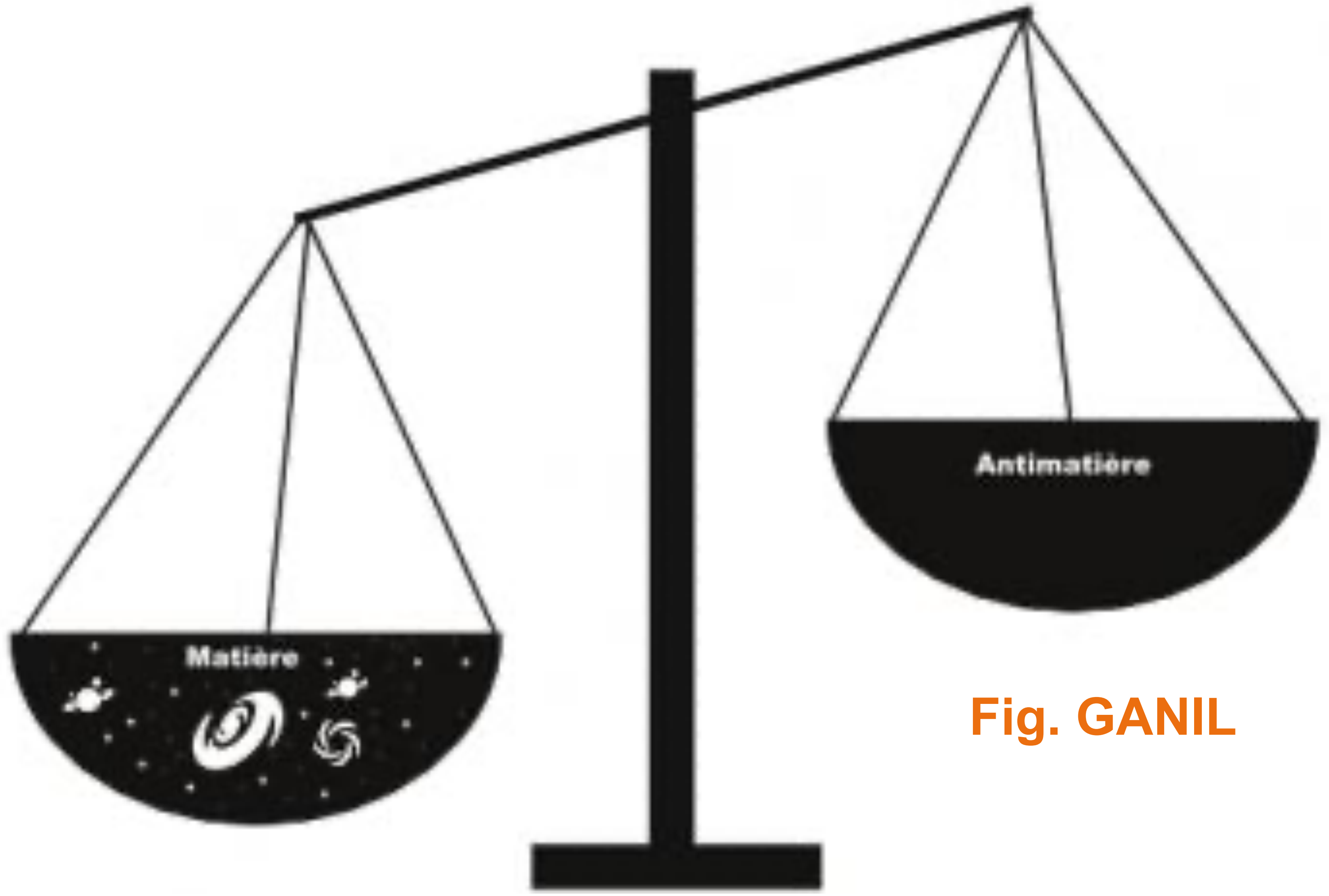
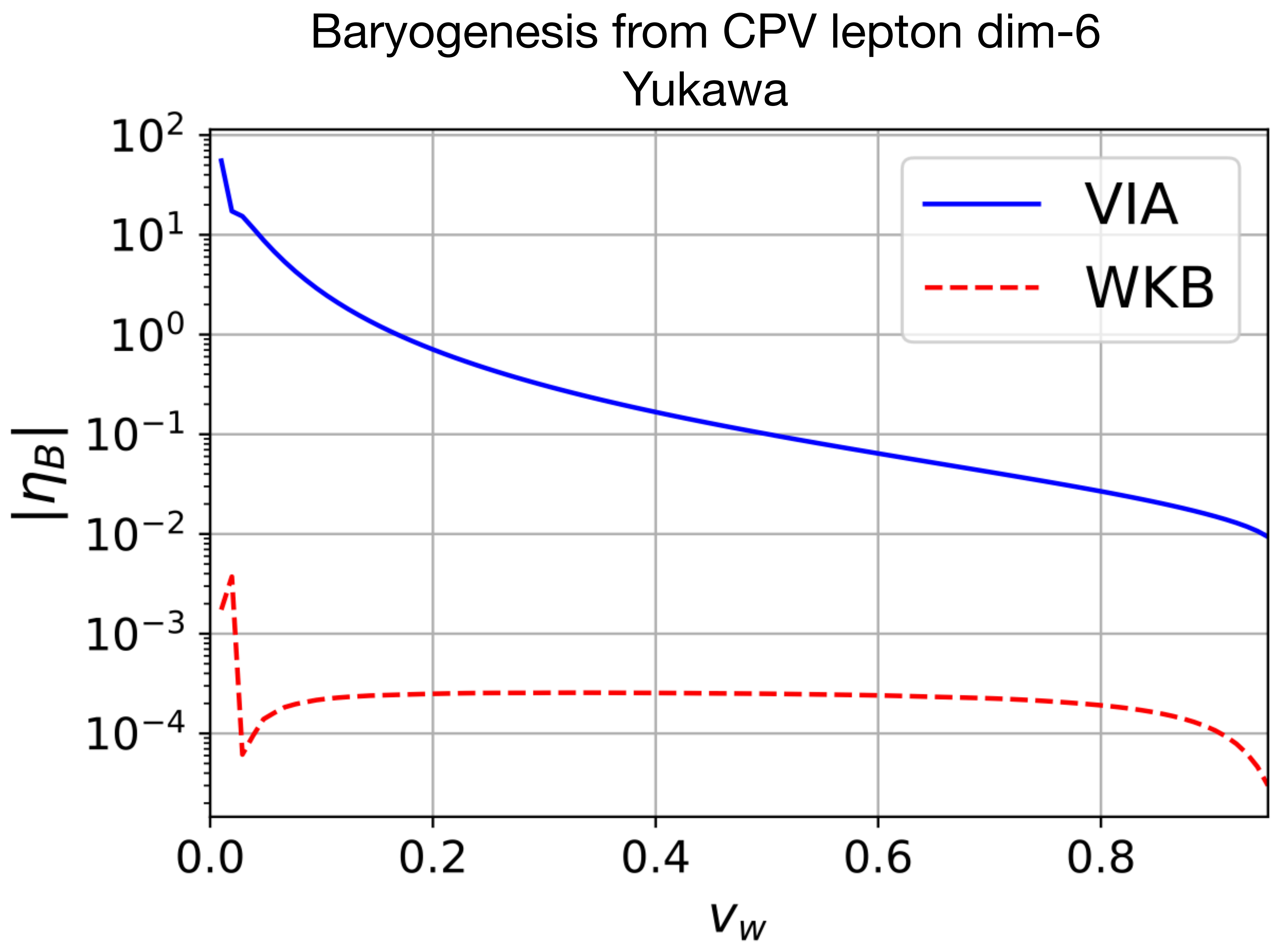
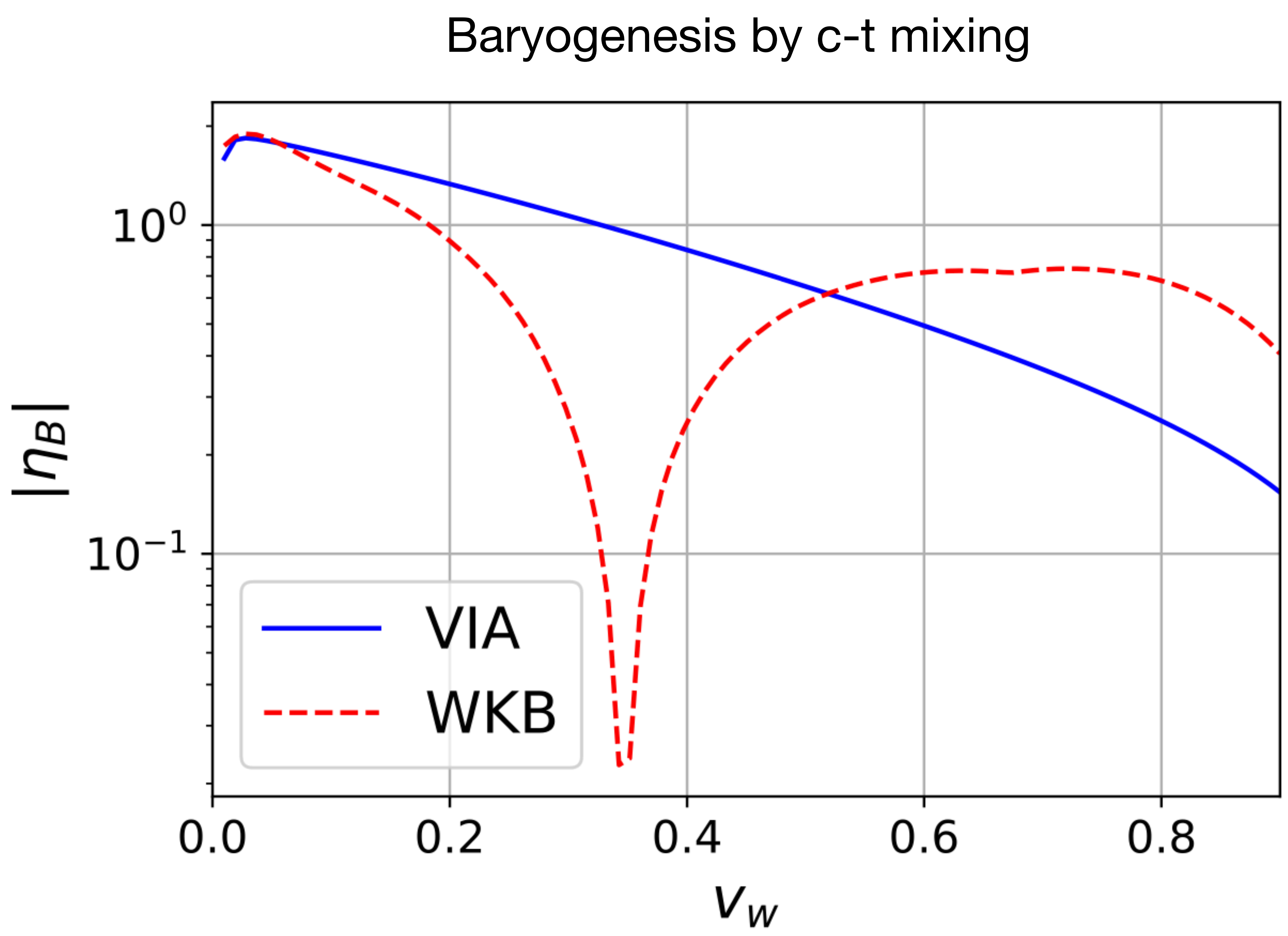


Fig. GANIL

Generation of the matter-antimatter asymmetry





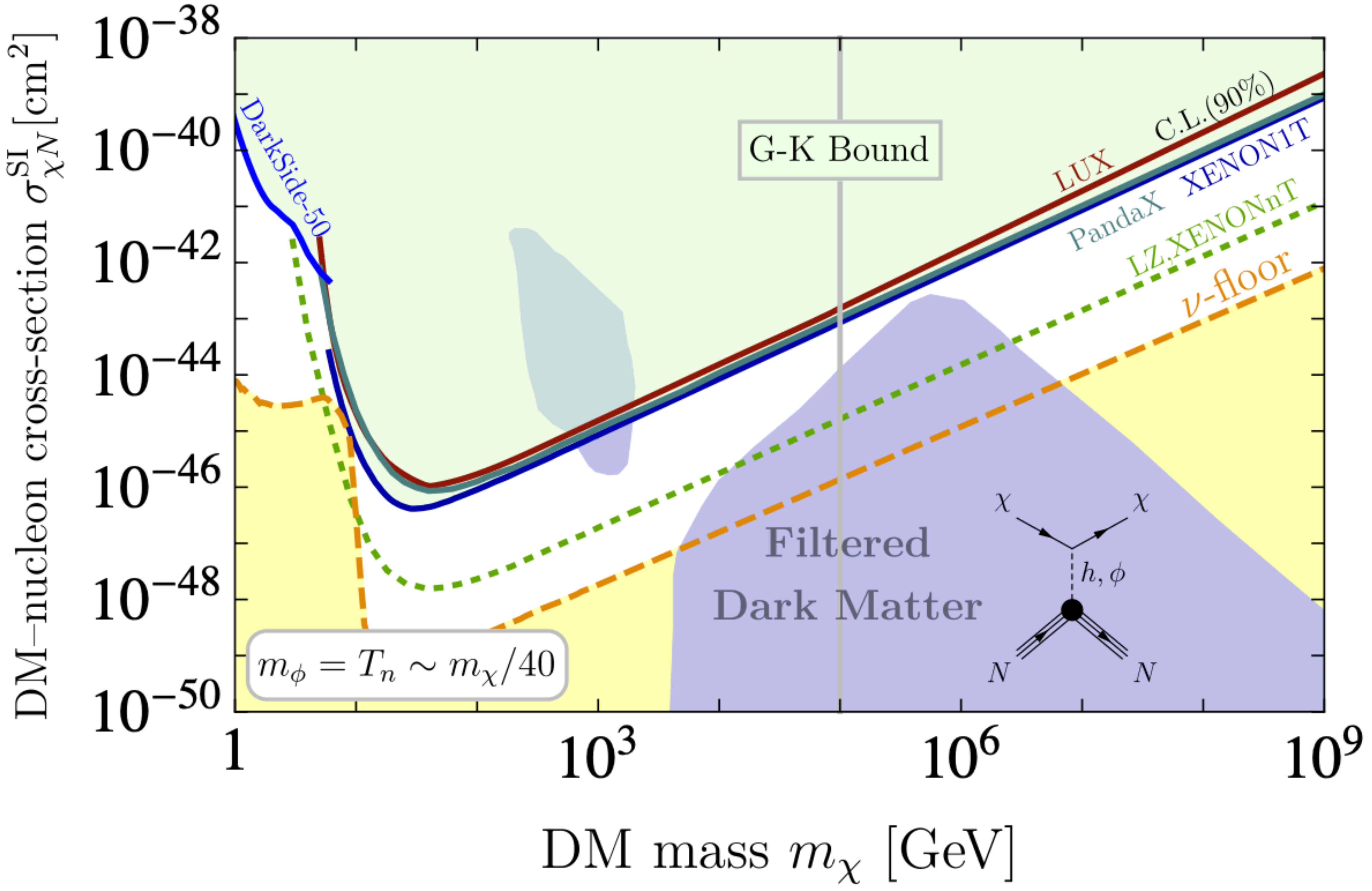
Figures from: Cline, Laurent 2021

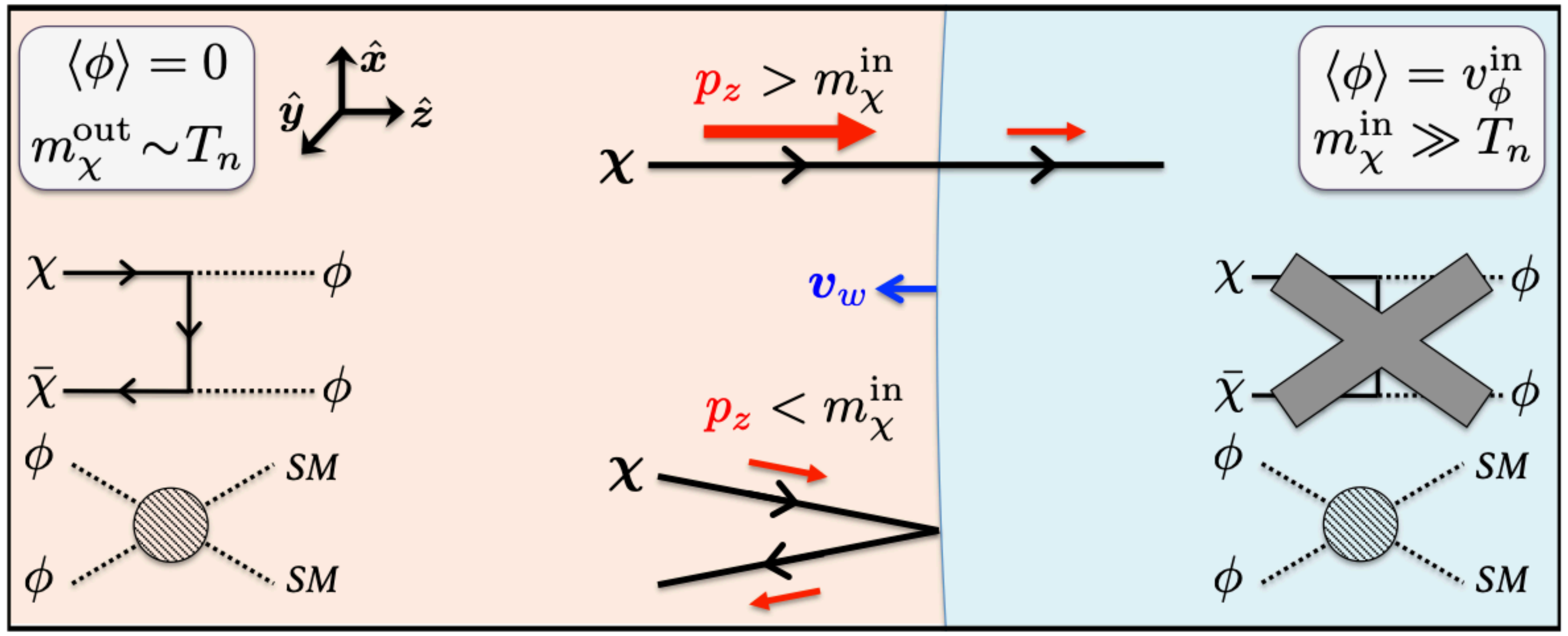
Why do we want to know the bubble wall velocity?

- First order phase transitions have many interesting possible phenomenological consequences:
  - Generation of the matter-antimatter asymmetry in electroweak baryogenesis or alternative scenarios
  - Generation of the dark matter abundance

Dark matter production

Slow bubbles: Filtered dark matter Baker, Kopp, Long 2019, Chway, Jung, Shin 2019, Marfatia, Tseng 2020

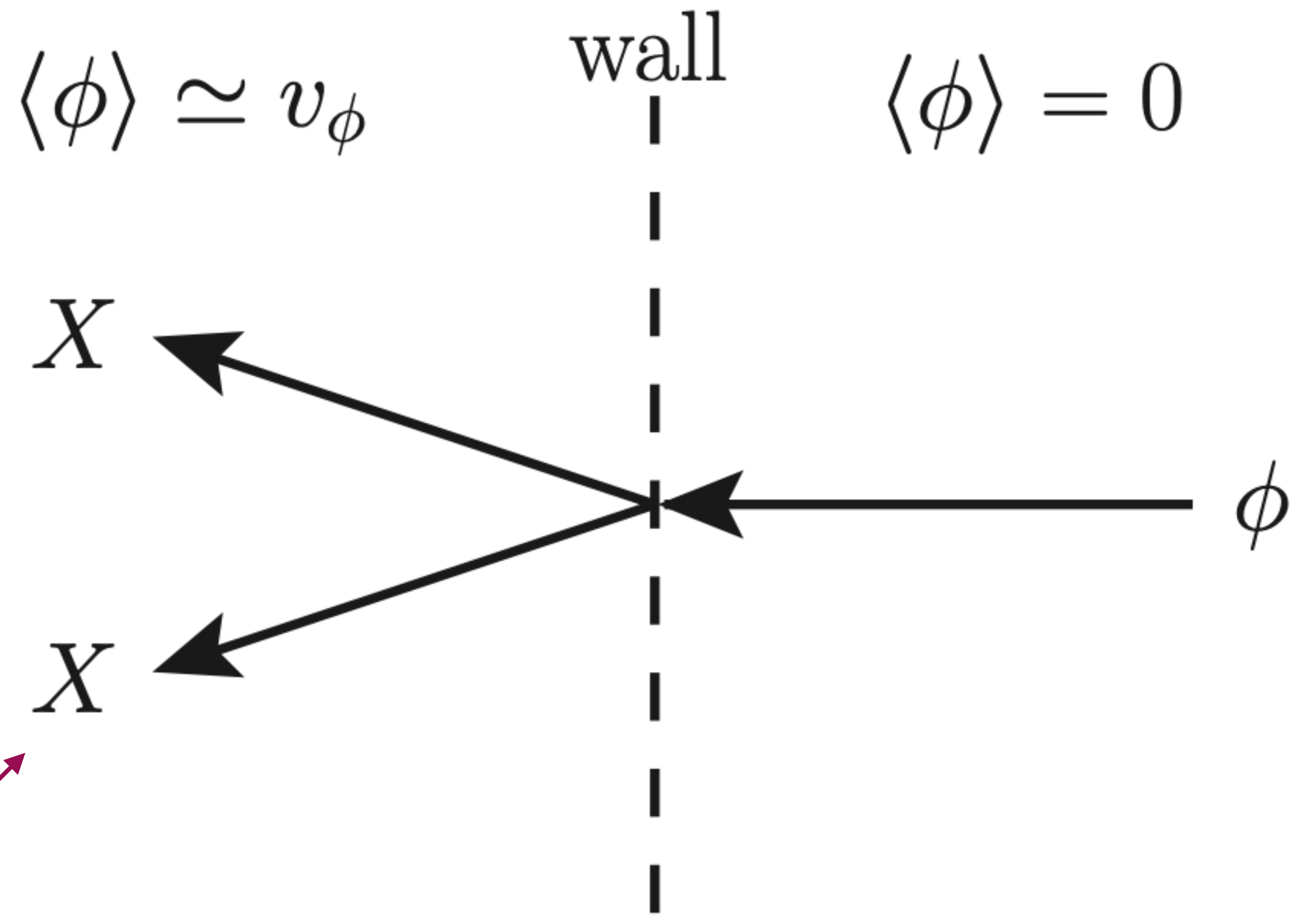




Figures from: Baker, Kopp, Long 2019

Dark matter production

Fast bubbles: Azatov, Vanvlasselaer, Yin 2021; Baldes, Gouttenoire, Sala 2022



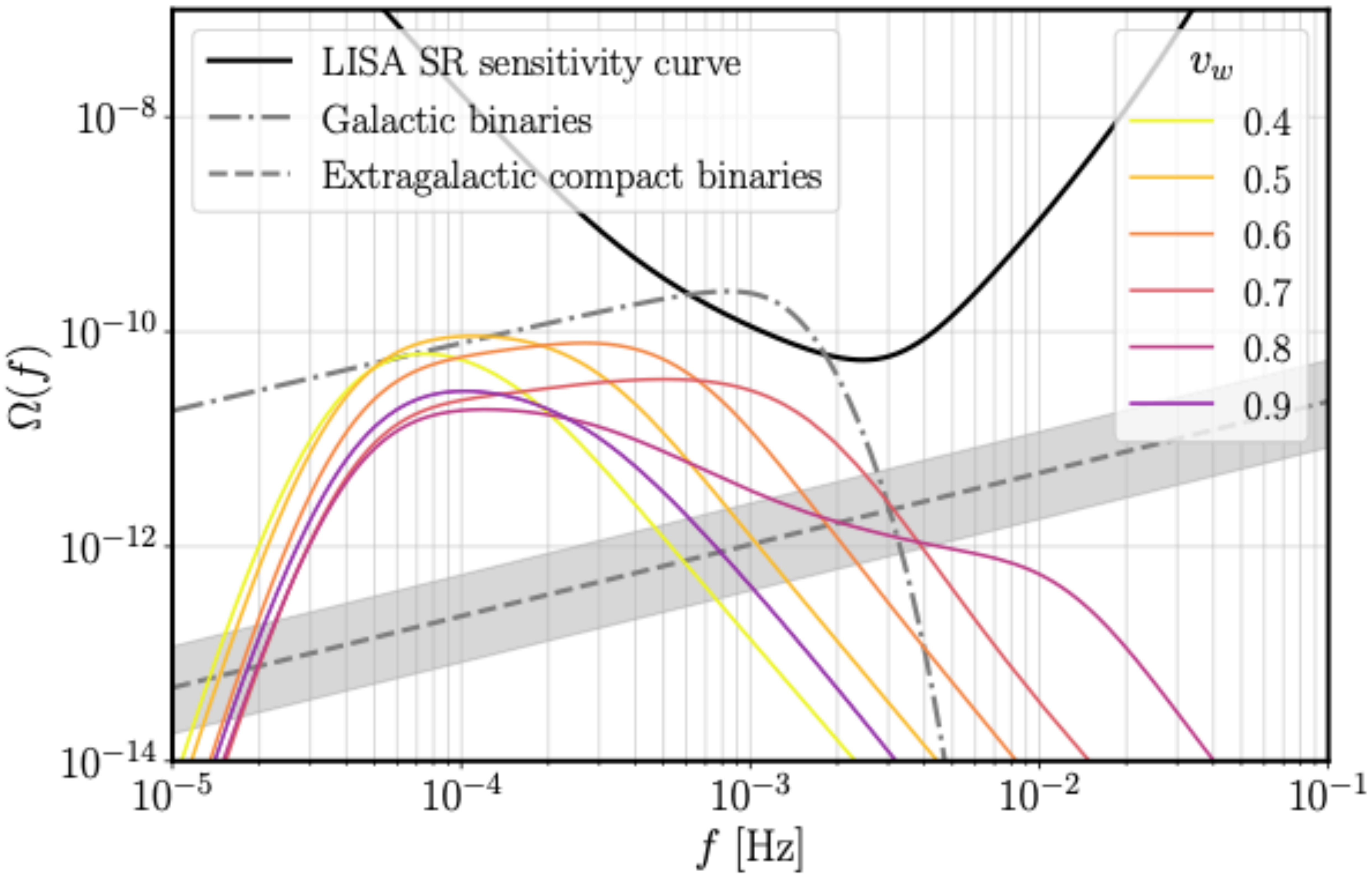
Heavy, out-of-equilibrium dark matter

Figure from: Baldes, Gouttenoire, Sala 2022

Why do we want to know the bubble wall velocity?

- First order phase transitions have many interesting possible phenomenological consequences:
  - Generation of the matter-antimatter asymmetry in electroweak baryogenesis or alternative scenarios
  - Generation of the dark matter abundance
  - Formation of a stochastic gravitational wave background

GW spectrum from sound waves



(a) Fixed: $\alpha = 0.2, \quad r_* = 0.1, \quad T_n = 100$ GeV.

Figure from: Gowling, Hindmarsh 2021
GW spectrum computed in the sound shell model

Why do we want to know the bubble wall velocity?

- First order phase transitions have many interesting possible phenomenological consequences:
    - Generation of the matter-antimatter asymmetry in electroweak baryogenesis or alternative scenarios
    - Generation of the dark matter abundance
    - Formation of a stochastic gravitational wave background
- In all cases, the value of the wall velocity affects the predictions

Problems

- The bubble wall velocity is not easy to compute
- The computation is model-dependent

Problems

- The bubble wall velocity … asy to compute
- The computation is mo…

Many papers in the literature

How fast does the

Ekstedt, Gould, Hirvonen, Laurent, Niemi, Schicho, JvdV: 2411.04970

Publicly available code for the computation of the wall velocity with out-of-equilibrium contributions

Wall velocity: theory

Some assumptions

- We can compute the wall velocity when the bubble still expands in isolation $t_{\text{formation}} \ll t_{\text{const}v_w} \ll t_{\text{collision}}$
- The bubbles are much smaller than horizon size, $(R_{\text{bubble}}H \ll 1)$, so we assume Minkowski spacetime
- The plasma surrounding the bubble will be at the nucleation temperature
- The theory is weakly-coupled

Scalar field equation of motion

Effective potential including temperature-corrections

Contribution from out-of-equilibrium particles

$$\partial^2\phi_i + \frac{\partial V_{\text{eff}}(\vec{\phi}, T)}{\partial \phi_i} + \sum_a \frac{\partial m_a^2}{\partial \phi_i} \int_{\vec{p}} \frac{1}{2E} \delta f^a(p^\mu, \xi) = 0$$

Scalar field(s) undergoing the phase transition

Distance from the wall

$\xi = -\bar{u}_w^\mu x_\mu$

$\bar{u}_w^\mu = \gamma_w(v_w, 0, 0, 1)$

Scalar field equation of motion

Friction force
Dominant contribution
from heavy particles
(e.g. top)

$$\partial^2\phi_i + \frac{\partial V_{\text{eff}}(\overrightarrow{\phi}, T)}{\partial \phi_i} + \sum_a \frac{\partial m_a^2}{\partial \phi_i} \int_{\vec{p}} \frac{1}{2E} \delta f^a(p^\mu, \xi) = 0$$

Driving force and
hydrodynamic
backreaction

Scalar field equation of motion

Balance of forces Balaji, Spannowski, Tamarit 2020; Ai, Garbrecht, Tamarit 2021

$$\int dz \frac{d\phi}{dz}\left(\partial^2\phi + \frac{\partial V_{\rm eff}(\phi,T)}{\partial\phi} + \sum_a \frac{\partial m_a^2}{\partial\phi}\int_{\vec{p}}\frac{1}{2E}\delta f^a(p^\mu,\xi)\right) = 0$$

$$\int dz \frac{d\phi}{dz}\frac{\partial V_{\rm eff}(\phi,T)}{\partial\phi} = \int dz\left(\frac{dV_{\rm eff}}{dz} - \frac{\partial V_{\rm eff}}{\partial T}\frac{dT}{dz}\right) = \Delta V_{\rm eff} - \int dz \frac{\partial V_{\rm eff}}{\partial T}\frac{dT}{dz}$$

Scalar field equation of motion

Balance of forces Balaji, Spannowski, Tamarit 2020; Ai, Garbrecht, Tamarit 2021

$$\int dz \frac{d\phi}{dz}\left(\partial^2\phi + \frac{\partial V_{\text{eff}}(\phi,T)}{\partial\phi} + \sum_a \frac{\partial m_a^2}{\partial\phi}\int_{\vec{p}} \frac{1}{2E}\delta f^a(p^\mu,\xi)\right) = 0$$

$$\int dz \frac{d\phi}{dz}\frac{\partial V_{\text{eff}}(\phi,T)}{\partial\phi} = \int dz\left(\frac{dV_{\text{eff}}}{dz} - \frac{\partial V_{\text{eff}}}{\partial T}\frac{dT}{dz}\right) = \Delta V_{\text{eff}} - \int dz \frac{\partial V_{\text{eff}}}{\partial T}\frac{dT}{dz}$$

$$\Delta V_{\text{eff}} - \int dz \frac{\partial V_{\text{eff}}}{\partial T}\frac{dT}{dz} + \sum_a \int dz \frac{dm_a^2}{dz}\int_{\vec{p}} \frac{1}{2E}\delta f^a(p^\mu,\xi) = 0$$

Driving force

Hydrodynamic backreaction

Friction force
Dominant contribution from heavy particles (e.g. top)

Also for vanishing δf^a, the wall can stop to accelerate!

Ignatius, Kajantie, Kurki-Suonio, Laine 1993; Konstandin, No 2011; Barroso Mancha, Prokopec, Swiezewska 2020; Balaji, Spannowski, Tamarit 2020; Ai, Garbrecht, Tamarit 2021; Ai, Laurent, JvdV 2023; Ai, Laurent, JvdV 2024

- Hydrodynamic effects also provide a backreaction force in local thermal equilibrium ($\delta f^a = 0$)
- This approximation provides an *upper bound* on the wall velocity
- Entropy conservation provides a third hydrodynamic matching condition: the wall velocity can be determined from hydrodynamics only

Scalar field equation of motion

$$\partial^2\phi_i + \frac{\partial V_{\text{eff}}(\overrightarrow{\phi}, T)}{\partial \phi_i} + \sum_a \frac{\partial m_a^2}{\partial \phi_i} \int_{\vec{p}} \frac{1}{2E} \delta f^a(p^\mu, \xi) = 0$$

The scalar field equation of motion depends on the temperature profile

Temperature and fluid profile

Energy-momentum tensor of the (perfect) fluid and scalar field

Fluid velocity

$$T_{\mu\nu} = w u_\mu u_\nu - p g_{\mu\nu} + \partial_\mu \phi_i \partial_\nu \phi_i - g_{\mu\nu} \left(\frac{1}{2} \partial_\sigma \phi_i \partial^\sigma \phi_i \right)$$

Enthalpy

$$w = T \frac{dp}{dT}$$

Pressure

$$p = -V_{\text{eff}}(\vec{\phi}, T)$$

(Including field-independent terms)

Temperature and fluid profile

Energy-momentum conservation for a planar wall, moving in the z-direction

$$T_{\mu\nu} = w u_\mu u_\nu - p g_{\mu\nu} + \partial_\mu \phi_i \partial_\nu \phi_i - g_{\mu\nu}\left(\frac{1}{2}\partial_\sigma \phi_i \partial^\sigma \phi_i\right)$$

$$T^{30} = w\gamma_{\text{pl}}^2 v_{\text{pl}} + T^{30}_{\text{out}} = c_1$$

$$T^{33} = \frac{1}{2}(\partial_z \phi_i)^2 - V_{\text{eff}}(\vec{\phi}, T) + w\gamma_{\text{pl}}^2 v_{\text{pl}}^2 + T^{33}_{\text{out}} = c_2$$

Contribution from out-of-equilibrium particles

Temperature and fluid profile

Energy-momentum conservation for a planar wall, moving in the z-direction

$$T_{\mu\nu} = w u_\mu u_\nu - p g_{\mu\nu} + \partial_\mu \phi_i \partial_\nu \phi_i - g_{\mu\nu}\left(\frac{1}{2}\partial_\sigma \phi_i \partial^\sigma \phi_i\right)$$

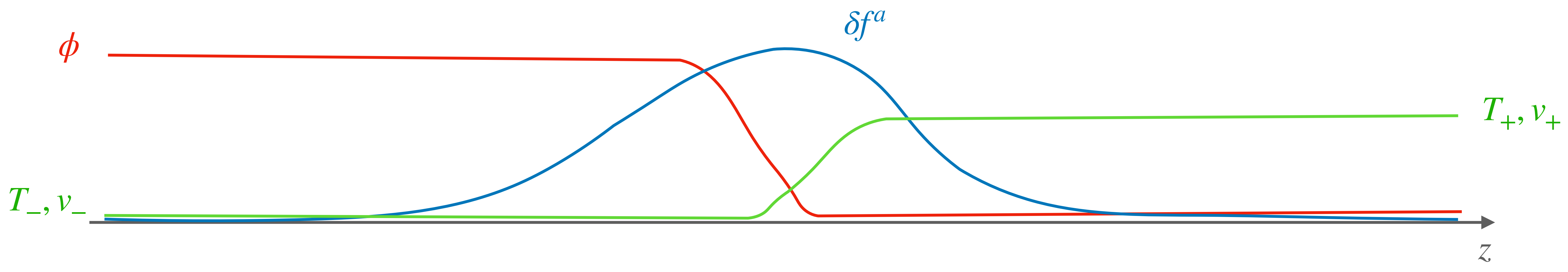
$$T^{30} = w\gamma_{\rm pl}^2 v_{\rm pl} + T^{30}_{\rm out} = c_1$$

$$T^{33} = \frac{1}{2}(\partial_z \phi_i)^2 - V_{\rm eff}(\vec{\phi}, T) + w\gamma_{\rm pl}^2 v_{\rm pl}^2 + T^{33}_{\rm out} = c_2$$

Constants obtained from hydrodynamic solution

Boundary conditions from hydrodynamics

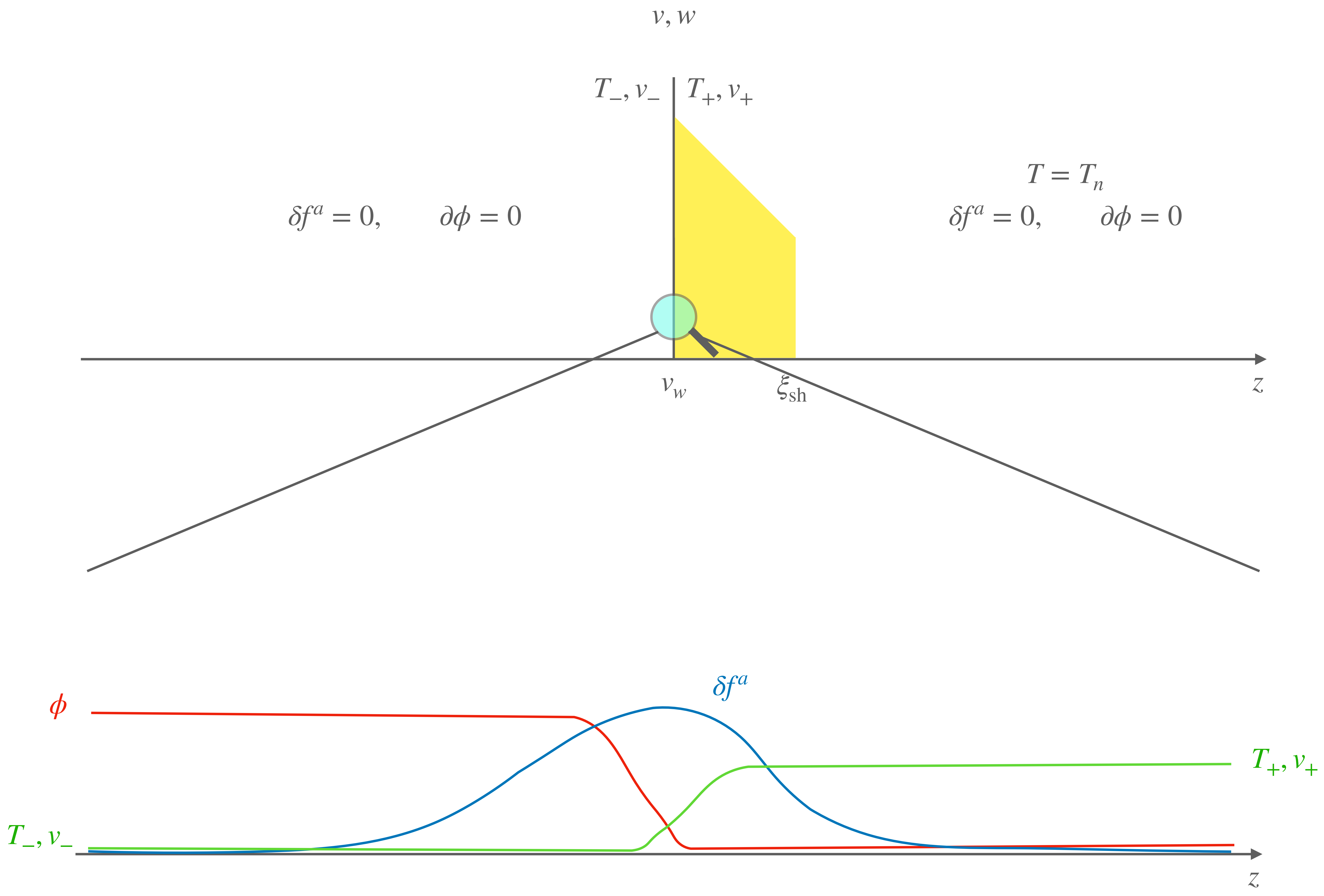
Bubble wall scale: scalar field of motion (and Boltzmann equations)



Units and scale
are arbitrary

Boundary conditions from hydrodynamics

On hydrodynamic scales, the wall corresponds to a discontinuity in T, v



Units and scale are arbitrary

Boundary conditions from hydrodynamics

- Spherically symmetric solutions to $\partial_\mu T^{\mu\nu} = 0$ for $\partial_\mu \phi_i = 0$ and equation of state given by $p_{\rm HT} = -V_{\rm eff}(v_{\rm HT}, T)$ and $p_{\rm LT} = -V_{\rm eff}(v_{\rm LT}, T)$
- Three types of solutions

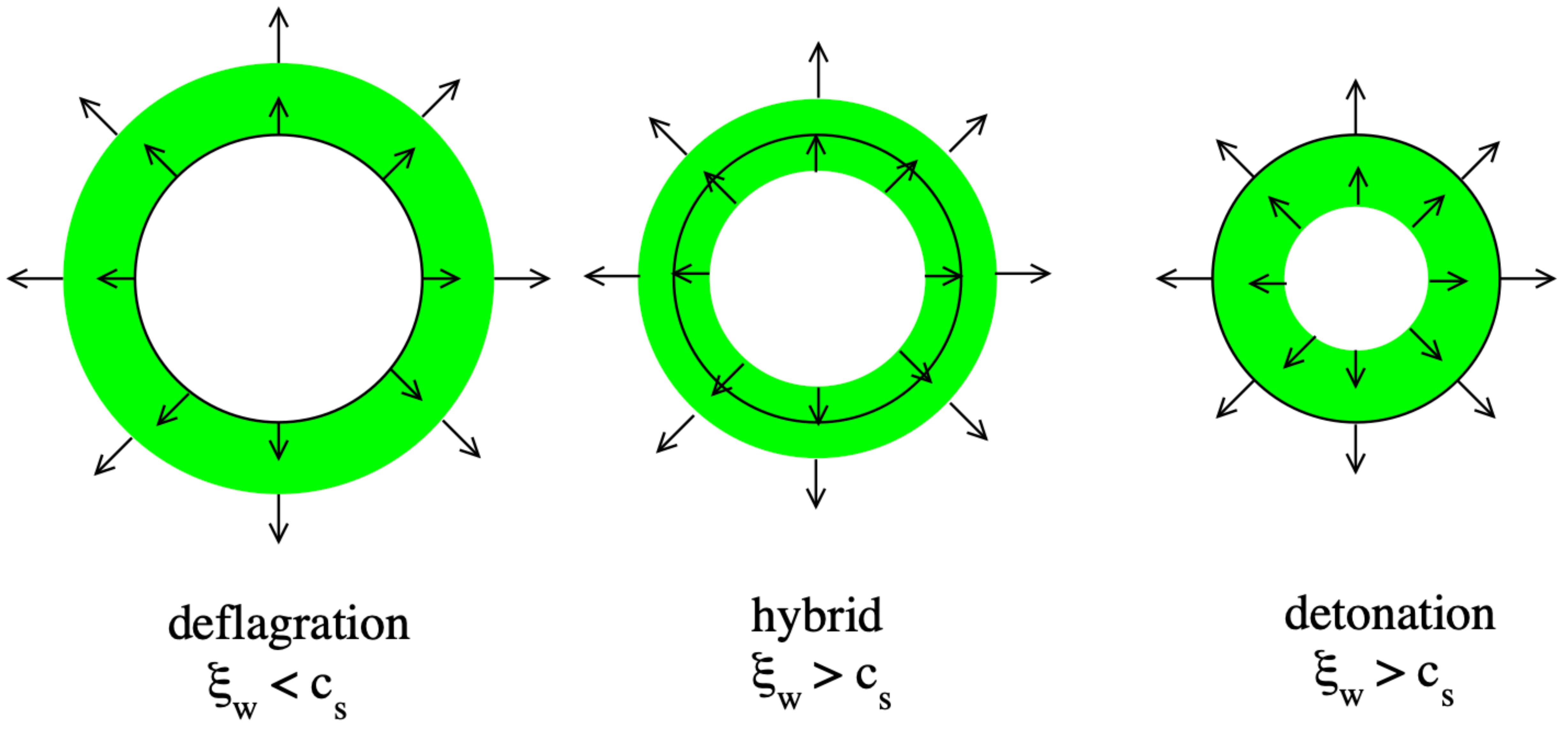
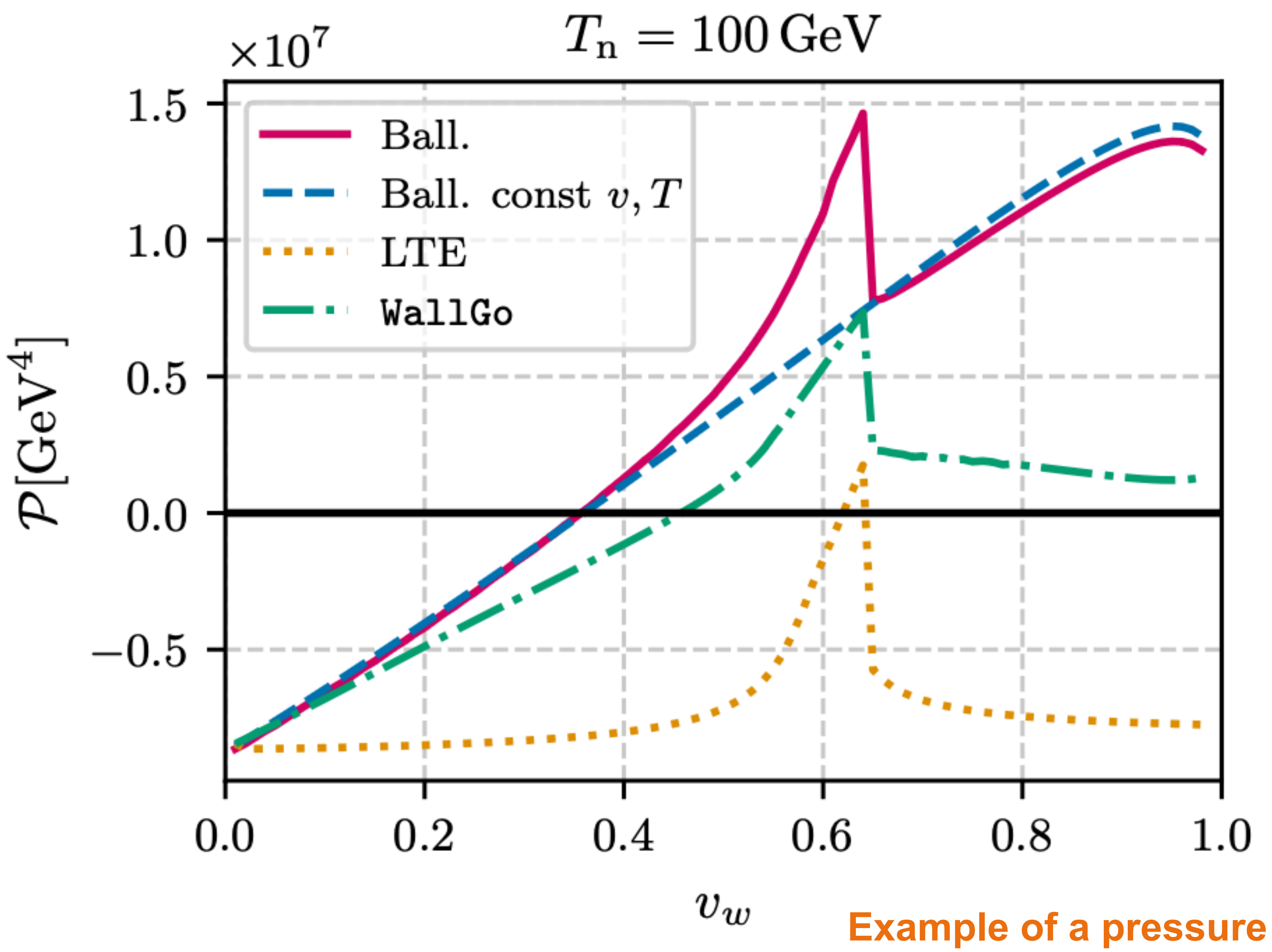


Figure from: Espinosa, Konstandin, No, Servant 2010

Hydrodynamic backreaction

- Due to the hydrodynamic backreaction, the pressure of the deflagration and hybrid solution always increase with v_w
- For detonations the hydrodynamic backreaction *decreases* with v_w



Example of a pressure profile
Fig. Ai, Laurent, JvdV 2024

Boundary conditions from hydrodynamics

- Spherically symmetric solutions to $\partial_\mu T^{\mu\nu} = 0$ for $\partial_\mu \phi_i = 0$ and equation of state given by $p_{\rm HT} = -V_{\rm eff}(v_{\rm HT}, T)$ and $p_{\rm LT} = -V_{\rm eff}(v_{\rm LT}, T)$
- Three types of solutions
- Fixing the nucleation temperature and v_w determines $T_+, T_-, v_+, v_- \quad \rightarrow \quad c_1, c_2$

Obtaining ϕ_i, v, T and v_w

- Let us assume for now that we know δf_a
- We can solve ϕ_i, v, T for a given v_w
- From this we determine the pressure on the wall P.
  $P(v_w) = 0$ gives us the wall velocity

Obtaining ϕ_i, v, T and v_w

- Often (also in `WallGo`) the ϕ_i are approximated as
$$\phi_i(z) = v_{\mathrm{HT},i} + \frac{v_{\mathrm{LT},i} - v_{\mathrm{HT},i}}{2}\left[1 - \tanh\left(\frac{z}{L_i} + \delta_i\right)\right]$$
- The ϕ_i are now not exact solutions of their equations of motion
- The L_i, δ_i can be obtained by taking moments of the equation of motion or by minimizing the action (`WallGo`)

The out-of-equilibrium contribution

- Until now we assumed that we already knew the out-of-equilibrium condition
- In practice we solve for ϕ_i, v, T first, and compute δf^a assuming that background solution
- We plug the solution of δf^a into the equations for ϕ_i, v, T until it converges

The out-of-equilibrium contribution

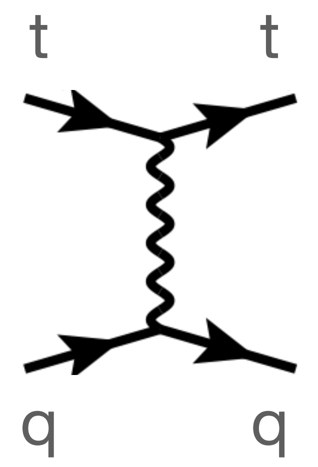
Boltzmann equation

$$\left(p^\mu \partial_\mu + \frac{1}{2}\vec{\nabla} m_a^2 \cdot \nabla_{\vec{p}}\right) f^a(\vec{p}, \xi) = -\,\mathscr{C}_a[\vec{f}]$$

Force term

Collision term: Nine-dimensional integral over distributions of 4 contributing species

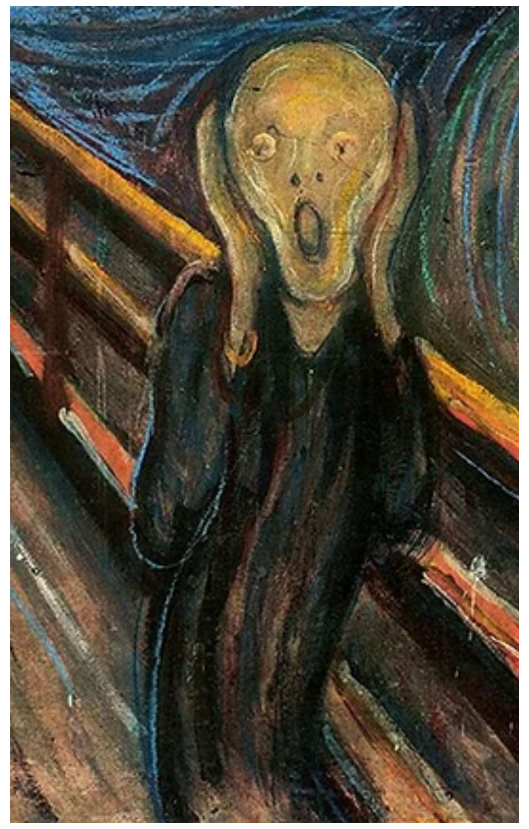
E.g.



The out-of-equilibrium contribution

Boltzmann equation

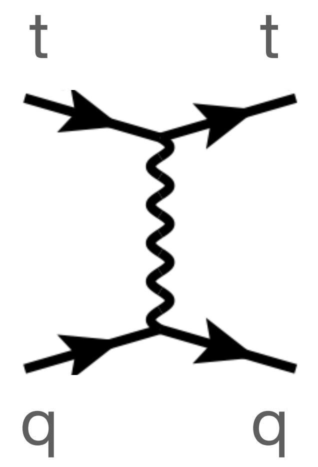
$$\left(p^\mu \partial_\mu + \frac{1}{2}\vec{\nabla} m_a^2 \cdot \nabla_{\vec{p}}\right) f^a(\vec{p}, \xi) = -\mathscr{C}_a[\vec{f}]$$



Force term

Collision term: Nine-dimensional integral over distributions of 4 contributing species

E.g.



The out-of-equilibrium contribution

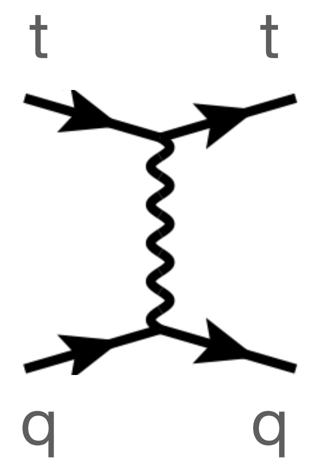
Boltzmann equation

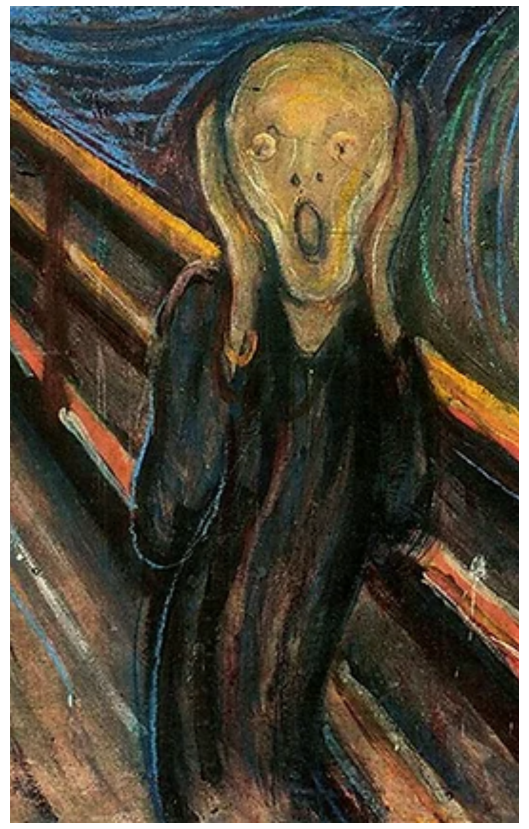
$$\left(p^\mu \partial_\mu + \frac{1}{2}\vec{\nabla} m_a^2 \cdot \nabla_{\vec{p}}\right) f^a(\vec{p}, \xi) = -\mathscr{C}_a[\vec{f}]$$

Force term

Collision term: Nine-dimensional integral over distributions of 4 contributing species

E.g.





The Boltzmann equations can be solved by linearizing in the perturbation from equilibrium and using that $\mathscr{C}[f_{\rm eq}] = 0$

Putting the Boltzmann equation in a solvable form

Taking moments Prokopec Moore 1995; Dorsch, Huber, Konstandin 2021

$$f = f_{\rm eq}(E+\delta), \qquad \delta = -\left[\mu + \mu_{\rm bg} + \frac{E}{T}(\delta T + \delta T_{\rm bg}) + p_z(v + v_{\rm bg})\right]$$

$$-f'_{\rm eq}\left(\frac{p_z}{E}[\partial_z(\mu+\mu_{\rm bg}) + \frac{E}{T}\partial_z(\delta T + \delta T_{\rm bg}) + p_z\partial_z(v+v_{\rm bg})] + \partial_t(\mu+\mu_{\rm bg}) + \frac{E}{T}\partial_t(\delta T + \delta T_{\rm bg}) + p_z\partial_t(v+v_{\rm bg})\right) + C(\mu, \delta T, v) = -f'_{\rm eq}\frac{\partial_t m^2}{2E}$$

Putting the Boltzmann equation in a solvable form

Taking moments Prokopec Moore 1995; Dorsch, Huber, Konstandin 2021

$$-f'_{\rm eq}\left(\frac{p_z}{E}[\partial_z(\mu+\mu_{\rm bg})+\frac{E}{T}\partial_z(\delta T+\delta T_{\rm bg})+p_z\partial_z(v+v_{\rm bg})]+\partial_t(\mu+\mu_{\rm bg})+\frac{E}{T}\partial_t(\delta T+\delta T_{\rm bg})+p_z\partial_t(v+v_{\rm bg})\right)+C(\mu,\delta T,v)=-f'_{\rm eq}\frac{\partial_t m^2}{2E}$$

Taking moments $\int d^3p/(2\pi)^3, \quad \int E d^3p/(2\pi)^3, \quad \int p_z d^3p/(2\pi)^3$

$$\begin{aligned}
c_2\partial_t(\mu+\mu_{bg})+c_3\partial_t(\delta T+\delta T_{bg})+\frac{c_3 T}{3}\partial_z(v+v_{bg})+\int\frac{d^3p}{(2\pi)^3T^2}C[f] &= \frac{c_1}{2T}\partial_t m^2\\
c_3\partial_t(\mu+\mu_{bg})+c_4\partial_t(\delta T+\delta T_{bg})+\frac{c_4 T}{3}\partial_z(v+v_{bg})+\int\frac{E d^3p}{(2\pi)^3T^3}C[f] &= \frac{c_2}{2T}\partial_t m^2\\
\frac{c_3}{3}\partial_z(\mu+\mu_{bg})+\frac{c_4}{3}\partial_t(\delta T+\delta T_{bg})+\frac{c_4 T}{3}\partial_t(v+v_{bg})+\int\frac{p_z d^3p}{(2\pi)^3T^3}C[f] &= 0
\end{aligned}$$

$$c_i T^{i+1}\equiv\int E^{i-2}(-f'_{\rm eq})\frac{d^3p}{(2\pi)^3}$$

Putting the Boltzmann equation in a solvable form

Taking moments Prokopec Moore 1995; Dorsch, Huber, Konstandin 2021

Advantages

- Numerically relatively easily manageable
- Collision terms become very simple;
  E.g. $\int \frac{d^3p}{(2\pi)^3 T^2} C[f] = \mu \Gamma_{\mu 1 f} + \delta T \Gamma_{T1f}$
  for top quarks, with
  $\Gamma_{\mu 1 f} = 0.00899T, \qquad \Gamma_{T1f} = 0.01752T$
- Different moments correspond to conservation of particle number, energy and momentum

Disadvantages

- A singularity appears around $v_w = c_s$, see also Laurent, Cline 2022; Dorsch, Konstandin, Perboni, Pinto 2024
- Not clear if three moments is sufficient for convergence
- Mixing between different out-of-equilibrium particles is neglected

Putting the Boltzmann equation in a solvable form

Expand δf^a in polynomials Laurent, Cline 2022; WallGo

$$\left(p^\mu \partial_\mu + \frac{1}{2} \vec{\nabla} m_a^2 \cdot \nabla_{\vec{p}}\right) f^a(\vec{p}, x^\mu) = -\mathscr{C}_a[\vec{f}]$$

$$f^a(\vec{p}, \xi) = f^a_{\rm eq}(\vec{p}, \xi) + \delta f^a(\vec{p}, \xi)\,, \qquad f^a_{\rm eq} = \left.\frac{1}{\exp[p_\mu u^\mu_{\rm pl}(\xi)/T(\xi)] \pm 1}\right|_{E_a = \vec{p}^2 + m_a^2}$$

$$\left(-p_\mu \bar{u}^\mu_w \partial_\xi - \frac{1}{2}\partial_\xi(m_a^2)\bar{u}^\mu_w \partial_{p^\mu}\right)\delta f^a = -\mathscr{C}^{\rm lin}_{ab}[\delta f^b] + \mathscr{S}_a, \qquad \mathscr{S}_a = \left(p_\mu \bar{u}^\mu_w \partial_\xi + \frac{1}{2}\partial_\xi(m_a^2)\bar{u}^\mu_w \partial_{p^\mu}\right) f^a_{\rm eq}$$

Mixing between different out-of-eq particles

Putting the Boltzmann equation in a solvable form

Expand δf^a in polynomials Laurent, Cline 2022; WallGo

Rescaled coordinates

Restricted Chebyshev polynomials

$$\delta f^a(\chi, \rho_z, \rho_\parallel) = \sum_{i=2}^{M} \sum_{j=2}^{N} \sum_{k=1}^{N-1} \delta f^a_{ijk} \bar{T}_i(\chi) \bar{T}_j(\rho_z) \tilde{T}_k(\rho_\parallel)$$

Algebraic equation

$$\sum_{i,j,k} \left\{ \partial_\xi \chi \left[\mathscr{P}_w \partial_\chi - \frac{\gamma_w}{2} \partial_\chi (m^2)(\partial_{p_z} \rho_z) \partial_{\rho_z} \right] \bar{T}_i(\chi) \bar{T}_j(\rho_z) \tilde{T}_k(\rho_\parallel) \delta f^a_{ijk} + \bar{T}_i(\chi) \mathscr{C}^{\text{lin}}_{ab} \left[\bar{T}_j(\rho_z) \tilde{T}_k(\rho_\parallel) \right] \delta f^b_{ijk} \right\} = \mathscr{S}_a(\chi, \rho_z, \rho_\parallel)$$

Putting the Boltzmann equation in a solvable form

Expand δf^a in polynomials Laurent, Cline 2022; WallGo

Rescaled coordinates

Restricted Chebyshev polynomials

$$\delta f^a(\chi, \rho_z, \rho_\parallel) = \sum_{i=2}^{M} \sum_{j=2}^{N} \sum_{k=1}^{N-1} \delta f^a_{ijk} \bar{T}_i(\chi) \bar{T}_j(\rho_z) \tilde{T}_k(\rho_\parallel)$$

Algebraic equation

$$\sum_{i,j,k} \left\{ \partial_\xi \chi \left[\mathscr{P}_w \partial_\chi - \frac{\gamma_w}{2} \partial_\chi(m^2)(\partial_{p_z}\rho_z)\partial_{\rho_z} \right] \bar{T}_i(\chi)\bar{T}_j(\rho_z)\tilde{T}_k(\rho_\parallel)\delta f^a_{ijk} + \bar{T}_i(\chi)\mathscr{C}^{\text{lin}}_{ab}\left[\bar{T}_j(\rho_z)\tilde{T}_k(\rho_\parallel)\right]\delta f^b_{ijk} \right\} = \mathscr{S}_a(\chi, \rho_z, \rho_\parallel)$$

Introduce a grid to convert it to a matrix equation

$$\left(\mathscr{L}[\alpha, \beta, \gamma; i, j, k]\delta_{ab} + \bar{T}_i(\chi^{(\alpha)})\mathscr{C}_{ab}[\beta, \gamma; j, k] \right) \delta f^b_{ijk} = \mathscr{S}_a[\alpha, \beta, \gamma]$$

Grid indices

Putting the Boltzmann equation in a solvable form

Expand δf^a in polynomials Laurent, Cline 2022; WallGo

$$\mathscr{C}^{\text{lin}}_{ab}[\bar{T}_j(\rho_z)\tilde{T}_k(\rho_\parallel)] = \frac{1}{4}\sum_{cde}\int_{\vec{p}_2,\vec{p}_3,\vec{p}_4} \frac{1}{2E_2 2E_3 2E_4}(2\pi)^4\delta^4(P_1+P_2-P_3-P_4)\times$$
$$|M_{ac\to de}(P_1,P_2;P_3,P_4)|^2 f^a f^c f^d f^e \left(\delta_{ab}F^c_a + \delta_{cb}F^a_c - \delta_{db}F^e_d - \delta_{eb}F^d_e\right)\bar{T}_j(\rho_z)\tilde{T}_k(\rho_\parallel)$$

$$F^a_b = \frac{e^{E_a/T}}{(f^b)^2}$$

Nine-dimensional integral for $n_p^2(N-1)^4$ components;

Four integrations are trivial because of the δ-function

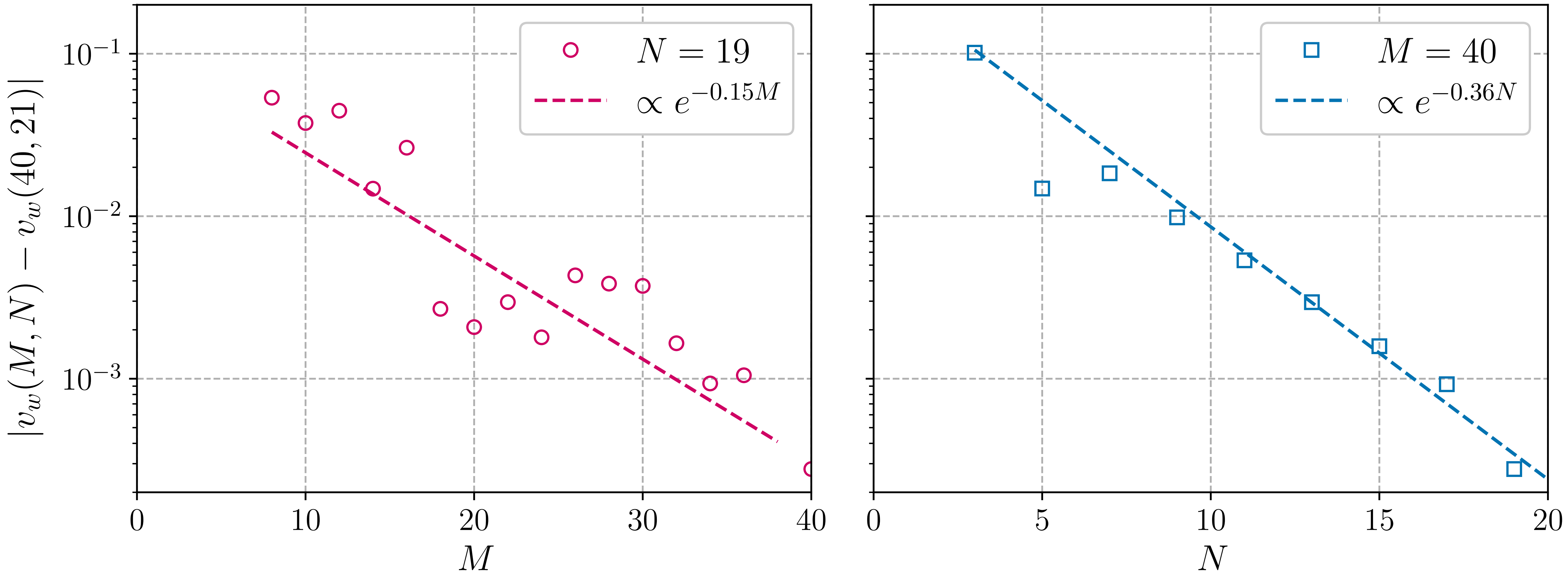
Putting the Boltzmann equation in a solvable form

Expand δf^a in polynomials Laurent, Cline 2022; WallGo

Advantages

Disadvantages

- Controlled convergence in number of polynomials



Putting the Boltzmann equation in a solvable form

Expand δf^a in polynomials Laurent, Cline 2022; WallGo

Advantages

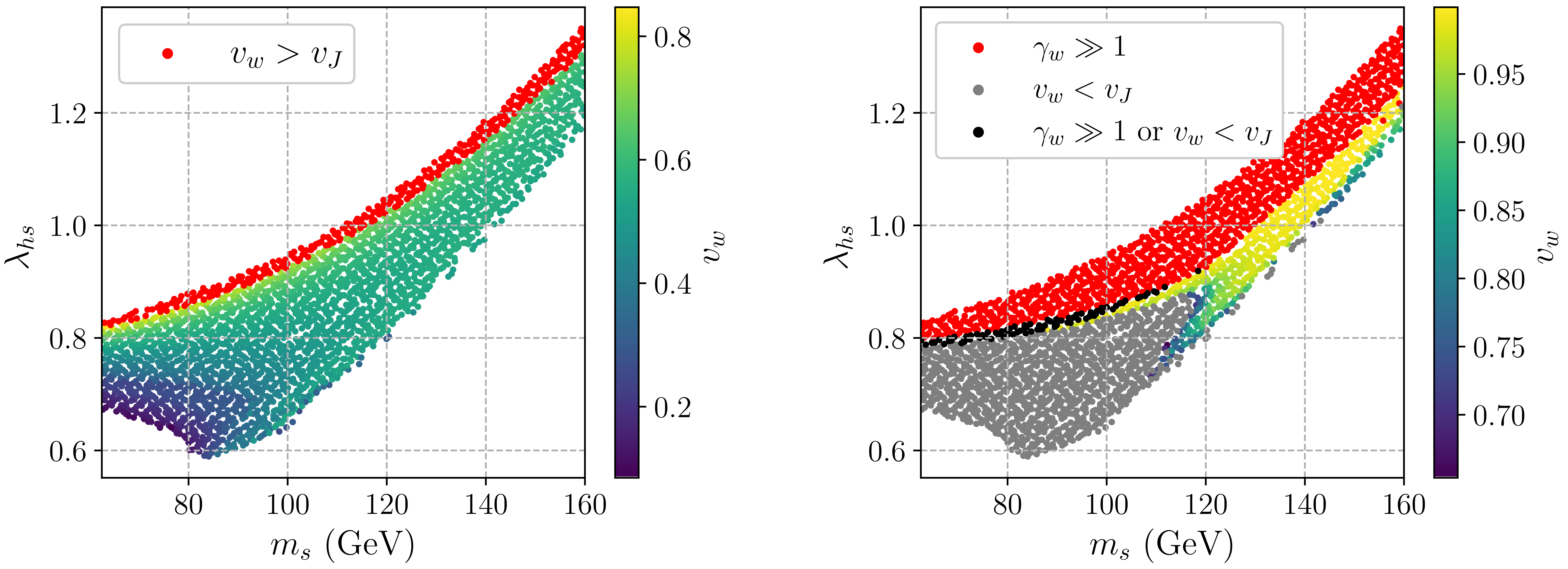
- Controlled convergence in number of polynomials
- No singularity appearing in the pressure

Disadvantages

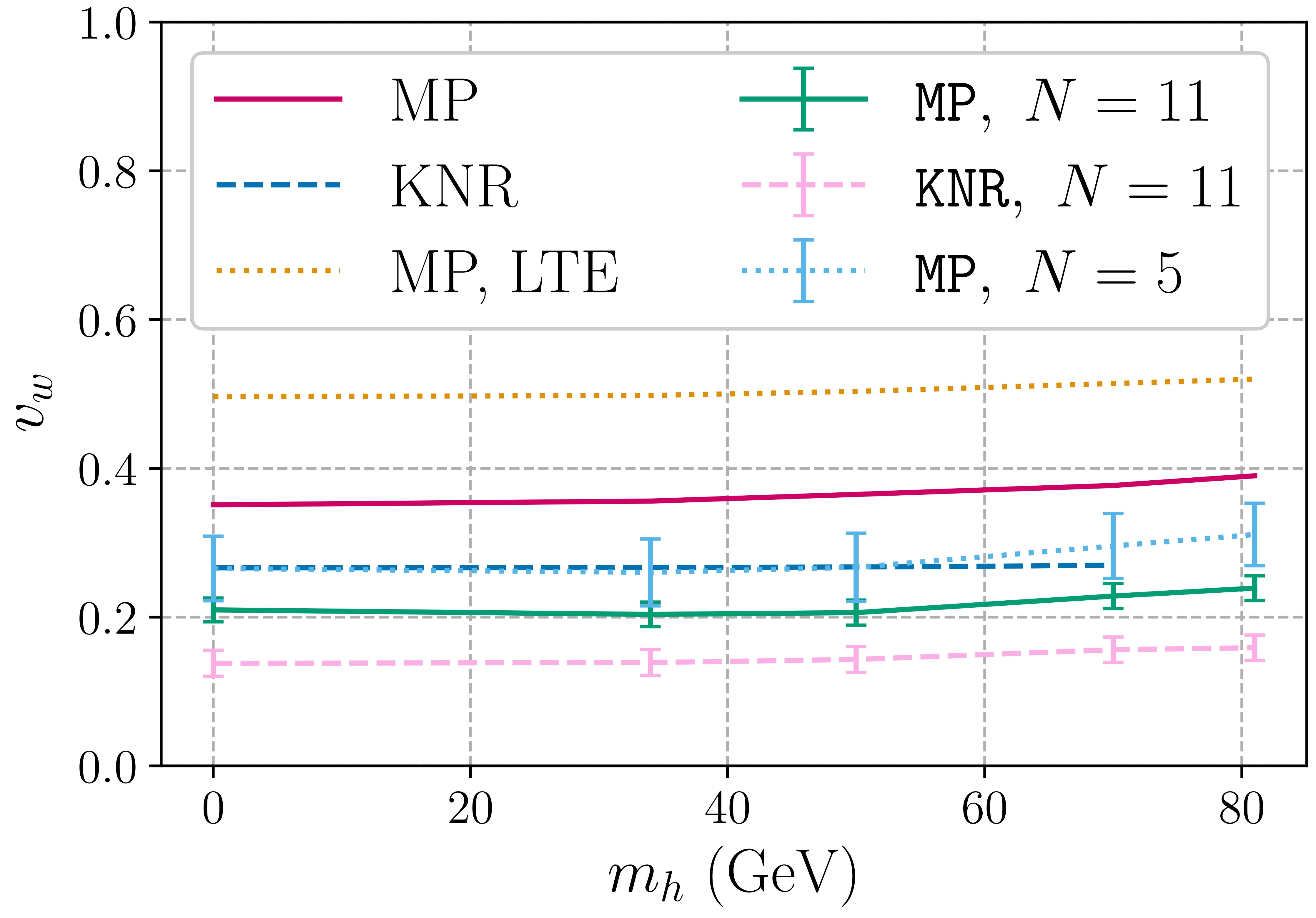
- Numerically more intensive than moments
- Individual moments have no clear physical interpretation

Some results from WAGGO

Parameter scan of the xSM



Standard Model with a light Higgs mass



Inert Doublet Model

Comparison with Jiang, Huang, Wang 2022

BM	$\bar{m}_H$ [GeV]	$\bar{m}_A, \bar{m}_{H^\pm}$ [GeV]	λ_L	T_c [GeV]	T_n [GeV]	v_w [49]	v_w [WallGo]
A	62.66	300	0.0015	118.3	117.1	0.165	0.191 ± 0.024
B	65.00	300	0.0015	118.6	117.5	0.164	0.180 ± 0.025
C	63.00	295	0.0015	119.4	118.4	0.164	0.182 ± 0.024

Questions?