Lecture 3
Physics of Dense Matter

Applications, Uncertainties, Prospects

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General Relativity

hydrostatic equilibrium

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2} \qquad \frac{dP}{dr} = -\frac{GM\rho}{r^2}\left(1+\frac{P}{\rho c^2}\right)\left(1+\frac{4\pi P r^3}{Mc^2}\right)\left(1-\frac{2GM}{rc^2}\right)^{-1}$$

$$\frac{dM}{dr} = 4\pi r^2 \rho \qquad \frac{dM}{dr} = 4\pi r^2 \left(\frac{\epsilon}{c^2}\right)$$

include all energy sources

physics of dense nuclear matter (strong interaction)

Without general relativity

n : polytropic index

ideal gas $p \propto \rho^{\gamma} \propto \rho^{(n+1)/n}$

nonrelativistic

$$\gamma = \frac{5}{3} \qquad n = \frac{3}{2} \qquad\qquad P_N = \left(\frac{3}{\pi}\right)^{2/3} \left(\frac{h^2}{20 m_N}\right) \left(\frac{\rho}{m_N}\right)^{5/3}$$

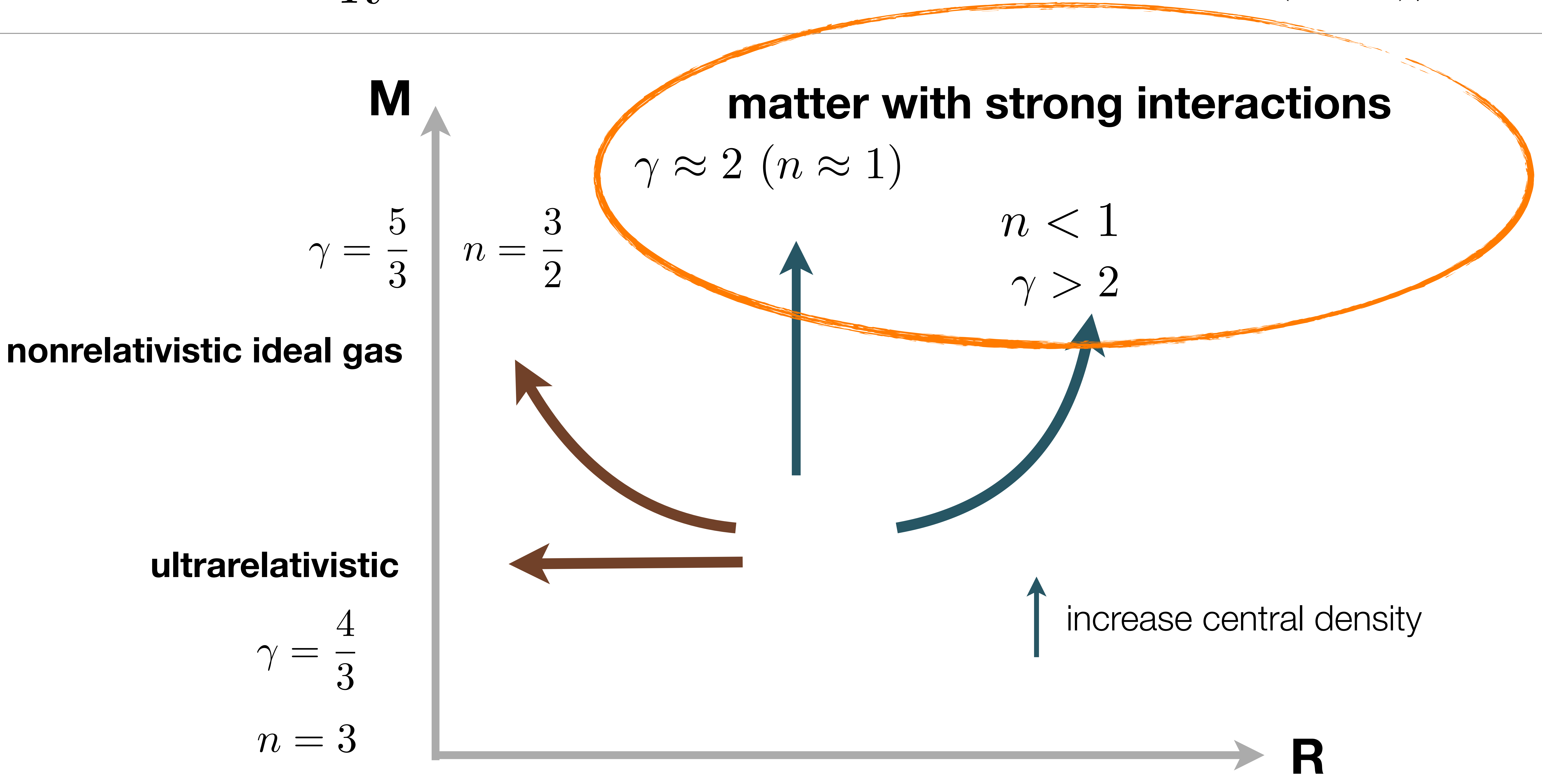
ultrarelativistic

$$\gamma = \frac{4}{3} \qquad n = 3 \qquad\qquad P_N = \left(\frac{3}{8\pi}\right)^{1/3} \left(\frac{hc}{4}\right) \left(\frac{\rho}{m_N}\right)^{4/3}$$

$$M \propto R^{(\gamma-2)/(3\gamma-4)}$$

$$n = 1/(\gamma - 1)$$

$$\gamma = (n+1)/n$$



Nuclear matter is not an ideal gas

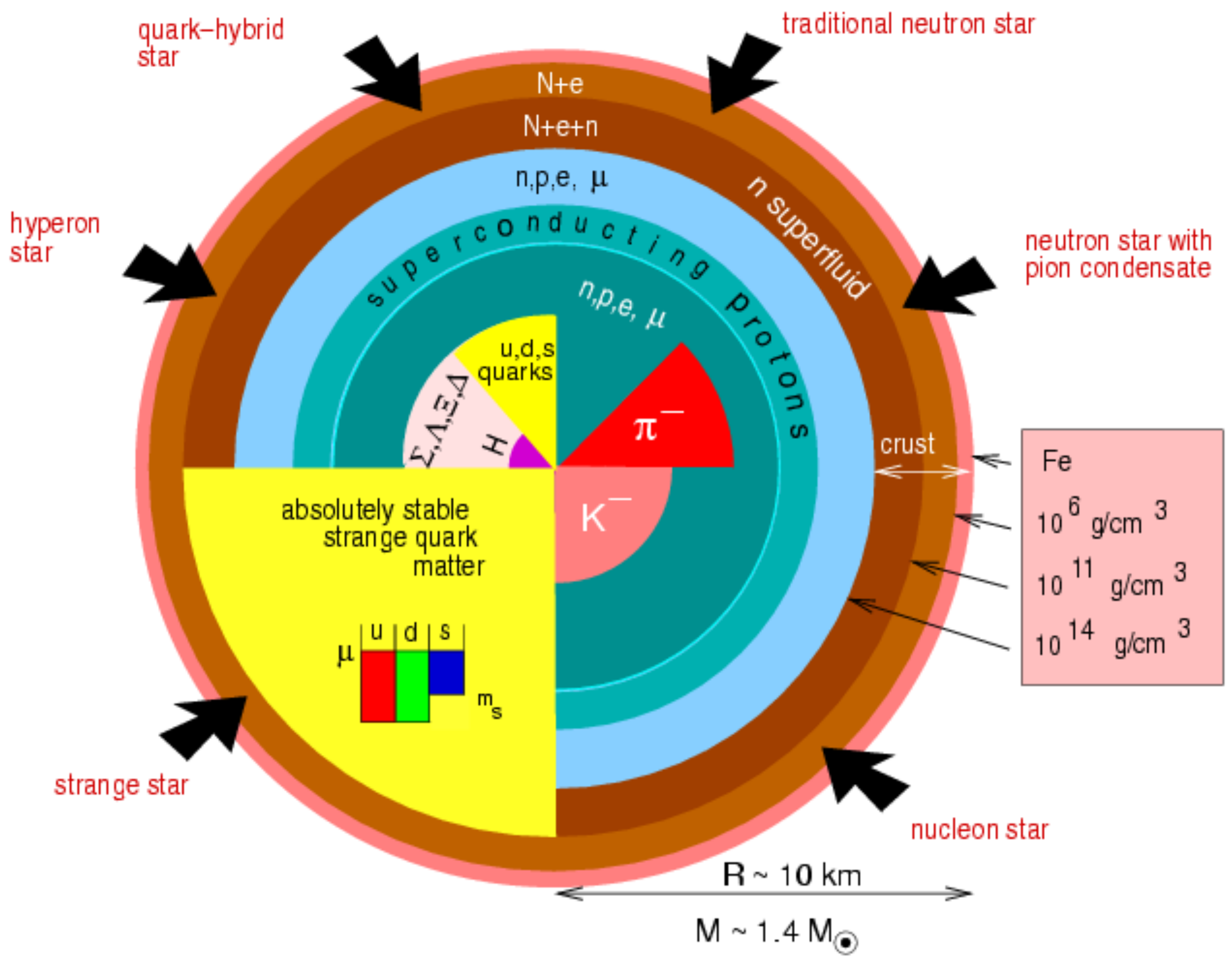
F. Weber 2005

nonrelativistic $\gamma = \frac{5}{3}$

ultrarelativistic $\gamma = \frac{4}{3}$

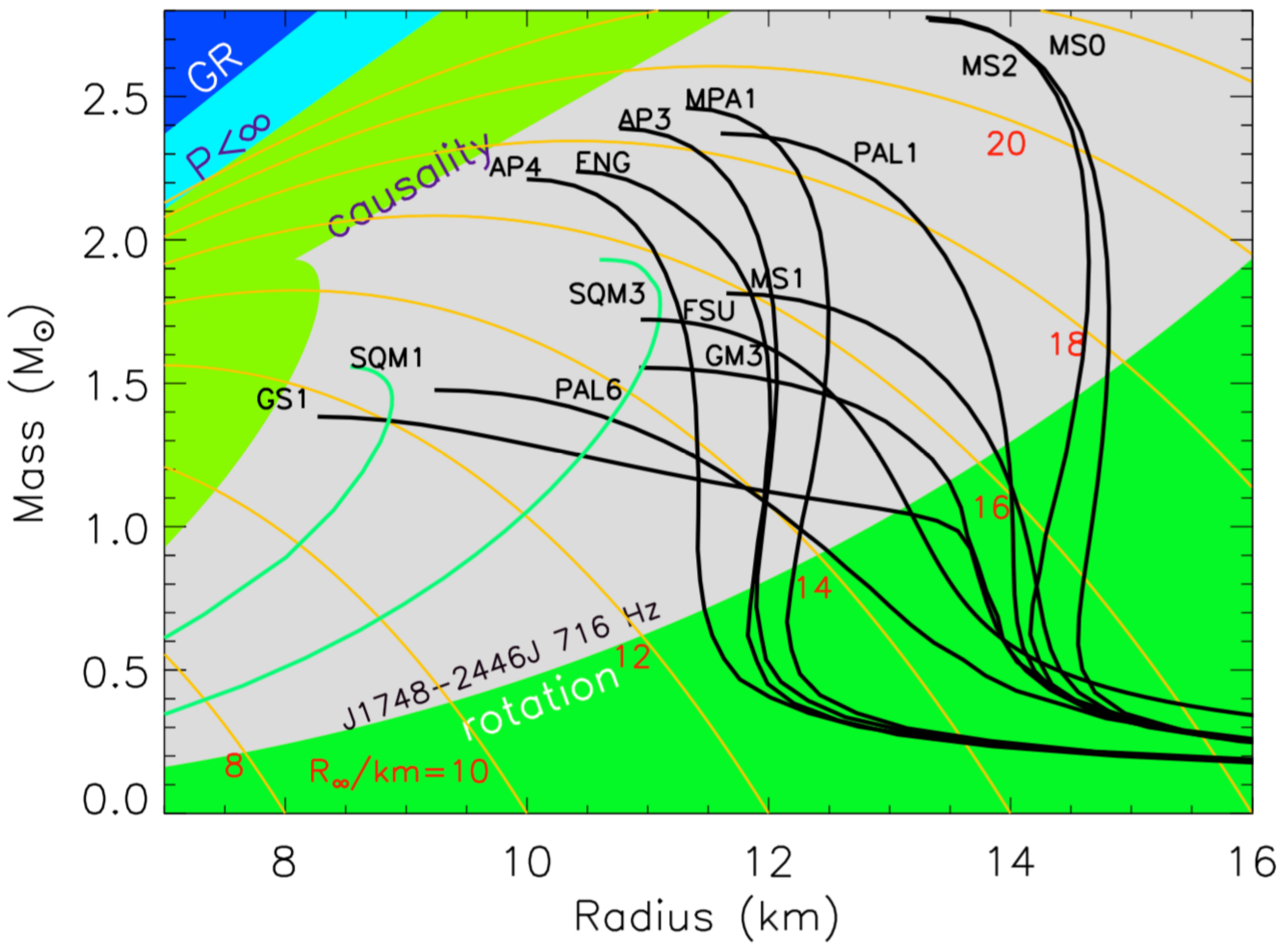
NS EOS includes

$$\gamma \approx 2 \ (n \approx 1)$$



- **still uncertain due to the nature of strong interactions**
- **introduction of 3 body forces**
- **exotic states with strangeness**
- **... ...**

Which EOS is consistent with nature ?



$$p \propto \rho^{\gamma} \propto \rho^{(n+1)/n}$$

with general relativity & strong interactions !

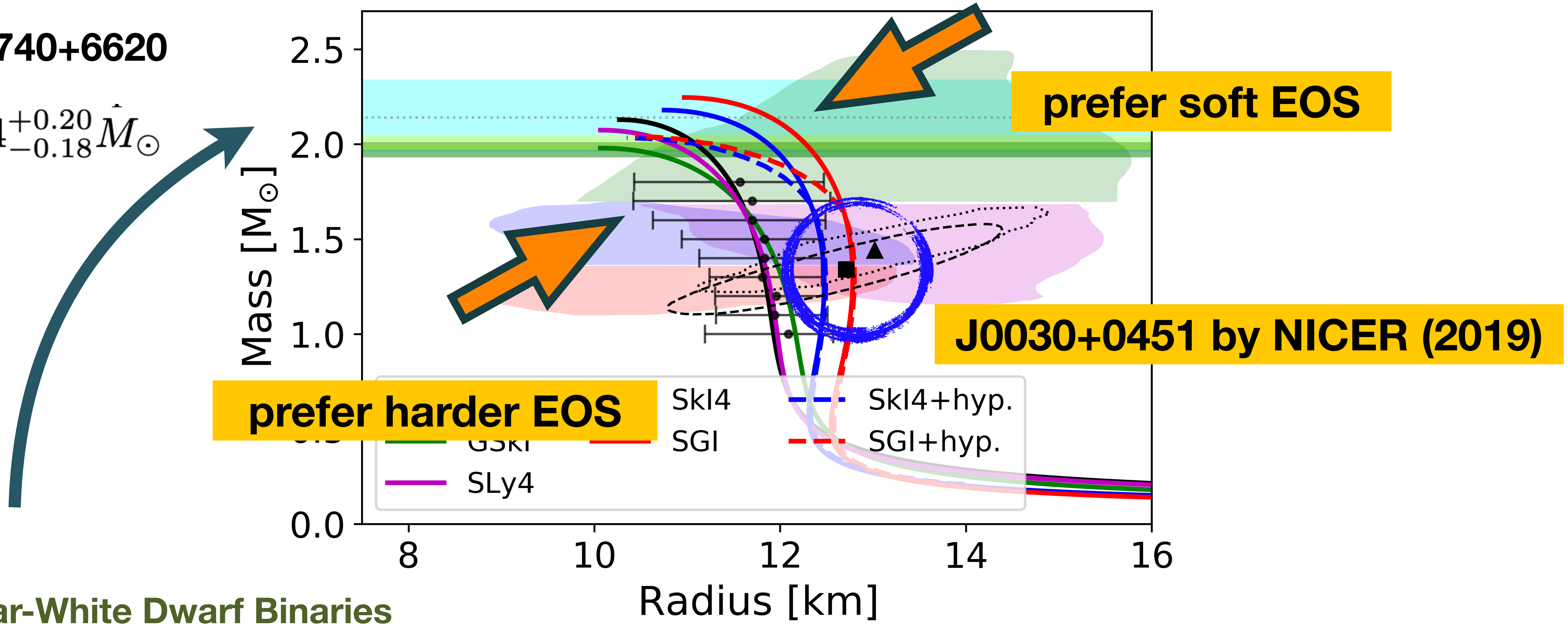
Q) Which EOS ?

H. T. Cromartie, *et al.* Nature Astronomy (2019).

MSP J0740+6620

$2.14^{+0.20}_{-0.18} M_\odot$

Tidal deformability of NS from GW
Strangeness in NS



Neutron Star-White Dwarf Binaries

1.97 solar mass NS : Nature 467 (2010) 1081

2.01 solar mass NS : Science 340 (2013) 6131

IJMPE (2020) Kim, Lee, Kim, Kwak, Lim, Hyun

Dense Matter EOS

Nuclear & Particle Physics

Example: quark stars

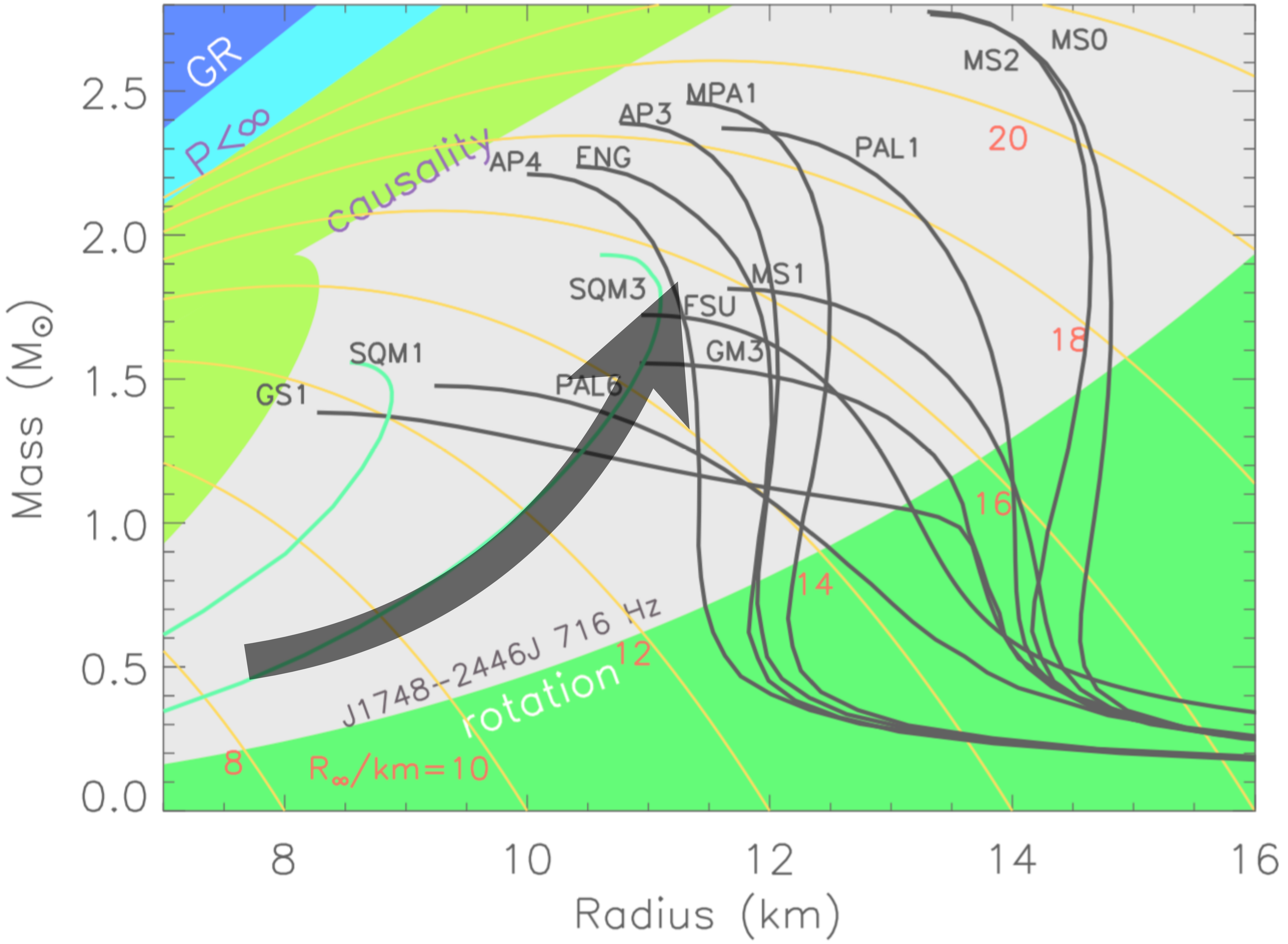
$$p \propto \rho^{\gamma} \propto \rho^{(n+1)/n}$$

strong interaction dominates

$$(\mathrm{u}, \mathrm{d}, \mathrm{s}) \text{ quarks} \Rightarrow \text{charge neutrality without electron}$$

$n < 1$

$\gamma > 2$



Example: Nuclear Equation of States

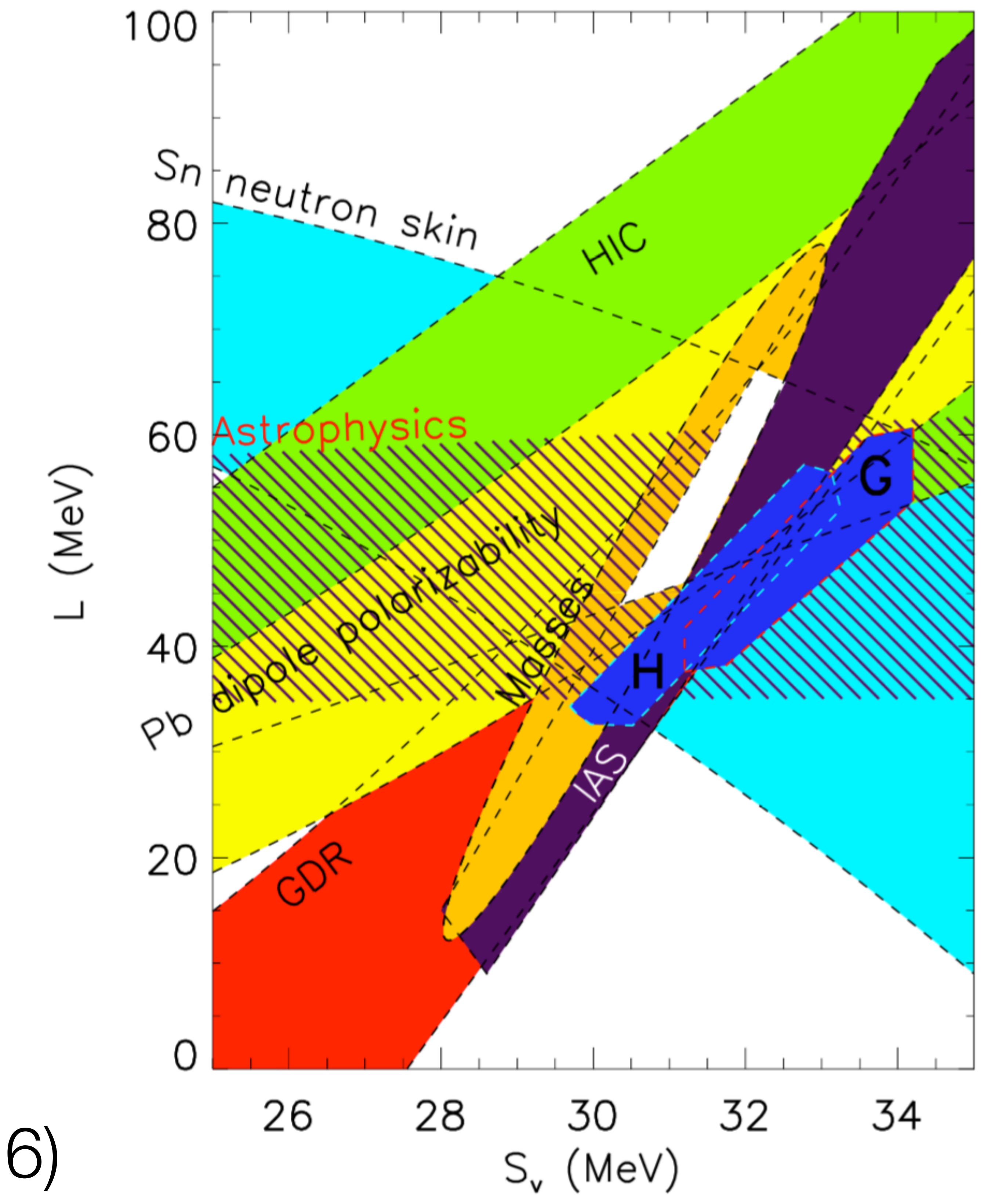
$$x = \frac{\rho_p}{\rho_p + \rho_n}$$

$$E(\rho, x) = -B + \frac{K_0}{18}\left(\frac{\rho}{\rho_0} - 1\right)^2 + \frac{K_0'}{162}\left(\frac{\rho}{\rho_0} - 1\right)^3 + E_{\rm sym}(\rho)(1-2x)^2 + \cdots$$

Incompressibility	$K_0 \simeq 230\ \mathrm{MeV}$
Skewness	$K_0' \sim -2000\ \mathrm{MeV}$

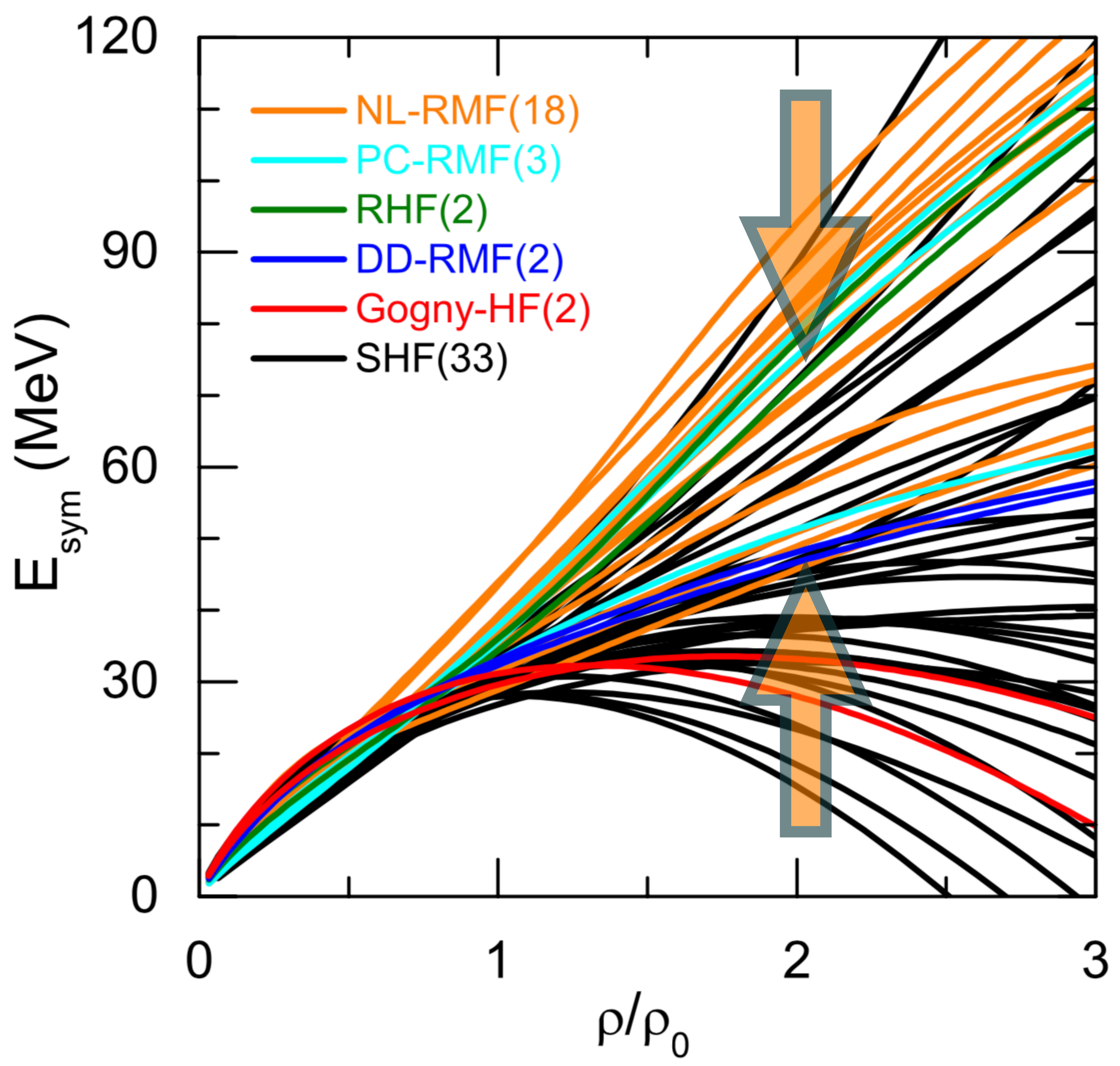
$$S_0 \equiv E_{\rm sym}(\rho_0)$$ **Symmetry Energy**

$$L \equiv 3\rho \left.\frac{\partial E_{\rm sym}}{\partial \rho}\right|_{\rho_0}$$



J.M. Latter, Y. Lim (2016)

Symmetry energy / Theoretical uncertainties



Courtesy of Lie-Wen Chen

waiting for RAON@IBS

Our works on NS EoS

From finite nuclei to neutron stars

IJMPE (2020) Kim, Lee, Kim, Kwak, Lim, Hyun
EPJA 56:157 (2020) Kim, Kwak, Hyun, Gil, Lee
PRC 98, 065805 (2018) Kim, Lim, Kwak, Hyun, Lee
… …

An Example

KIDS nuclear energy density functional

Courtesy of Chang Ho Hyun (Daegu)

PRC 97, 014312 (2018)

- **Motivation**

  Construct models for nuclear structures on a basis with systematic expansion scheme.

$$\mathscr{E}(\rho, \delta) = \mathscr{T}(\rho, \delta) + \sum_{i=0}^{N-1} c_i(\delta)\rho^{1+i/3},$$

$$c_i(\delta) = \alpha_i + \beta_i \delta^2$$

$$\delta = (\rho_n - \rho_p)/\rho$$

- **Fitting**
  - α_i : ρ_0 = 0.16 fm^{-3}, BE = 16.0 MeV, K_0 = 240 MeV,

    Q_0 = -360, -390, -420 MeV (skewness)
  - β_i : pure neutron matter EoS of APR, QMC and etc
  - Parameters for closed-shell magic nuclei
    - E/A, R_c of ^{40}Ca, ^{48}Ca, and ^{208}Pb (only 6)
    - Specific values of isoscalar and isovector effective masses m^*_s and m^*_v

Courtesy of Chang Ho Hyun (Daegu)

KIDS nuclear energy density functional

PRC 97, 014312 (2018)

$$\mathcal{E}(\rho,\delta) = \mathcal{T}(\rho,\delta) + \sum_{i=0}^{N-1} c_i(\delta)\rho^{1+i/3},$$

$$c_i(\delta) = \alpha_i + \beta_i\delta^2$$

$$\delta = (\rho_n - \rho_p)/\rho$$

$$\mathcal{T} = \mathcal{T}_p + \mathcal{T}_n\,,$$

$$\mathcal{T}_{p,n} = \frac{3}{5}\frac{\hbar^2}{2m_{p,n}} x_{p,n}^{5/3}(3\pi^2\rho)^{2/3}$$

$$x_{p,n} \equiv \rho_{p,n}/\rho$$

$$p = \rho^2 \frac{\partial(\mathcal{E}/\rho)}{\partial\rho}$$

Constraints on Nuclear EoS

- 11 experimental/empirical data for nuclear matter around saturation density [PRC 85, 035201 (2012)]

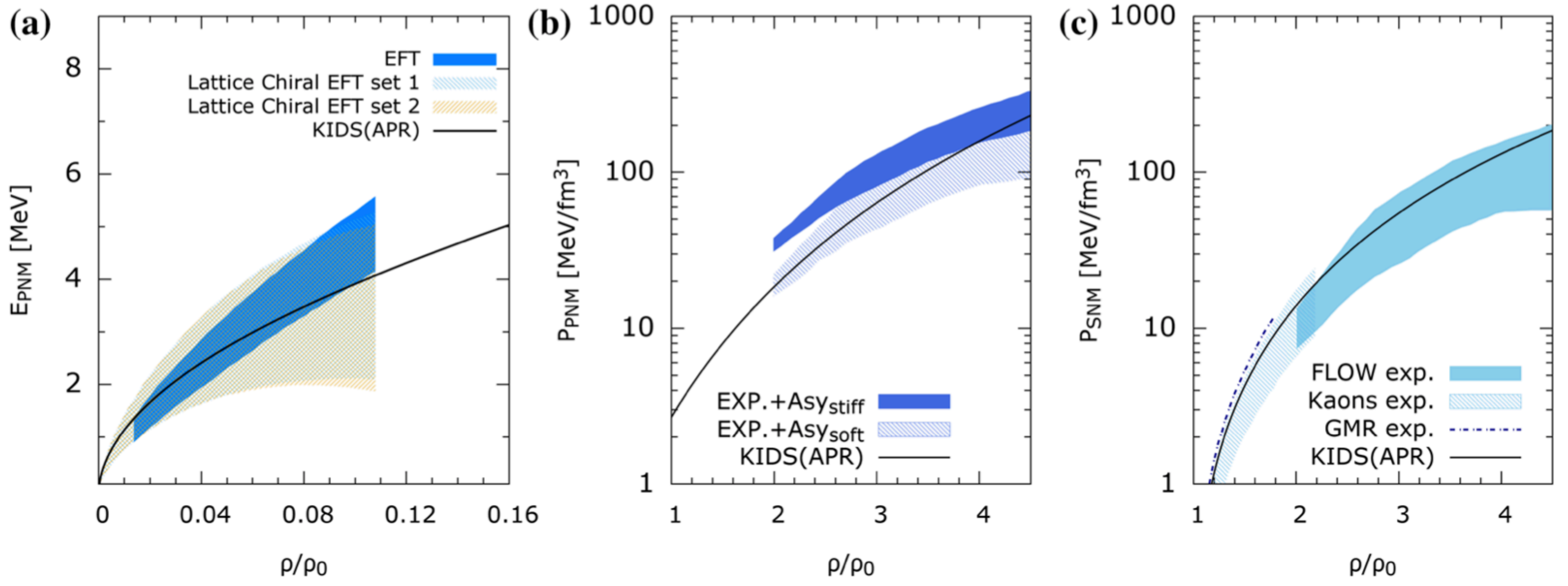
Constraint	Quantity	Eq.	Density Region	Range of constraint exp/emp	Range of constraint from CSkP	Ref.
SM1	K_o	(7),(15)	ρ_o (fm^{-3})	200 – 260 MeV	202.0 – 240.3 MeV	[64]
SM2	$K' = -Q_o$	(8),(16)	ρ_o (fm^{-3})	200 – 1200 MeV	362.5 – 425.6 MeV	[65]
SM3	$P(\rho)$	(6)	$2 < \frac{\rho}{\rho_o} < 3$	Band Region	see Fig. 1	[78]
SM4	$P(\rho)$	(6)	$1.2 < \frac{\rho}{\rho_o} < 2.2$	Band Region	see Fig. 2	[80]
PNM1	$\frac{E_{PNM}}{E^o_{PNM}}$	(31)	$0.014 < \frac{\rho}{\rho_o} < 0.106$	Band Region	see Fig. 3	[39, 40]
PNM2	$P(\rho)$	(6)	$2 < \frac{\rho}{\rho_o} < 3$	Band Region	see Fig. 5	[78]
MIX1	J	(9)	ρ_o (fm^{-3})	30 – 35 MeV	30.0 – 35.5 MeV	[44]
MIX2	L	(10)	ρ_o (fm^{-3})	40 – 76 MeV	48.6 – 67.1 MeV	[101]
MIX3	$K_{\tau,v}$	(21)	ρ_o (fm^{-3})	-760 – -372 MeV	-407.1 – -360.1 MeV	[107]
MIX4	$\frac{S(\rho_o/2)}{J}$	-	ρ_o (fm^{-3})	0.57 – 0.86	0.61 – 0.67	[110]
MIX5	$\frac{3P_{PNM}}{L\rho_o}$	(41)	ρ_o (fm^{-3})	0.90 – 1.10	1.02 – 1.10	[112]

Table 1 Properties of nuclear matter calculated with KIDS model. The saturation density ρ_0 is in units of fm^{-3}. Exp/Emp values are quoted from Ref. [15]. E/A, K_0, and Q_0 are binding energy per particle, compression modulus, and skewness (the third derivative of energy per particle) at the saturation density in the symmetric nuclear matter, respectively. J, L, and K_τ are parameters in the symmetry energy of nuclear matter. E_0, K_0, Q_0, J, L, and K_τ are in units of MeV

	ρ_0	E/A	K_0	$-Q_0$,	J	L	$-K_\tau$
KIDS	0.160	16.00	240.0	372.7	32.8	49.1	375.1
Exp/Emp	$\simeq$ 0.16	$\simeq$ 16.0	200–260	200–1200	30–35	40–76	372–760

Table 4 Four basic nuclear matter properties obtained from models in comparison. The saturation density ρ_0 is in fm^{-3}, and E/A, K_0 and J are in units of MeV. The MS1b model gives the same results as MS1

	GSkI	SLy4	SkI4	SGI	MS1
ρ_0	0.159	0.160	0.160	0.154	0.148
E/A	16.02	15.97	15.95	15.89	15.75
K_0	230.2	229.9	248.0	261.8	250
J	32.0	32.0	29.5	28.3	35



Binding energy

Table 2 Binding energy per nucleon in units of MeV. The deviation is defined as |Exp − KIDS|/Exp × 100. Data are taken from [26,27]

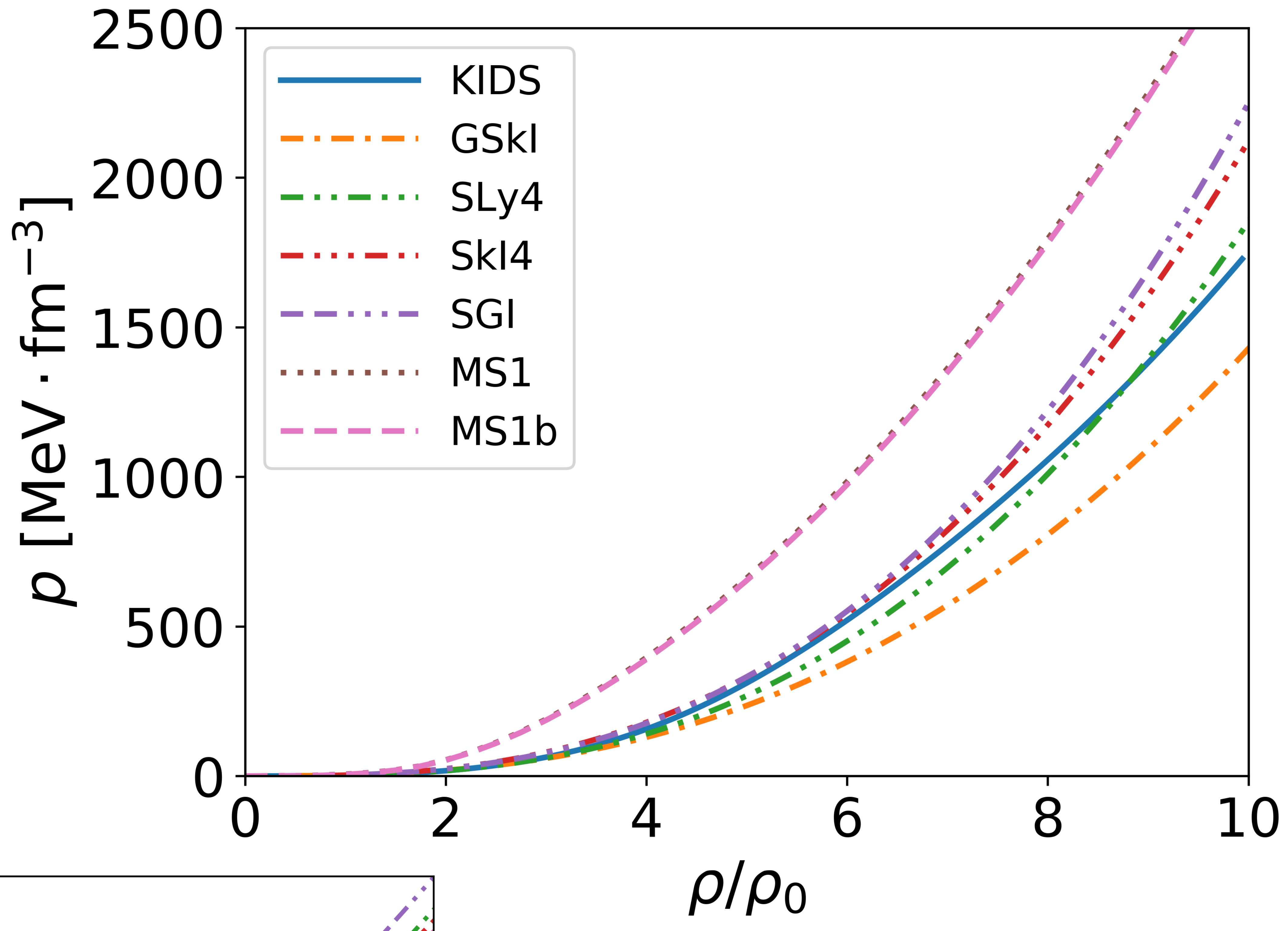
	^{16}O	^{40}Ca	^{48}Ca	^{90}Zr	^{132}Sn	^{208}Pb
Exp	7.9762	8.5513	8.6667	8.7100	8.3550	7.8675
KIDS	7.8684	8.5565	8.6564	8.7328	8.3563	7.8809
Deviation (%)	1.35	0.06	0.12	0.26	0.02	0.17

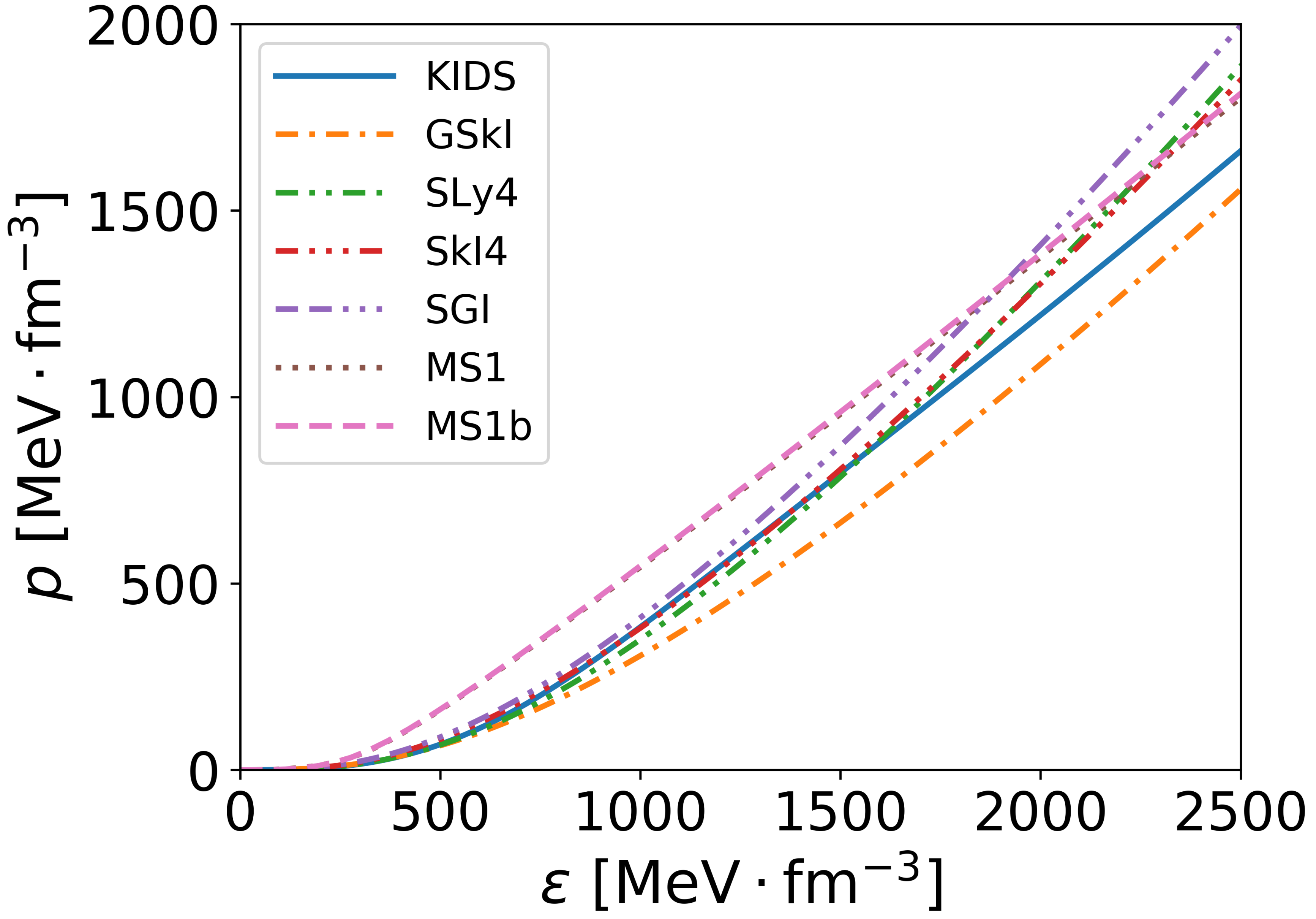
Charge radius

Table 3 Charge radius in units of fm. The deviation is defined as |Exp − KIDS|/Exp × 100. Data are taken from [26,27]

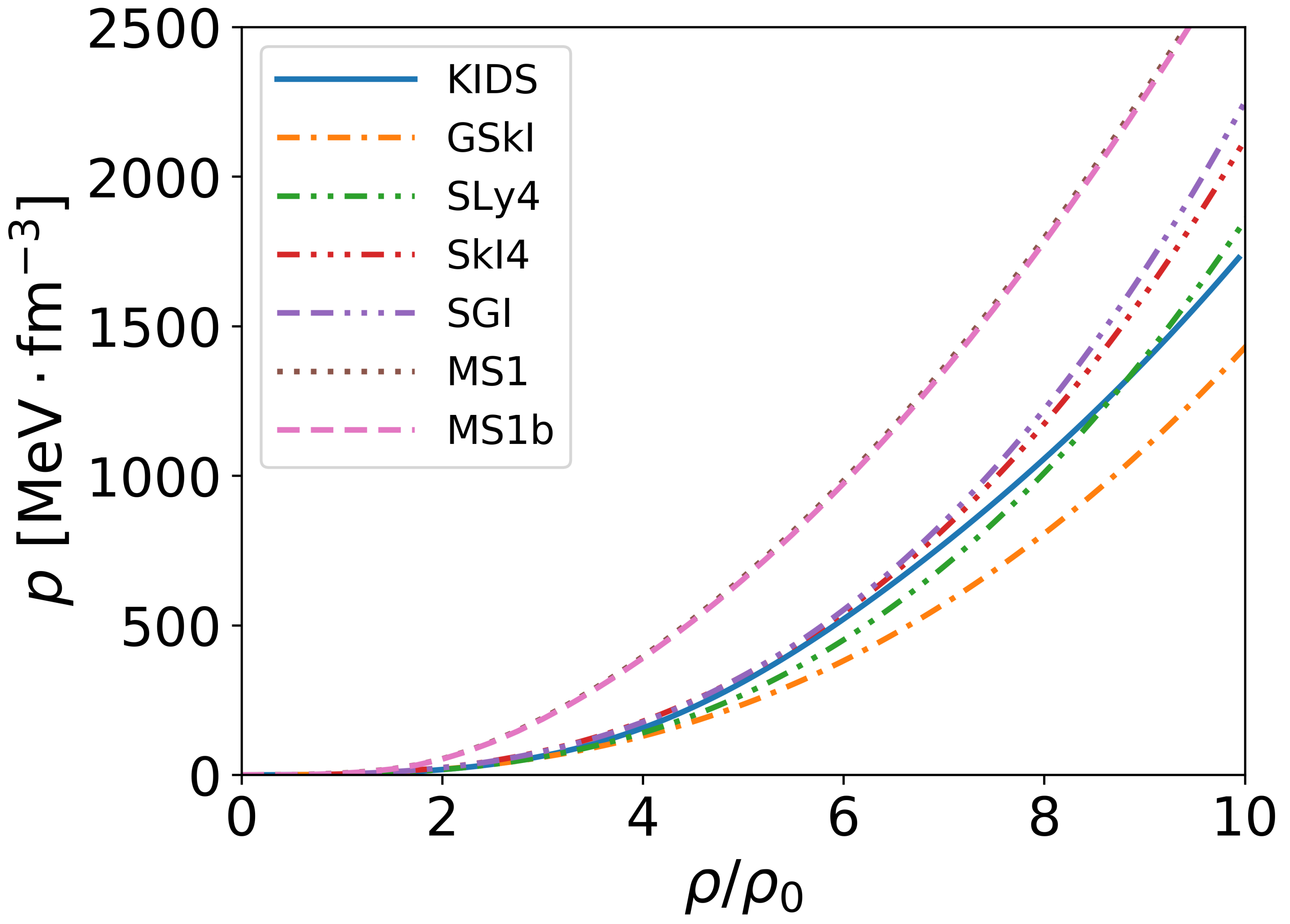
	^{16}O	^{40}Ca	^{48}Ca	^{90}Zr	^{132}Sn	^{208}Pb
Exp	2.6991	3.4776	3.4771	4.2694	4.7093	5.5012
KIDS	2.7618	3.4781	3.4867	4.2476	4.7089	5.4887
Deviation (%)	2.32	0.01	0.28	0.51	0.01	0.23

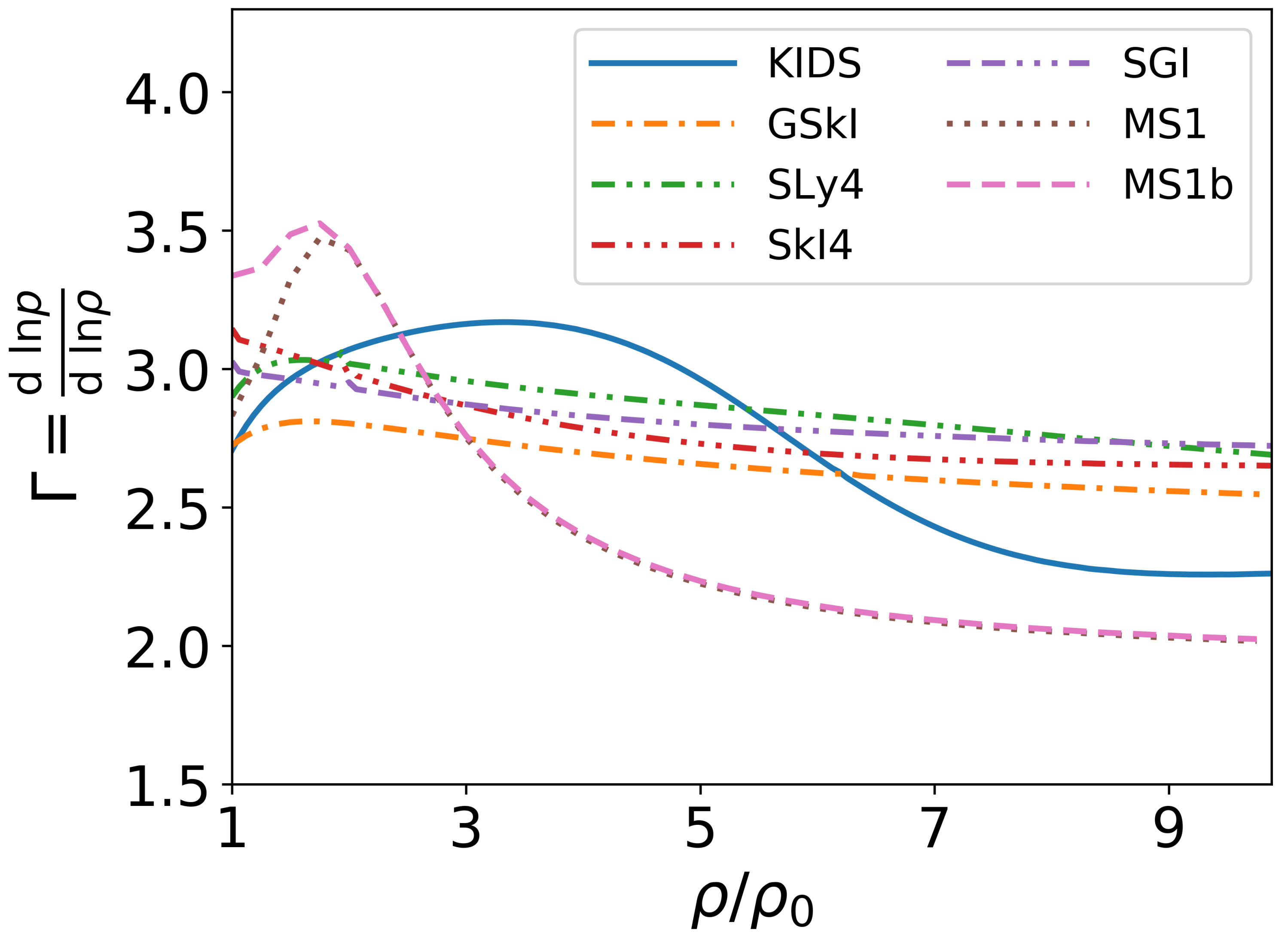
$$p = \rho^2 \frac{\partial(\mathcal{E}/\rho)}{\partial \rho}$$





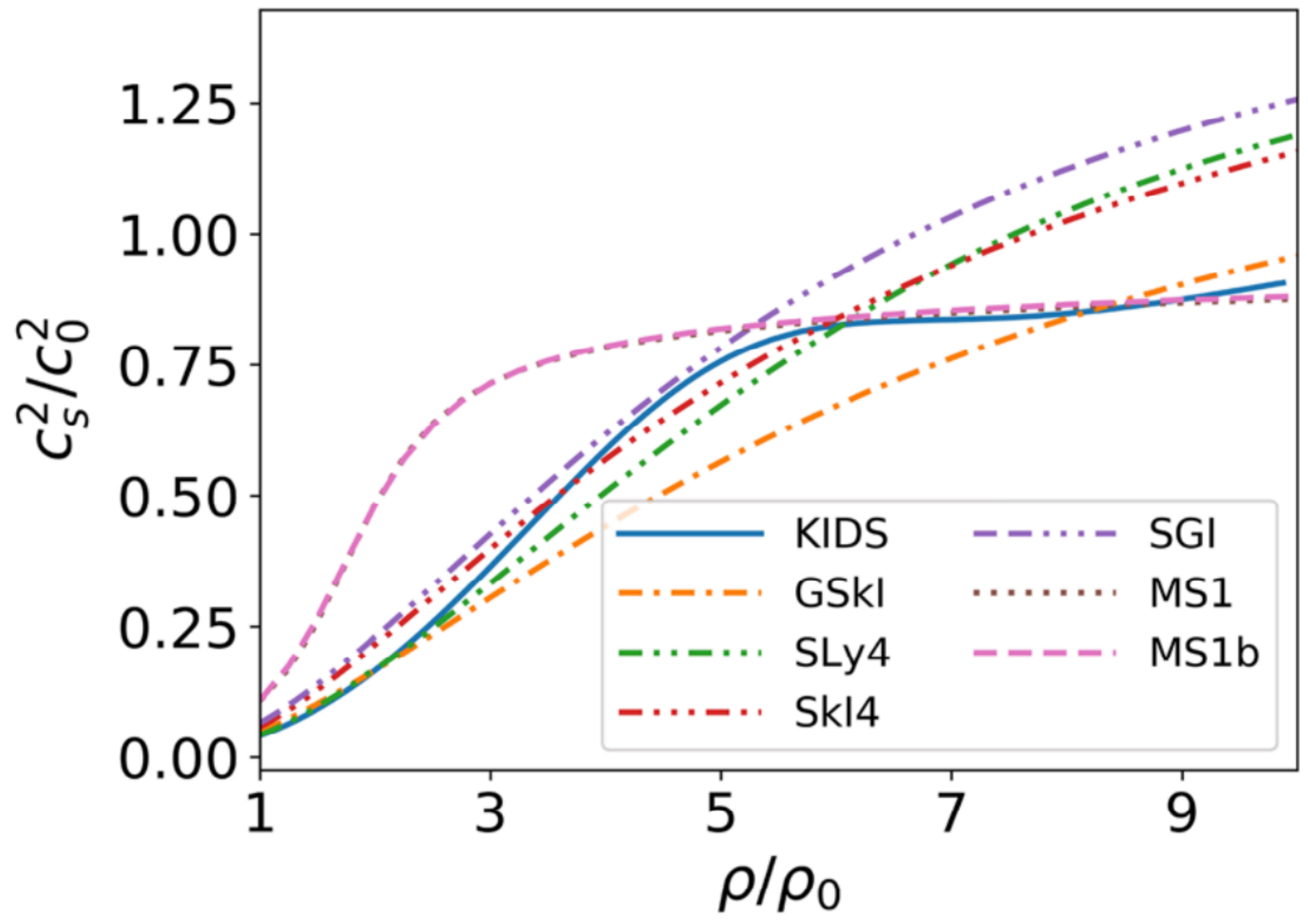
$$P = K\rho^{\gamma}$$



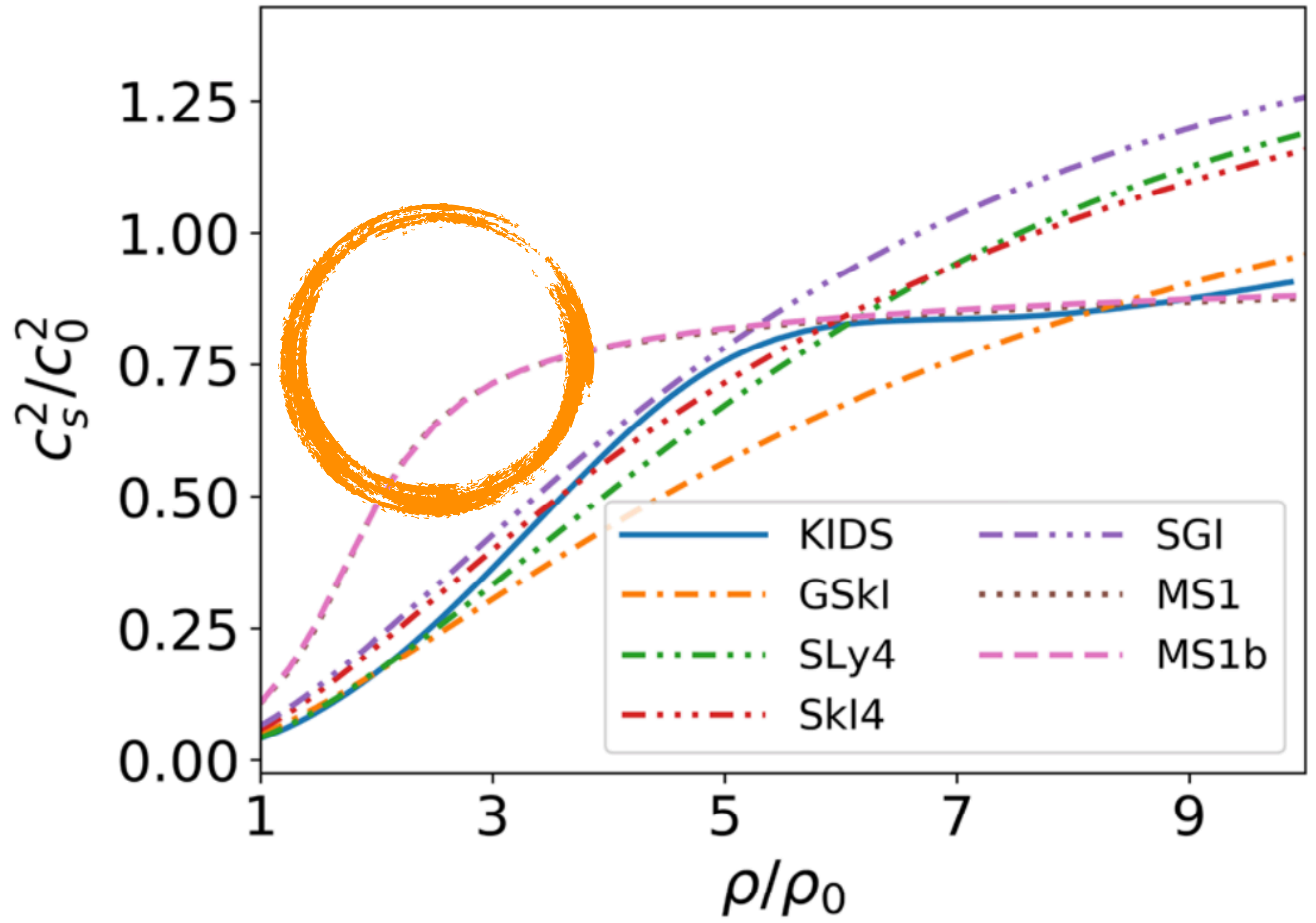


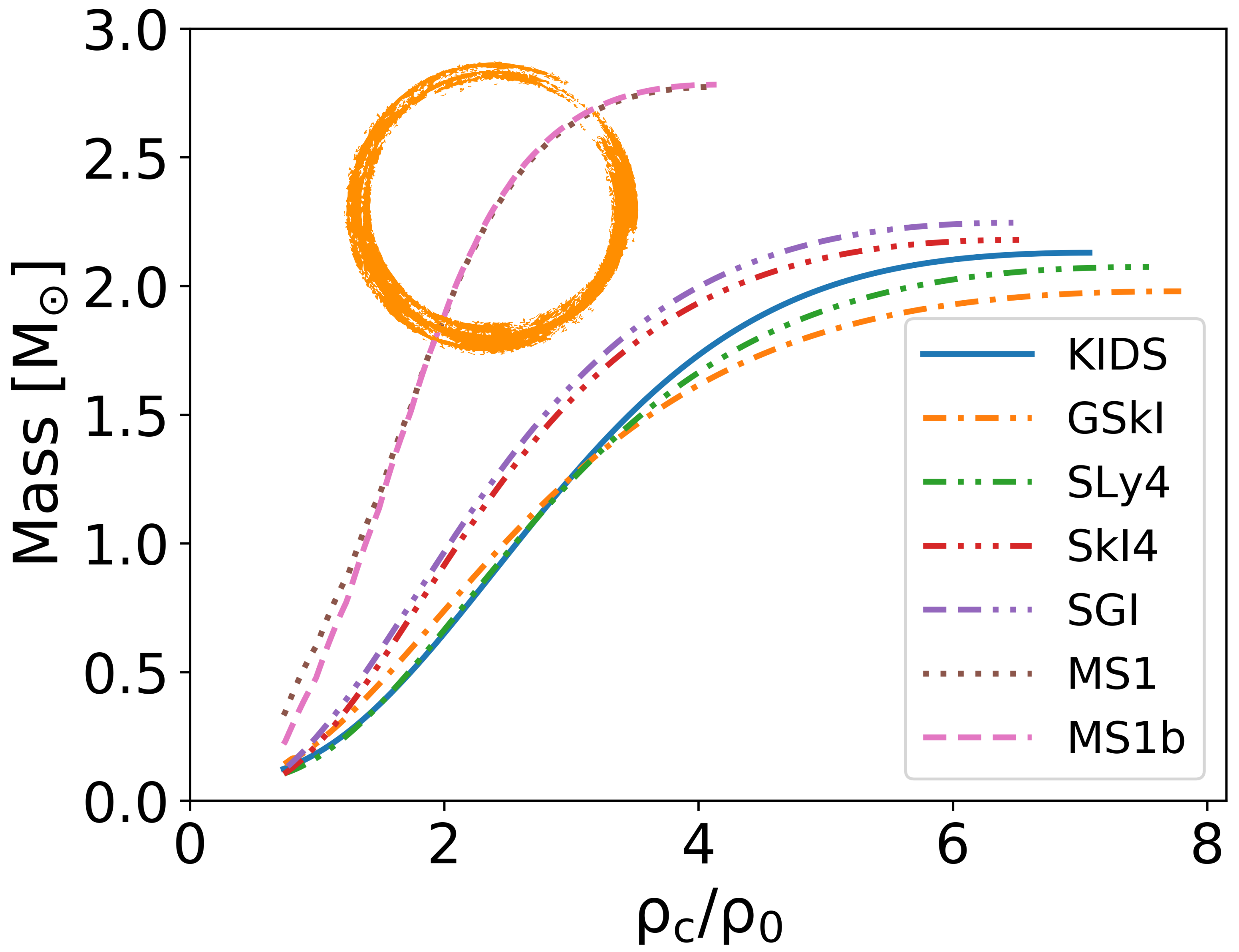
$$\Gamma(p) \equiv \frac{d(\ln p)}{d(\ln \rho)}$$

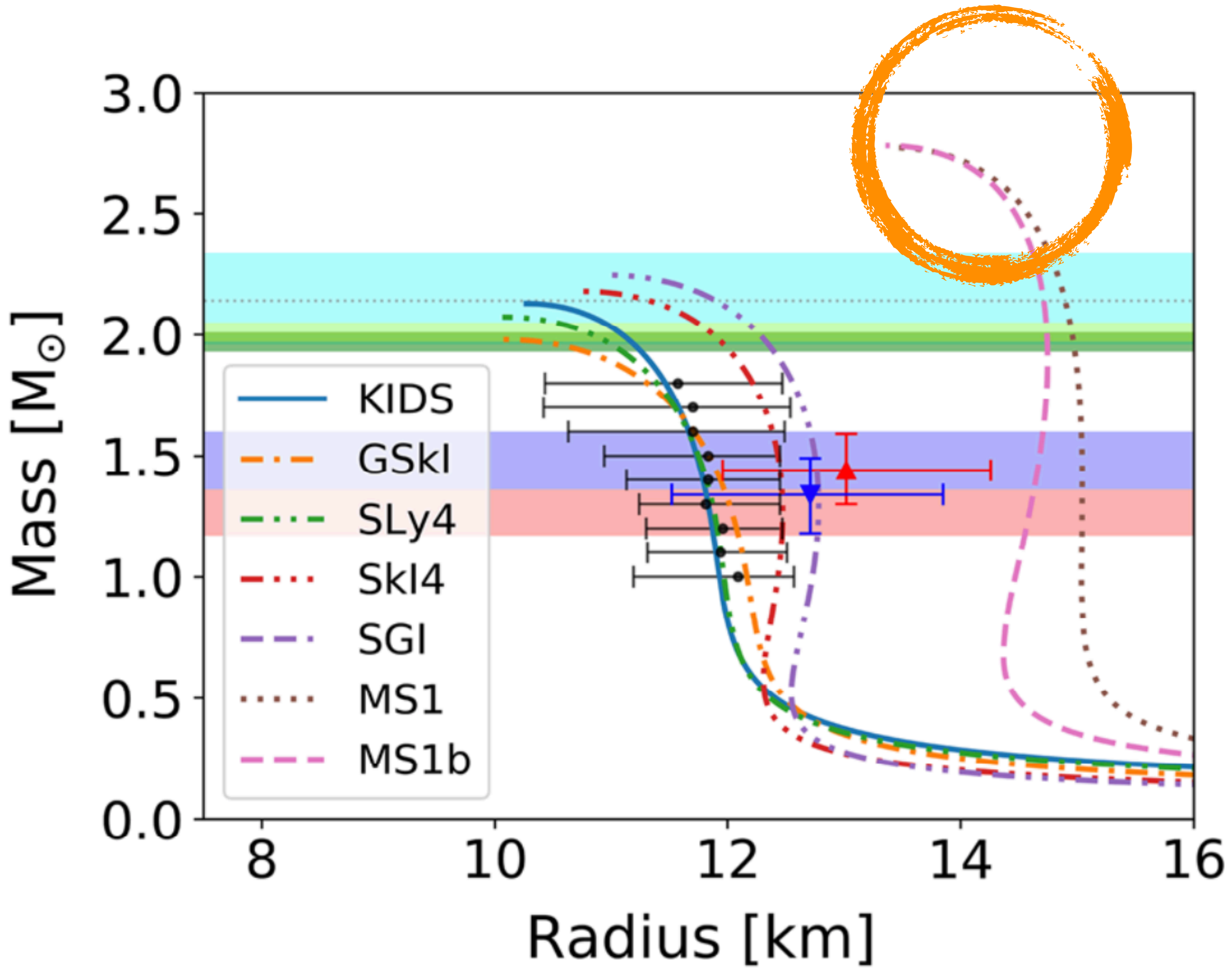
$$\Gamma(p) = \frac{\epsilon + p}{p}\frac{dp}{d\epsilon}$$



Sound velocity



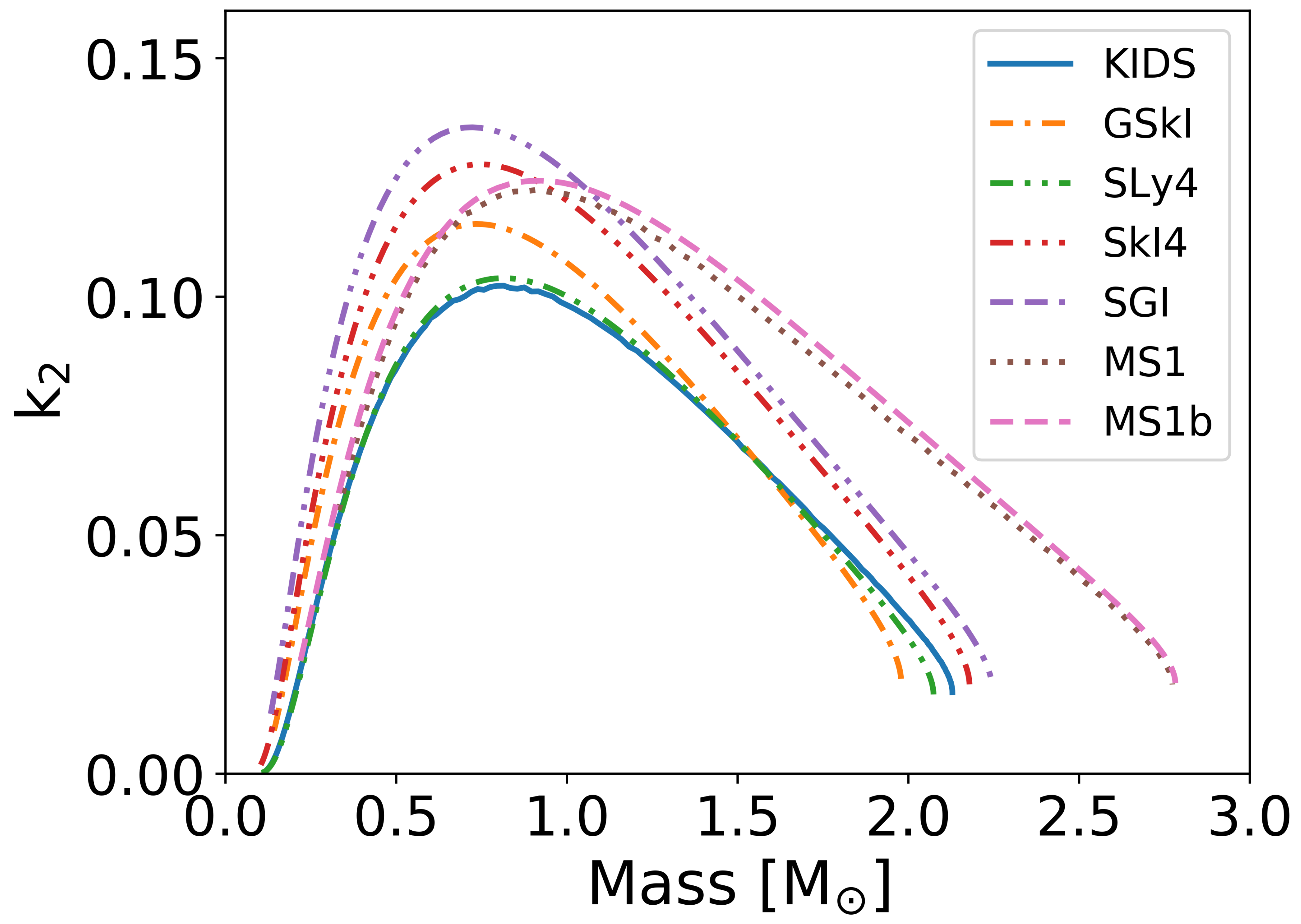


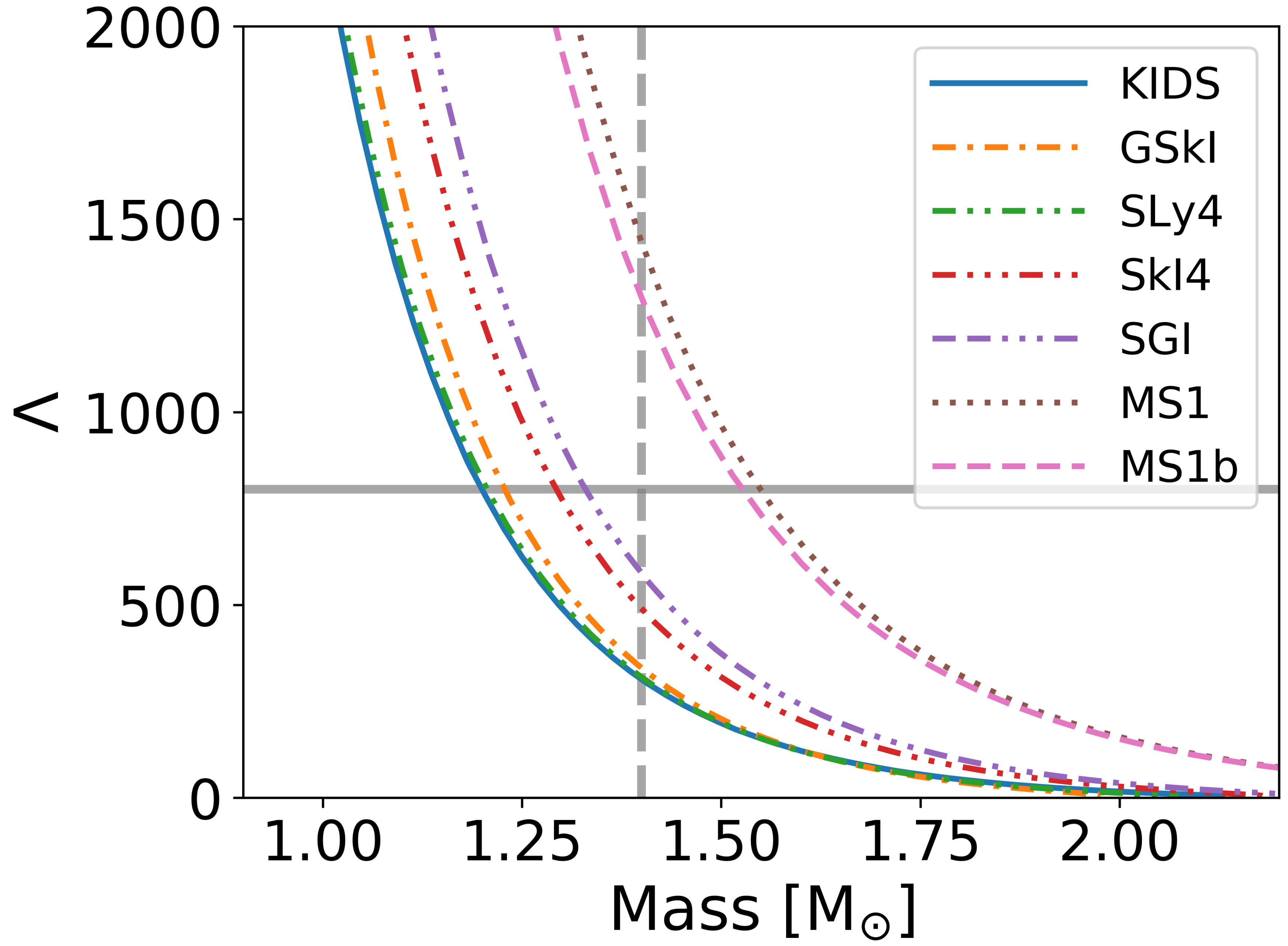


Nature seems to prefer

- *low sound velocity at low density*
- *high sound velocity at high density*

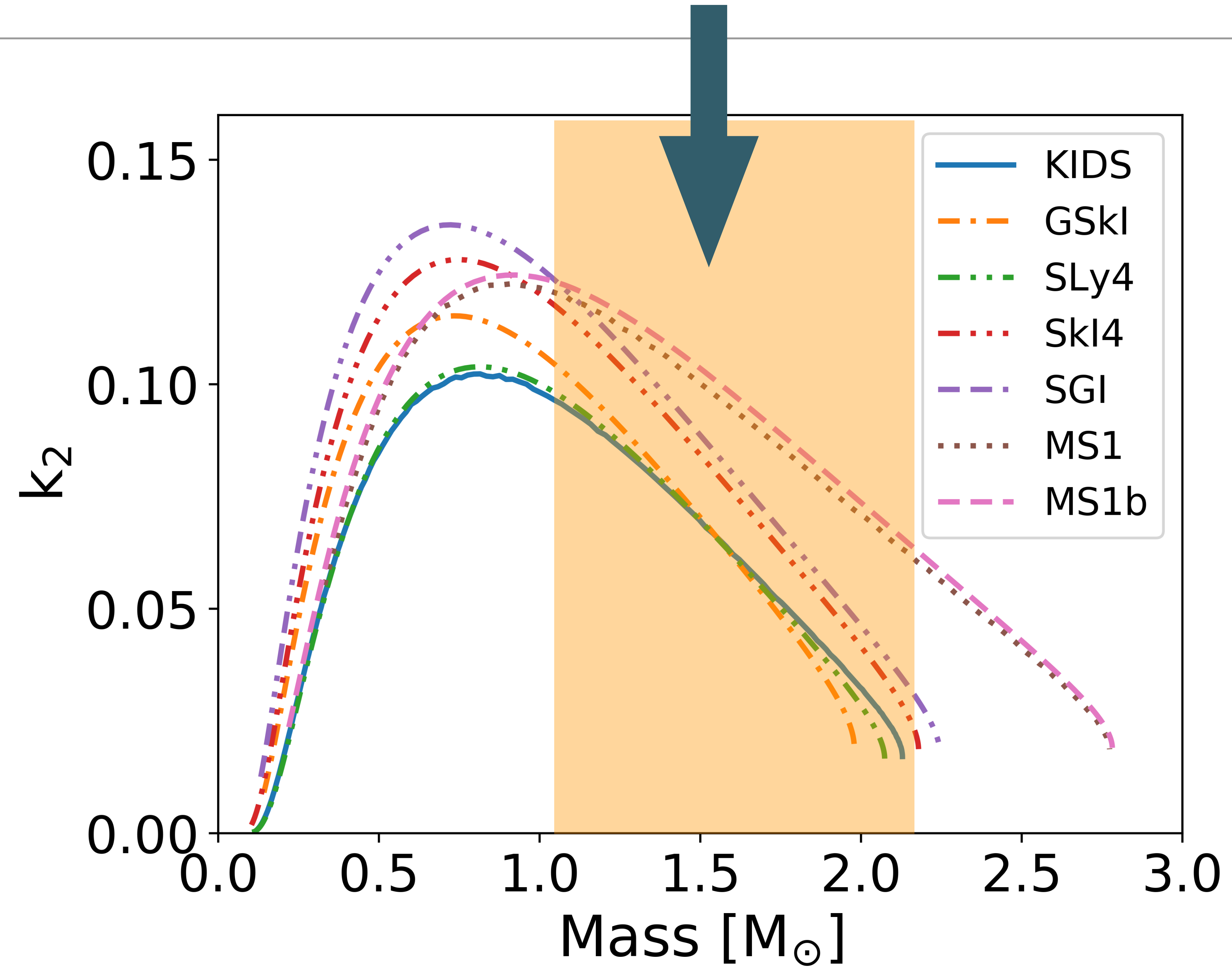
Tidal deformability

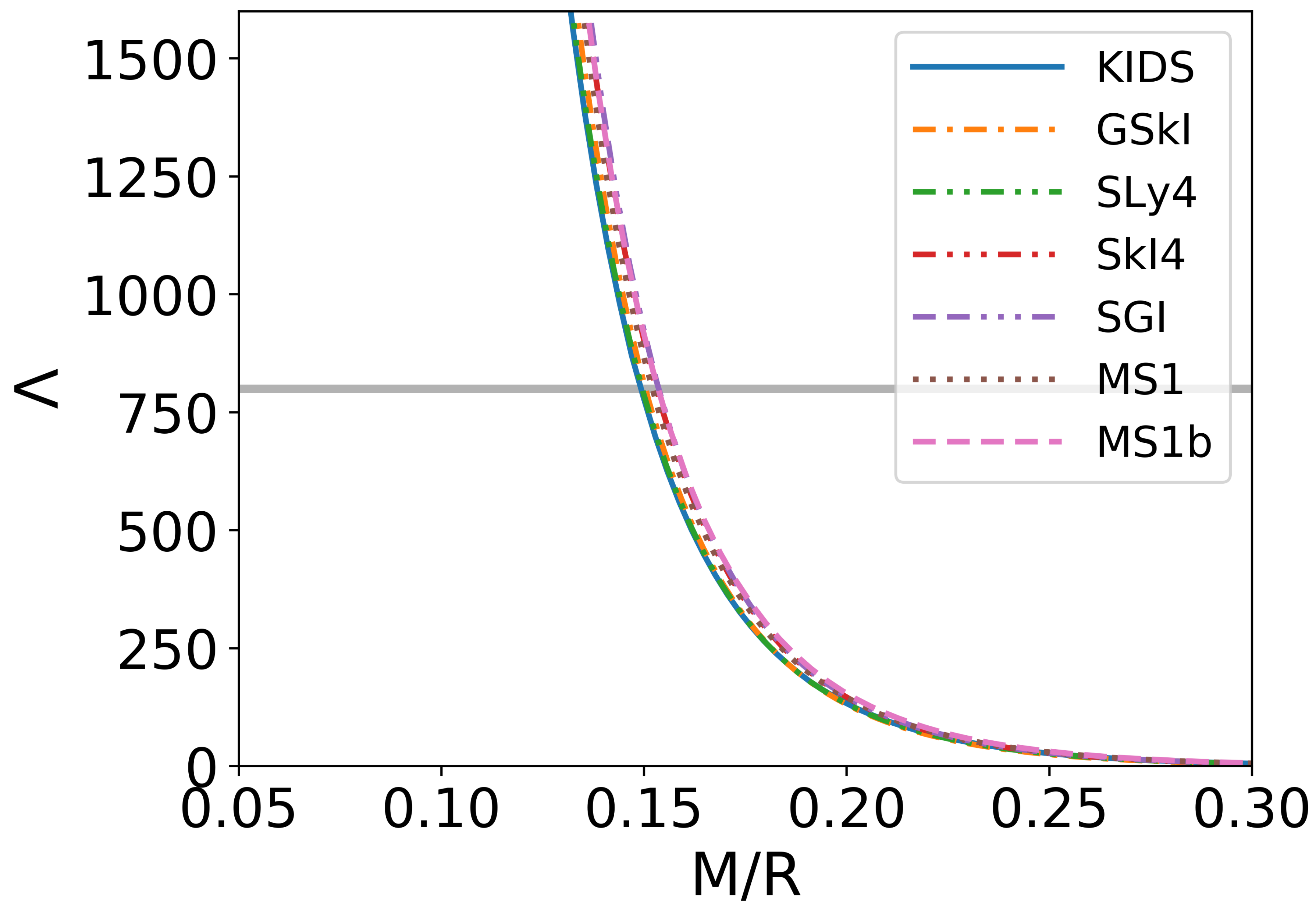




$$k_2 \propto \left(\frac{M}{R}\right)^{-1}$$

$$\Lambda = \frac{2}{3}\left(\frac{M}{R}\right)^{-5} k_2$$





scaling behavior in tidal deformability

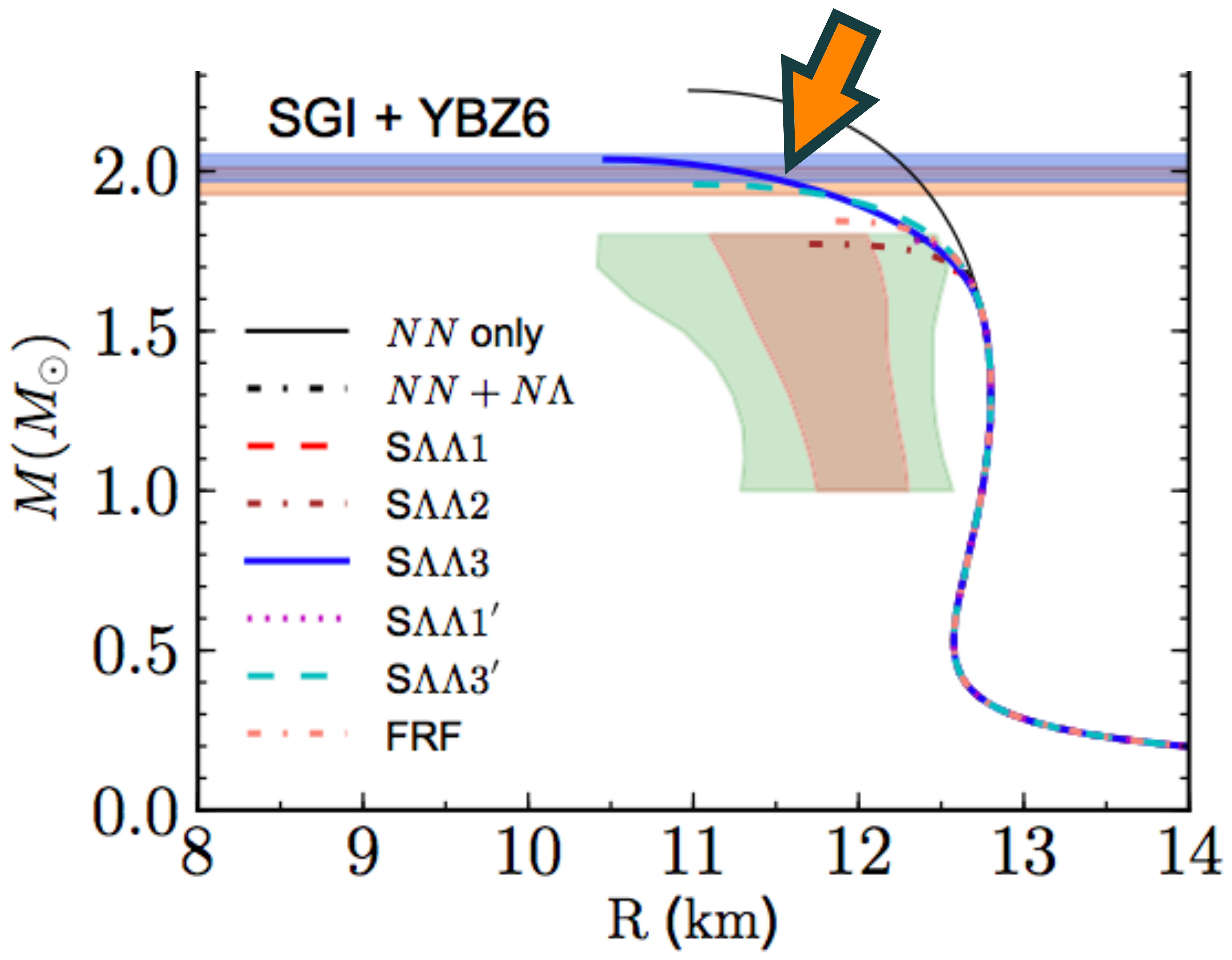
$$\Lambda \propto \left(\frac{M}{R}\right)^{-6}$$

Strangeness in neutron star

Role of strangeness in the core

IJMPE 12, 1550100 (2015)

reduction of degeneracy pressure



Skyrme-type forces : NN interaction

$$
\begin{aligned}
v_{NN}(\boldsymbol{r}_{ij}) = {} & t_0(1+x_0P_\sigma)\delta(\boldsymbol{r}_{ij}) + \frac{1}{2}t_1(1+x_1P_\sigma)[\boldsymbol{k}'^2_{ij}\delta(\boldsymbol{r}_{ij}) + \delta(\boldsymbol{r}_{ij})\boldsymbol{k}^2_{ij}] \\
& + t_2(1+x_2P_\sigma)\boldsymbol{k}'_{ij}\cdot\delta(\boldsymbol{r}_{ij})\boldsymbol{k}_{ij} + \frac{1}{6}t_3(1+x_3P_\sigma)\rho_N^\epsilon(\boldsymbol{R})\delta(\boldsymbol{r}_{ij}) \\
& + iW_0\boldsymbol{k}'_{ij}\cdot\delta(\boldsymbol{r}_{ij})(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\times\boldsymbol{k}_{ij},
\end{aligned}
$$

$\boldsymbol{r}_{ij} = \boldsymbol{r}_i - \boldsymbol{r}_j$, $\boldsymbol{R} = (\boldsymbol{r}_i + \boldsymbol{r}_j)/2$, $\boldsymbol{k}_{ij} = -i(\overrightarrow{\nabla}_i - \overrightarrow{\nabla}_j)/2$, $\boldsymbol{k}'_{ij} = i(\overleftarrow{\nabla}_i - \overleftarrow{\nabla}_j)/2$,

$P_\sigma = (1 + \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)/2$,

NN model	ρ_0	B	S_v	L	K	m_N^*/m_N	$M_{\max}/M_\odot$
SLy4	0.160	16.0	32.0	45.9	230	0.694	2.07
SkI4	0.160	16.0	29.5	60.4	248	0.649	2.19
SGI	0.155	15.9	28.3	63.9	262	0.608	2.25

N-Lambda interaction

$$v_{N\Lambda} = u_0(1 + y_0 P_\sigma)\delta(\boldsymbol{r}_{N\Lambda}) + \frac{1}{2}u_1[\boldsymbol{k}'^2_{N\Lambda}\delta(\boldsymbol{r}_{N\Lambda}) + \delta(\boldsymbol{r}_{N\Lambda})\boldsymbol{k}^2_{N\Lambda}]$$

$$+ u_2\boldsymbol{k}'_{N\Lambda} \cdot \delta(\boldsymbol{r}_{N\Lambda})\boldsymbol{k}_{N\Lambda} + \frac{3}{8}u'_3(1 + y_3 P_\sigma)\rho_N^\gamma\left(\frac{\boldsymbol{r}_N + \boldsymbol{r}_\Lambda}{2}\right)\delta(\boldsymbol{r}_{N\Lambda}),$$

Table 2. Parameters for the $N\Lambda$ interactions. u_0 is in unit of MeV$\cdot$ fm^3, u_1 and u_2 in unit of MeV$\cdot$ fm^5, and u'_3 in unit of MeV$\cdot$ fm$^{3+3\gamma}$. y_0 and y_3 are dimensionless. The last column U_Λ^{opt} is the depth of the Λ-nucleus optical potential in unit of MeV at the saturation density.

$N\Lambda$ model	γ	u_0	u_1	u_2	u'_3	y_0	y_3	U_Λ^{opt}
HPΛ2	1	−399.946	83.426	11.455	2046.818	−0.486	−0.660	−31.23
OΛ2	1/3	−417.7593	1.5460	−3.2671	1102.2221	−0.3854	−0.5645	−28.27
YBZ6	1	−372.2	100.4	79.60	2000	−0.107	0	−29.73

Lambda-Lambda interaction

$$v_{\Lambda\Lambda}(\boldsymbol{r}_{ij}) = \lambda_0\delta(\boldsymbol{r}_{ij}) + \frac{1}{2}\lambda_1[\boldsymbol{k}'^2_{ij}\delta(\boldsymbol{r}_{ij}) + \delta(\boldsymbol{r}_{ij})\boldsymbol{k}^2_{ij}]$$
$$+ \lambda_2\boldsymbol{k}'_{ij}\cdot\delta(\boldsymbol{r}_{ij})\boldsymbol{k}_{ij} + \lambda_3\rho^{\alpha}_{N}(\boldsymbol{R})\delta(\boldsymbol{r}_{ij})$$

$$v^{\mathrm{FRF}}_{\Lambda\Lambda}(r) = \sum_{i=1}^{3}(v_i + v_i^{\sigma}\boldsymbol{\sigma}_\Lambda\cdot\boldsymbol{\sigma}_\Lambda)e^{-\mu_i r^2}.$$

Table 3. Parameters of the $\Lambda\Lambda$ interactions in the Skyrme-type force. λ_0 is in unit of $\mathrm{MeV}\cdot\mathrm{fm}^3$, and λ_1 in unit of $\mathrm{MeV}\cdot\mathrm{fm}^5$. Note that all models considered here do not have λ_2 momentum interaction and λ_3, α density dependent interaction.

$\Lambda\Lambda$ model	λ_0	λ_1	λ_2	λ_3	α
S$\Lambda\Lambda$1	−312.6	57.5	0	0	—
S$\Lambda\Lambda$2	−437.7	240.7	0	0	—
S$\Lambda\Lambda$3	−831.8	922.9	0	0	—
S$\Lambda\Lambda$1′	−37.9	14.1	0	0	—
S$\Lambda\Lambda$3′	−156.4	347.2	0	0	—

Lambda-Lambda interaction / Binding energy

$$v_{\Lambda\Lambda}(\boldsymbol{r}_{ij}) = \lambda_0\delta(\boldsymbol{r}_{ij}) + \frac{1}{2}\lambda_1[\boldsymbol{k}'^2_{ij}\delta(\boldsymbol{r}_{ij}) + \delta(\boldsymbol{r}_{ij})\boldsymbol{k}^2_{ij}]$$

$$+ \lambda_2\boldsymbol{k}'_{ij} \cdot \delta(\boldsymbol{r}_{ij})\boldsymbol{k}_{ij} + \lambda_3\rho_N^\alpha(\boldsymbol{R})\delta(\boldsymbol{r}_{ij})$$

$$v_{\Lambda\Lambda}^{\mathrm{FRF}}(r) = \sum_{i=1}^{3}(v_i + v_i^\sigma \boldsymbol{\sigma}_\Lambda \cdot \boldsymbol{\sigma}_\Lambda)e^{-\mu_i r^2}.$$

Nuclei	$B_{\Lambda\Lambda}$(SΛΛ1)	$B_{\Lambda\Lambda}$(SΛΛ2)	$B_{\Lambda\Lambda}$(SΛΛ3)	$B_{\Lambda\Lambda}$(Exp.)
${}^{6}_{\Lambda\Lambda}$He	11.88	9.25	7.60	6.93 ± 0.16[22]
${}^{10}_{\Lambda\Lambda}$Be	19.78	18.34	15.19	14.94 ± 0.13[23,a]
${}^{11}_{\Lambda\Lambda}$Be	20.55	19.26	16.27	20.49 ± 1.15[22]
${}^{12}_{\Lambda\Lambda}$Be	21.10	19.97	17.18	22.23 ± 1.15[22]
${}^{13}_{\Lambda\Lambda}$B	21.21	20.26	17.76	23.30 ± 0.70[22]

Maximum mass of NS

NN model (without Λ)†	SLy4 (2.07)		SkI4 (2.19)			SGI (2.25)		
$N\Lambda$ model (no $\Lambda\Lambda$)††	HPΛ2* (1.51)	OΛ2 (1.08)	HPΛ2 (1.52)	OΛ2 (1.19)	YBZ6* (1.80)	HPΛ2 (1.52)	OΛ2 (1.22)	YBZ6* (1.79)
S$\Lambda\Lambda$1	*1.40*	1.00	1.41	1.12	*1.70*	1.42	1.16	*1.69*
S$\Lambda\Lambda$2	*1.58*	1.28	1.57	1.30	*1.79*	1.57	1.28	*1.77*
S$\Lambda\Lambda$3	*1.85*	1.57	1.87	1.62	*2.03*	1.88	1.65	*2.04*
S$\Lambda\Lambda$1′	*1.51*	1.08	1.51	1.18	*1.79*	1.51	1.21	*1.78*
S$\Lambda\Lambda$3′	*1.76*	1.43	1.76	1.47	*1.97*	1.77	1.49	*1.96*
FRF	*1.61*	1.22	1.60	1.25	*1.86*	1.59	1.26	*1.84*

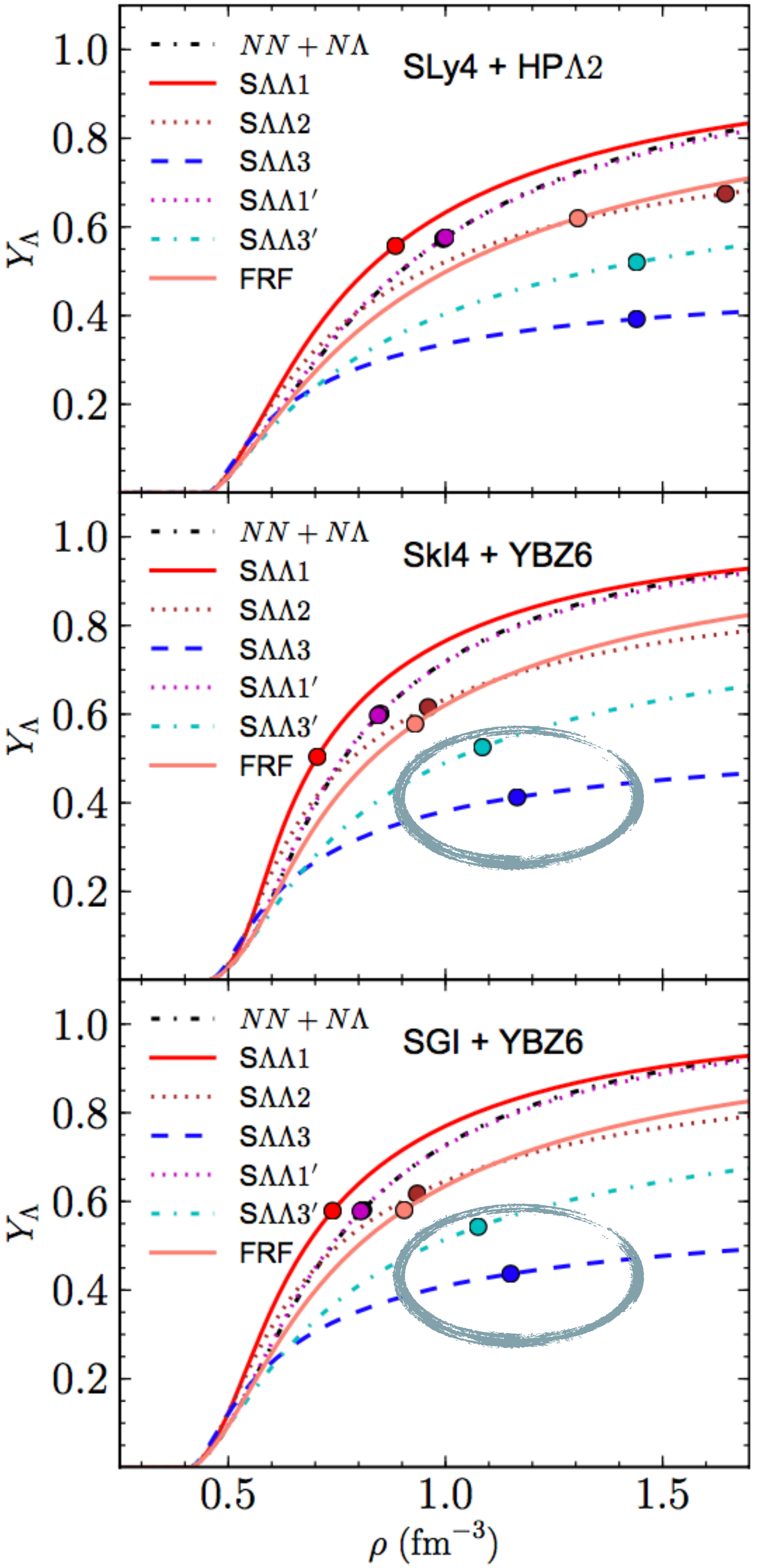
*Numbers in italic correspond to three selected models, SLy4-HPΛ2, SkI4-YBZ6 and SGI-YBZ6, which give relatively large maximum NS mass.
†Maximum NS mass without both $N\Lambda$ and $\Lambda\Lambda$ interactions.
††Maximum NS mass with $N\Lambda$ but without $\Lambda\Lambda$ interactions.

Central densities

Table 6. Density in fm^{-3} at the center of stars corresponding to the locations of filled circles in Fig. 1.

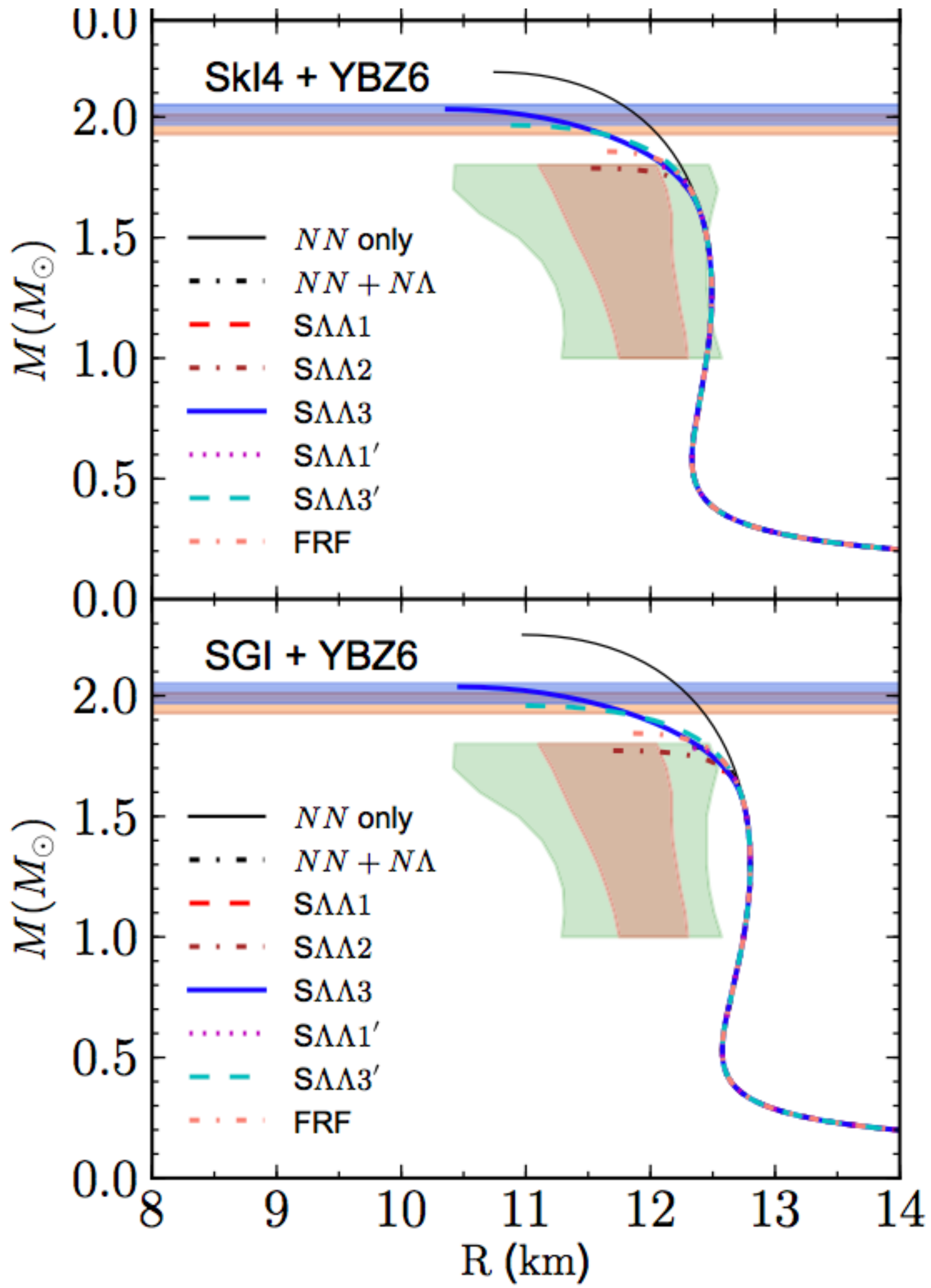
	SLy4+HLΛ2	SkI4+YBZ6	SGI+YBZ6
Without ΛΛ	1.00	0.85	0.81
SΛΛ1	0.89	0.71	0.74
SΛΛ2	1.65	0.96	0.94
SΛΛ3	1.44	1.17	1.15
SΛΛ1′	1.00	0.85	0.81
SΛΛ3′	1.44	1.09	1.08
FRF	1.31	0.93	0.91

O : central densities for maximum mass



Hyperons in Skyrme force models

IJMPE 12, 1550100 (2015)



reduction of degeneracy pressure

Kaons with Skyrme-type models

Y Lim, K Kwan, C H Hyun, C-H Lee, PRC 89, 055804 (2014)

Kaons with Skyrme force models

PRC 89, 055804 (2014)

$$
\begin{aligned}
\mathcal{L} = & \frac{f_\pi^2}{4}\mathrm{Tr}\partial_\mu U \partial^\mu U^\dagger + c\mathrm{Tr}[m_q(U+U^\dagger-2)] \\
& + i\mathrm{Tr}\bar{B}\gamma^\mu\partial_\mu B + i\mathrm{Tr}B^\dagger[V_0,B] - D\mathrm{Tr}B^\dagger\boldsymbol{\sigma}\cdot\{\mathbf{A},B\} \\
& - F\mathrm{Tr}B^\dagger\boldsymbol{\sigma}\cdot[\mathbf{A},B] \\
& + a_1\mathrm{Tr}B^\dagger(\xi m_q\xi + \mathrm{H.c.})B + a_2\mathrm{Tr}B^\dagger B(\xi m_q \xi + \mathrm{H.c.}) \\
& + a_3\mathrm{Tr}B^\dagger B\mathrm{Tr}(m_q U + \mathrm{H.c.})
\end{aligned}
$$

$$
M = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}
$$

$$
B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda \end{pmatrix}
$$

Kaons

$$\langle K^- \rangle = v_K e^{-i\mu_K t}$$

$$\mathcal{L}_K = f_\pi^2 \frac{\mu_K^2}{2} \sin^2\theta - 2m_K^2 f_\pi^2 \sin^2\frac{\theta}{2}$$
$$- n^\dagger n[-\mu_K + (2a_2 + 4a_3)m_s] \sin^2\frac{\theta}{2}$$
$$- p^\dagger p[-2\mu_K + (2a_1 + 2a_2 + 4a_3)m_s] \sin^2\frac{\theta}{2}$$

$$\mathcal{H}_K = -f_\pi^2 \frac{\mu_K^2}{2} \sin^2\theta + 2m_K^2 f_\pi^2 \sin^2\frac{\theta}{2} + \mu_K \rho_p$$
$$-\mu_K(\rho + \rho_p) \sin^2\frac{\theta}{2} + a_{K1}\rho_p \sin^2\frac{\theta}{2} + a_{K2}\rho \sin^2\frac{\theta}{2}$$

Particle fraction

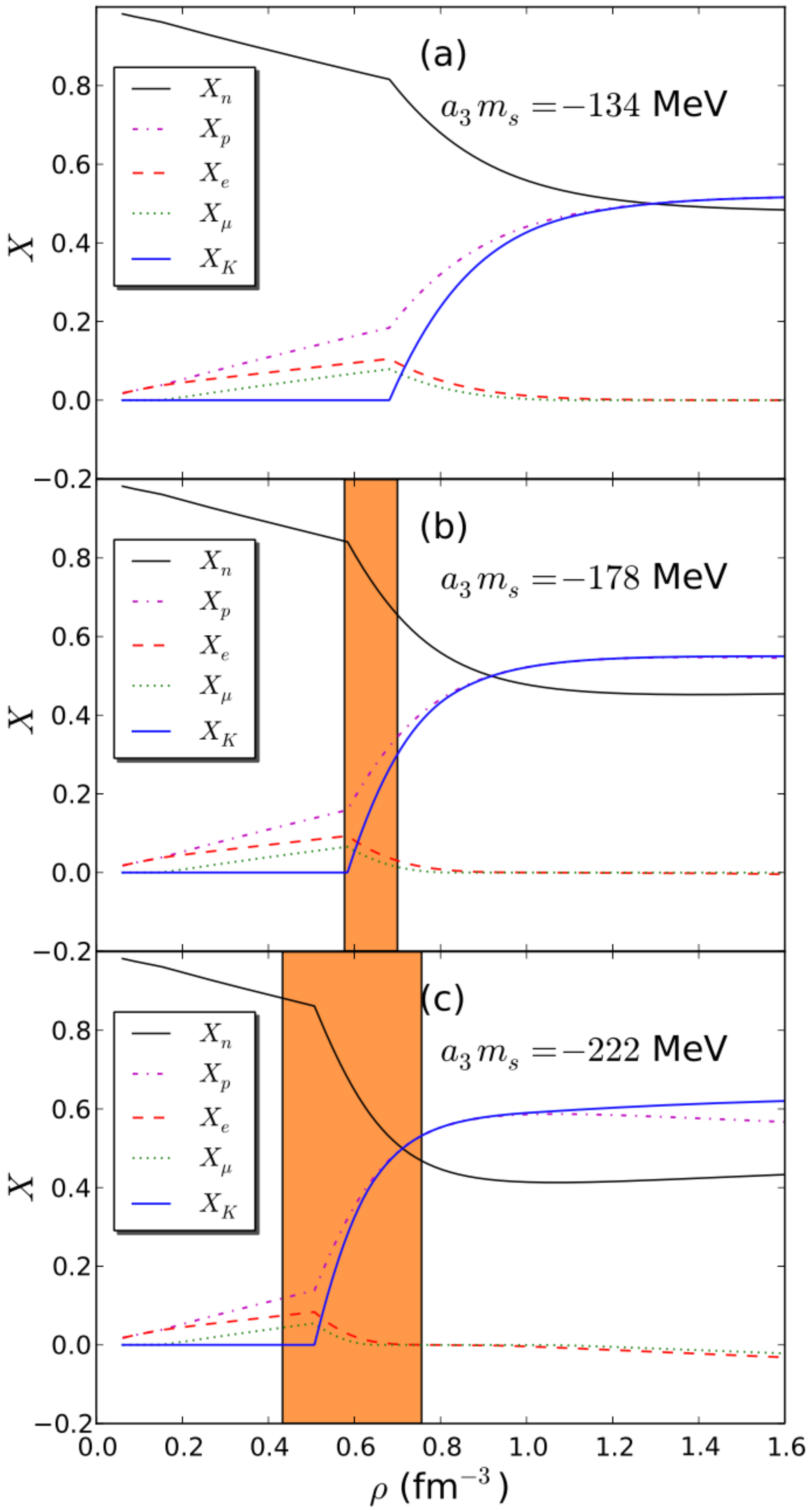
$$m_s \langle \bar{s}s \rangle_p = -2(a_2 + a_3)m_s,$$

$$\Sigma^{KN} = -\frac{1}{2}(a_1 + 2a_2 + 4a_3)m_s$$

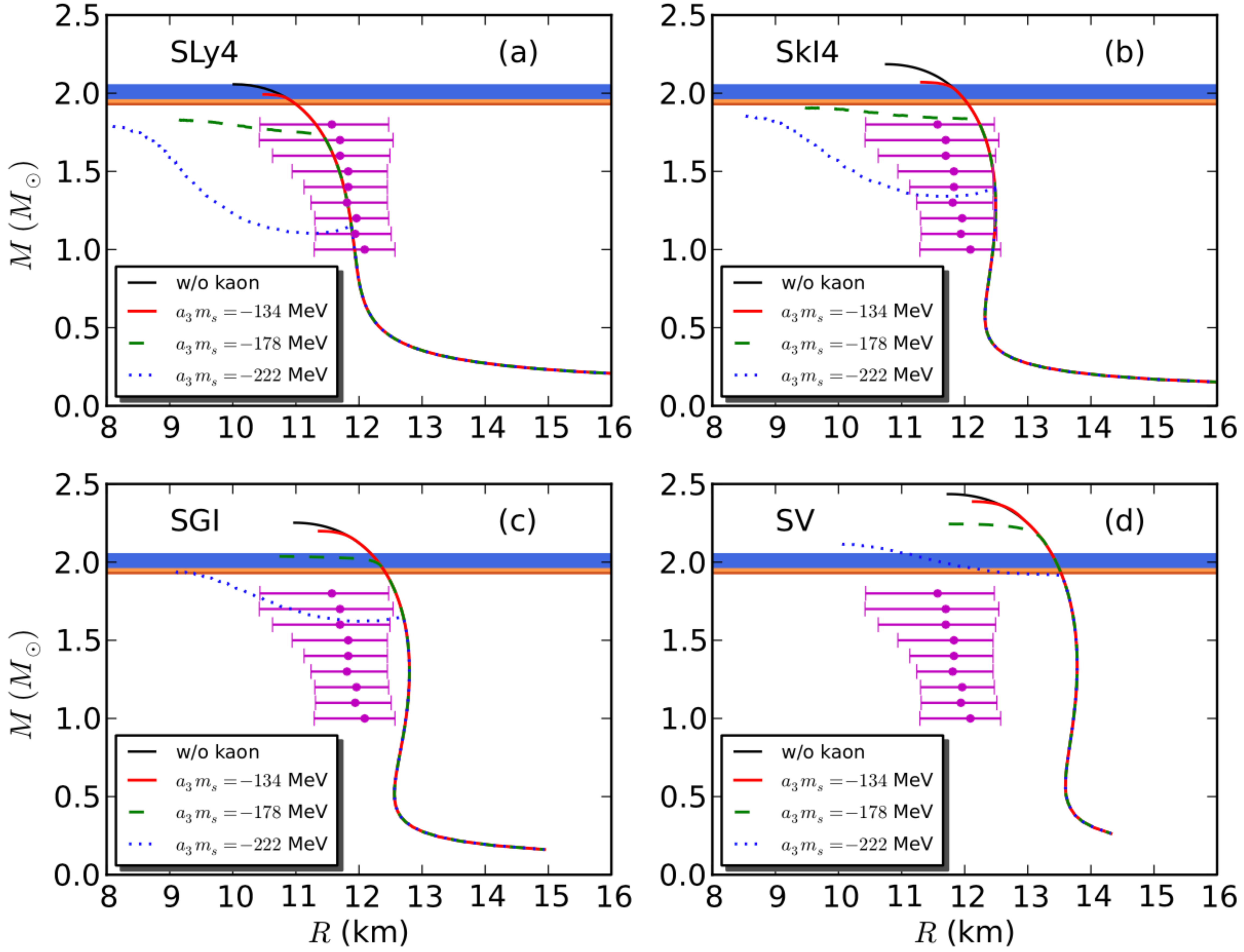
$$a_1 m_s = -67 \text{ MeV},$$

$$a_2 m_s = 134 \text{ MeV}.$$

Orange region: maxwell construction



NS mass with kaons



NS with strangeness

- **In general, strangeness reduces maximum NS mass**
- **But, there still remain possibilities for the strangeness**

masses with Lambda hyperons

SLy4	SkI4	SGI
$1.15 \sim 1.85$	$1.47 \sim 1.97$	$1.44 \sim 2.04$

masses with kaon condensation

consistent with 2 Msun NS

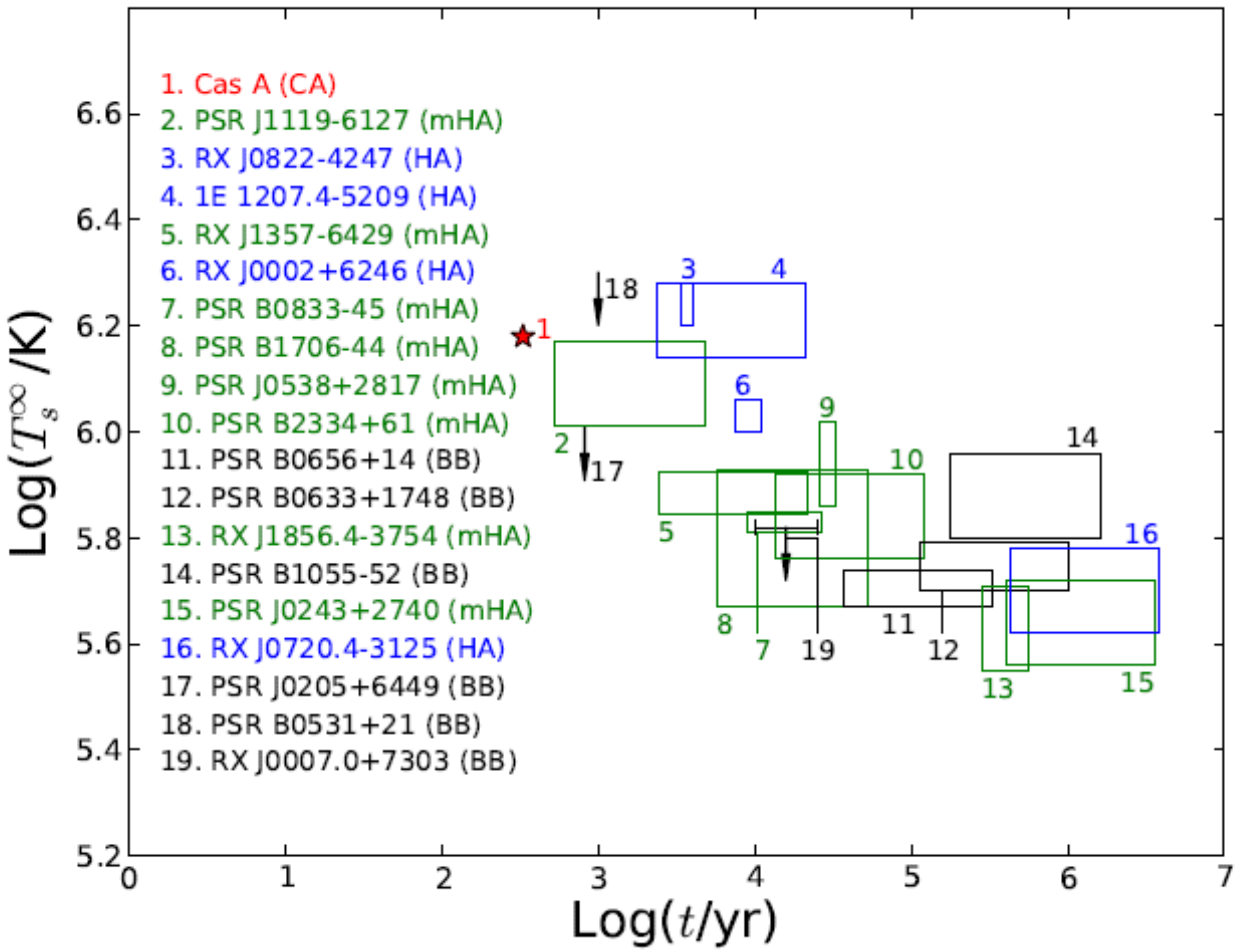
$a_3 m_s$	SLy4	SkI4	SGI
-134	$1.94 \sim 1.99$	$2.00 \sim 2.06$	$2.12 \sim 2.19$
-178	$1.74 \sim 1.83$	$1.84 \sim 1.90$	$1.95 \sim 2.03$
-222	$1.49 \sim 1.79$	$1.64 \sim 1.85$	$1.75 \sim 1.93$

NS cooling

Q) Can **NS EOS with/without strangeness** be consistent with
both **NS maximum mass & NS cooling**?

arXiv:1608.02078

Neutron Star Cooling



Y.Lim, C H Hyun, CHL, IJMPE (2017)

Name	Process	Emissivity[b] (erg cm^{-3} s^{-1})	
Modified Urca (neutron branch)	$n+n \to n+p+e^- +\bar{\nu}_e$ $n+p+e^- \to n+n+\nu_e$	$\sim 2\times10^{21}\, \mathcal{R}\, T_9^8$	Slow
Modified Urca (proton branch)	$p+n \to p+p+e^- +\bar{\nu}_e$ $p+p+e^- \to p+n+\nu_e$	$\sim 10^{21}\, \mathcal{R}\, T_9^8$	Slow
Bremsstrahlung	$n+n \to n+n+\nu\bar{\nu}$ $n+p \to n+p+\nu\bar{\nu}$ $p+p \to p+p+\nu\bar{\nu}$	$\sim 10^{19}\, \mathcal{R}\, T_9^8$	Slow
Cooper pair formations	$n+n \to [nn]+\nu\bar{\nu}$ $p+p \to [pp]+\nu\bar{\nu}$	$\sim 5\times10^{21}\, \mathcal{R}\, T_9^7$ $\sim 5\times10^{19}\, \mathcal{R}\, T_9^7$	
Direct Urca	$n \to p+e^- +\bar{\nu}_e$ $p+e^- \to n+\nu_e$	$\sim 10^{27}\, \mathcal{R}\, T_9^6$	Fast
π^- condensate	$n+<\pi^->\to n+e^- +\bar{\nu}_e$	$\sim 10^{[illegible]}\, \mathcal{R}\, T_9^6$	Fast
K^- condensate	$n+<K^->\to n+e^- +\bar{\nu}_e$	$\sim 10^{25}\, \mathcal{R}\, T_9^6$	Fast

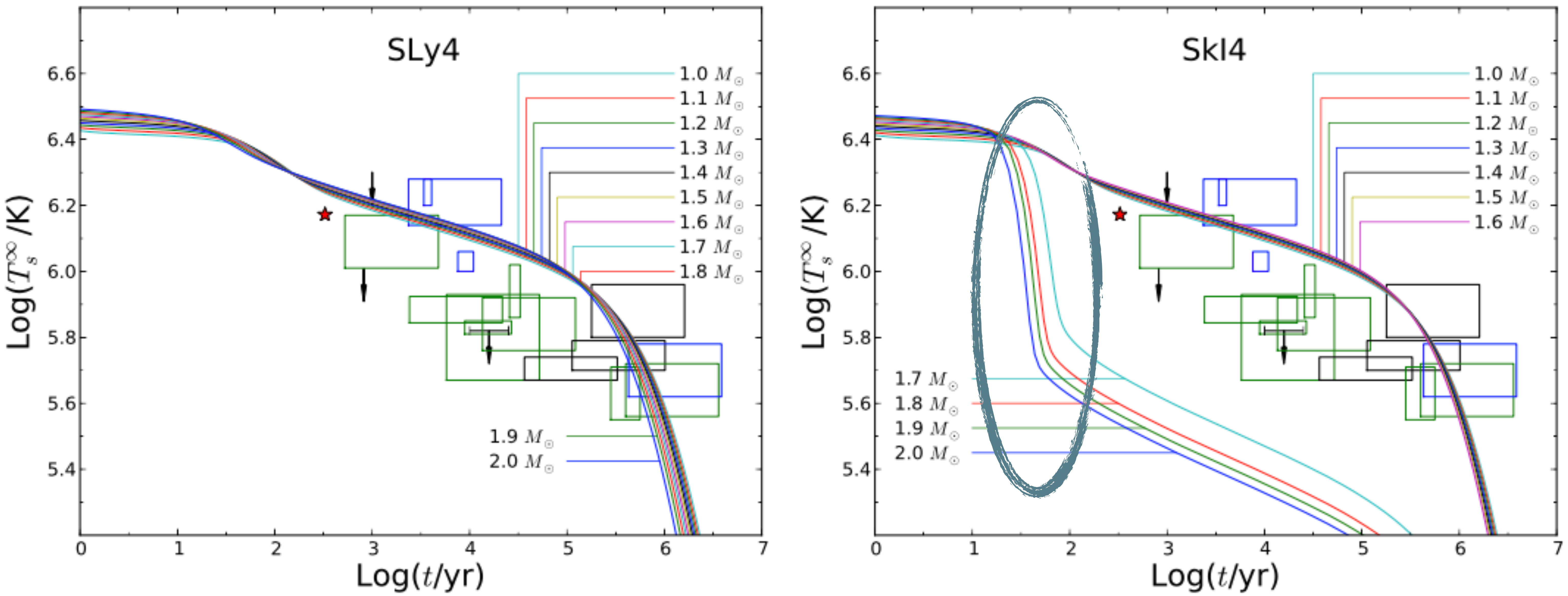
depends on

- particle fraction
- elements in the envelope
- nuclear superfluidity
- … …

Role of nucleon direct Urca

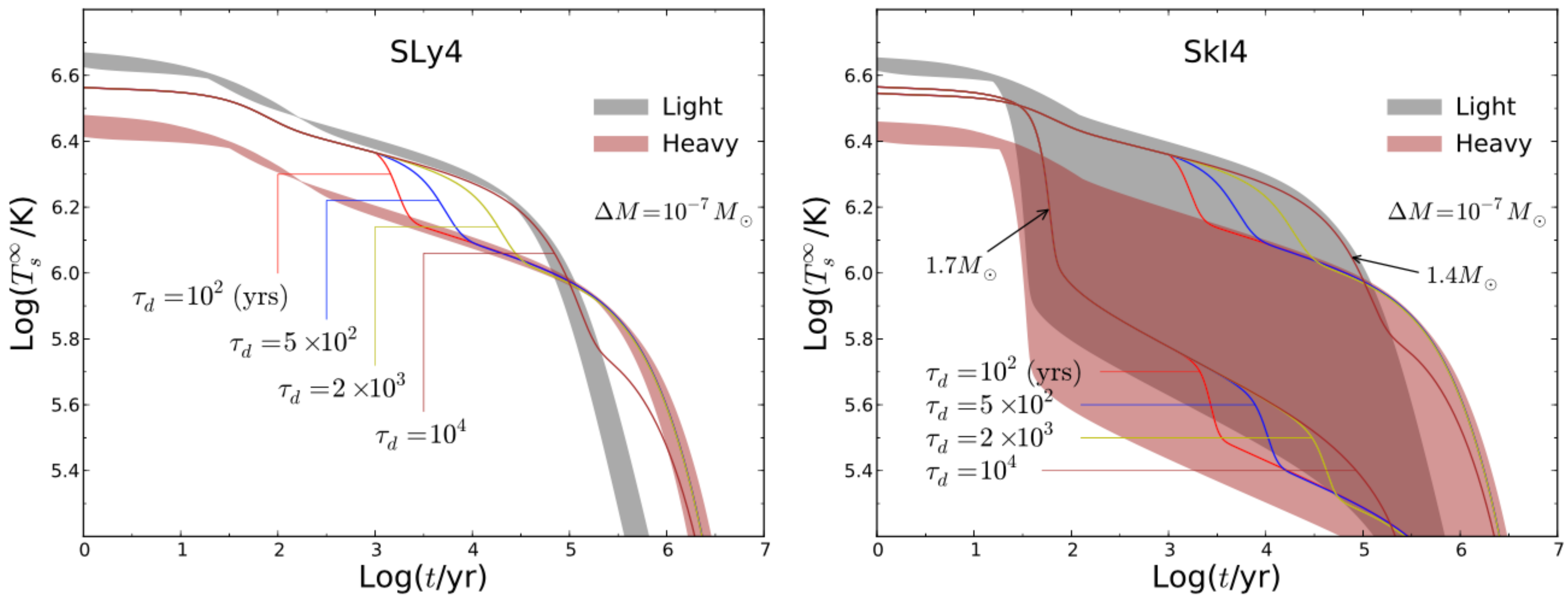
	SLy4	SkI4	SGI
$M_{\rm max}$	2.07 (-)	2.19 (1.63)	2.25 (1.72)

maximum mass without strangeness (critical mass for nucleon direct Urca)



effect of direct Urca

effect of surface element

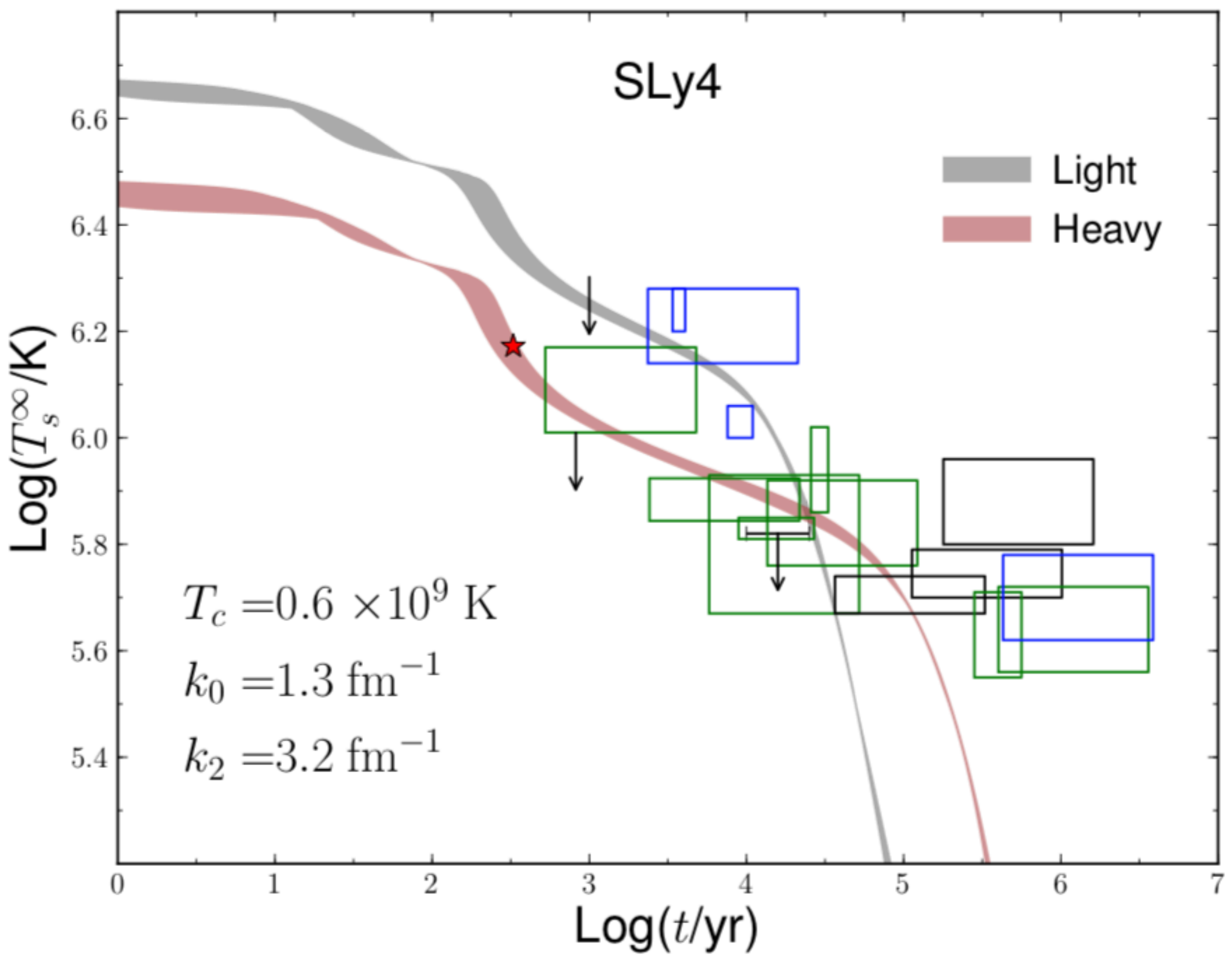


light elements burn into heavier elements

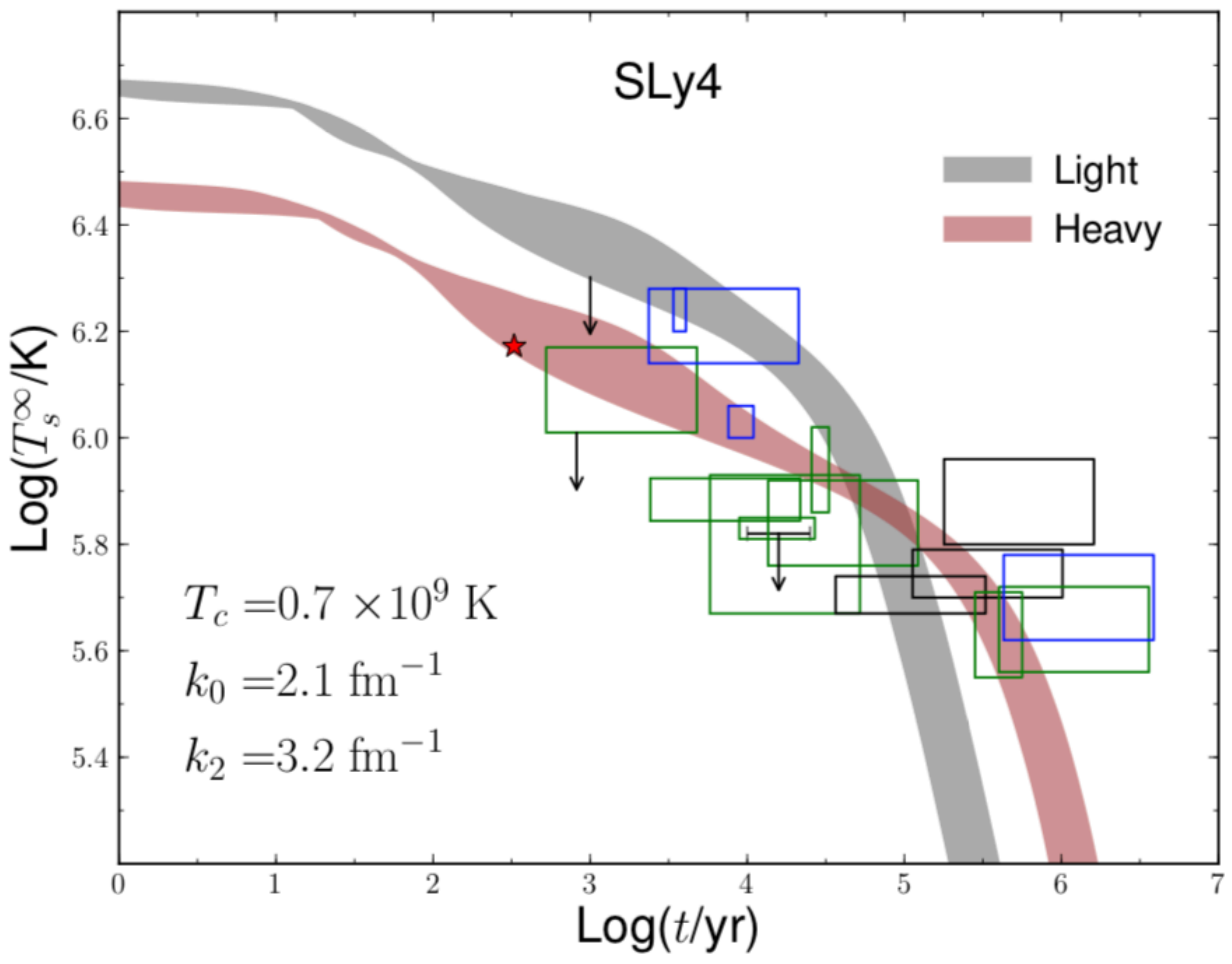
light element : H, He, C, O

Superfluidity

$^1S_0$ neutron superfluidity



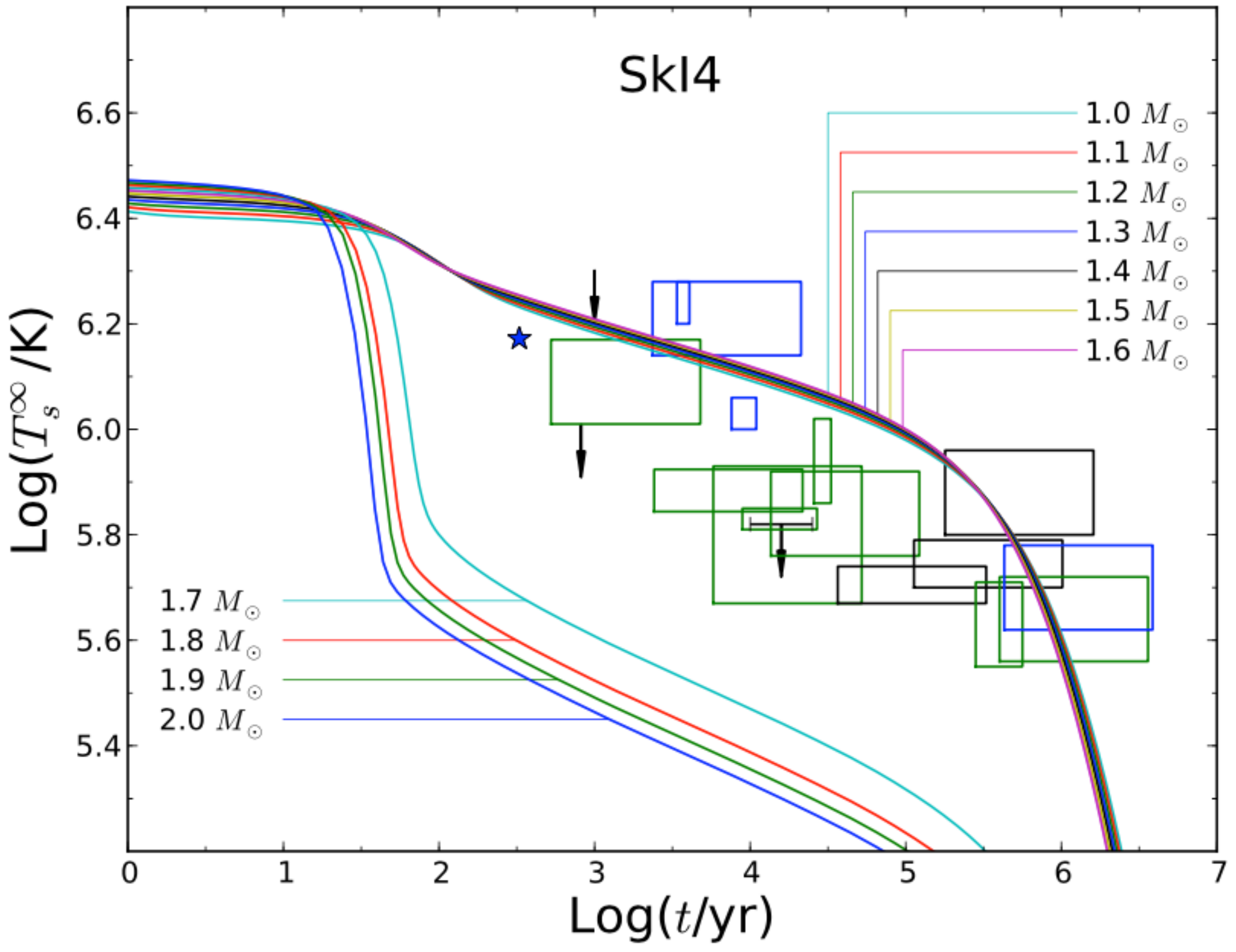
$^1S_0$ proton and $^3P_2$ neutron superfluid states



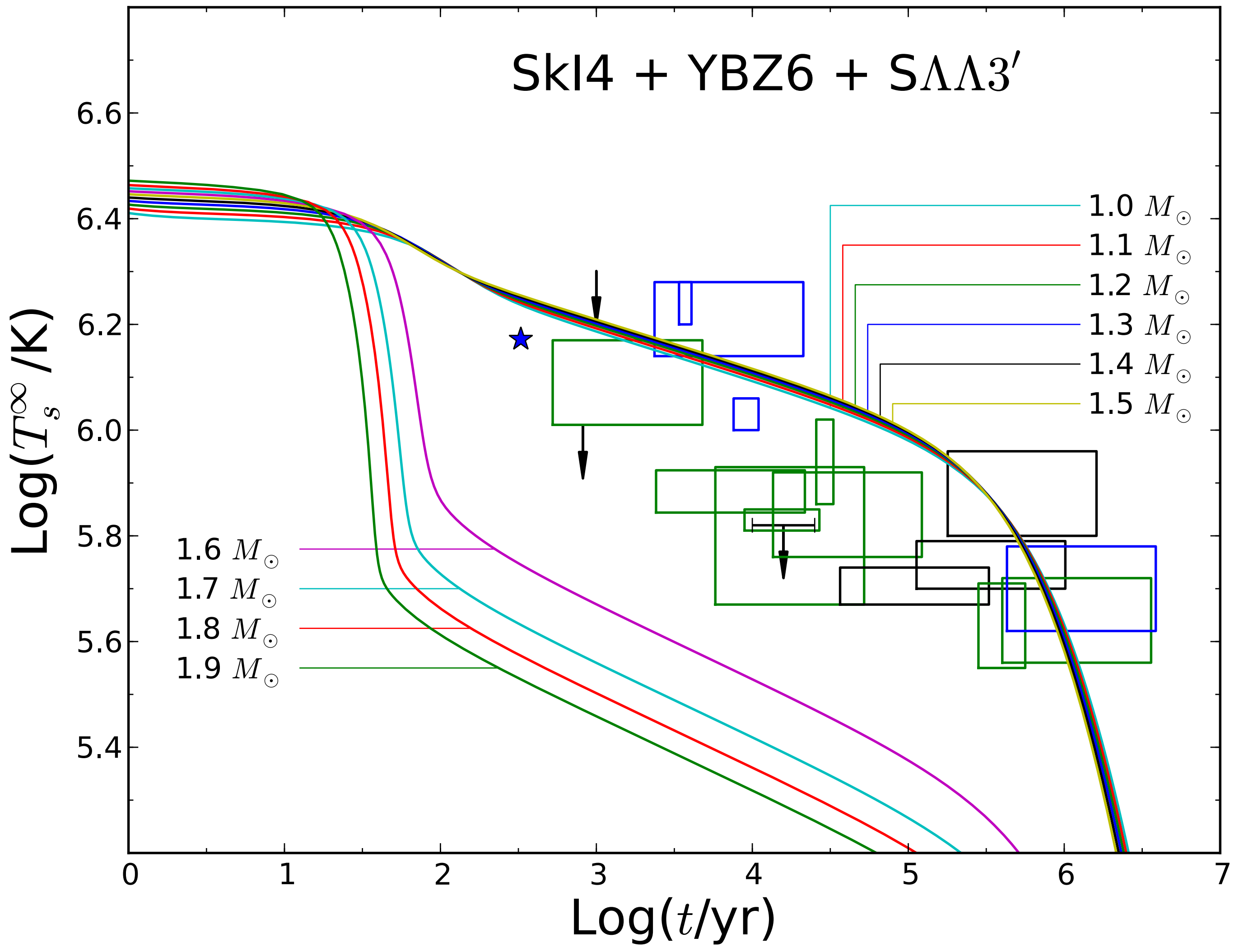
Cooling with hyperons (Skyrme force model **SkI4**)

$$\Lambda \to p + l + \bar{\nu}_l$$
$$p + l \to \Lambda + \nu_l \,,$$

$$Q_\Lambda = 4.0 \times 10^{27} \frac{m_\Lambda^* m_p^*}{m_\Lambda m_p} \left(\frac{n_e}{n_0}\right)^{1/3} R\, T_9^6 \times \Theta_t \,\mathrm{erg\,cm^{-3}\,s^{-1}} \,,$$



without hyperons
(sudden drop between 1.6~1.7)



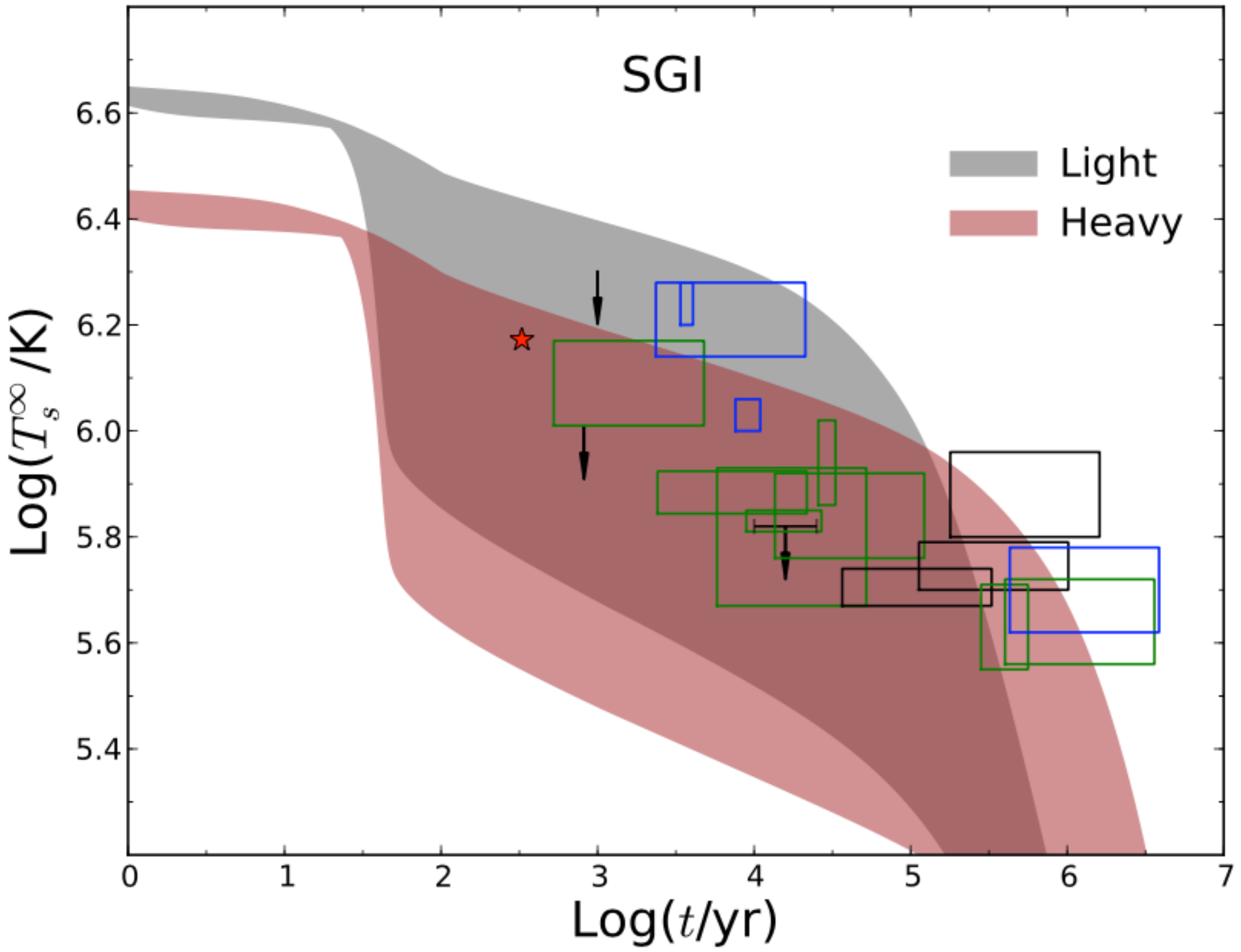
with hyperons
(sudden drop between 1.5~1.6)

NS mass : 1.0 – 2.0 $M_\odot$

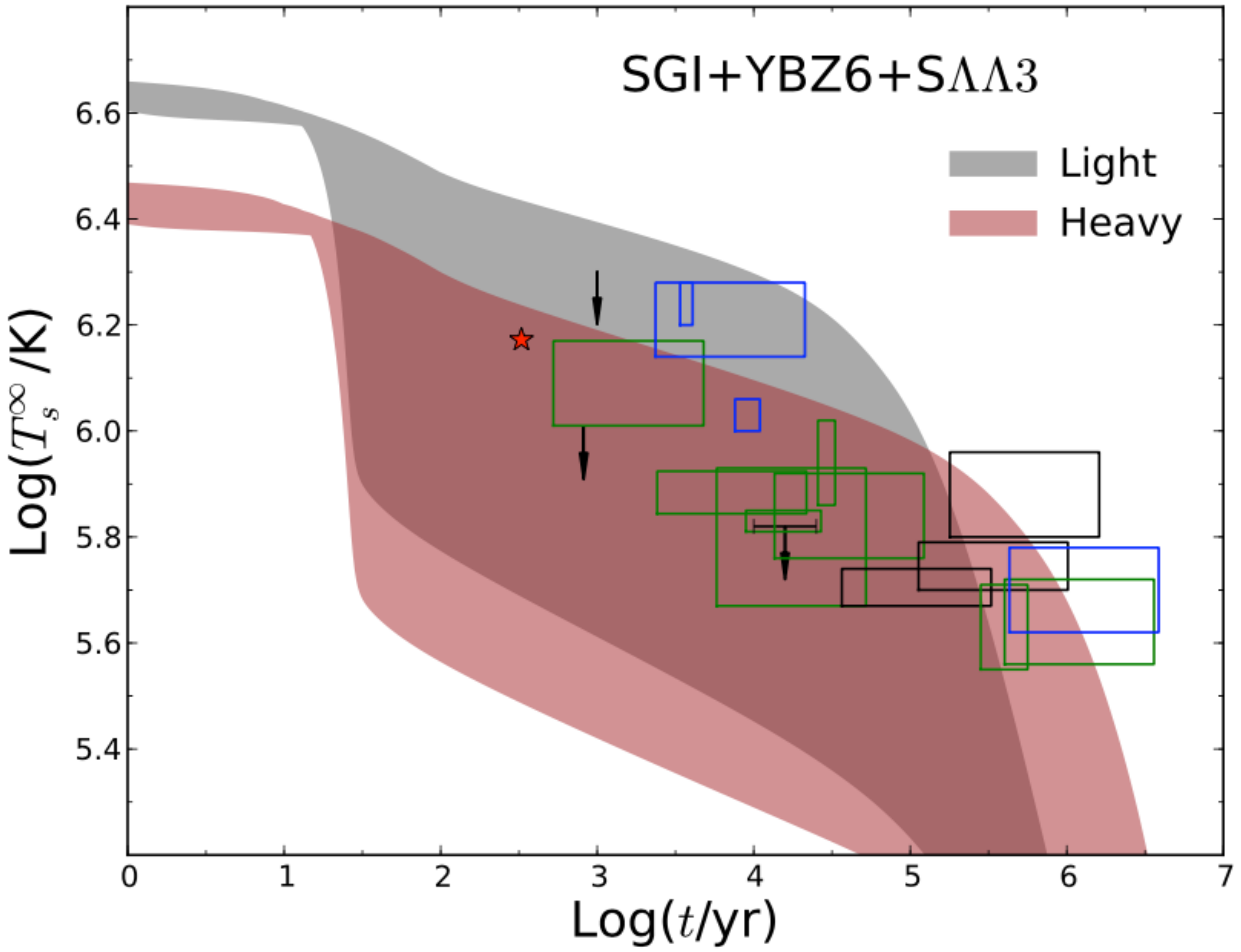
- abrupt drop: ingnition of direct URCA
- stiffer EoS allows early direct Urca
- no calculated-curve can explain middle-age data
- **require real fine-tuning**

Dependence on the surface elements

without hyperons



with hyperons

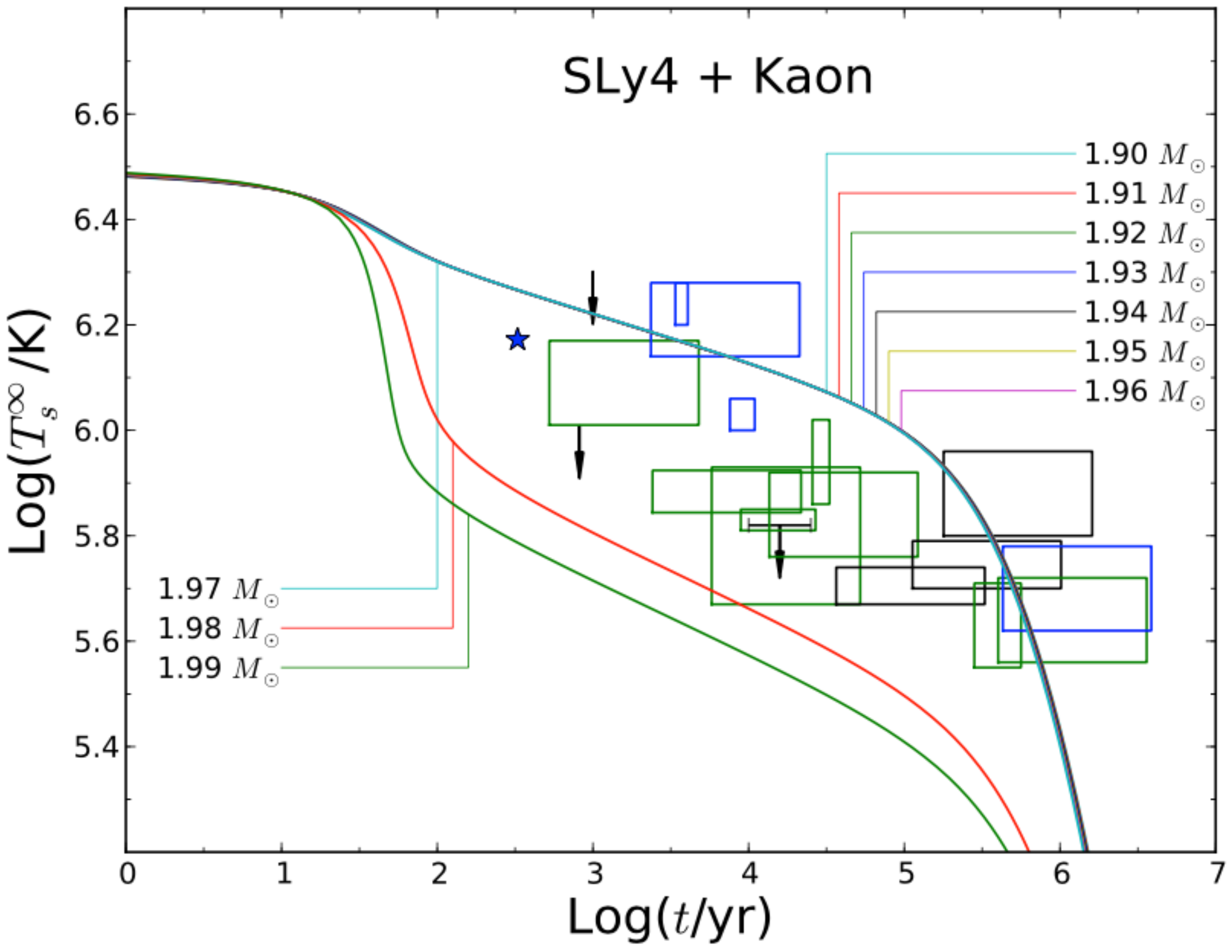


- chemical evolution from light to heavy elements
(earlier plots are with heavy elements)
- pulsar injection of light elements into magnetosphere

Negligible contribution with kaons

density for nucleon direct Urca < density for kaon condensation

Nucleon direct Urca is the dominant neutrino emission process



$$n + \langle K^- \rangle \rightarrow n + l + \bar{\nu}_l$$

$$Q_K = 2.5 \times 10^{26} \frac{m_n^{*2}}{m_n^2} \left(\frac{n_e}{n_0}\right)^{1/3} T_9^6\, \theta_K^2 \times \tan^2\theta_C \,\mathrm{erg\,cm^{-3}\,s^{-1}},$$

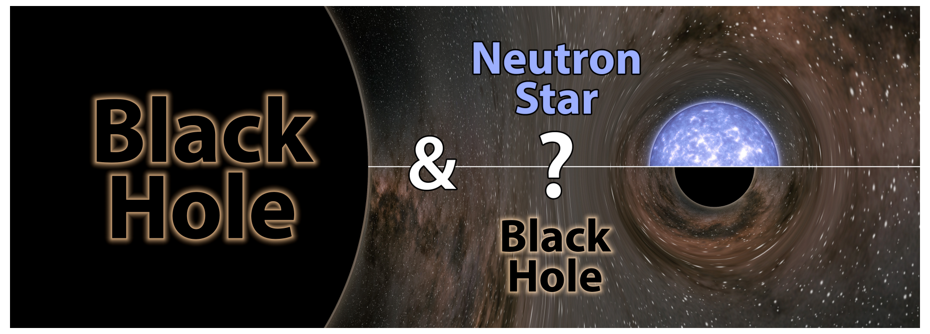
Neutron stars in new era of multi-messenger astrophysics

- **Nuclear Physics**
  - dense matter equation of states
  - neutron star cooling
  - heavy ion collisions / hot & dense matter
  - hadronic physics
  - rare isotope physics

- **Astrophysics/Particle Division**
  - gravitational waves
  - gamma-ray bursts
  - neutrino observations

Prospect

- **GW** from NS mergers
  - GW190425 : NS-NS merger candidate (500 Mly, 153 Mpc)
  - GW190426 : NS-BH merger candidate (1.2 Gly, 368 Mpc)
  - GW190814 : NS-BH or BH-BH merger candidate (700 Mly, 240 Mpc)

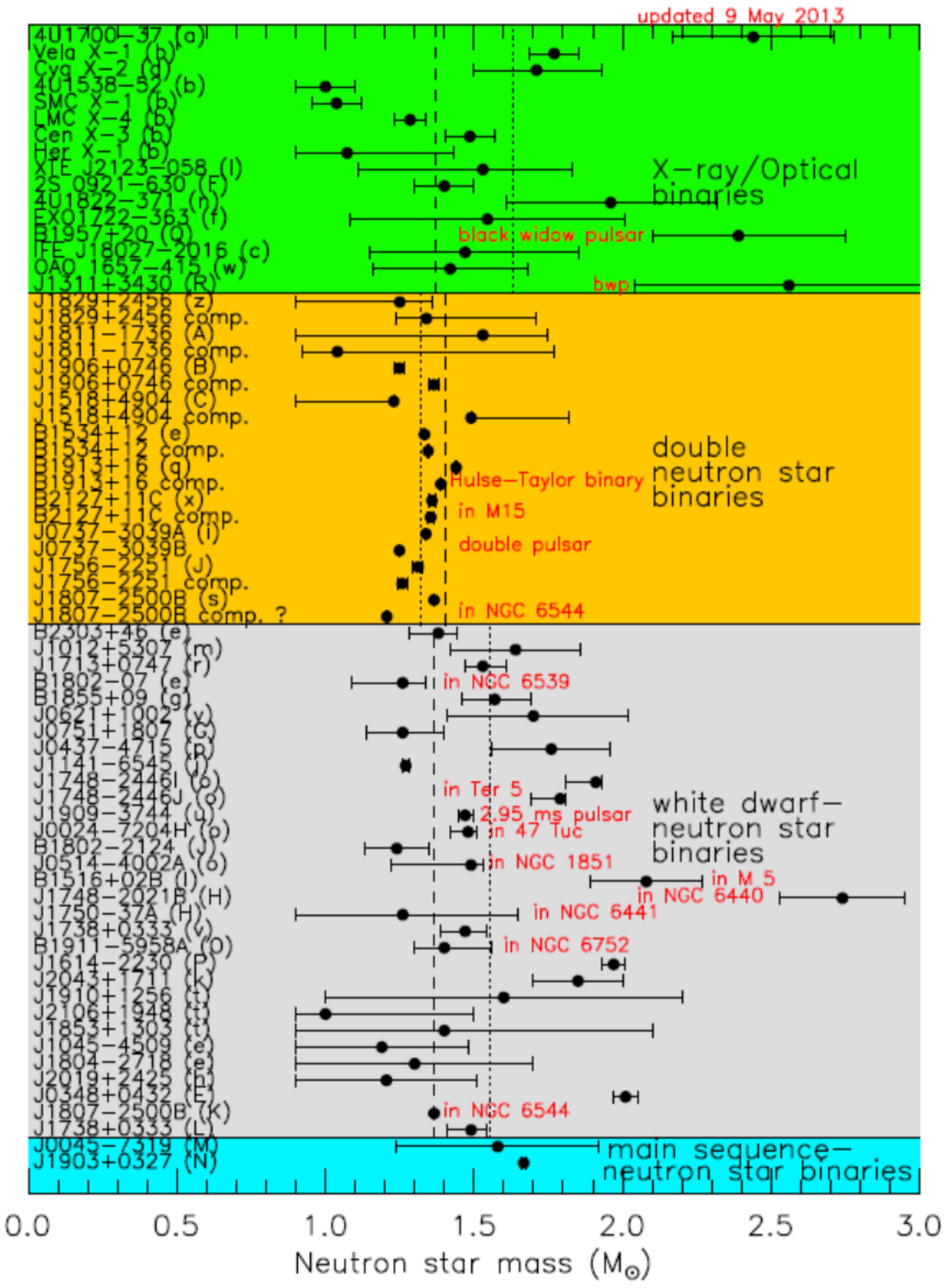


- **BUD^2-McGill Collaboration**
  - **DJBUU (new transport code) for RAON**
  - Dense Matter & Neutron Star EOS

NS Binary Evolution

NS Binary Evolution

NS mass may depend on the evolution

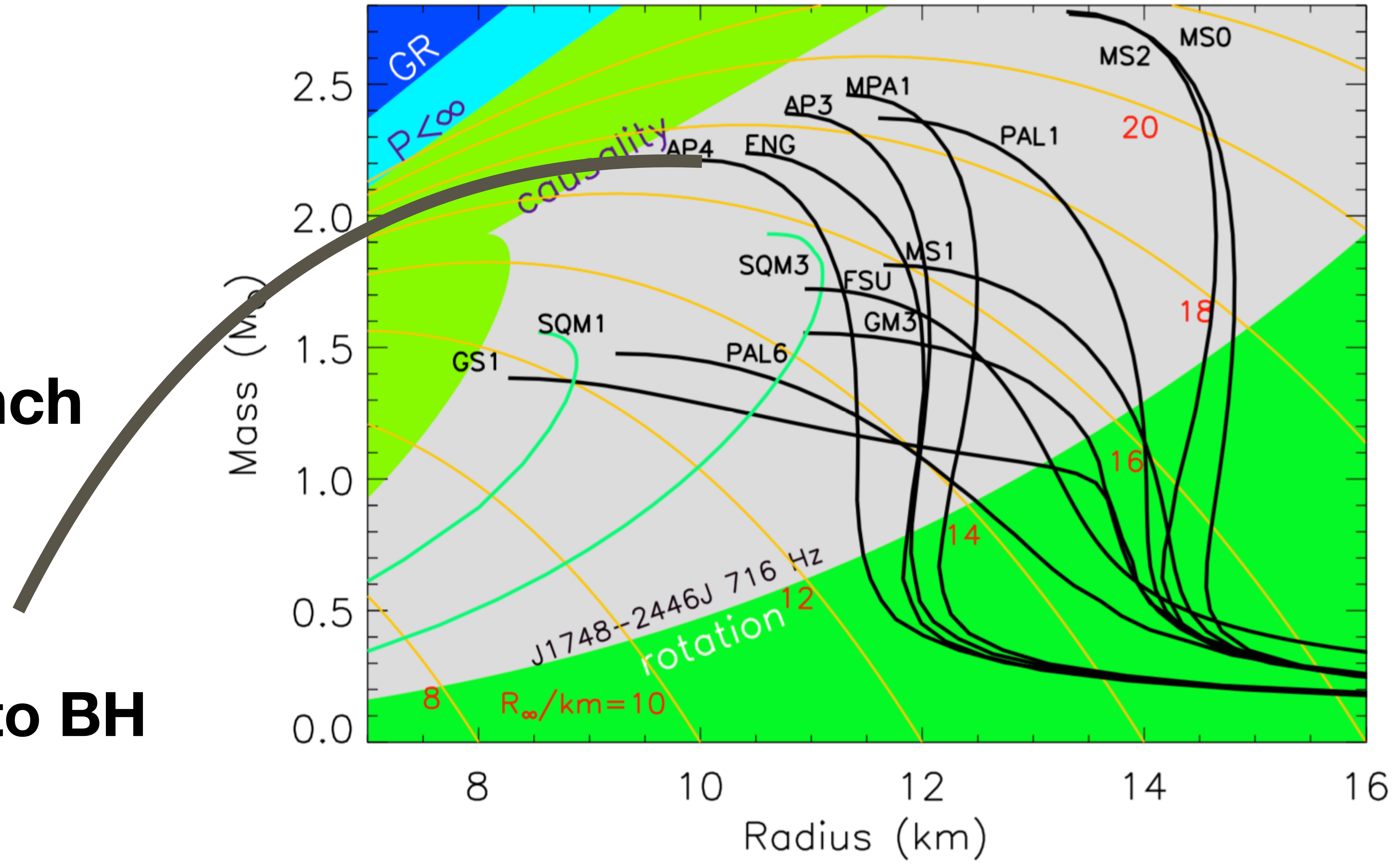


Prakash 2013

Black Hole Formation from **Core Collapse of Giants** or **Accretion onto Neutron Stars**

unstable branch

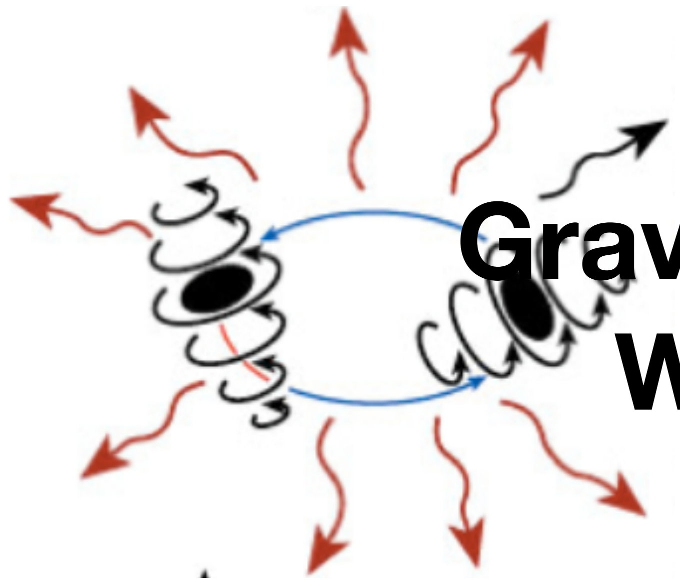
collapse to BH



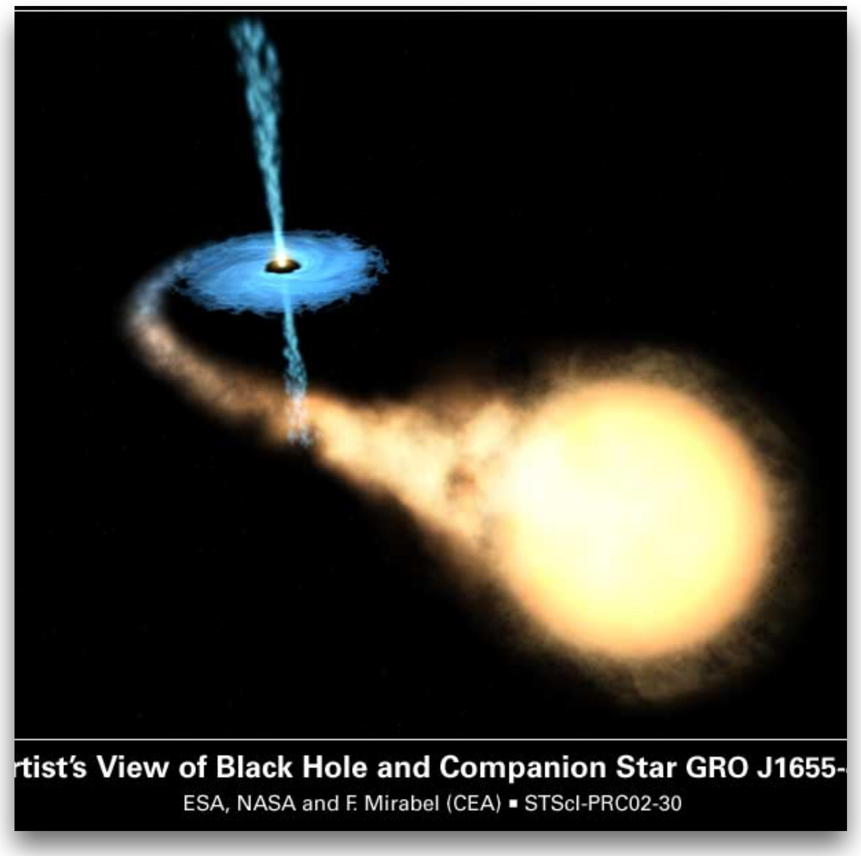
as central density increases equilibrium mass decreases	as central density increases equilibrium mass increases

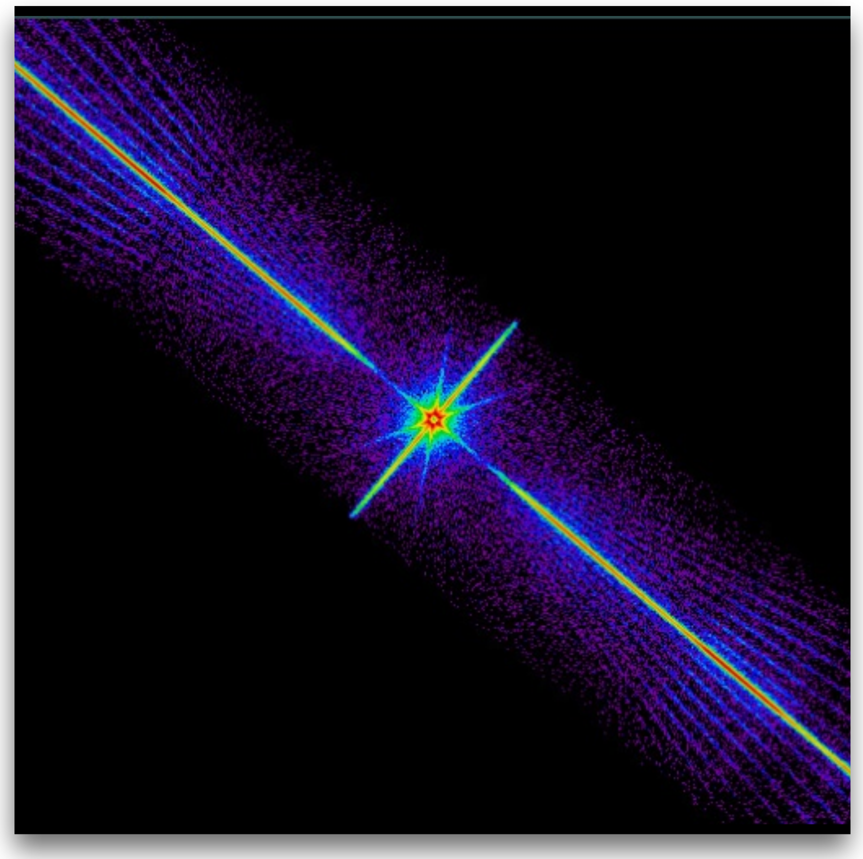
Black Hole Observation

Stellar Mass Black Holes



Gravitational Waves

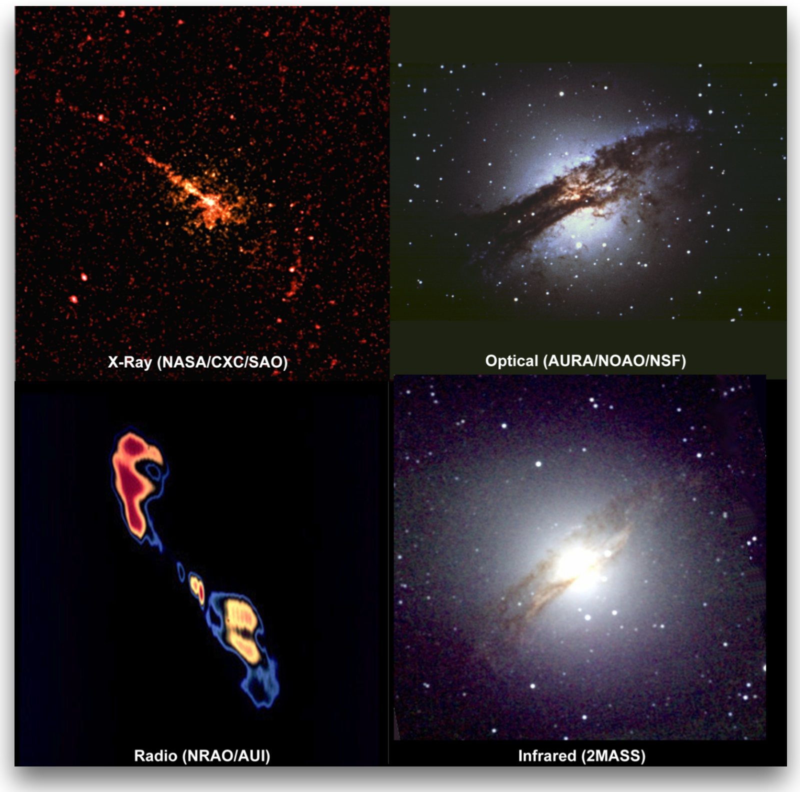




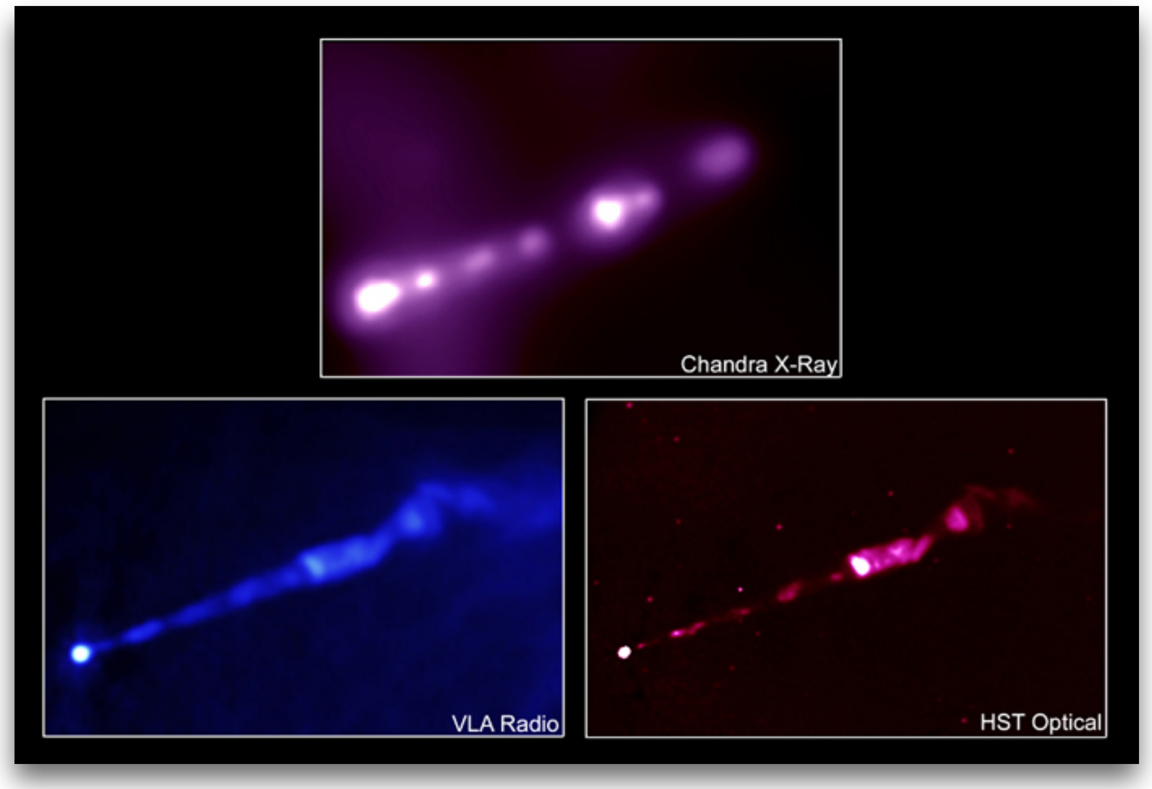
X-ray Binaries

XTEJ1118+480

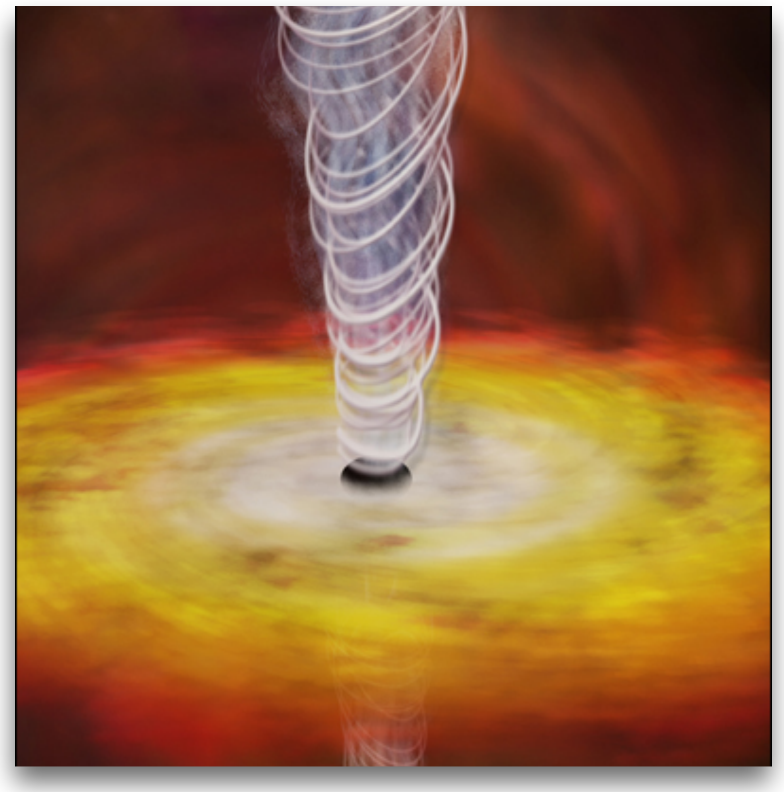
Galactic Center



Centaurus A

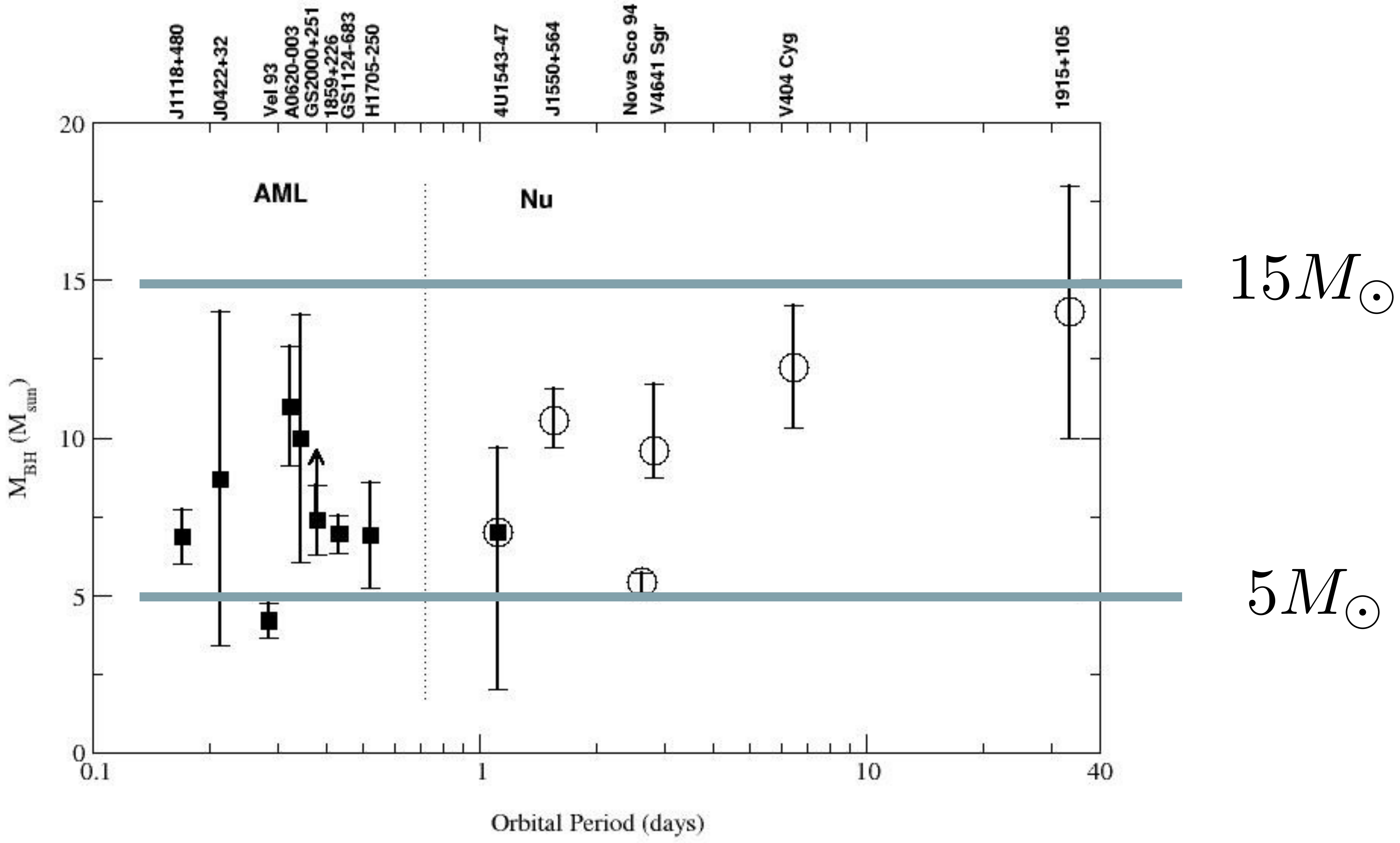


M87



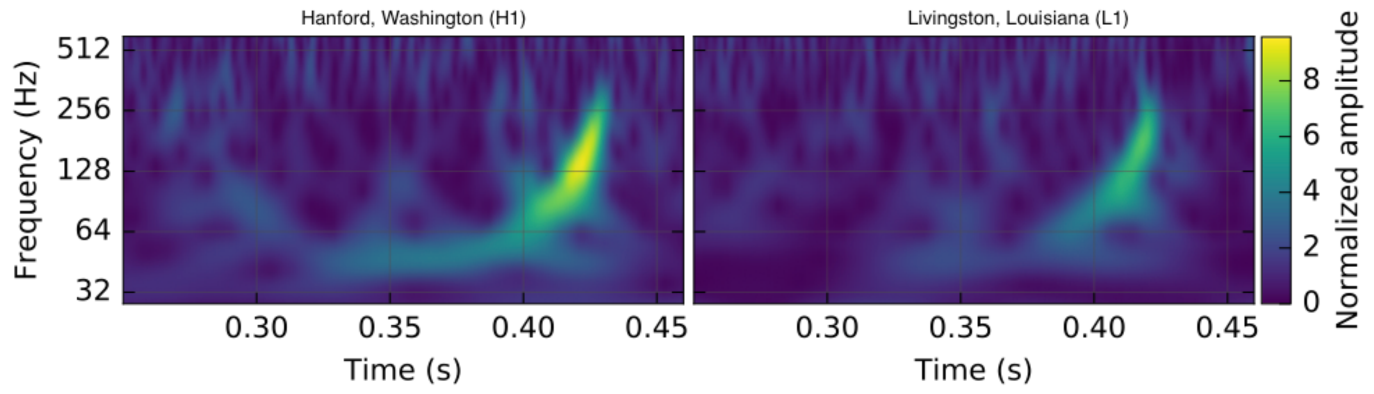
Soft X-ray Transients

ApJ 575, 996 (2002)
ApJ 670, 741 (2007)
NPA 928, 296 (2014)



Main Sequence Companion | **Evolved Companion**

New Class of Black Hole Binaries



$36M_\odot + 29M_\odot$

$z = 0.09 \ (1.3 \times 10^9 \text{ light year})$

Formation of massive black holes
in low-metallicity galaxies
(from pop-III stars)

Evaporation of BH / Hawking Radiation

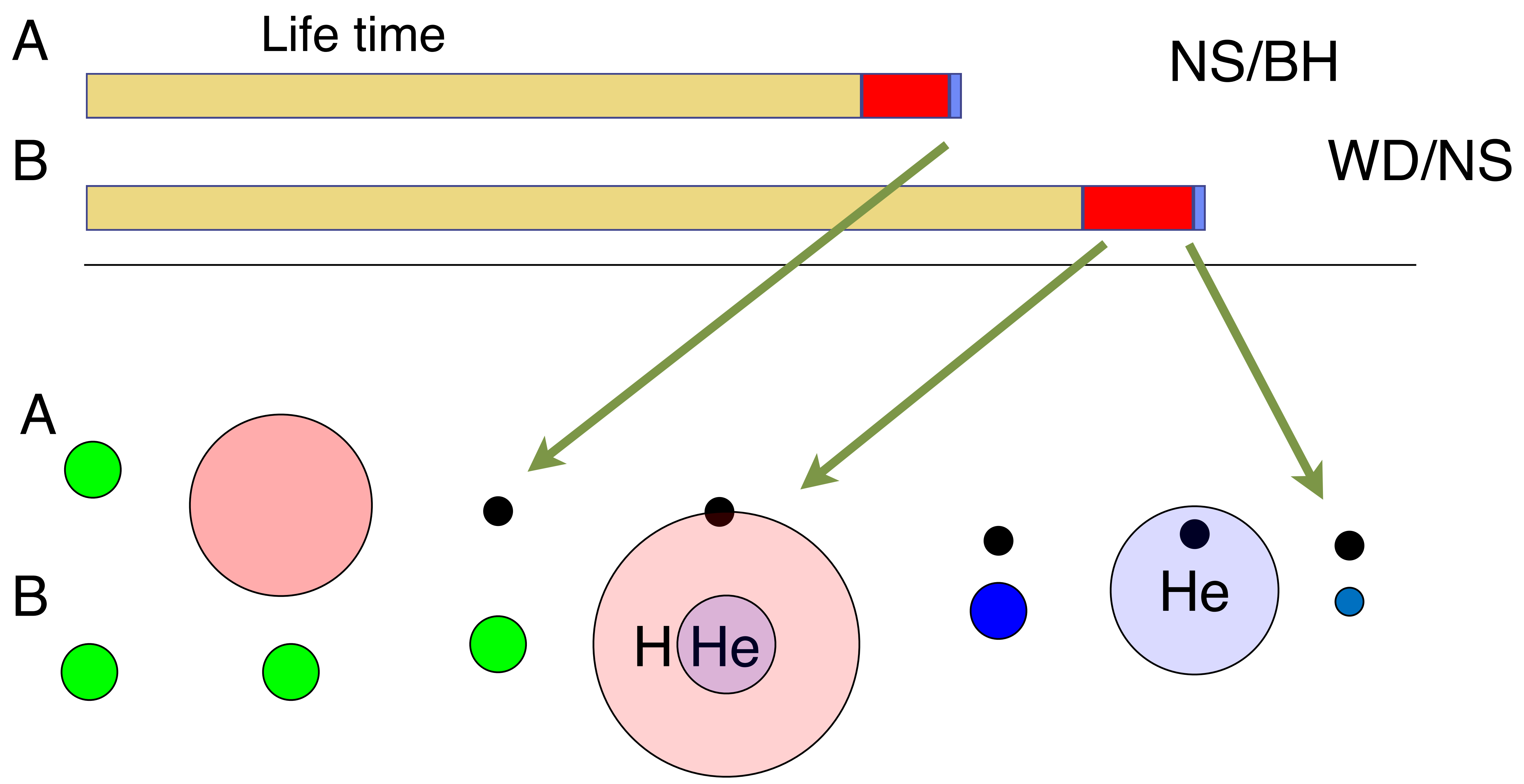
$$\tau_{\text{Universe}} \sim 1.37 \times 10^{10} \text{ year}$$

Mass	R_event	Temp	Evaporation Time
Sun / $2*10^{33}$g	3 km	10^{-7} K	10^{62} year
Each / $6*10^{27}$g	1 cm	0.01 K	10^{35} year
10^{16}g	10^{-12} cm	10^{10} K	10^{10} year

Cosmic Microwave Background (CMB) : 2.7 K

Binary Evolution

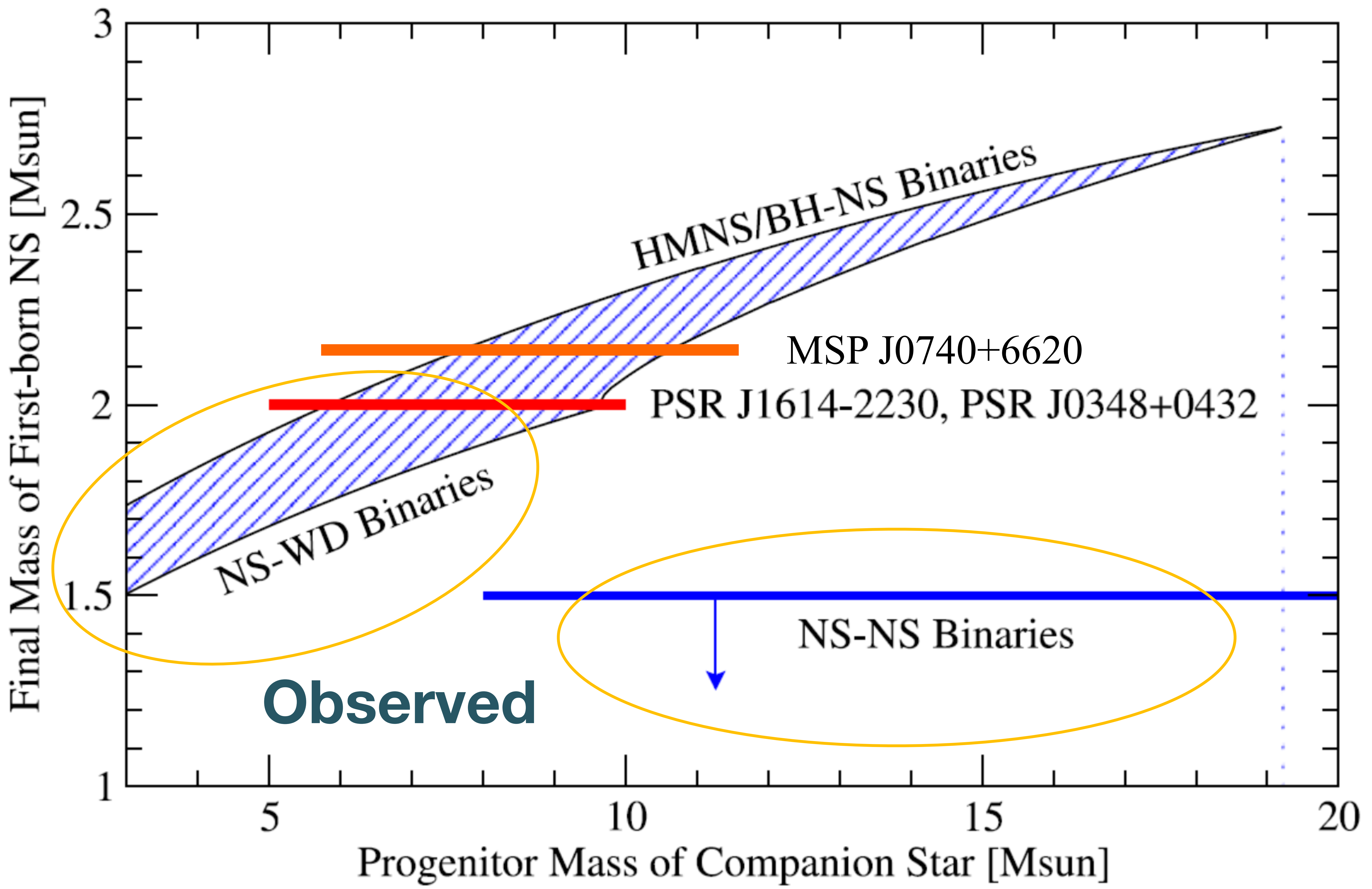
- NS/BH mass depends on the host galaxy
- NS/BH mass depends on the binary evolution



Key ingredients

- Mass accretion rate vs Eddington limit
- Conservative mass transfer vs orbital evolution
- Evolution time scale vs mass transfer time scale
- Roche lobe overflow
- Tidal interaction
- … …

Mass Accretion after NS formation



$2.14^{+0.20}_{-0.18} M_\odot$

BH-NS binaries

$$\tau_{\text{recycled}} \gg \tau_{\text{fresh}}$$

$$\tau_{\text{pulsar}} \propto \frac{1}{B}$$

$$B_{\text{fresh}} \sim 10^{12}\ \text{G}$$

$$B_{\text{recycled}} \sim 10^{8}\ \text{G}$$

NS-NS

- if first-born NS is recycled by accretion
- longer pulsar life time, larger beaming angle

BH-NS

- no recycled pulsar
- smaller chances to be observed

Gravitational Wave Sources

Binary interactions are always interesting !!