Phase-transition thermodynamics and matrix element generation

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Sizing the walls:
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Seoul, 02/2025

Introduction to DRalgo and WallGoMatrix Part I

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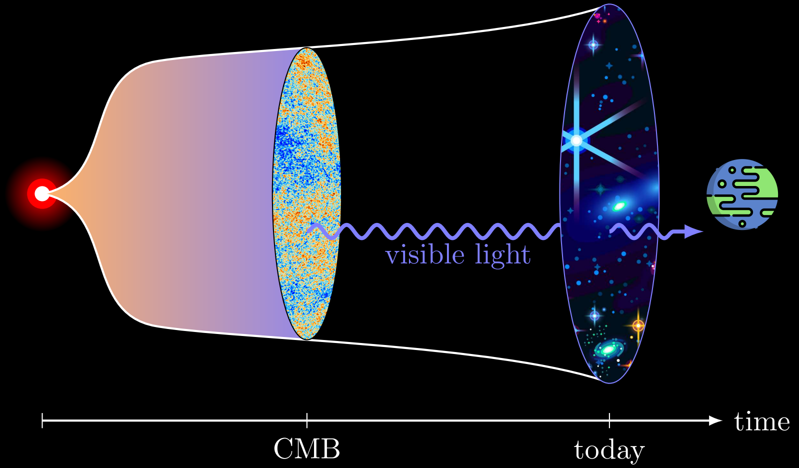
Gravitational Waves (GWs) are distortions of spacetime

sourced by early-universe phase transitions caused by new physics.



Cosmic Microwave Background (CMB)

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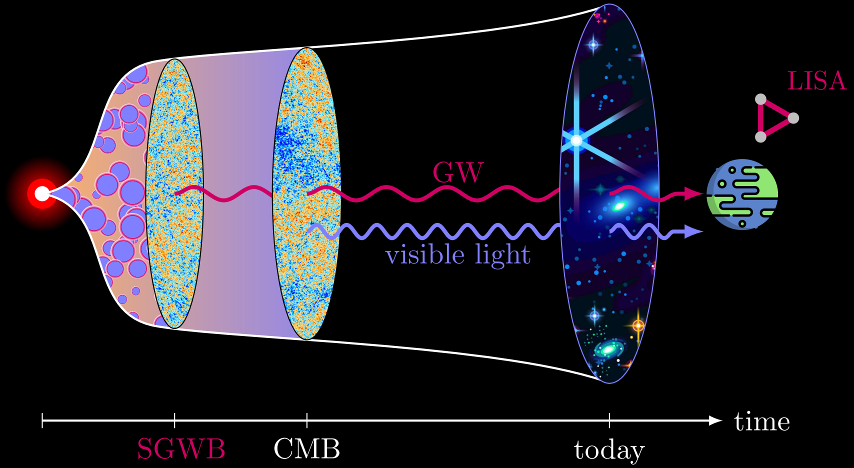


Cosmic Microwave Background (CMB)
Stochastic GW Background (SGWB)
Laser Interferometer Space Antenna (LISA)

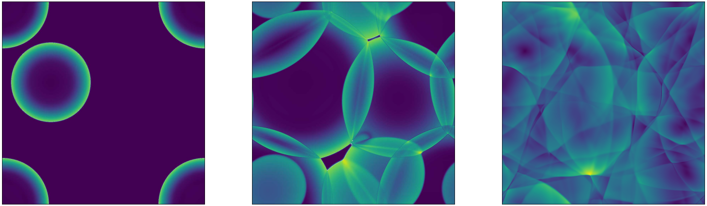
The thermal history of electroweak symmetry breaking

Strong first-order cosmic phase transition at EW scale $T_c \sim 100$ GeV:

- Baryogenesis — Baryon asymmetry of the universe
- Colliding bubbles — Gravitational wave (GW) production

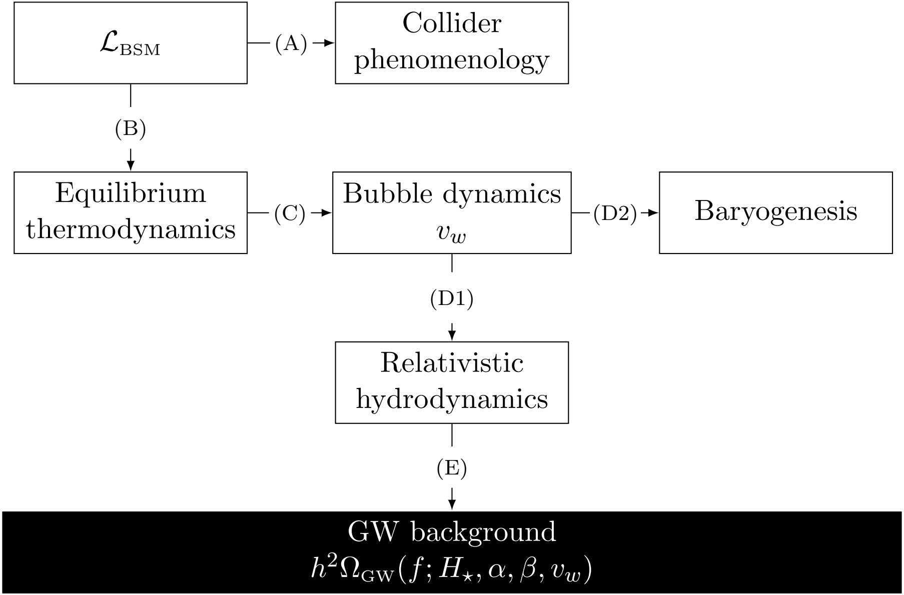
In Standard Model EWSB occurs via a smooth crossover but possible that it is first-order in Beyond the Standard Model (BSM) extensions. Study **BSM physics** near EW scale in context of phase transitions:

- Light fields strongly coupled to Higgs
- Collider targets. BSM testing pipeline: Collider phenomenology



figures by D. Cutting, M. Hindmarsh, and D. J. Weir, *Vorticity, kinetic energy, and suppressed gravitational wave production in strong first order phase transitions*, Phys. Rev. Lett. **125** (2020) 021302 [1906.00480]

Uncertainties of the gravitational wave pipeline



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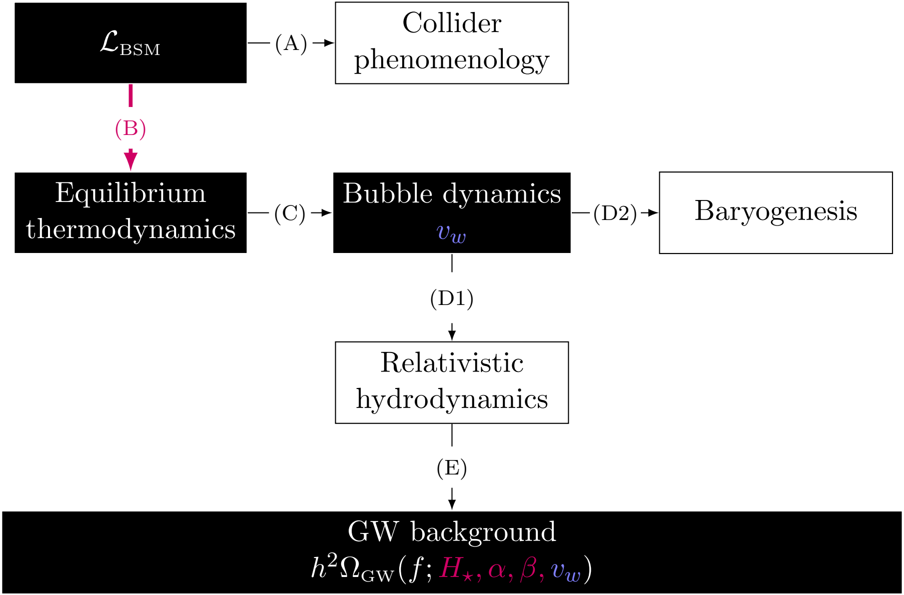

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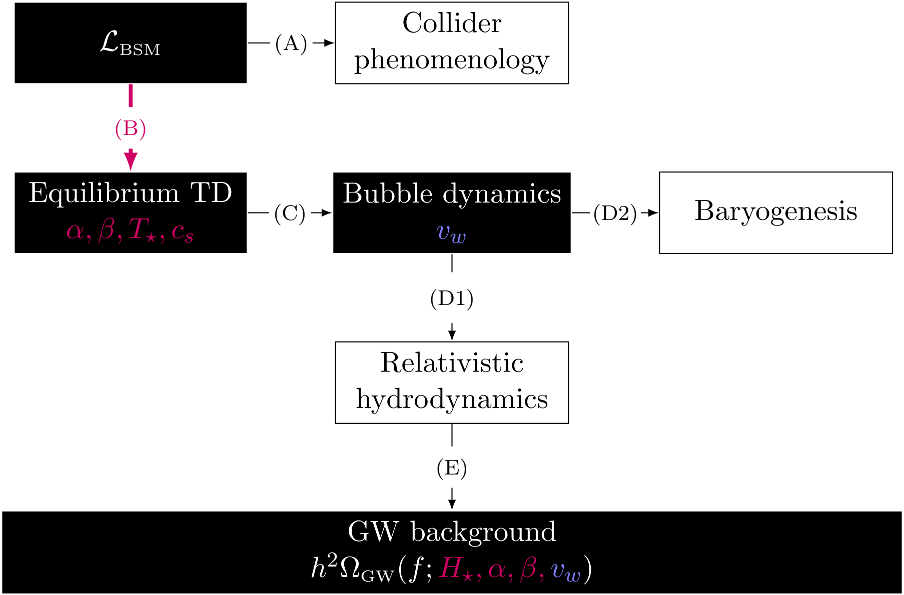
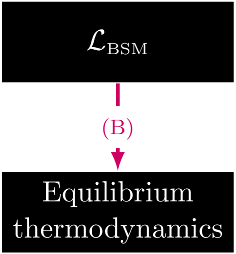
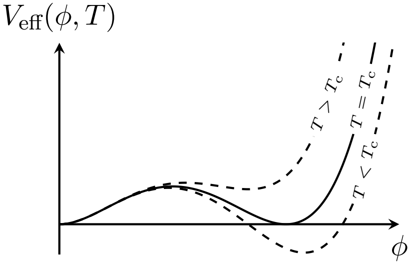


(B): The effective potential V_{eff} in perturbation theory[1]

encodes equilibrium thermodynamics as function of BSM parameters.
Origin of uncertainty.





[1] R. Jackiw, *Functional evaluation of the effective potential*, Phys. Rev. D **9** (1974) 1686

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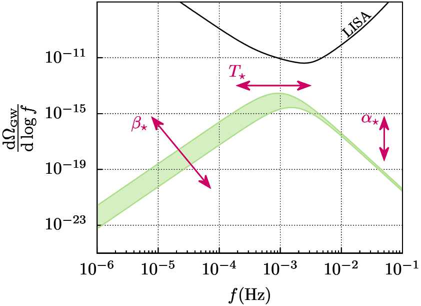
Origin of uncertainty.

- $T_\star$ reference temperature of the transition ($T_\star = T_{\text{n}}, T_{\text{p}}$),
- α phase transition strength,
- β/H inverse duration of the transition,



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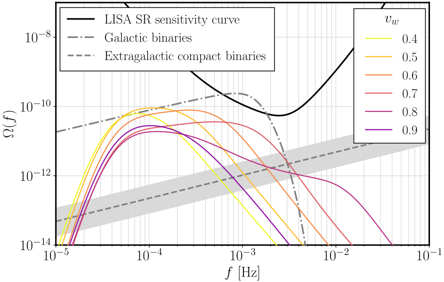
Origin of uncertainty.

- $T_\star$ reference temperature of the transition ($T_\star = T_{\text{n}}, T_{\text{p}}$),
- α phase transition strength,
- β/H inverse duration of the transition,
- v_w Iteratively solve coupled fluid, scalar field, Boltzmann equations[2]



[2] Numerical package for v_w: Ekstedt, Gould, Hirvonen, Laurent, Niemi, Schicho, van de Vis: [2411.04970], B. Laurent and J. M. Cline, *First principles determination of bubble wall velocity,* Phys. Rev. D **106** (2022) 023501 [2204.13120], C. Gowling and M. Hindmarsh, *Observational prospects for phase transitions at LISA: Fisher matrix analysis,* JCAP **10** (2021) 039 [2106.05984]

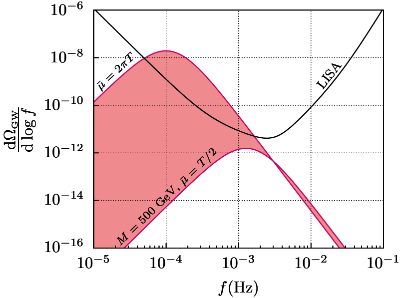
Theoretical predictions are not robust

$\mathcal{O}(10^4)$ uncertainty even for purely perturbative regimes[3] as $\Omega_{\rm GW}$ depends strongly on the transition temperature, $T_\star$, in simulation fits:

$$\Omega_{\rm GW} \propto \frac{(\Delta V_\star)^2}{T_\star^8} \,.$$

Vary RG scale $\bar{\mu}$ in SM extensions:

- ▷ SMEFT: SM $+ \frac{1}{M^2}(\phi^\dagger \phi)^3$



[3] D. Croon, O. Gould, P. Schicho, T. V. I. Tenkanen, and G. White, *Theoretical uncertainties for cosmological first-order phase transitions,* JHEP **04** (2021) 055 [2009.10080], O. Gould and T. V. I. Tenkanen, *On the perturbative expansion at high temperature and implications for cosmological phase transitions,* JHEP **06** (2021) 069 [2104.04399]

[4] S. Biondini, P. Schicho, and T. V. I. Tenkanen, *Strong electroweak phase transition in t-channel simplified dark matter models,* JCAP **10** (2022) 044 [2207.12207]

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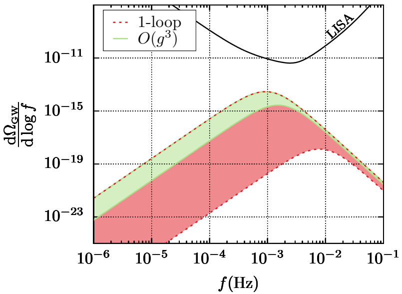
Vary RG scale $\bar{\mu}$ in SM extensions:

- SMEFT: SM + $\frac{1}{M^2}(\phi^\dagger\phi)^3$
- xSM: SM + singlet
- DM models[4]



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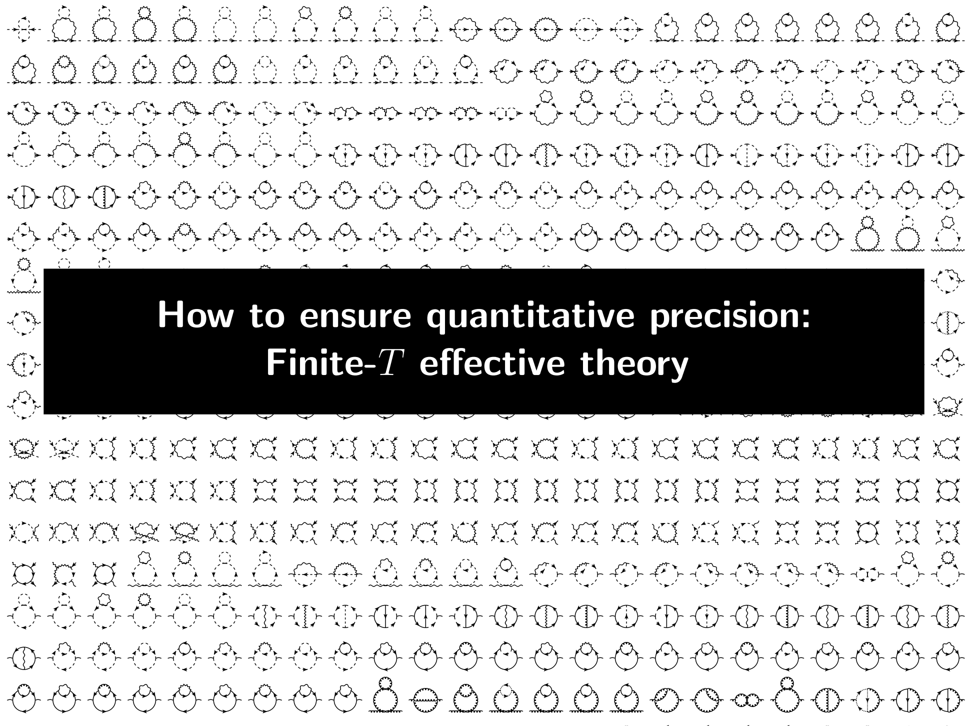


How to ensure quantitative precision: Finite-T effective theory

Perturbative phase transitions need scale hierarchies

for quantum effects ΔV_{fluct} to influence the tree-level potential

$$V_{\text{eff}} = V_{\text{tree}} + \Delta V_{\text{fluct}}\ .$$

Assume particle χ couples to the SM via $g^2 \Phi^\dagger \Phi \chi^\dagger \chi$. If $M_\chi \gg m_\Phi$, integrating out χ introduces Higgs-mass corrections of the form:

$$(\Delta m_\Phi^2)\Phi^\dagger\Phi =$$

$$\sim g^2 M_\chi^2 \Phi^\dagger \Phi\,, \qquad \frac{(\Delta m_\Phi^2)}{m_\Phi^2} = g^2 \left[\frac{M_\chi}{m_\Phi}\right]^2 .$$

Relevant operators ($\sigma > 0$) in the IR get large UV contributions and

$$\frac{\Delta V_{\text{fluct}}}{V_{\text{tree}}} \sim g^2 \left[\frac{\Lambda_{\text{fluct}}}{\Lambda_{\text{tree}}}\right]^\sigma \overset{!}{\sim} 1 \quad \Rightarrow \begin{cases} \text{strong coupling} & g^2 \gtrsim 1 \\ \text{scale hierarchy} & \frac{\Lambda_{\text{fluct}}}{\Lambda_{\text{tree}}} \sim \left[\frac{1}{g^2}\right]^{\frac{1}{\sigma}} \gg 1 \end{cases}$$

Equilibrium thermodynamics: Imaginary time formalism

$\rho(\beta) = e^{-\beta\mathcal{H}} \to \mathcal{U}(t) = e^{-i\mathcal{H}t}$. Relating density operator to time evolution corresponds to path integral over imaginary-time $t \to -i\tau$,

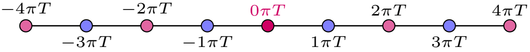
$$\mathcal{Z} = C \int_{\text{b.c.}} \mathcal{D}\phi \, \exp\left[- \int_0^{\beta=1/T} \mathrm{d}\tau \int_{\mathbf{x}} \mathcal{L}_{\mathrm{E}}\right] , \quad \phi(0,\mathbf{x}) = \pm\phi(\beta,\mathbf{x}) \, .$$

(Anti-)periodic bosonic(fermionic) fields at boundaries → compact time direction: $\mathbb{R}^3 \times S^1_\beta$.

Finite-τ and (b.c.) induce a discrete Fourier sum for time component $P = (\omega_n, \mathbf{p})$ with **Matsubara frequencies**

$$\omega_n = \begin{cases} 2n\pi T & \text{bosonic} \\ (2n+1)\pi T & \text{fermionic} \end{cases}$$

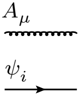
Absent zero mode $\omega_{n=0}$ for fermions:



Differences to zero temperature

$(d+1) \to d$-dimensional theory with infinite tower of massive modes.

Euclidean free-particle propagator for bosonic(fermionic) fields:



$$\Bigg\} \propto \frac{1}{P^2 + m^2} = \frac{1}{\mathbf{p}^2 + \omega_n^2 + m^2}\,, \quad P = (\omega_n, \mathbf{p})\,.$$

Same diagrams as in zero temperature QFT, go over to Euclidean space, and substitute Euclidean frequency integrals by sums

$$\int \frac{\mathrm{d}^{d+1}p}{(2\pi)^{d+1}} f(p) \to T \sum_n \int \frac{\mathrm{d}^d p}{(2\pi)^d} f(\omega_n, \mathbf{p}) = \sum\!\!\!\!\!\!\int_P f(\omega_n, \mathbf{p})\,.$$

- Ultraviolet (UV) contained at $T = 0$
- Infrared (IR) sensitivity stronger $\to$ field in reduced spacetime dimension

Resummation

Dynamically generated masses through collective plasma effects

$$m_T = g^n T + m \ .$$

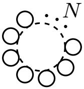
Evaluate Matsubara sums yielding Bose(Fermi) distribution. At asymptotically high-T and weak $g \ll 1$ the effective expansion parameter

$$g^2 n_{\mathrm{B}}(|p|) = \frac{g^2}{e^{|p|/T} - 1} \approx \frac{g^2 T}{|p|} \geq \frac{g^2 T}{m}$$

differs from the weak coupling g^2. Fermions are IR-safe $g^2 n_{\mathrm{F}} |p| \sim g^2/2$.

Cure IR-sensitive contributions at $m_T \sim gT$ by thermal resummation:

$$V_{\text{eff}} \supset$$



$$\propto \quad g^{2N} \left[{m_T}^{3-2N} T \right] \left[\frac{T^2}{12} \right]^N \propto m^3 T \left[\frac{gT}{m_T} \right]^{2N}$$

For $m_T \leq g^2 T$ weak expansion breaks down. At finite T, light bosons are non-perturbative.

Multi-scale hierarchy in hot classicalizing gauge theories

Evaluated **Matsubara sums** yield Bose(Fermi) distribution. Asymptotically high T and weak $g \ll 1$: effective expansion parameter

$$\epsilon_{\rm B} = g^2 n_{\rm B}(E) = \frac{g^2}{e^{E/T} - 1} \approx \frac{g^2 T}{E} \,.$$

Differs from weak coupling g^2. Fermions are IR-safe $g^2 n_{\rm F}(E) \sim g^2/2$.

$$E \sim \begin{cases} \pi T & \textit{hard} \text{ scale} \\ gT & \textit{soft} \text{ scale} \\ g^{3/2}T & \textit{supersoft} \text{ scale} \\ g^2 T/\pi & \textit{ultrasoft} \text{ scale} \end{cases}$$

quantum theory

symmetry breaking

Limit: Confinement-like behavior in ultrasoft sector $g^2 n_{\rm B}(g^2 T) \sim \mathcal{O}(1)$. Ultrasoft bosons are non-perturbative at finite T: Linde IR problem.[5]

[5] A. Linde, *Infrared problem in the thermodynamics of the Yang-Mills gas*, Phys. Lett. B **96** (1980) 289, O. Gould and T. V. I. Tenkanen, *Perturbative effective field theory expansions for cosmological phase transitions*, JHEP **01** (2024) 048 [2309.01672]

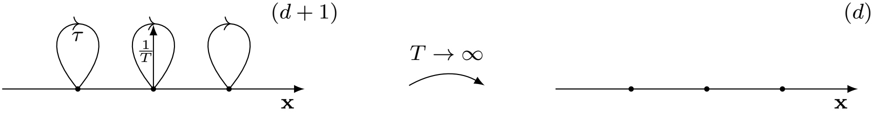
Effective Theory (EFT): Definition

Framework for theory with scale hierarchy: Effective Field Theory.

1. Identify soft degrees of freedom.
2. Construct most general low-energy Lagrangian.
3. *Match* Green's functions $\to$ determine EFT coupling constants.

Perturbative and IR safe: Matching in the IR (regime of mutual validity), UV incorporated in EFT coefficients.

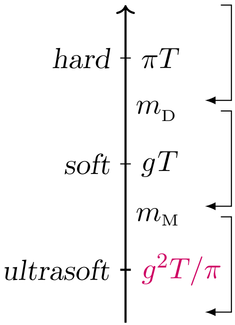
Modes with wavelengths $|\mathbf{x}|, |x_0| \gg \beta$ or $\omega_n^2 + m^2 \ll T^2$ *effectively* live in 3-dimensions.



Effective Field Theory (EFT): Dimensional Reduction (DR)

Integrate out hard modes perturbatively → EFT for static modes.

Precision thermodynamics of non-Abelian gauge theories as QCD and (EW) phase transition[6] using e.g. `DRalgo`.[7] Two `step` procedure:



$\mathcal{L}_{\rm QCD}$, hot QCD, $(d+1)$-dim

`step1`: Dimensional Reduction

$\mathcal{L}_{\rm EQCD}$, $\mathrm{SU}(N_{\rm c})$ + adj. Higgs, d-dim

`step2`: Integrate out soft temporal scalars, A_0

$\overline{\mathcal{L}}_{\rm MQCD}$, Yang-Mills, d-dim

[6] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model,* Nucl. Phys. B **458** (1996) 90 [`hep-ph/9508379`]

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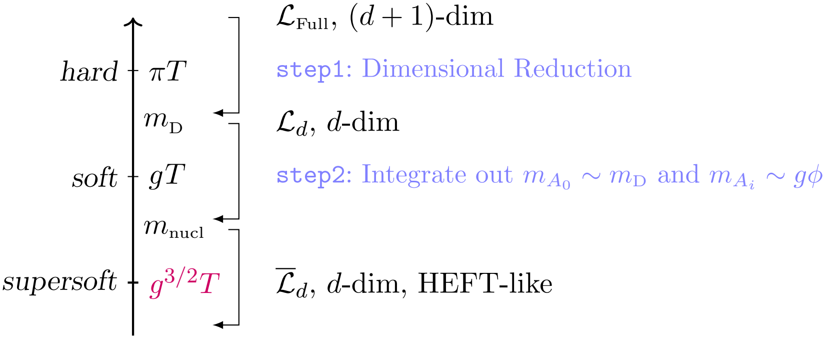

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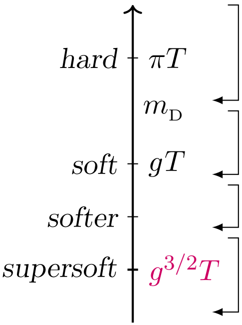

$\mathcal{L}_{\mathrm{Full}}$, $(d+1)$-dim

`step1`: Dimensional Reduction

$\mathcal{L}_d$, d-dim

`step2.1`: Integrate out $m_{A_0} \sim m_{\mathrm{D}}$

`step2.2`: Integrate out $m_{A_i} \sim g\phi$

$\overline{\mathcal{L}}_d$, d-dim, HEFT-like

[6] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model,* Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379]

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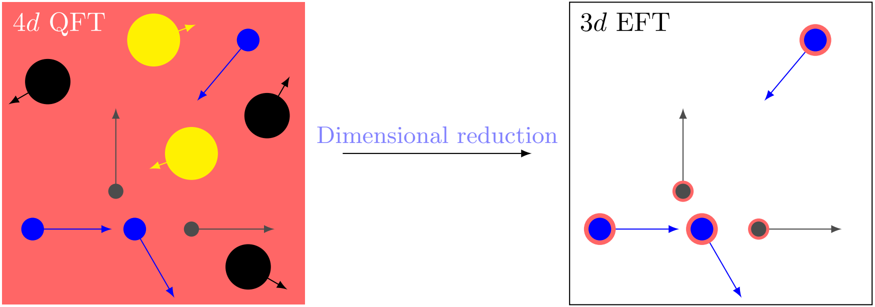

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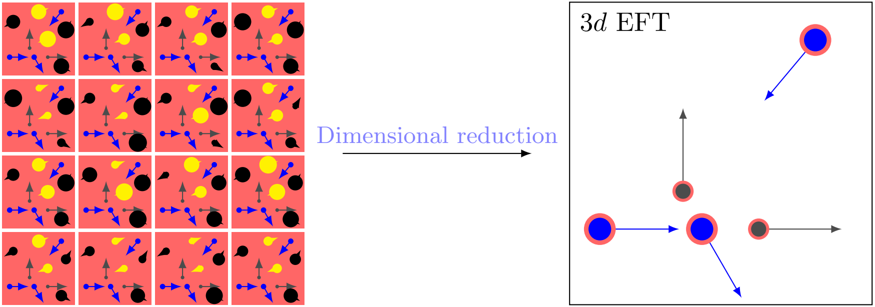

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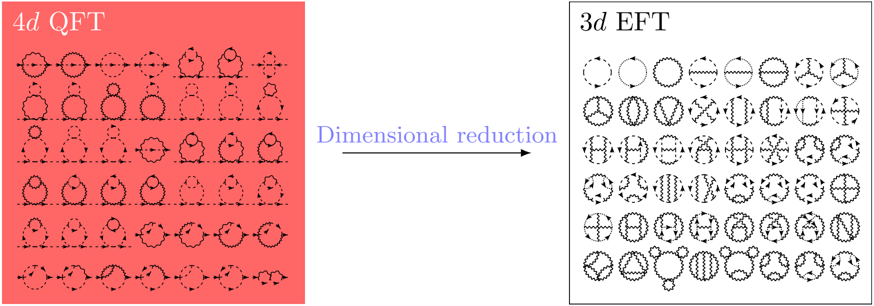

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Constructing the supersoft EFT

Two approaches to a *final* (nucleation) EFT

$$\mathcal{L}_{4\text{d}} \to \mathcal{L}_{3\text{d}}^{\text{soft}} \to \mathcal{L}_{3\text{d}}^{\text{softer}} \to V_{\text{eff}}^{\text{supersoft}} \,,$$

In the broken phase, utilize different hierarchies among Lorentz scalars, temporal scalars, and vectors:

$$m_{A_0}^2\big[\sim (gT)^2\big] \overset{\texttt{step2.1}}{\gg} m_{A_i}^2\big[\sim (g_3\phi)^2\big] \overset{\texttt{step2.2}}{\gg} m_3^2\big[\sim \lambda_3\phi^2\big] \,,$$

Constructing the supersoft EFT

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$$m_{A_0}^2 \big[\sim (gT)^2\big] = m_{A_i}^2 \big[\sim (g_3\phi)^2\big] \overset{\texttt{step2}}{\gg} m_3^2 \big[\sim \lambda_3\phi^2\big] \,.$$

Constructing the supersoft EFT

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$$\mathcal{L}_{\rm 4d} \to \mathcal{L}_{\rm 3d}^{\rm soft} \to \mathcal{L}_{\rm 3d}^{\rm softer} \to V_{\rm eff}^{\rm supersoft}\,,$$
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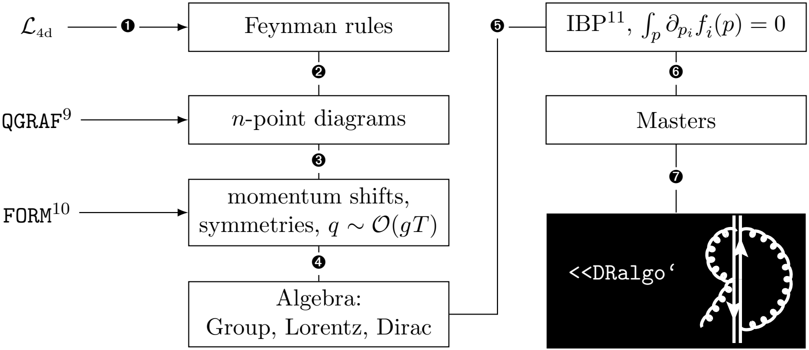
$$m^2_{A_0}\big[\sim (gT)^2\big] = m^2_{A_i}\big[\sim (g_3\phi)^2\big] \overset{\texttt{step2}}{\gg} m^2_{3}\big[\sim \lambda_3\phi^2\big]\,.$$

In the symmetric phase, the hierarchy is flipped

$$m^2_{A_0}\big[\sim (gT)^2\big] \gg m^2_{3}\big[\sim (g^{3/2}T)^2\big] \overset{\texttt{step2}}{\gg} m^2_{A_i}\big[\sim (g^2T)^2\big]\,.$$

The Dimensional Reduction algorithm (DRalgo)

State-of-the-art Mathematica package DRalgo.[8]



[8] github.com/DR-algo/DRalgo, A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, DRalgo*: A package for effective field theory approach for thermal phase transitions,* Comput. Phys. Commun. **288** (2023) 108725 [2205.08815]

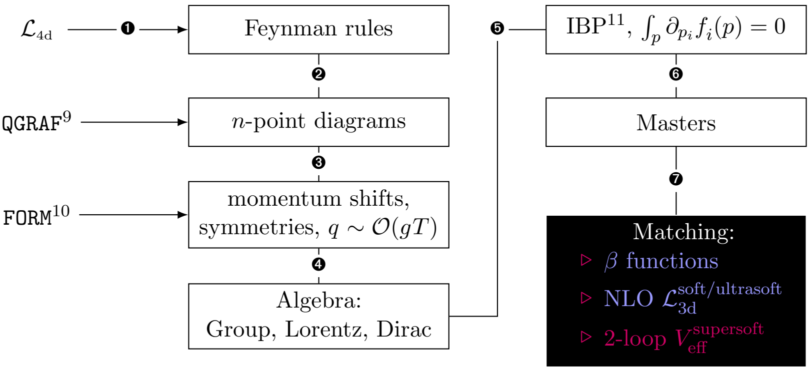
[9] P. Nogueira, *Automatic Feynman Graph Generation,* J. Comput. Phys. **105** (1993) 279

[10] B. Ruijl, T. Ueda, and J. Vermaseren, *FORM version 4.2* arXiv (2017) [1707.06453]

[11] S. Laporta, *High precision calculation of multiloop Feynman integrals by difference equations,* Int. J. Mod. Phys. A **15** (2000) 5087 [hep-ph/0102033]

The Dimensional Reduction algorithm (DRalgo)

State-of-the-art Mathematica package DRalgo.[8]



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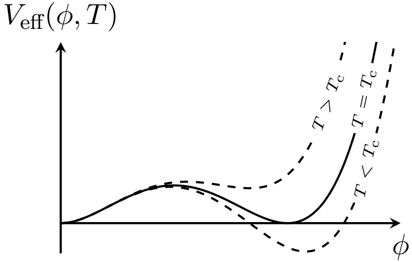
Predictions of GWs sourced by cosmological phase transitions

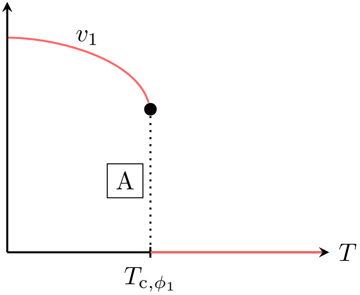
The effective potential in perturbation theory

receives thermal corrections $\Pi_T \sim \gamma T^2$ with $\gamma \sim g^n$. Dynamical generation of scales close to critical temperature T_c:

$$V_{\text{eff}} \simeq \frac{1}{2}(-\mu^2 + \Pi_T)\phi^2 + \frac{1}{2}\lambda\phi^4 - \frac{g^3 T}{16\pi}\phi^3 + \dots$$

$$(-\mu^2 + g^n T^2) \sim \underbrace{0 \times (gT)^2}_{\text{soft}} + \underbrace{0 \times (g^{3/2}T)^2}_{\text{supersoft}} + \underbrace{\#(g^2T)^2}_{\text{ultrasoft}}.$$





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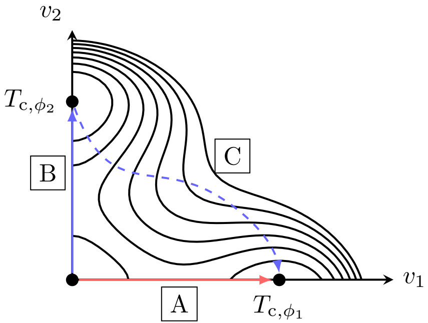
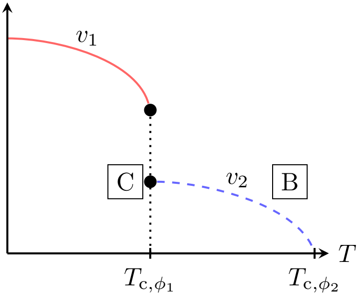
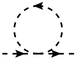
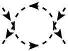
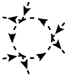




The thermal effective potential at LO

$$V_{\text{eff}} = V_{\text{eff}}^{\text{tree}} + V_{\text{eff}}^{1\ell} \,.$$

At 1-loop sum over n-point functions at $Q_i = 0$ external momenta

$V_{\text{eff}}^{1\ell} =$  $+ \frac{1}{2}$  $+ \frac{1}{3}$  $+ \dots \Big|_{Q_i=0}$

$$= \frac{1}{2} \sum\!\!\!\!\!\!\!\int_P \ln\left(P^2 + m^2\right)$$

$$V_{\text{eff}}^{1\ell} = \underbrace{\frac{1}{2}\int_P \ln(P^2 + m^2)}_{\equiv V_{\text{CW}}(m)} \underbrace{- T\int_p \ln\left(1 \mp n_{\text{B/F}}(E_p, T)\right)}_{\equiv V_{T,b/f}\left(\frac{m^2}{T^2}\right)}$$

$$= \underbrace{\frac{T}{2}\int_p \ln(p^2 + m^2)}_{\equiv T V_{\text{soft}}(m)} + \underbrace{\frac{1}{2} \sum\!\!\!\!\!\!\!\int_{P/\{P\}}' \ln(P^2 + m^2)}_{\equiv V_{\text{hard}}(m)} \,.$$

Renormalization scale (in)dependence at finite T

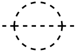
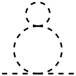
At zero temperature

$$V_{\text{eff}}(\phi,\bar{\mu}) = \boxed{\begin{matrix} V_{\text{eff}}^{\text{tree}} \\ \mathcal{O}(g^2) \end{matrix}} + \boxed{\begin{matrix} V_{\text{CW},1\ell} \\ \mathcal{O}(g^4) \end{matrix}}\,, \quad \mu\frac{\mathrm{d}}{\mathrm{d}\mu}\left(V_{\text{eff}}^{\text{tree}} + V_{\text{CW},1\ell}\right) = 0\,.$$

At finite temperature[12]

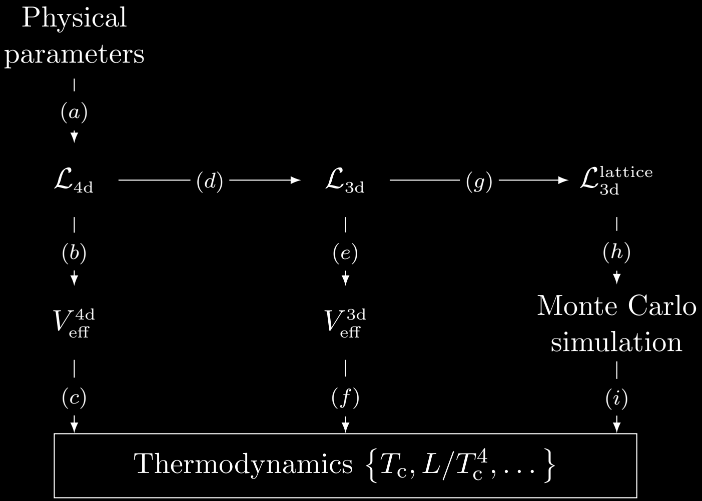
$$V_{\text{eff}}^{\text{res.}}(\phi,T,\bar{\mu}) = \boxed{\begin{matrix} V_{\text{eff}}^{\text{tree}} \\ \mathcal{O}(g^2) \end{matrix}} + \boxed{\begin{matrix} V_{\text{res.,soft}} \\ \mathcal{O}(g^3) \end{matrix}} + \boxed{\begin{matrix} V_{\text{hard}} \\ \mathcal{O}(g^2T^2)+\mathcal{O}(g^4) \end{matrix}}\,,$$

running of 1-loop thermal masses is of the same order as 2-loop thermal-mass logarithms.
Automatically included in dimensionally reduced 3d EFT:

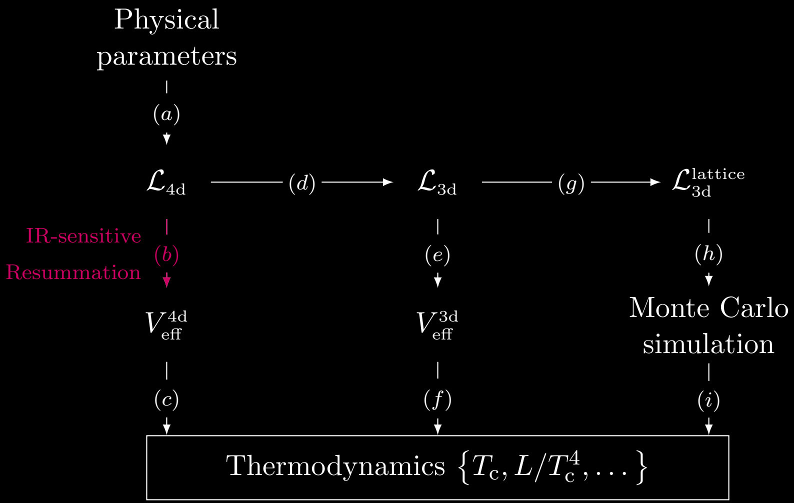
$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}$  $\sim \mu\frac{\mathrm{d}}{\mathrm{d}\mu}$  $\sim$  $\sim$  $\sim \mathcal{O}(g^4T^2)$

[12] O. Gould and T. V. I. Tenkanen, *On the perturbative expansion at high temperature and implications for cosmological phase transitions*, JHEP **06** (2021) 069 [2104.04399]

Thermodynamics of electroweak phase transition

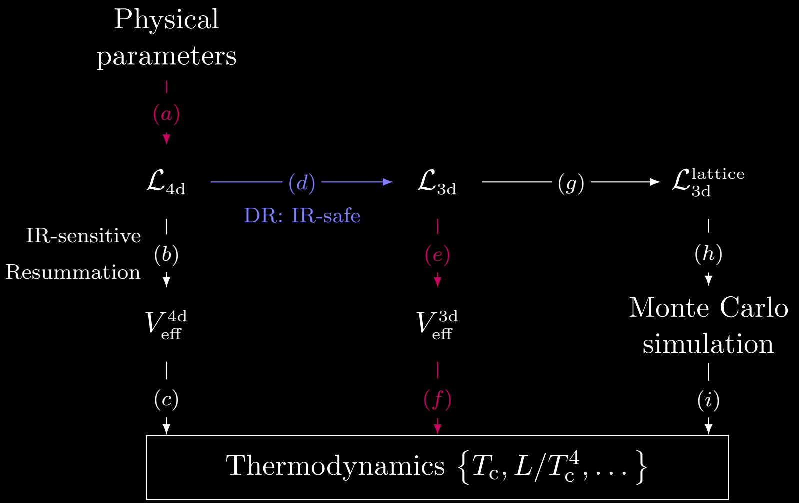


Thermodynamics of electroweak phase transition



▷ $4d$ approach: Classical predictions within quantum theory

Thermodynamics of electroweak phase transition



- $4d$ approach: Classical predictions within quantum theory
- $3d$ approach: Classical predictions within classical EFT

Completing the perturbative program for phase transitions

❶ $\mathcal{L}^{3d}$ (hard-to-soft matching)

❷ V_{eff}^{3d} (soft-to-supersoft matching)

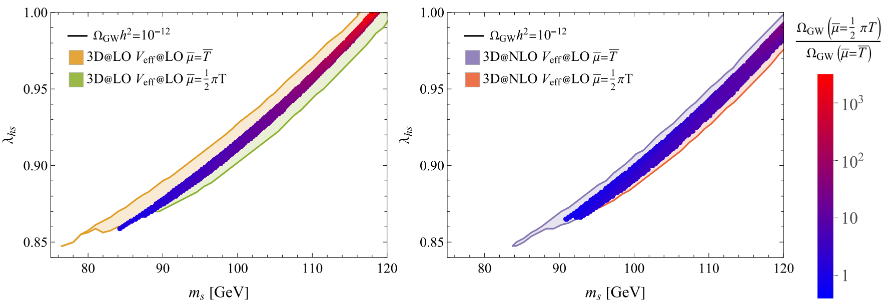
❸ Thermodynamics

❶ $\mathcal{L}_{3d}$ (hard-to-soft matching)

$3d$ EFT at NLO (gauge-invariant) with `DRalgo`. Todo: NNLO.

Monitor Higgs (v) and **real singlet** (x) VEV after shift $s \to x + s$.
2-loop corrections significantly affect GW signal.[13]

Parameters	m_s λ_{hs} $\lambda_s = 1$	$\alpha < 1$ $\mathrm{SNR_{LISA}} > 10$



[13] M. Lewicki, M. Merchand, L. Sagunski, P. Schicho, and D. Schmitt, *Impact of theoretical uncertainties on model parameter reconstruction from GW signals sourced by cosmological phase transitions*, Phys. Rev. D **110** (2024) 023538 [2403.03769], L. Niemi, P. Schicho, and T. V. I. Tenkanen, *Singlet-assisted electroweak phase transition at two loops*, Phys. Rev. D **103** (2021) 115035 [2103.07467]

Computational diligence at $\mathcal{O}(g^4)$

Monitor Higgs (v) and **real singlet** (x) VEV after shift $s \to x + s$.
2-loop corrections move "Bananas": significant effect on GW signal.[14]

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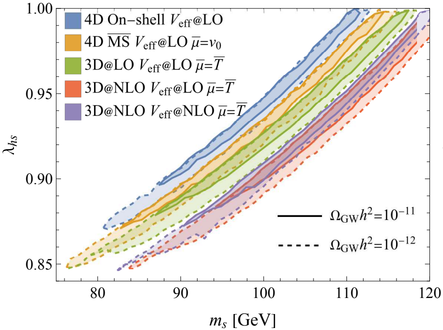

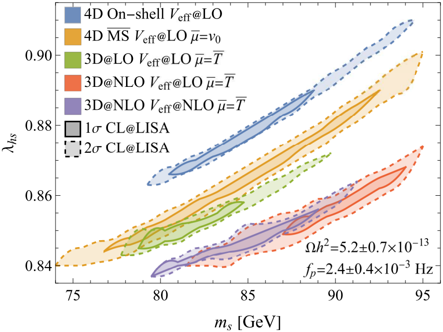

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Does the banana move?

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2-loop corrections move "Bananas": significant effect on GW signal.[14]

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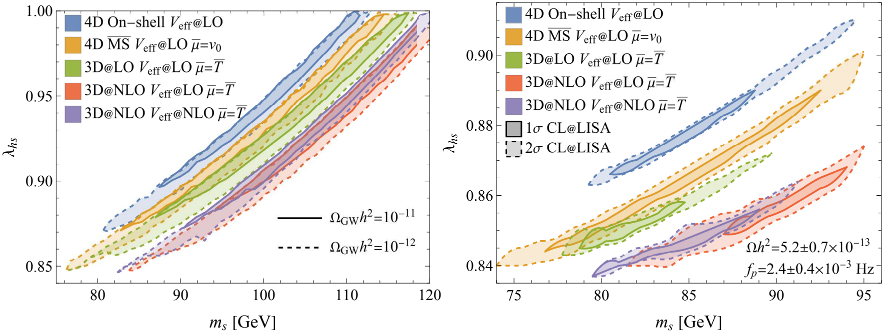

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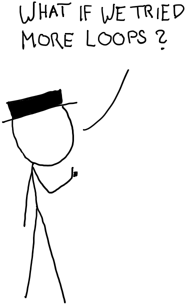
[14] M. Lewicki, M. Merchand, L. Sagunski, P. Schicho, and D. Schmitt, *Impact of theoretical uncertainties on model parameter reconstruction from GW signals sourced by cosmological phase transitions*, Phys. Rev. D **110** (2024) 023538 [2403.03769], L. Niemi, P. Schicho, and T. V. I. Tenkanen, *Singlet-assisted electroweak phase transition at two loops*, Phys. Rev. D **103** (2021) 115035 [2103.07467]

❷ V_{eff}^{3d} (soft-to-supersoft matching)

Integrating out vector bosons in two steps up to 2-loops with `DRalgo`

$$m_{A_0}^2[\sim (gT)^2] \overset{\texttt{step2.1}}{\gg} m_{A_i}^2[\sim (g_3\phi)^2] \overset{\texttt{step2.2}}{\gg} m_3^2[\sim \lambda_3\phi^2] \ .$$

Focus on `step2.2` and add last perturbative orders N^{3}LO and N^{4}LO.[15]



[15] A. Ekstedt, O. Gould, and J. Löfgren, *Radiative first-order phase transitions to next-to-next-to-leading order,* Phys. Rev. D **106** (2022) 036012 [2205.07241], A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, *Cosmological phase transitions at three loops: The final verdict on perturbation theory,* Phys. Rev. D **110** (2024) 096006 [2405.18349]

❷ $V_{\text{eff}}^{3\text{d}}$ (soft-to-supersoft matching)

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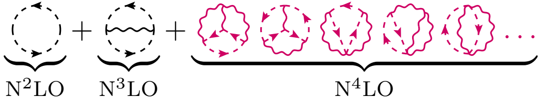
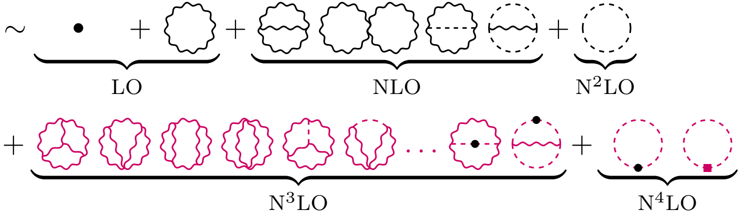
$$m^2_{A_0}\left[\sim (gT)^2\right] \overset{\texttt{step2.1}}{\gg} m^2_{A_i}\left[\sim (g_3\phi)^2\right] \overset{\texttt{step2.2}}{\gg} m^2_3\left[\sim \lambda_3\phi^2\right] .$$

Focus on `step2.2` and add last perturbative orders N^{3}LO and N^{4}LO.[15]

$V_{\text{eff}}^{\text{sym}} \sim$



$V_{\text{eff}}^{\text{bro}} \sim$



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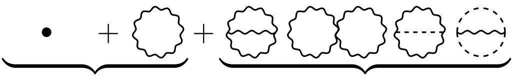
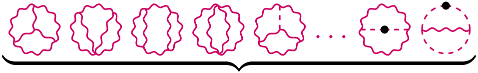
$$m_{A_0}^2 \left[\sim (gT)^2\right] \overset{\texttt{step2.1}}{\gg} m_{A_i}^2 \left[\sim (g_3\phi)^2\right] \overset{\texttt{step2.2}}{\gg} m_3^2 \left[\sim \lambda_3 \phi^2\right] .$$

Focus on `step2.2` and add last perturbative orders N^{3}LO and N^{4}LO.[15]
In strict EFT expansion this organization can also be understood as:

$$S_{\text{supersoft}}^{\text{tree}} = \int_{\mathbf{x}} \frac{1}{2} (\partial_i s)^2 Z_s + $$

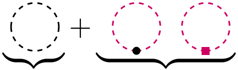


LO NLO

+  ,

N^{3}LO

$$S_{\text{supersoft}}^{\text{1-loop}} = $$

 .

N^{2}LO N^{4}LO

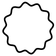
[15] A. Ekstedt, O. Gould, and J. Löfgren, *Radiative first-order phase transitions to next-to-next-to-leading order*, Phys. Rev. D **106** (2022) 036012 [2205.07241], A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, *Cosmological phase transitions at three loops: The final verdict on perturbation theory*, Phys. Rev. D **110** (2024) 096006 [2405.18349]

The SM-like $3d$ EFT (SU(2)+Higgs)

describes the thermodynamics[16] of several parent $4d$ theories:

$$\mathcal{L}_{\rm 3d} = \frac{1}{4}F^a_{ij}F^a_{ij} + (D_i\Phi)^\dagger(D_i\Phi) + V(\Phi)\ ,$$
$$V(\Phi) = m_3^2\Phi^\dagger\Phi + \lambda_3(\Phi^\dagger\Phi)^2\ .$$

If $m_A \sim g_3\phi \gg m_3$, integrating out vector boson introduces LO barrier

$$V_{\rm LO}(\Phi) = \quad \bullet \quad +$$  .

[16] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model*, Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379]

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$$V_{\mathrm{LO}}(\Phi) = m_3^2\Phi^\dagger\Phi + \lambda_3(\Phi^\dagger\Phi)^2 - \frac{g_3^3}{2\pi}\Big(\frac{\Phi^\dagger\Phi}{2}\Big)^{3/2}\ .$$

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$$V_{\mathrm{LO}}(\Phi) \to y\Phi^\dagger\Phi + x(\Phi^\dagger\Phi)^2 - \frac{1}{2\pi}\Big(\frac{\Phi^\dagger\Phi}{2}\Big)^{3/2}\ .$$

Since $x \sim \frac{m_3^2}{m_A^2} \ll 1$, and at the phase transition $y \sim 1/x$, we strictly[17] x-pand the perturbative series using 3D EFT dimensionless couplings

$$x \equiv \frac{\lambda_3}{g_3^2}\ , \qquad y \equiv \frac{m_3^2}{g_3^4}\ .$$

[16] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model,* Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379]

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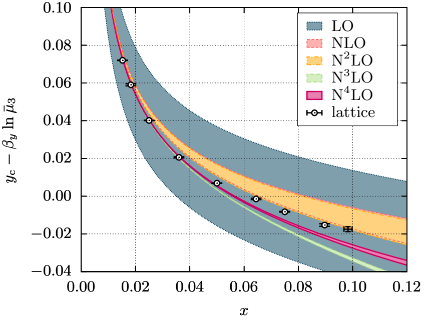
❸ Thermodynamics

By using $F \sim V_{\text{eff}}(\phi_{\text{min}})$, determine the critical mass y_{c} (or T_{c})

$$\Delta F(y_{\text{c}}(x), x) = [F_{\text{bro}} - F_{\text{sym}}](y_{\text{c}}(x), x) = 0\,,$$

and the scalar *condensates*[18]

$$\Delta\langle\Phi^\dagger\Phi\rangle \equiv \frac{\partial}{\partial y}\Delta F\,, \qquad \Delta\langle(\Phi^\dagger\Phi)^2\rangle \equiv \frac{\partial}{\partial x}\Delta F\,.$$



[18] Lattice data from: K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *The Electroweak phase transition: A Nonperturbative analysis,* Nucl. Phys. B **466** (1996) 189 [hep-lat/9510020], O. Gould, S. Güyer, and K. Rummukainen, *First-order electroweak phase transitions: a nonperturbative update,* [2205.07238]

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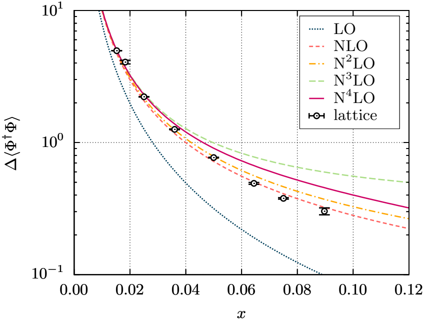

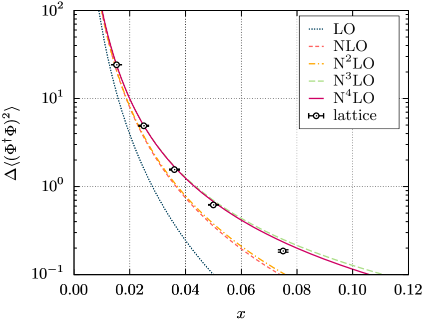

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Predicting Gravitational Waves

Thermodynamics enters the GW spectrum through the strength and inverse duration of the transition, $h^2\Omega_{\rm GW}(f; H_\star, \alpha, \beta, v_w)$:

$$\alpha \sim \frac{\mathrm{d}\,\Delta F(y_{\rm c}, x)}{\mathrm{d}\ln T} = \left(\frac{\mathrm{d}\,y_{\rm c}}{\mathrm{d}\ln T}\right)\Delta\langle\Phi^\dagger\Phi\rangle + \left(\frac{\mathrm{d}\,x}{\mathrm{d}\ln T}\right)\Delta\langle(\Phi^\dagger\Phi)^2\rangle\,,$$

$$\frac{\beta}{H} = -\frac{\mathrm{d}\ln\Gamma}{\mathrm{d}\ln T}\,.$$

$\implies$ [UV] $\times$ [IR] factorization.[19]

Completed perturbative predictions for: α at N^{4}LO, β/H at N^{2}LO.

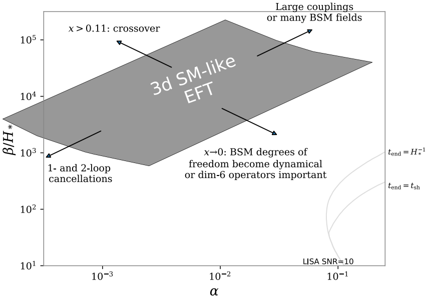
Todo: final perturbative correction for thermal bubble nucleation rate.

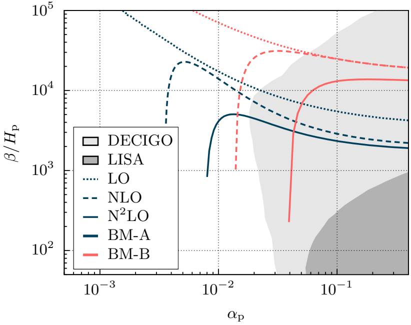
[19] O. Gould, J. Kozaczuk, L. Niemi, M. J. Ramsey-Musolf, T. V. I. Tenkanen, and D. J. Weir, *Nonperturbative analysis of the gravitational waves from a first-order electroweak phase transition,* Phys. Rev. D **100** (2019) 115024 [1903.11604]

Impact on Gravitational Waves

$\alpha/(\beta/H)$ rhombus of SM-like EFT with no prospect for large SNR.[20]
Better access interesting LISA SNR by increasing loop order:

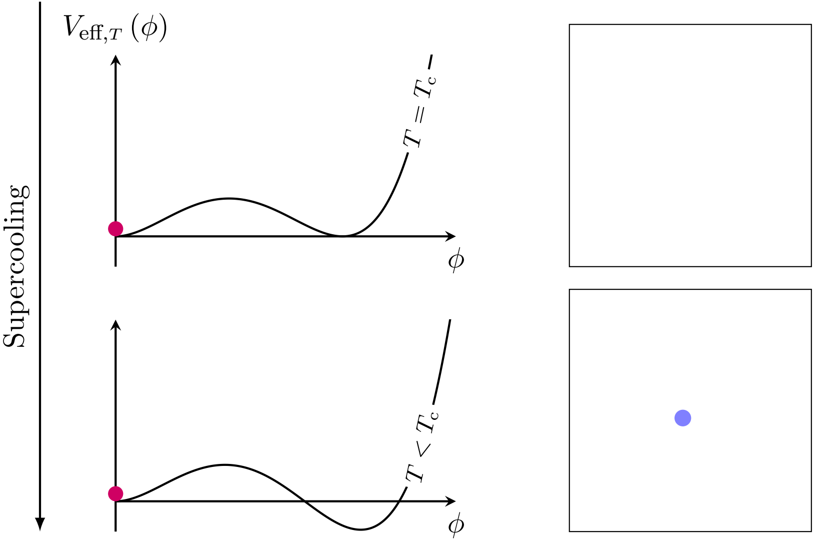
BM-A xSM with weakly portal-coupled singlet (decoupled)
BM-B xSM with strongly portal-coupled singlet





[20] O. Gould, J. Kozaczuk, L. Niemi, M. J. Ramsey-Musolf, T. V. I. Tenkanen, and D. J. Weir, *Nonperturbative analysis of the gravitational waves from a first-order electroweak phase transition*, Phys. Rev. D **100** (2019) 115024 [1903.11604]

Nucleation rate and transition reference scale



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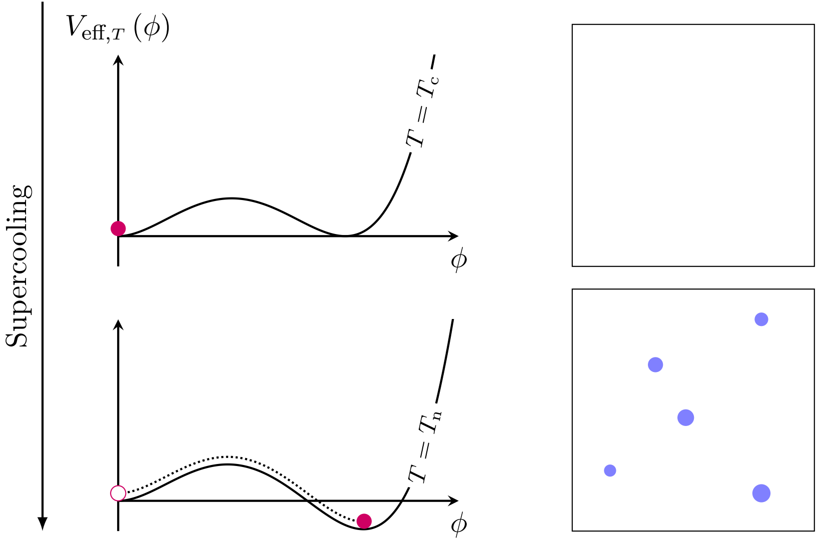

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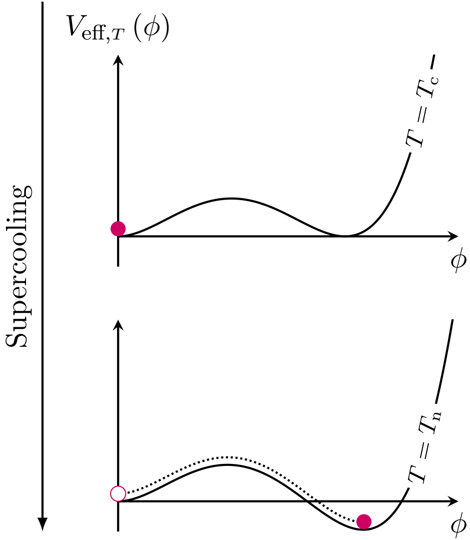

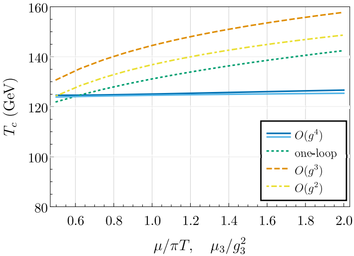

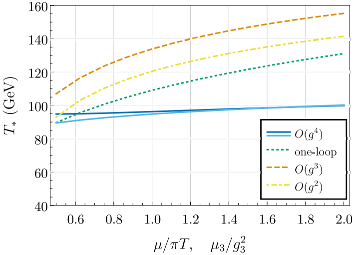

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The semiclassical bubble nucleation rate

$$\Gamma \sim A e^{-B} \,,$$

is known to N^{2}LO and factors into:

- B *Internal energy* of critical bubble. Solve one non-linear ODE

$$\frac{\mathrm{d}^2\phi_b}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}\phi_b}{\mathrm{d}r} - V'(\phi_b) = 0$$

- A log *entopy* of fluctuations about bubble. Solve infinite number of linear ODEs

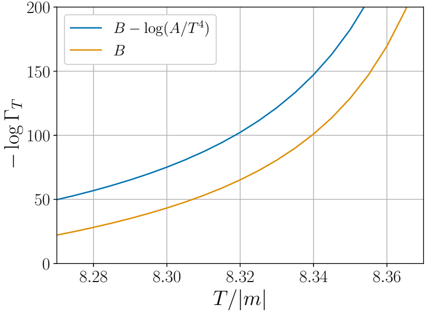
$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}}{\mathrm{d}r} - \frac{l(l+1)}{r^2} + V''(\phi_b)\right]\psi_l^b(r) = 0$$

Need both A and B for **reliable** rate computation

$$\Gamma \sim \left[m^4 e^S\right] \times e^{-E_b/T} \sim m^4 e^{-(E_b - ST)/T} \,.$$

Determining the Bubble determinant

`BubbleDet`[21] computes term A. Here: Yukawa model.



Nucleation not just energy and entropy → out-of-equilibrium effects[22]

$$\Gamma \sim \frac{\kappa_{\mathrm{dyn}}}{2\pi} m^3 e^{-(E_b - ST)/T} \,.$$

[21] A. Ekstedt, O. Gould, and J. Hirvonen, *BubbleDet: A Python package to compute functional determinants for bubble nucleation,* [2308.15652]

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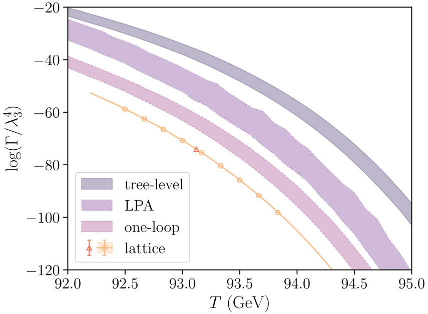

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Matrix elements for bubble wall velocities

How fast does the...?

How fast does the...?

Solve system of coupled equations

to compute v_w w/ out-of-equilibrium contributions at weak coupling.[23]

In scalar field EOM, $f = f_{\rm eq} + \delta f$ and V_T contains $f_{\rm eq}$:

$$\Box\phi + V_T'(\phi) + \sum \frac{\mathrm{d}m^2}{\mathrm{d}\phi} \int_{\mathbf{p}} \frac{1}{2E} \delta f(p,x) = 0\,. \qquad (1)$$

Particles in the plasma:

$$\hat{\mathbf{L}} f = \left[\partial_t + \dot{\mathbf{x}}\cdot\partial_{\mathbf{x}} + \dot{\mathbf{p}}\cdot\partial_{\mathbf{p}}\right] f = -\mathcal{C}[f]\,. \qquad (2)$$

Temperature $T(\xi)$ and fluid $v(\xi)$ via energy-momentum conservation.

In practice determine v_w:
Vary wall parameters until all eqs. (1) + (2) + Hydro are satisfied.

[23] G. D. Moore and T. Prokopec, *How fast can the wall move? A Study of the electroweak phase transition dynamics*, Phys. Rev. D **52** (1995) 7182 [hep-ph/9506475]

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Multiple out-of-equilibrium particles in the plasma:

$$\hat{\mathbf{L}} f^a = \Big[\partial_t + \dot{\mathbf{x}} \cdot \partial_{\mathbf{x}} + \dot{\mathbf{p}} \cdot \partial_{\mathbf{p}}\Big] f^a = -\mathcal{C}_a[\boldsymbol{f}] \,.$$

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Why matrix elements?

After linearization Botlzmann equaiton depends on collision matrix

$$\hat{\mathbf{L}}\delta f^a = -\mathcal{C}^{\text{lin}}_{ab}[\delta f^b] + \mathcal{S}_a \ ,$$

which mixes between different distributions and the source $\mathcal{S}_a = \hat{\mathbf{L}} f^a_{\text{eq}}$.

Matrix elements of the first-index-b particles are required

$$\mathcal{C}^{\text{lin}}_{ab}[\delta f^b] = \frac{1}{4}\sum_{cde}\int \frac{\mathrm{d}^3p_2\,\mathrm{d}^3p_3\,\mathrm{d}^3p_4}{(2\pi)^5 2E_2 2E_3 2E_4}\delta^4(P_1+P_2-P_3-P_4)|M|^2\mathcal{P}^{\text{lin}}[\delta f^b]\,.$$

These are the matrix-elements averaged over the degrees of freedom the first index a

$$\begin{aligned}|M|^2 &= |M_{ac\to de}(P_1,P_2;P_3,P_4)|^2\\ &\equiv \frac{1}{N_a}\sum_{a_i\in a}\sum_{c_i\in c}\sum_{d_i\in d}\sum_{e_i\in e}|\mathcal{T}_{a_ic_i\to d_ie_i}(P_1,P_2;P_3,P_4)|^2\,.\end{aligned}$$

What is WallGo ?

In: Model,
Set of out-of-equilibrium particles.

Out: v_w.

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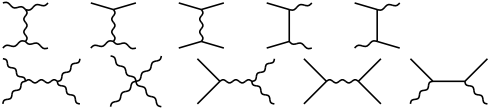
(1) `WallGoMatrix` (`Wolfram`, `DRalgo`)
computes all out-of-equilibrium particle $2 \to 2$ matrix elements:

$\mathcal{T} \supset$



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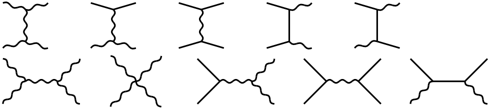

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(2) `WallGoCollision` (`c++`)
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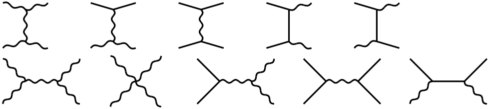

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(2) `WallGoCollision` (`c++`)
computes the corresponding collision elements.

(3) `WallGo` (`python`)
solves EOM for the scalar field(s), fluid equations, and
Boltzmann equations for out-of-equilibrium particles.

Out: v_w.

Limitations of current (out-of)-equilibrium effects

Contributions to the Boltzmann equation from collisions:

- Focus on leading logarithmic (LL) contributions to collision integral. Implemented some next-to-leading log terms.
- Dominant channels are $2 \to 2$ scatterings.
  E.g. $1 \to 2$ is beyond LL, $2 \to 3$ is coupling suppressed.
- All momentum dependence in Mandelstam variables `s_`, `t_`, `u_`,
- Physical parameters must be constant in units of T

Contributions to the Boltzmann equation from equilbrium:

- Higher order effects from equilibrium thermodynamics[24]. Soft effects contributing to $V_T \to V_{\text{eff}}$ require a modified EOM.

[24] A. Dashko and A. Ekstedt, *Bubble-wall speed with loop corrections,* [2411.05075]

Conclusions

Precision thermodynamics of BSM theories using 3d EFT:

- reliably describe cosmological FOPT and GW production,
- practical approach: **Effective Theories** (**universality**),
- Nucleation rate.

Final perturbative steps:

- higher dimensional operators at $\mathcal{O}(g^6)$ in the $3d$ EFT,
- purely perturbative $\mathcal{O}(g^6)$ contributions from hard scale πT,
- final perturbative corrections for bubble nucleation rate.

Precision determination of out-of-equilibrium effects:

WALLGO