Tutorial for DRalgo and WallGoMatrix[*]

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Sizing the walls:
Yonsei-Konkuk-Sogang Mini-Workshop on FOPT
Seoul, 02/2025

[*] Introduction to DRalgo and WallGoMatrix Part II

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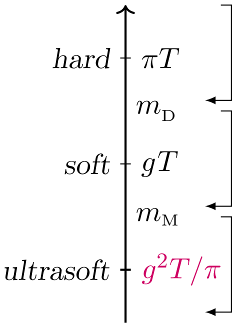
Precision thermodynamics of non-Abelian gauge theories as QCD and (EW) phase transition[1] using e.g. `DRalgo`.[2] Two `step` procedure:



$\mathcal{L}_{\mathrm{QCD}}$, hot QCD, $(d+1)$-dim

`step1`: Dimensional Reduction

$\mathcal{L}_{\mathrm{EQCD}}$, $\mathrm{SU}(N_{\mathrm{c}})$ + adj. Higgs, d-dim

`step2`: Integrate out soft temporal scalars, A_0

$\overline{\mathcal{L}}_{\mathrm{MQCD}}$, Yang-Mills, d-dim

[1] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model*, Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379]

[2] A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, `DRalgo`*: A package for effective field theory approach for thermal phase transitions*, Comput. Phys. Commun. **288** (2023) 108725 [2205.08815]

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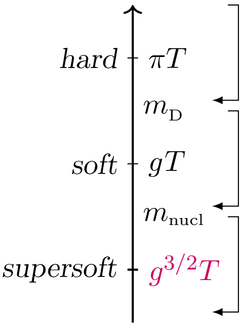

$\mathcal{L}_{\mathrm{Full}}$, $(d+1)$-dim

`step1`: Dimensional Reduction

$\mathcal{L}_d$, d-dim

`step2`: Integrate out $m_{A_0} \sim m_{\mathrm{D}}$ and $m_{A_i} \sim g\phi$

$\overline{\mathcal{L}}_d$, d-dim, HEFT-like

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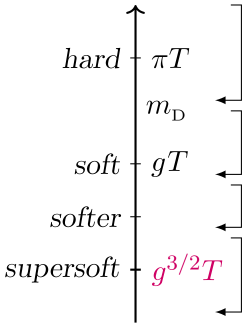

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`step2.1`: Integrate out $m_{A_0} \sim m_{\mathrm{D}}$

`step2.2`: Integrate out $m_{A_i} \sim g\phi$

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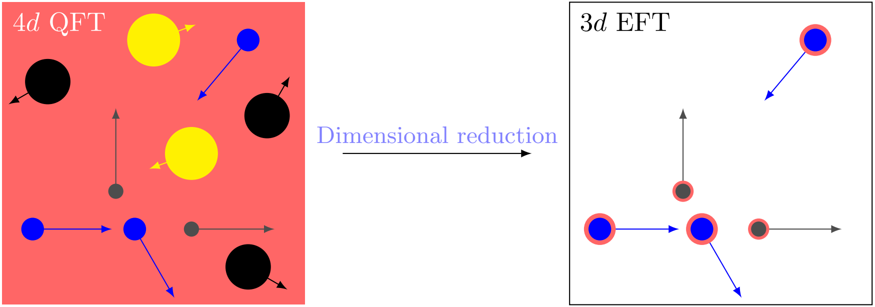

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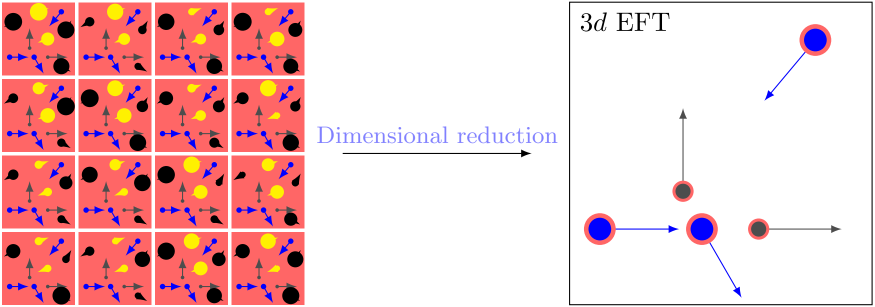

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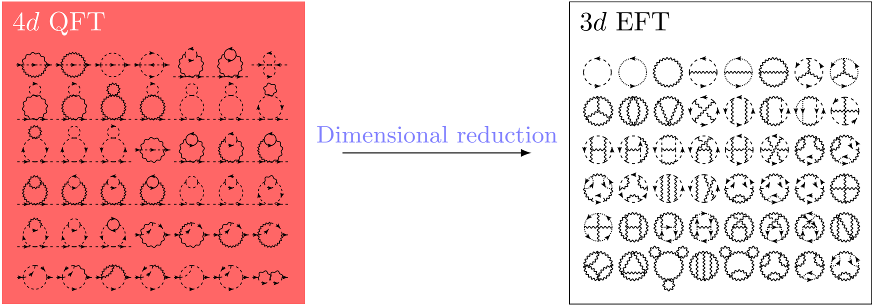

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Constructing the supersoft EFT

Two approaches to a *final* (nucleation) EFT

$$\mathcal{L}_{4d} \to \mathcal{L}_{3d}^{\text{soft}} \to \mathcal{L}_{3d}^{\text{softer}} \to V_{\text{eff}}^{\text{supersoft}} \,,$$

In the broken phase, utilize different hierarchies among Lorentz scalars, temporal scalars, and vectors:

$$m_{A_0}^2 \big[\sim (gT)^2\big] \overset{\texttt{step2.1}}{\gg} m_{A_i}^2 \big[\sim (g_3\phi)^2\big] \overset{\texttt{step2.2}}{\gg} m_3^2 \big[\sim \lambda_3 \phi^2\big] \,,$$

Constructing the supersoft EFT

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$$m_{A_0}^2\big[\sim (gT)^2\big] = m_{A_i}^2\big[\sim (g_3\phi)^2\big] \overset{\texttt{step2}}{\gg} m_3^2\big[\sim \lambda_3\phi^2\big] \,.$$

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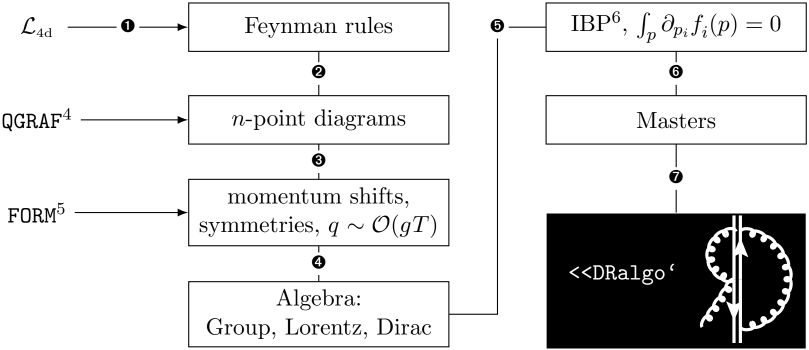
In the symmetric phase, the hierarchy is flipped

$$m_{A_0}^2 \big[\sim (gT)^2\big] \gg m_3^2 \big[\sim (g^{3/2}T)^2\big] \overset{\texttt{step2}}{\gg} m_{A_i}^2 \big[\sim (g^2T)^2\big] \,.$$

The Dimensional Reduction algorithm (DRalgo)

State-of-the-art Mathematica package DRalgo.[3]



[3] github.com/DR-algo/DRalgo, A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, DRalgo*: A package for effective field theory approach for thermal phase transitions,* Comput. Phys. Commun. **288** (2023) 108725 [2205.08815]

[4] P. Nogueira, *Automatic Feynman Graph Generation,* J. Comput. Phys. **105** (1993) 279

[5] B. Ruijl, T. Ueda, and J. Vermaseren, *FORM version 4.2* arXiv (2017) [1707.06453]

[6] S. Laporta, *High precision calculation of multiloop Feynman integrals by difference equations,* Int. J. Mod. Phys. A **15** (2000) 5087 [hep-ph/0102033]

The Dimensional Reduction algorithm (`DRalgo`)

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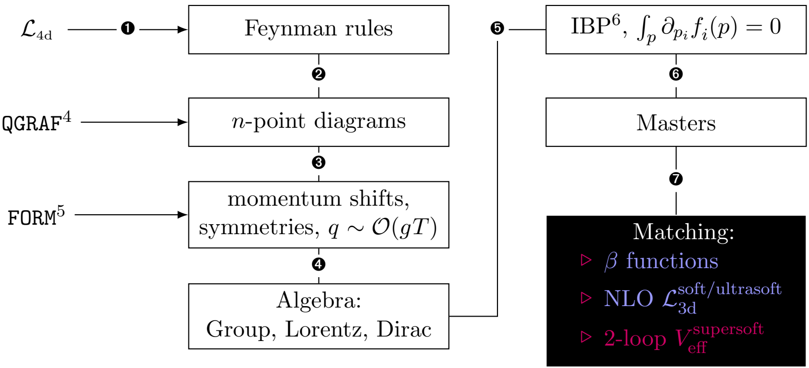

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DRalgo **basics**

Where is `DRalgo`?

Maintained on GitHub:

github.com/DR-algo/DRalgo

Manual published:

A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, `DRalgo`*: A package for effective field theory approach for thermal phase transitions*, Comput. Phys. Commun. **288** (2023) 108725 [`2205.08815`]

Installation

To load DRalgo, place the content of the package folder in

$UserBaseDirectory/Applications/DRalgo

and execute the following commands:

1

```
SetDirectory[NotebookDirectory[]];
$LoadGroupMath=True;
<<DRalgo`
```

Alternatively place the package content into /DRalgo and load it within the root directory.

Generalisation of coupling tensors

Most general Lagrangian in Euclidean signature:

$$\mathcal{L} = \frac{1}{2} R_i (\delta_{ij} \partial_\mu \partial^\mu + \mu_{ij}) R_j + \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu,a} \delta_{ab} + \frac{1}{2\xi_a} (\partial_\mu A^{a,\mu})^2$$
$$+ \partial^\mu \overline{\eta}^a \partial_\mu \eta^a + \psi^\dagger_I \overline{\sigma}^\mu \partial_\mu \psi^I + \frac{1}{2} (M^{IJ} \psi_I \psi_J + \text{h.c.}) + \mathcal{L}_\text{int} \ ,$$

with interaction

$$\mathcal{L}_\text{int} = \lambda_i R_i + \frac{1}{3!} \lambda_{ijk} R_i R_j R_k + \frac{1}{4!} \lambda_{ijkl} R_i R_j R_k R_l$$
$$+ \frac{1}{2} (Y^{iIJ} R_i \psi_I \psi_J + \text{h.c.}) + i g^a_{IJ} A^a_\mu \psi^\dagger_I \overline{\sigma}^\mu \psi_J$$
$$+ g^a_{jk} A^a_\mu R_j \partial^\mu R_k + \frac{1}{2} g^a_{jn} g^b_{kn} A^a_\mu A^{\mu,b} R_j R_k$$
$$+ g^{abc} A^{\mu,a} A^{\nu,b} \partial_\mu A^c_\nu + \frac{1}{4} g^{abe} g^{cde} A^{\mu a} A^{\nu b} A^c_\mu A^d_\nu$$
$$- g^{abc} A^a_\mu \eta^b \partial^\mu \overline{\eta}^c \ .$$

Generalisation of coupling tensors

Most general Lagrangian in Euclidean signature:

$$\mathcal{L} = \frac{1}{2} R_{\mathtt{i}} (\delta_{\mathtt{ij}} \partial_\mu \partial^\mu + \mathtt{\mu ij[i,j]}) R_{\mathtt{j}} + \frac{1}{4} F^{\mathtt{a}}_{\mu\nu} F^{\mu\nu,\mathtt{a}} \delta_{\mathtt{ab}} + \frac{1}{2\xi_{\mathtt{a}}} (\partial_\mu A^{\mathtt{a},\mu})^2$$

$$+ \partial^\mu \overline{\eta}^{\mathtt{a}} \partial_\mu \eta^{\mathtt{a}} + \psi^\dagger_{\mathtt{I}} \overline{\sigma}^\mu \partial_\mu \psi^{\mathtt{I}} + \frac{1}{2} (\mathtt{\mu IJ[I,J]} \psi_{\mathtt{I}} \psi_{\mathtt{J}} + \text{h.c.}) + \mathcal{L}_{\text{int}} \ ,$$

with interaction tensors used in `DRalgo`

$$\mathcal{L}_{\text{int}} = \mathtt{\lambda 1[i]} R_{\mathtt{i}} + \frac{1}{3!} \mathtt{\lambda 3[i,j,k]} R_{\mathtt{i}} R_{\mathtt{j}} R_{\mathtt{k}} + \frac{1}{4!} \mathtt{\lambda 4[i,j,k,l]} R_{\mathtt{i}} R_{\mathtt{j}} R_{\mathtt{k}} R_{\mathtt{l}}$$

$$+ \frac{1}{2} (\mathtt{Ysff[i,I,J]} R_{\mathtt{i}} \psi_{\mathtt{I}} \psi_{\mathtt{J}} + \text{h.c.}) + i \mathtt{gvff[I,J,a]} A^{\mathtt{a}}_\mu \psi^\dagger_{\mathtt{I}} \overline{\sigma}^\mu \psi_{\mathtt{J}}$$

$$+ \mathtt{gvss[j,k,a]} A^{\mathtt{a}}_\mu R_{\mathtt{j}} \partial^\mu R_{\mathtt{k}} + \frac{1}{2} \mathtt{gvss[j,n,a]} \mathtt{gvss[k,n,b]} A^{\mathtt{a}}_\mu A^{\mu \mathtt{b}} R_{\mathtt{j}} R_{\mathtt{k}}$$

$$+ \mathtt{gvvv[a,b,c]} A^{\mu \mathtt{a}} A^{\nu \mathtt{b}} \partial_\mu A^{\mathtt{c}}_\nu + \frac{1}{4} \mathtt{gvvv[a,b,e]} \mathtt{gvvv[c,d,e]} A^{\mu \mathtt{a}} A^{\nu \mathtt{b}} A^{\mathtt{c}}_\mu A^{\mathtt{d}}_\nu$$

$$- \mathtt{gvvv[a,b,c]} A^{\mathtt{a}}_\mu \eta^{\mathtt{b}} \partial^\mu \overline{\eta}^{\mathtt{c}} \ .$$

A minimal example of the EW sector

Dimensional reduction of SU(2) + Higgs

$$\mathcal{L}_{\mathrm{SU(2)}} = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + (D_\mu \Phi)^\dagger (D_\mu \Phi) + V(\Phi) \; ,$$

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 \; ,$$

with self coupling λ.

Phases at finite T:

- ϕ condenses, SM-like Higgs regime
- Symmetric phase

Define a model in DRalgo

Using SU(2) in adjoint representation $R = \mathbf{3}$ and Dynkin index $\{2\}$:

2

```
Group={"SU2"};
RepAdjoint={{2}};
CouplingName={gw};
```

Adding a scalar in the fundamental $R = \mathbf{2}$ with index $\{1\}$:

3

```
HiggsDoublet={{{1}},"C"};
RepScalar={HiggsDoublet};
RepFermion={};
```

Model-input from the gauge sector is directly generated by:

4

```
{gvvv,gvff,gvss,λ1,λ3,λ4,μij,μIJ,μIJC,Ysff,YsffC}=
  AllocateTensors[Group,RepAdjoint,CouplingName,RepFermion,RepScalar];
```

Specify scalar potential

Finally specify mass term in scalar potential

5

```
InputInv={{1,1},{True,False}};
MassTerm1=CreateInvariant[Group,RepScalar,InputInv][[1]]//Simplify;
VMass=msq*MassTerm1; (* corresponds to a term ½m²φφ† *)
μij=GradMass[VMass];
```

and add a quartic term:

6

```
VQuartic=λ*MassTerm1^2;
λ4=GradQuartic[VQuartic]
```

Finally load the model:

7

```
ImportModelDRalgo[Group,gvvv,gvff,gvss,λ1,λ3,λ4,μij,
    μIJ,μIJC,Ysff,YsffC,Verbose->False];
```

Dimensionally reduced effective theory

Describe theory by $3d$ EFT[7]. Super-renormalisable "Electrostatic SU(2) + Higgs" (E-SU2+H) to study high-T thermodynamics

$$\mathcal{L}^{\rm 3d}_{\rm SU2} = \frac{1}{4}F^a_{ij}F^a_{ij} + (D_i\Phi)^\dagger(D_i\Phi) + V(\Phi) \ ,$$

$$V^{\rm 3d}(\Phi) = \mu_3^2\Phi^\dagger\Phi + \lambda_3(\Phi^\dagger\Phi)^2 \ .$$

Broken Lorentz symmetry induces temporal-scalar coupling to singlet

$$\mathcal{L}^{\rm 3d}_{\rm temp} = \frac{1}{2}m^2_{\rm D}\, A^a_0 A^a_0 + \frac{1}{2}\kappa\,(A^a_0 A^a_0)^2 + \frac{1}{2}h_3\,(A^a_0 A^a_0)\Phi^\dagger\Phi + \dots \ .$$

Truncating operators at high T (can compare to lattice):

$$S^{\rm 3d}_{\rm SU2} = \frac{1}{T}\int_{\mathbf{x}}\left\{\mathcal{L}^{\rm 3d}_{\rm SU2} + \sum_{n\geq 5}\frac{\mathcal{O}_n}{(\pi T)^n}\right\} .$$

[7] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model,* Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379]

EFT step 1: SU2+H $\to$ E-SU2+H

Inspect Higgs potential: $V(\phi) \supset \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$.

DR step 1 fixes high-T E-SU2+H. EFT for Electrostatic modes ($D_i = \partial_i - ig_3 A_i - ig_3' B_i$). Describes SU2 + Higgs IR dynamics and contains UV in matching coefficients:

$$\mu_3^2 = \underbrace{\boxed{\overset{\text{tree-level}}{\mu^2} + \overset{\text{1-loop}}{\#g^2T^2}}}_{\mathcal{O}(g^2)} + \underbrace{\boxed{\overset{\text{1-loop}}{\#g^2\mu^2} + \overset{\text{2-loop}}{\#g^4T^2}}}_{\mathcal{O}(g^4)} + \mathcal{O}(g^6)\ ,$$

$$\lambda_3 = \underbrace{\boxed{\overset{\text{tree-level}}{T\lambda}}}_{\mathcal{O}(g^2)} + \underbrace{\boxed{\overset{\text{1-loop}}{\#g^4}}}_{\mathcal{O}(g^4)} + \mathcal{O}(g^6)\ .$$

EFT step 2: E-SU2+H $\to$ M-SU2+H

Inspect Higgs potential: $V(\phi) \supset \mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2$.

DR step 2 fixes high-T M-SU2+H. EFT for Magnetostatic modes aka 3d pure gauge with dynamical Higgs ($D_i = \partial_i - i\bar{g}_3 A_i - i\bar{g}'_3 B_i$). Describes E-SU2+H IR dynamics and contains UV in matching coefficients:

$$\bar{\mu}_3^2 = \underbrace{\boxed{\underset{\text{tree-level}}{\mu^2} + \underset{\text{1-loop}}{\# g^2 T^2}}}_{\mathcal{O}(g^2)} + \underbrace{\boxed{\underset{\text{1-loop}}{\# g^2 \mu^2} + \underset{\text{2-loop}}{\# g^4 T^2}}}_{\mathcal{O}(g^4)} + \mathcal{O}(g^6)$$

$$+ \underbrace{\boxed{\underset{\text{1-loop}}{\# g^2 m_\text{D}}}}_{\mathcal{O}(g^3)} + \underbrace{\boxed{\underset{\text{2-loop}}{\# g^4}}}_{\mathcal{O}(g^4)} + \mathcal{O}(g^5) \; ,$$

$$\bar{\lambda}_3 = \underbrace{\boxed{\underset{\text{tree-level}}{T\lambda}}}_{\mathcal{O}(g^2)} + \underbrace{\boxed{\underset{\text{1-loop}}{\# g^4}}}_{\mathcal{O}(g^4)} + \mathcal{O}(g^6) + \underbrace{\boxed{\underset{\text{1-loop}}{\# \frac{g^4}{m_\text{D}}}}}_{\mathcal{O}(g^3)} + \underbrace{\boxed{\underset{\text{2-loop}}{\# \frac{g^6}{m_\text{D}^2}}}}_{\mathcal{O}(g^4)} + \mathcal{O}(g^5) \; .$$

EFT step 2: E-SU2+H $\to$ M-SU2+H

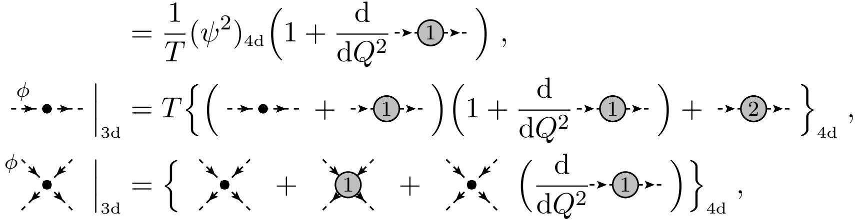
Inspect Higgs potential: $V(\phi) \supset \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$.

DR step 2 fixes high-T M-SU2+H. EFT for Magnetostatic modes aka 3d pure gauge with dynamical Higgs ($D_i = \partial_i - i\bar{g}_3 A_i - i\bar{g}'_3 B_i$). Describes E-SU2+H IR dynamics and contains UV in matching coefficients:

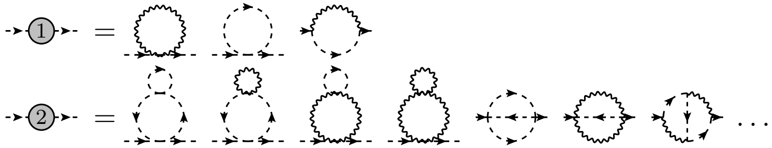
$$\mathcal{L}_{\rm 3d}^{\rm softer} \equiv \frac{1}{4}F^a_{ij}F^a_{ij} + (D_i\phi)^\dagger(D_j\phi) + \bar{\mu}_3^2\phi^\dagger\phi + \bar{\lambda}_3(\phi^\dagger\phi)^2 \ .$$

EFT setup: Matching correlators at NLO

$$(\psi^2)_{3\mathrm{d}} = \frac{1}{T}(\psi^2)_{4\mathrm{d}} Z_\psi^{-1}$$

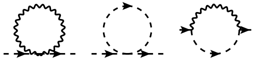


where



Computing the thermal scalar mass

The 1-loop contributions to μ_3 consists of three contributions:

 $=$ 

$$
\begin{aligned}
&= \frac{d}{2}(g^a_{in}g^a_{nj} + g^a_{jn}g^a_{ni})\mathcal{Z}_1 + \frac{1}{2}\lambda_{ijnn}\mathcal{Z}_1 + p^2 g^a_{in}g^a_{jn}\mathcal{Z}_2 \\
&= \delta_{ij}\Big[dC_{\mathrm{F}}g^2\mathcal{Z}_1 + 2(C_{\mathrm{A}}+1)\lambda\mathcal{Z}_1 + p^2[\dots]\mathcal{Z}_2\Big]
\end{aligned}
$$

where $\mathcal{Z}_1 = \SumInt_P \frac{1}{[P^2]} = \frac{T^2}{12} + \mathcal{O}(\epsilon)$. Final result is:

$$
\mu_3 = \mu^2 + T^2\Big[\frac{3}{16}g^2 + \frac{\lambda}{2}\Big]\,.
$$

Kinetic contribution is given by p^2 part of .

Dimensional reduction in DRalgo

Perform the first step in dimensional reduction from $\mathcal{L}_{\text{hard}} \to \mathcal{L}_{\text{soft}}$

8

```
PerformDRhard[];
```

Perform the second step in dimensional reduction from $\mathcal{L}_{\text{soft}} \to \mathcal{L}_{\text{softer}}$

9

```
PerformDRsoft[{i_1,...,i_n}];
```

Indices over scalars d.o.f. that are soft.

Renormalisation-scale evolution

- Evolve λ_{4d} from $\mu = \mu_0$ to $\mu = T$
- Insert result into λ_{3d} with $\mu = T$ (renders logarithms small)
- Calculate the effective potential
- Change T until the two minima coincide at T_{c}
- Calculate observables at T_{c}

The nucleation rate in the EFT

Similar strategy as for $T_{\rm c}$:

- Evolve $\lambda_{\rm 4d}$ from $\mu = \mu_0$ to $\mu = T$
- Calculate the three-dimensional bounce action S_3
- Change T until $e^{-S_3} \sim e^{-140}$
- Calculate the inverse-duration $\beta/H = \frac{\rm d}{{\rm d}\log T} S_3$

Optionally use factorization into UV and IR (condensate):

$$\frac{\rm d}{{\rm d}\log T} S_3 = \underbrace{\frac{\partial \lambda_3}{\partial \log T}}_{\eta_\lambda} \underbrace{\frac{\partial}{\partial \lambda_3} S_3}_{\langle (\phi^\dagger \phi)^2 \rangle} \ .$$

DRalgo 2.0 (HEFTFire)

Recent updates and future features

Construct HEFT-like EFT at super-soft scale where

$$m_{\text{vectors}} \gg m_{\text{scalars}}$$

Kinetic Z-factors of EFT is needed for bounce action in derivative expansion:[8]

$$S_{\text{eff}} = \int_{\mathbf{x}} \left[\frac{Z}{2} (\partial_i \phi_b)^2 + V_{\text{eff}}(\phi_b) + \dots \right] .$$

Todo: Higher-dimensional operators of the thermal EFT affect validity window of the EFT.

[8] This is an approximation of evaluating the functional determinant for vectors.

Constructing the supersoft EFT $\mathcal{L}_{3d}^{\text{soft}} \to \mathcal{L}_{3d}^{\text{softer}} \to V_{\text{eff}}^{\text{supersoft}}$

Define vacuum expectation value (VEV) for scalar d.o.f. of the EFT:

10

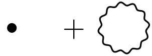
```
UseUltraSoftTheory[];
vev={0,sphi,0,0}//SparseArray;
DefineVEVS[vev];
```

To integrate out vectors use internal function

`PrepareHET[{Scalar_indices},{vector_indices}]`

11

```
PrepareHET[{},{1,2,3}]
CalculatePotentialHET[]
PrintActionHET["LO"]
```

$V_{\text{LO}} =$ 

$$= \frac{1}{2}\bar{\mu}_3^2\phi^2 + \frac{1}{2}\bar{\lambda}_3\phi^2 - C_{\text{A}}(d-1)\frac{(m_{A_i}^2)^{3/2}}{12\pi}$$

Define vacuum expectation value (VEV) for scalar d.o.f. of the EFT:

12

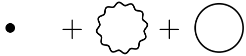
```
UseSoftTheory[];
vev={0,sphi,0,0}//SparseArray;
DefineVEVS[vev];
```

To integrate out vectors use internal function

`PrepareHET[{Scalar_indices},{vector_indices}]`

13

```
PrepareHET[{1,2,3},{1,2,3}]
CalculatePotentialHET[]
PrintActionHET["LO"]
```

$V_{\text{LO}} =$ 

$$= \frac{1}{2}\mu_3^2\phi^2 + \frac{1}{2}\lambda_3\phi^2 - C_{\text{A}}(d-1)\frac{(m_{A_i}^2)^{3/2}}{12\pi} - C_{\text{A}}\frac{(m_{A_0}^2)^{3/2}}{12\pi}$$

Derivative contributions to the action

Gauge bosons also change the scalar field kinetic term

$$\mathcal{L}_{\text{supersoft}} \supset (\delta_{ij} + 2\delta Z_{ij})\partial_\mu R_i \partial_\mu R_j \ ,$$

which is obtained by

$$\texttt{PrintScalarKineticHET}[\,][\texttt{i},\texttt{j}] = \delta Z_{ij} \ .$$

For softer → supersoft

$$\delta Z_{\phi\phi} = -\frac{11}{16\pi}\frac{\bar{g}_3}{\phi_3}$$

For soft → supersoft get contribution from temporal scalars

$$\delta Z_{\phi\phi} = -\frac{11}{16\pi}\frac{\bar{g}_3}{\phi_3} + \frac{1}{64\pi}\frac{\phi_3^2 h_3^2}{(m_{B_0}^2)^{3/2}} \ .$$

WallGoMatrix

Where is WallGo Matrix ?

Maintained on GitHub:

github.com/Wall-Go/WallGoMatrix

Manual:

A. Ekstedt, O. Gould, J. Hirvonen, *et al.*, *How fast does the WallGo? A package for computing wall velocities in first-order phase transitions*, [2411.04970]

Installation

To load `WallGoMatrix`

14

```
PacletInstall["WallGo/WallGoMatrix"];
```

and execute the following commands:

15

```
SetDirectory[NotebookDirectory[]];
WallGo`WallGoMatrix`$InstallGroupMath=True;
<<WallGo`WallGoMatrix`
```

Alternatively place the package content into `/WallGoMatrix` and load it within the root directory.

Defining a simple model: Yukawa model

Scalar field ϕ may undergo a phase transition. Fermion field ψ contributes to the friction for the bubble wall growth:

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \sigma\phi - \frac{m^2}{2}\phi^2 - \frac{g}{3!}\phi^3 - \frac{\lambda}{4!}\phi^4 \\ - i\bar{\psi}\not{\partial}\psi - m_f\bar{\psi}\psi - y\phi\bar{\psi}\psi\,.$$

Same model implementation as in `DRalgo`

16

```
Group={"U1"};
RepAdjoint={0};
RepScalar={{{0},"R"}};
CouplingName={g1};
RepScalar = {{{0},"R"}};
```

A Dirac fermion is constructed from two Weyl fermions:

17

```
RepFermion={{{0},"L"},{{0},"R"}};
```

Implement the interactions

Cubic interactions

18

```
InputInv={{1,1,1},{True,True,True}};
CubicTerm=CreateInvariant[Group,RepScalar,InputInv][[1]];
VCubic=gamma/6 CubicTerm
λ3=GradCubic[VCubic];
```

Yukawa interactions

19

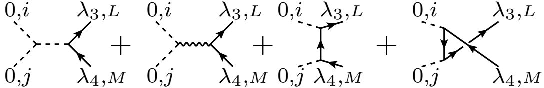
```
InputInv={{1,2,1},{True,True,True}};
YukawaDoublet1=CreateInvariantYukawa[
  Group,RepScalar,RepFermion,InputInv][[1]]//Simplify;
InputInv={{1,1,2},{True,False,False}};
YukawaDoublet2=CreateInvariantYukawa[
  Group,RepScalar,RepFermion,InputInv][[1]]//Simplify;

Ysff= y*GradYukawa[YukawaDoublet1];
YsffC=y*GradYukawa[YukawaDoublet2];
```

Matrix elements at leading logarithm (LL)

Using spinor helicity formalism, focus on the $S_1 S_2 \to F_1 F_2$ channel:

$$- \mathcal{T}_{00\lambda_3\lambda_4} =$$



$$= \lambda_{ijk} \, Y_M^{k\,L} \mathcal{A}^s_{00\lambda_3\lambda_4} + g^a_{ij} \, g^{a\,L}_M A^s_{00\lambda_3\lambda_4} + Y_i^{IM} Y_{jIL} A^t_{00\lambda_3\lambda_4} + Y_i^{IM} Y_{jIL} A^u_{00\lambda_3\lambda_4} \, .$$

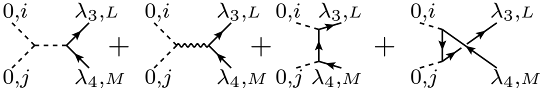
The final result now depends on contractions of the coupling tensors in the model definition:

$$\left|\mathcal{T}_{00+-}\right|^2 \supset 4\mathrm{Tr}\left[\left(g^a_{ij}\, g^{a\,L}_M\right)^2\right] \frac{s_{13}s_{14}}{s_{12}^2} + \mathrm{Tr}\left[\left(Y_i^{IM} Y_{jIL}\right)^2\right] \frac{s_{13}s_{14}}{s_{13}^2} + \mathrm{Tr}\left[\left(Y_i^{IM} Y_{jIL}\right)^2\right] \frac{s_{13}s_{14}}{s_{14}^2} \, .$$

Matrix elements at leading logarithm (LL)

Using spinor helicity formalism, focus on the $S_1 S_2 \to F_1 F_2$ channel:

$-\mathcal{T}_{00\lambda_3\lambda_4} =$



$$= \lambda\mathtt{3}[\mathtt{ijk}]\,\mathtt{Ysff}[\mathtt{k,M,L}]\mathcal{A}^{s}_{00\lambda_3\lambda_4} + \mathtt{gvss}[\mathtt{i,j,a}]\,\mathtt{gvff}[\mathtt{a,L,M}]A^{s}_{00\lambda_3\lambda_4}$$
$$+\,\mathtt{Ysff}[\mathtt{i,I,M}]\mathtt{Ysff}[\mathtt{j,I,L}]A^{t}_{00\lambda_3\lambda_4} + \mathtt{Ysff}[\mathtt{i,I,M}]\mathtt{Ysff}[\mathtt{j,I,L}]A^{u}_{00\lambda_3\lambda_4}\,.$$

The final result now depends on contractions of the coupling tensors in the model definition:

$$\left|\mathcal{T}_{00+-}\right|^2 \supset 4\mathrm{Tr}\left[\left(g^{a}_{ij}\, g^{a\,L}_{M}\right)^2\right]\frac{s_{13}s_{14}}{s_{12}^2}$$
$$+\,\mathrm{Tr}\left[\left(Y_i^{IM} Y_{jIL}\right)^2\right]\frac{s_{13}s_{14}}{s_{13}^2} + \mathrm{Tr}\left[\left(Y_i^{IM} Y_{jIL}\right)^2\right]\frac{s_{13}s_{14}}{s_{14}^2}\,.$$

Translation to matrix element model

Set VEV to discriminate particles by mass after symmetry breaking

20

```
vev={v};
SymmetryBreaking[vev]
```

Now create particles from given representations:

21

```
RepScalar = CreateParticle[{1}, "S", ms2, "Phi"];
RepFermionL = CreateParticle[{1}, "F", mf2, "PsiL"];
RepFermionR = CreateParticle[{2}, "F", mf2, "PsiR"];
RepZ = CreateParticle[{1}, "V", mv2, "Z"];
```

Collecting out-of-equilibrium and light particles:

22

```
ParticleList={RepScalar,RepFermionL,RepFermionR};
LightParticleList = {RepZ};
```

Generate the matrix elements

23

```
OutputFile="output/matrixElements.yukawa";
MatrixElements=ExportMatrixElements[
        OutputFile,
        ParticleList,
        LightParticleList,
        {
                TruncateAtLeadingLog->True,
                Format->{"json","txt"}}]
```